

# Stabilization of Walls in Ferromagnetic Nanowires

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**Joint works with Abdelkader Al Sayed (UPPA), Stéphane Labbé (UJF-Grenoble)  
and David Sanchez (INSA-Toulouse)**

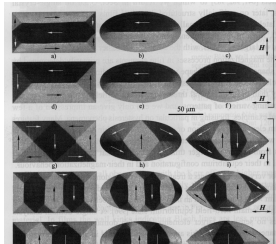
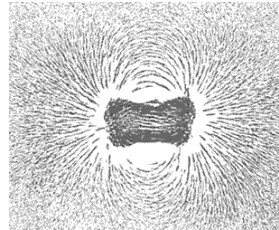
# Plan

- 1 Physical Context
- 2 Walls in constant-section wire
- 3 One wall configurations for pinched nanowires
- 4 Bent wires and junctions
- 5 Conclusion

## Ferromagnetic materials

### Properties

- Spontaneous magnetization
- Magnetic field generated by the magnetization
- Formation of domains and domain walls

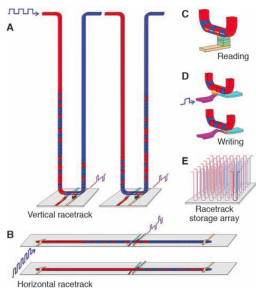


### Industrial Applications

- Electromagnets
- Transformers
- Radar absorbing paints
- Mobile phones
- Modern Recording Devices

## Ferromagnetic Nanowires : Applications

data storage



S. Parkin, M. Hayashi, L. Thomas, *Magnetic Domain-Wall Racetrack Memory*, *Science* **320** (2008)

logic

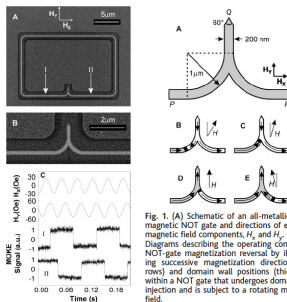


Fig. 1. (A) Schematic of an all-metallic ferromagnetic NOT gate and directions of elliptical magnetic field components,  $H_x$  and  $H_y$ . (B to E) Diagrams describing the operating concept of NOT-gate magnetization reversal by illustrating successive magnetization directions (arrows) and domain wall positions (thick line) within a NOT gate that undergoes domain wall injection and is subject to a rotating magnetic field.

D. A. Allwood, Gang Xiong, M. D. Cooke, C. C. Faulkner, D. Atkinson, N. Vernier, R. P. Cowburn, *Submicrometer Ferromagnetic NOT Gate and Shift Register*, *Science* **296** (2002)



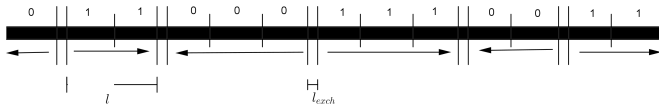
# Ferromagnetic nanowires : properties

## Easy axis in the direction of the wire $\Rightarrow$ Formation of domains

For logic devices : presence of a domain wall or not  $\sim$  information T or F

For data storage :

- Length of the domains  $\sim$  length of the sequences of zeros or ones in the data



- Walls positions must be unchanged (reliability of the storage)
- Information is transported to the reading head by application of an electric current

### Objectives

- Obtention and justification of equivalent simpler 1d-models for nanowires
- Description of walls configurations and walls dynamics
- Stability of walls configurations
- Study of geometrical effects : pinching, bend, junctions

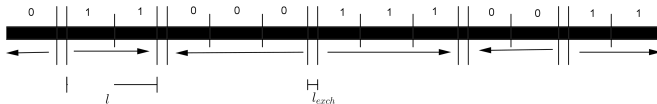
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## Starting Point : 3d model

## Magnetic Moment

- Magnetic domain :  $\Omega \subset \mathbb{R}^3$
- Magnetization described by the magnetic moment  $m : \mathbb{R}_t^+ \times \Omega_x \rightarrow \mathbb{R}^3$

$$|m(t, x)| = 1 \text{ p. p.}$$

- Constitutive Relation :  $B = H + m$

## Landau-Lifschitz Equation

$$\frac{\partial m}{\partial t} = -m \times \mathcal{H} - m \times (m \times \mathcal{H}) - (v \cdot \nabla)m - m \times (v \cdot \nabla)m$$

$$\mathcal{H} = -\partial_m \mathcal{E}(m) = A\Delta m + h_d(m) + H_a$$

$$\mathcal{E}(m) = \frac{A}{2} \int_{\Omega} |\nabla m|^2 + \frac{1}{2} \int_{\mathbb{R}^3} |h_d(m)|^2 - \int_{\Omega} H_a \cdot m$$

$$\text{curl } h_d(m) = 0, \quad \text{div } (h_d(m) + \bar{m}) = 0$$

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**Slonczewski model** :  $v$  is related to the current density

# Ferromagnetic nanowires : 1d model

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$$m : \mathbb{R}_t^+ \times \Omega_\eta \rightarrow \mathbb{S}^2$$

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$$\Omega_\eta = [0, L] \times B(0, \eta)$$

- 1 rescaling in the transverse variable in the weak formulation : fixed domain  
 $\Omega_1 = [0, L] \times B(0, 1)$
- 2  $\eta \rightarrow 0$



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- Wire assimilated to  $[0, L]e_1$
- 1d Exchange
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## Ferromagnetism Energy

$$\mathcal{E}(m) = \frac{A}{2} \int_I |\partial_x m|^2 dx + \frac{1}{4} \int_I (|m_2|^2 + |m_3|^2) dx - \int_I h_a m_1$$

# Plan

- 1 Physical Context
- 2 Walls in constant-section wire**
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# Exact Static Solutions

## Model of Nanowire

$$m : \mathbb{R}_t^+ \times I_x \rightarrow \mathcal{S}^2, I = \mathbb{R} \text{ or } I = [-L, L]$$

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## Static Solutions : Domains

$$\text{Domains : } m = \pm \vec{e}_1.$$

# Exact Static Solutions

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- $\varphi$  is constant
- $\theta'' + \cos \theta \sin \theta = 0$
- $\theta \rightarrow \pm\pi/2$  when  $x \rightarrow \pm\infty$  or  $\theta'(-L) = \theta'(L) = 0$

## Static Solutions : Walls

$$\bullet M(x) = R_{\varphi(x)} \begin{pmatrix} \sin \theta(x) \\ \cos \theta(x) \\ 0 \end{pmatrix}$$

$$R_{\varphi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & -\sin \varphi \\ 0 & \sin \varphi & \cos \varphi \end{pmatrix}$$

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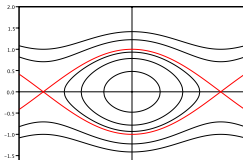
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# Stability in Infinite Nanowire

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Stability of the constant profiles but not asymptotic stability

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Controlability of the wall position by the mean of the applied field



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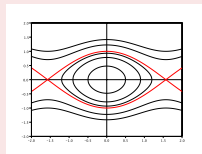
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- One wall in a finite wire : centered in the middle of the wire
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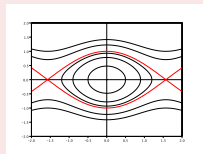
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## Stability in non pinched wires

**The position of walls is unstable or non asymptotically stable**



The data storage is not reliable

**Stabilization by Pinching**

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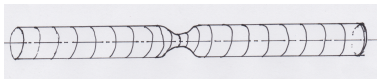
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# Pinched Wire



$$\Omega_\eta = \{(x, y, z), x \in I, y^2 + z^2 < \eta^2(\rho(x))^2\}$$

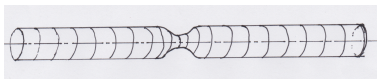
- By rescaling : fixed domain  $\Omega_1 = \{(x, y, z), x \in I, y^2 + z^2 < (\rho(x))^2\}$
- $\eta \rightarrow 0$  in the weak formulation

## Radius of the section : $\rho$

- $\rho : \mathbb{R} \rightarrow \mathbb{R}^+, \mathcal{C}^\infty,$
- $\rho = 1$  outside  $[-a, a],$
- $\rho(-x) = \rho(x),$
- $\rho$  non decreasing on  $[0, a]$
- $\mathbf{s}(x) = \text{area of the section}$   
 $= \pi(\rho(x))^2$



# Pinched Wire



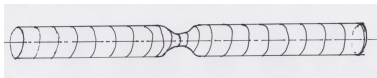
$$\Omega_\eta = \{(x, y, z), x \in I, y^2 + z^2 < \eta^2(\rho(x))^2\}$$

## Radius of the section : $\rho$

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- $\mathbf{s}(x) = \text{area of the section}$   
 $= \pi(\rho(x))^2$

- By rescaling : fixed domain  $\Omega_1 = \{(x, y, z), x \in I, y^2 + z^2 < (\rho(x))^2\}$
- $\eta \rightarrow 0$  in the weak formulation

# Pinched Wire



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 $= \pi(\rho(x))^2$

$$m : \mathbb{R}_t^+ \times I_x \rightarrow \mathcal{S}^2, \quad I = \mathbb{R} \text{ or } I = [-L, L], L > a$$

## Ferromagnetism Energy in pinched nanowire

$$E(m) = \frac{A}{2} \int_I \mathbf{s}(x) |\partial_x m|^2 dx + \frac{1}{4} \int_I \mathbf{s}(x) (|m_2|^2 + |m_3|^2) dx - \int_I \mathbf{s}(x) h_a m_1$$

## Dynamic model of pinched wire

- $m : \mathbb{R}_t^+ \times I_x \rightarrow S^2$ ,  $I = \mathbb{R}$  or  $I = [-L, L]$ ,  $L > a$
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### Model of pinched Nanowire

$$\mathbf{s}(x) \frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}}) - \mathbf{v} \partial_x m - \mathbf{v} m \times \partial_x m$$

$$H_{\text{eff}} = A \partial_x (\mathbf{s}(x) \partial_x m) - \frac{\mathbf{s}(x)}{2} (m_2 \vec{e}_2 + m_3 \vec{e}_3) + \mathbf{s}(x) h_a \vec{e}_1$$

$$\mathbf{v}(x) = \int_{y^2+z^2 \leq \rho(x)^2} v(x, y, z) \cdot \vec{e}_1 dy dz$$

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# Pinched Wire with vanishing applied field and current

## Model of Nanowire

$$m : \mathbb{R}_t^+ \times I \rightarrow \mathcal{S}^2,$$

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G. Carbou, D. Sanchez, *Stability of walls for pinched nano-wires*, in preparation

- Existence of a profile for one wall
- Asymptotically stable for the position

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G. Carbou, D. Sanchez, *Stability of walls for pinched nano-wires*, in preparation

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# Pinched Wire with vanishing applied field and current

## Model of Nanowire

$$m : \mathbb{R}_t^+ \times [-L, L] \rightarrow \mathbb{S}^2,$$

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G. Carbou, D. Sanchez, *Stability of walls for pinched nano-wires*, in preparation

- Existence of a profile for one wall for  $L \geq \pi/2$
- **Asymptotically stable for the position** if  $L$  is large enough



# Existence of profiles

$$\bullet M(x) = R_{\varphi(x)} \begin{pmatrix} \sin \theta(x) \\ \cos \theta(x) \\ 0 \end{pmatrix}$$

$$R_{\varphi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & -\sin \varphi \\ 0 & \sin \varphi & \cos \varphi \end{pmatrix}$$

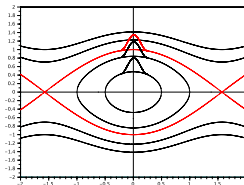
$$\bullet \text{BC} : \theta \rightarrow \pm \pi/2 \text{ when } x \rightarrow \pm \infty$$

$$\theta'(-L) = \theta'(L) = 0$$

- $\varphi$  is constant

$$\bullet \theta'' + \frac{\mathbf{s}'}{\mathbf{s}} \partial_x \theta + \cos \theta \sin \theta = 0$$

$$\bullet \left( (\theta')^2 - \cos^2 \theta \right)' = -2 \frac{\mathbf{s}'}{\mathbf{s}} (\theta')^2$$



# Stability

## Pinched Nanowire

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$$\frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}})$$

$$\mathcal{H}_{\text{eff}} = m_{xx} + \frac{\mathbf{s}'}{\mathbf{s}} m_x - m_2 \vec{e}_2 - m_3 \vec{e}_3$$

Boundary Conditions

## Wall Profile

$$M_0(x) = \begin{pmatrix} \sin \theta(x) \\ \cos \theta(x) \\ 0 \end{pmatrix}$$

$$\theta'' + \frac{\mathbf{s}'}{\mathbf{s}} \theta' + \cos \theta \sin \theta = 0$$

$$\text{BC} : \theta \rightarrow \pm \pi/2 \text{ when } x \rightarrow \pm \infty$$

$$\theta'(-L) = \theta'(L) = 0$$

## Stability of $M_0$ , asymptotic stability for the position of the wall ?

If  $\|m(t=0) - M_0\|_{H^1(\mathbb{R})}$  is small, then

- for all time,  $\|m(t) - M_0\|_{H^1(\mathbb{R})}$  remains small,
- $m(t)$  tends to  $R_{\theta_\infty}(M_0)$  when  $t$  tends to  $+\infty$ .

## Mobile frame

Geometrical constraint  $|m| = 1$

We describe the perturbations in the mobile frame

$$m(t, x) = \sqrt{1 - |r(t, x)|^2} M_0(x) + r_1(t, x) M_1(x) + r_2(t, x) M_2$$

$$M_0(x) = \begin{pmatrix} \sin \theta(x) \\ \cos \theta(x) \\ 0 \end{pmatrix} \quad M_1(x) = \begin{pmatrix} -\cos \theta(x) \\ \sin \theta(x) \\ 0 \end{pmatrix} \quad M_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

### New Equations for $r$

$$r : \mathbb{R}_t^+ \times I_x \rightarrow \mathbb{R}^2$$

$$\partial_t r_1 = -L_1(r_1) - L_2(r_2) + \text{non linear}$$

$$\partial_t r_2 = L_1(r_1) - L_2(r_2) + \text{non linear}$$

$$L_1(r) = -\partial_{xx} r - \frac{s'}{s} \partial_x r + (\sin^2 \theta - \cos^2 \theta) r$$

$$L_2(r) = -\partial_{xx} r - \frac{s'}{s} \partial_x r + (\sin^2 \theta - (\theta')^2) r$$

### Exact Solution

$M_0$  for LL  $\sim 0$  for the new system

Stability of 0 ?

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### Exact Solution

$M_0$  for LL  $\sim 0$  for the new system

Stability of 0 ?

## Properties of the linearized : $L_2$

On  $L^2(\mathbb{R})$ , weighted scalar product :

$$\langle u | v \rangle = \int_{\mathbb{R}} \mathbf{s}(x) u(x) v(x) dx,$$

$$L_2 = -\frac{1}{\mathbf{s}} \partial_x (\mathbf{s} \partial_x) + (\sin^2 \theta_0 - (\theta'_0)^2) \implies L_2 \text{ is self-adjoint for } \langle | \rangle.$$

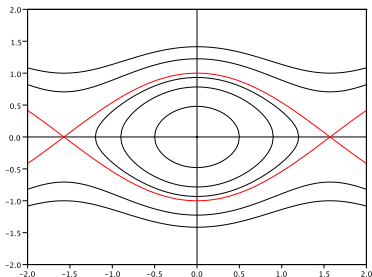
$$\langle L_2 u | v \rangle = \langle \ell u | \ell v \rangle, \quad \text{with } \ell = \partial_x + \theta' \tan \theta$$

$$L_2 \geq 0, \quad \text{Ker } L_2 = \mathbb{R}(\cos \theta)$$

- Valid whatever the configurations of the wire (several pinched zones)
- for one wall, on  $(\cos \theta)^\perp$ ,  $L_2 \geq 1$
- **Invariance by rotation**  $\implies 0$  is an eigenvalue of the linearized

## Properties of the linearized : $L_1$

$$L_1 = L_2 + (\theta')^2 - \cos^2 \theta$$

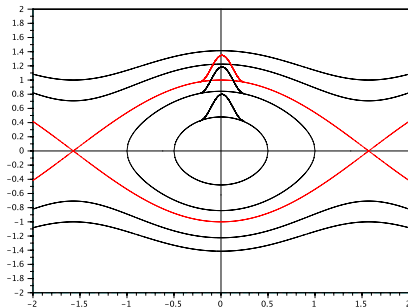


**Key point :  $L_1 \geq 0$  ?**

- Infinite nanowire without pinching :  $L_1 = L_2$ ,  $L_1 \geq 0$ . **Stability**
- Finite nanowire without pinching :  $L_1 = L_2 - c$ . **Bad sign, unstability for the position**

# Properties of the linearized : $L_1$

$$L_1 = L_2 + (\theta')^2 - \cos^2 \theta$$



- Infinite wire :  $(\theta')^2 - \cos^2 \theta > 0$  on  $\text{Ker } L_2 \implies L_1 > 0$
- Short finite wire :  $(\theta')^2 - \cos^2 \theta \leq 0 \implies$  Unstability
- Long finite wire :  $(\theta')^2 - \cos^2 \theta > 0$  on a sufficiently large zone  $\implies L_1 > 0$

## New unknown

Invariance by rotation  $\implies$  Zero is an eigenvalue of the linearized

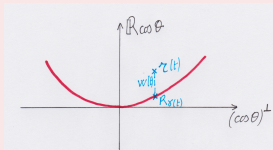
$\gamma \mapsto R_\gamma M_0(x)$  family of solutions for LL, where  $R_\gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & -\sin \gamma \\ 0 & \sin \gamma & \cos \gamma \end{pmatrix}$

$\mathcal{R}_\gamma(x) = \begin{pmatrix} R_\gamma M_0(x) \cdot M_1(x) \\ R_\gamma M_0(x) \cdot M_2 \end{pmatrix}$  static solutions for the  $r$ -equation

## New coordinates

$$r(t, x) = \mathcal{R}_\gamma(t) + w(t, x)$$

$$w_2 \text{ in } (\cos \theta)^\perp$$



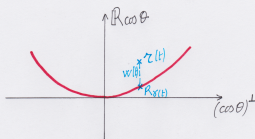


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Equations for  $w$  and  $\gamma$ 

$$\gamma : \mathbb{R}^+ \rightarrow \mathbb{R}, \quad w : \mathbb{R}^+ \rightarrow \mathbb{R} \times (\cos \theta)^\perp$$

$$\partial_t \gamma = A_1(t, \gamma, w)$$

$$\partial_t w_1 = -L_1(w_1) - L_2(w_2) + \text{non linear}$$

$$\partial_t w_2 = L_1(w_1) - L_2(w_2) + \text{non linear}$$

## To be proved

$$w \rightarrow 0$$

$$\gamma \text{ remains small}$$

$$\gamma \rightarrow \gamma_\infty$$

# Variational Estimates

## Equations for $w, \gamma$

$$\gamma : \mathbb{R}^+ \rightarrow \mathbb{R}, \quad w : \mathbb{R}^+ \rightarrow \mathbb{R} \times (\cos \theta)^\perp$$

$$\partial_t \gamma = A(t, \gamma, w)(w)$$

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By coercivity of the linearized,  $w$  is exponentially decreasing

$\partial_t \gamma$  is integrable

Stability of  $M_0$ , asymptotic stability modulo rotation

The wall is pinned at the pinched zone

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# Pinched wire with small applied field

## Pinched Nanowire

$$m : \mathbb{R}_t^+ \times I_x \rightarrow S^2$$

$$\frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}})$$

$$\mathcal{H}_{\text{eff}} = m_{xx} + \frac{\mathbf{s}'}{\mathbf{s}} m_x - m_2 \vec{e}_2 - m_3 \vec{e}_3 + h_a \vec{e}_1$$

### Boundary Conditions

Constant-section wire : precessional regime

$$m(t, x) = R_{h_a t} M_0(x + h_a t)$$

### Pinched wire :

For small  $h_a$  : the wall does not move, remains in the pinching (static solution), its profile is planar.

Construction of the solution by implicit function theorem

# Pinched wire with small applied field

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# Pinched wire with small current

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$$\frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}}) - \frac{\mathbf{v}}{\mathbf{s}} \partial_x m - \frac{\mathbf{v}}{\mathbf{s}} m \times \partial_x m$$

$$\mathcal{H}_{\text{eff}} = m_{xx} + \frac{\mathbf{s}'}{\mathbf{s}} m_x - m_2 \vec{e}_2 - m_3 \vec{e}_3$$

Boundary Conditions

### Pinched wire :

- the wall remains in the pinching
- non planar configuration
- the profile turns around the wire



# Pinched wire with small current

## Pinched Nanowire

$$m : \mathbb{R}_t^+ \times I_x \rightarrow \mathcal{S}^2$$

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Boundary Conditions

### Pinched wire :

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# Plan

- 1 Physical Context
- 2 Walls in constant-section wire
- 3 One wall configurations for pinched nanowires
- 4 Bent wires and junctions**
- 5 Conclusion

# Model of wires junction

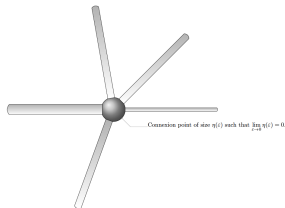


Figure 1: Conjunction of nanowires of size proportional to  $\epsilon$ .

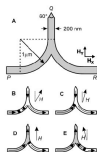
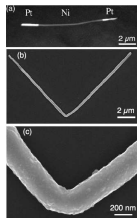


Fig. 3. (A) Schematic of an all-metallic ferromagnetic NOT gate and directions of elliptical magnetic field components,  $H_x^i$  and  $H_y^i$ . (B to E) Diagrams describing the operating concept of NOT-gate magnetization reversal by illustrating successive magnetization directions (arrows) and domain wall positions (thick line) within a NOT gate that undergoes domain wall injection and is subject to a rotating magnetic field.



## Model of junction of $k$ wires

Wire  $i$  :  $[0, L_i] \vec{u}_i$

$$m^i : \mathbb{R}_t^+ \times [0, L_i]_x \rightarrow S^2$$

$$\frac{\partial m^i}{\partial t} = -m^i \times H_{\text{eff}}^i(m^i) - m^i \times (m^i \times H_{\text{eff}}^i(m^i)),$$

$$H_{\text{eff}}^i(m^i) = \partial_{xx} m^i + (m^i \cdot \vec{u}_i) \vec{u}_i$$

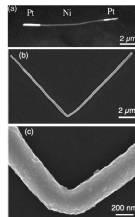
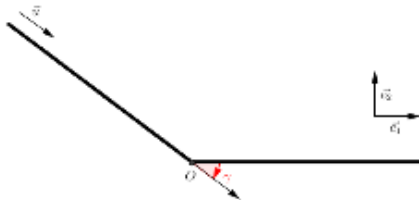
Homogeneous Neumann BC at the ends

Junction conditions :

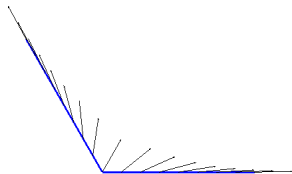
$$m^1(t, 0) = \dots = m^k(t, 0)$$

$$\sum_{i=1}^k \mathbf{s}_i \partial_x m^i(t, 0) = 0$$

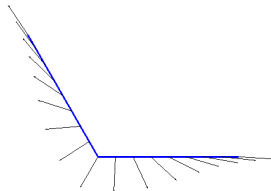
# Bent nanowire



# Bent nanowire

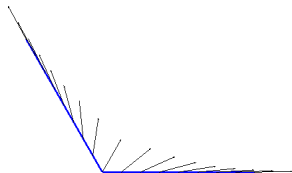


Asymptotically stable

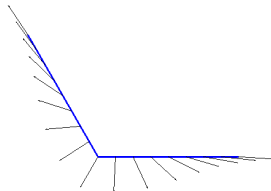


Unstable

# Bent nanowire

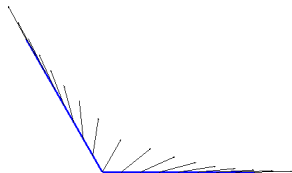


Asymptotically stable

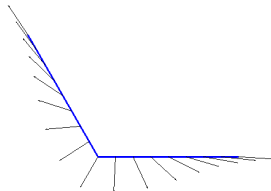


Unstable

## Bent nanowire



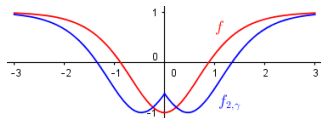
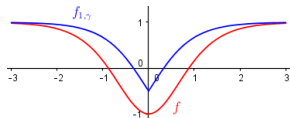
Asymptotically stable



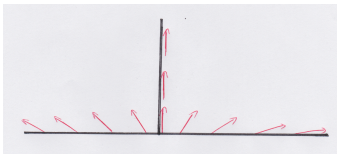
Unstable

Key Point : positiveness of  $L_1 = -\partial_{xx} + f_{,\gamma}$

$$L = -\partial_{xx} + f \geq 0, f(x) = 2 \tanh^2 x - 1$$



# Stabilizing junction



$$\text{On } \mathbb{R}\vec{e}_1, H_{\text{eff}}(m^h) = \partial_{xx} m^h + m_1^h \vec{e}_1$$

$$M^h = (\tanh x, 1/\cosh x, 0)$$

$$\text{On } [0, L]\vec{e}_2, H_{\text{eff}}(m^v) = \partial_{xx} m^v + m_2^v \vec{e}_2$$

$$M^v = (0, 1, 0)$$

## Asymptotic Stability

- On the vertical part : asymptotic stability (as for one domain)
- On the horizontal part : stability of a domain wall in an infinite wire
- Junction condition (continuity) : we gain asymptotic stability on the horizontal wire



# Plan

- 1 Physical Context
- 2 Walls in constant-section wire
- 3 One wall configurations for pinched nanowires
- 4 Bent wires and junctions
- 5 Conclusion**

## **Conclusion : notches, bends and junctions can stabilize walls positions**

### Open Questions :

- wire with several pinched zones
- threshold current for walls depinning ?
- Dynamics of walls in pinched wire ?