

# Walls Dynamics in Ferromagnetic Nanotubes

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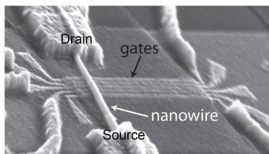
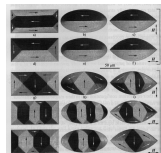
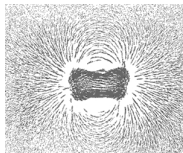
# Plan

- 1 Physical Context
- 2 Model for Ferromagnetic Nanotubes
- 3 Walls in Nanotubes
- 4 Stability Proof
- 5 Conclusion

# Ferromagnetic materials

## Properties

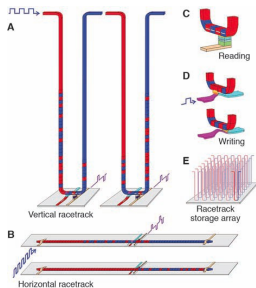
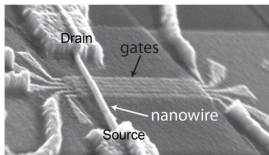
- Spontaneous magnetization
- Magnetic field generated by the magnetization
- Formation of domains and domain walls



## Industrial Applications

- Electromagnets
- Transformers
- Radar absorbing paints
- Mobile phones
- Modern Recording Devices

# Ferromagnetic nanowires

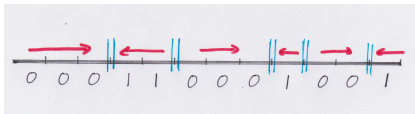


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- Mobile phones
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Stuart S. P. Parkin, Masamitsu Hayashi, Luc Thomas, Magnetic Domain-Wall Racetrack Memory, *Science* **320** (2008)

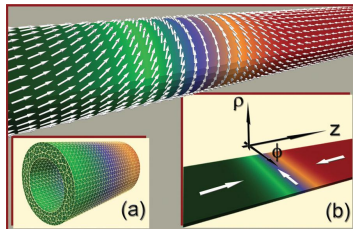
- 3d storage devices
- Fast access to the information



## Alternative : nanotubes

M. Yan, C. Andreas, A. Kákay, F. García-Sánchez and R. Hertel, Fast domain wall dynamics in magnetic nanotubes : Suppression of Walker breakdown and Cherenkov-like spin wave emission, Appl. Phys. Lett. **99** (2011)

- nanotubes instead of nanowires or nano stripes
- avoid boundary destabilizing effects

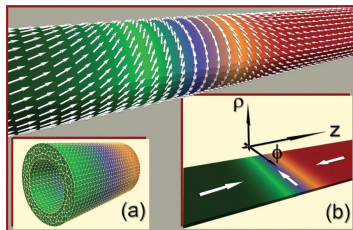


Goal : Modeling of nanotubes, description of walls dynamics

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## 3d model

### Magnetic Moment

- Magnetic material :  $\Omega \subset \mathbb{R}^3$
- Magnetization described by the magnetic moment  $m : \mathbb{R}_t^+ \times \Omega_x \rightarrow \mathbb{R}^3$

$$|m(t, x)| = 1 \text{ p. p.}$$

- Constitutive Relation :  $B = H + m$

### Landau-Lifschitz

$$\frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}})$$

$$H_{\text{eff}} = A\Delta m + H_d(m) + H_a$$

### Ferromagnetism Energy

$$E(m) = \frac{A}{2} \int_{\Omega} |\nabla m|^2 dx + \frac{1}{2} \int_{\mathbb{R}^3} |H_d(m)|^2 dx - \int_{\Omega} H_a \cdot m$$



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#### Exchange field

- Nonlinear parabolic behaviour
- Homogeneous Neumann boundary condition

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#### Demagnetizing field

- field generated by the magnetization
- $\text{curl } H_d(m) = 0$
- $\text{div } (H_d(m) + m) = 0$

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#### Zeeman term

$H_a$  : applied magnetic field

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## 2d-Model of nanotube

Characteristics : Length  $\gg$  diameter  $\gg$  thickness

$$\text{Length} = \infty, \quad \text{thickness} = 0$$

Asymptotic model of ferromagnetic thin film (Carbou, Gioia-James)

- Nanotube assimilated to  $\mathbb{R} \times \rho S^1$
- Demagnetizing field equivalent to a planar anisotropy

$$\begin{aligned} \bullet \operatorname{curl} H_d(m) &= 0 \\ \bullet \operatorname{div} (H_d(m) + m) &= 0 \end{aligned}$$

 $\implies$ 

$$\begin{aligned} \bullet H_d(m) &= -(m \cdot \nu)\nu \\ \bullet \nu &: \text{unitary normal} \\ \bullet E_d(m) &= \int_{\mathbb{R} \times \rho S^1} |m \cdot \nu|^2 \end{aligned}$$

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$$\partial_t m = -m \times H_{\text{eff}}(m) - m \times (m \times H_{\text{eff}}(m)),$$

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Thin film model for the nanotube

$m : (t, x, y) \rightarrow S^2$ ,  $2\pi$ -periodic in  $y$

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### 2d Model of nanotube

$\mathbf{m} : (t, x, y) \rightarrow S^2$ ,  $2\pi$ -periodic in  $y$

$$\partial_t \mathbf{m} = -\mathbf{m} \times \mathbf{h}(\mathbf{m}) - \mathbf{m} \times (\mathbf{m} \times \mathbf{h}(\mathbf{m}))$$

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# Model of Nanotube

## 2d Model of nanotube

$\mathbf{m} : (t, x, y) \rightarrow S^2, 2\pi$ -periodic in  $y$

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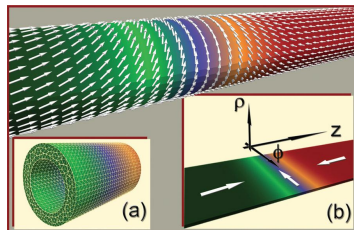
# One-Wall profiles

## 2d-model of nanotube

$\mathbf{m} : (t, x, y) \rightarrow S^2, 2\pi\text{-periodic in } y$

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# One-Wall profiles

## 2d-model of nanotube

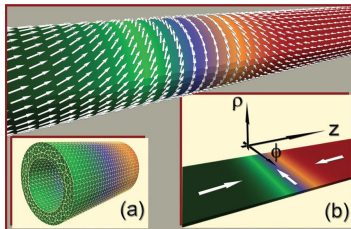
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$h_a = 0$  : static case

$$\mathbf{M}_0(x) = \begin{pmatrix} \tanh x \\ 1 \\ \frac{1}{\cosh x} \\ 0 \end{pmatrix}$$



# One-Wall profiles

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$h_a \neq 0$  : moving wall

$$\mathbf{M}^{h_a}(t, x, y) = \mathbf{R}_\theta(\mathbf{M}_0(\frac{x - ct}{\delta}))$$

$$\mathbf{R}_\theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}, \quad \mathbf{M}_0(x) = \begin{pmatrix} \tanh x \\ 1 \\ \cosh x \\ 0 \end{pmatrix}$$

$$h_a = \sin \theta \cos \theta, \quad \frac{c}{\delta} = -2h_a, \quad \frac{1}{\delta^2} = 1 + \sin^2 \theta$$



# Stability Result

## 2d-model of nanotube

$$\partial_t \mathbf{m} = -\mathbf{m} \times \mathbf{h}(\mathbf{m}) - \mathbf{m} \times (\mathbf{m} \times \mathbf{h}(\mathbf{m}))$$

$$\mathbf{h}(\mathbf{m}) = \Delta \mathbf{m} + 2\mathbf{e}_1 \times \partial_y \mathbf{m} + \mathbf{m}_1 \mathbf{e}_1 - \gamma \mathbf{m}_3 \mathbf{e}_3 + h_a \mathbf{e}_1$$

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## Theorem

If  $|h_a| \leq h_{max}$ ,  $\mathbf{M}^{h_a}$  is stable and asymptotically stable modulo translations, *i.e.* If  $\mathbf{m}(0, x, y)$  is close to  $\mathbf{M}^{h_a}(0, x, y)$ , then

- $\|\mathbf{m}(t, \cdot) - \mathbf{M}^{h_a}(t, \cdot)\|_{H^2}$  remains small
- there exists  $\sigma_\infty$  such that :

$$\|\mathbf{m}(t, x, y) - \mathbf{M}^{h_a}(t, x - \sigma_\infty, y)\|_{H^2} \rightarrow 0 \quad \text{when} \quad t \rightarrow +\infty.$$

# Comparison with other results

## 2d-model of nanotube

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## Results about 1d models for nanowires

### Round-section nanowires

$$\partial_t \mathbf{m} = -\mathbf{m} \times \mathbf{h}(\mathbf{m}) - \mathbf{m} \times (\mathbf{m} \times \mathbf{h}(\mathbf{m}))$$

$$\mathbf{h}(\mathbf{m}) = \partial_{xx} \mathbf{m} + \mathbf{m}_1 \mathbf{e}_1 + h_a \mathbf{e}_1$$

Carbou, Labbé, Jizzini, Privat, Trélat

$$\mathbf{M}(t, x) = \mathbf{R}_{h_a t}(\mathbf{M}_0(x + h_a t))$$

Asymptotic stability modulo translation and rotation

Instability threshold :  $|h_a| < 1$

Controlability

## Comparison with other results

### 2d-model of nanotube

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### Results about 1d models for nanowires

#### Elliptical-section wires

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#### K. Takasao

$$\mathbf{M}(t, x) = \mathbf{R}_\theta(\mathbf{M}_0(\frac{x-ct}{\delta}))$$

Asymptotic stability modulo translation  
 $h_a$  small enough

# Comparison with other results

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## Result in 3d

3d-model :  $\Omega = \mathbb{R}^3$

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### Asymptotic Stability

$\mathbf{e}_3$  easy-axis

Simplified model :  $h_d(m) \sim -\mathbf{m}_1 \mathbf{e}_1$

$h_a = 0$  (Only static walls)

$$\mathbf{M}(t, x, y, z) = \begin{pmatrix} 0 \\ 1 \\ \cosh x \\ -\tanh x \end{pmatrix}$$

# Comparison with other results

## 2d-Model

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# Difficulties

- Saturation constraint  $|\mathbf{m}| = 1$
- Invariances  $\implies 0$  is an eigenvalue of the linearized equation
- Sign of the other eigenvalues ? Coercivity ?
- Quasilinear equation - Variational estimates

## Sketch of the proof in the static case

## 2d-Model

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Saturation constraint  $|\mathbf{m}| = 1$  for the perturbations : **Mobile Frame**

$$M_0(x) = \begin{pmatrix} \tanh x \\ \frac{1}{\cosh x} \\ 0 \end{pmatrix}, \quad M_1(x) = \begin{pmatrix} -\frac{1}{\cosh x} \\ \tanh x \\ 0 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

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$$\mu(r) = \sqrt{1 - |r|^2} - 1$$



## Sketch of the proof in the static case

## 2d-Model

$$\partial_t \mathbf{m} = -\mathbf{m} \times \mathbf{h}(\mathbf{m}) - \mathbf{m} \times (\mathbf{m} \times \mathbf{h}(\mathbf{m}))$$

$$\mathbf{h}(\mathbf{m}) = \Delta \mathbf{m} + 2\mathbf{e}_1 \times \partial_y \mathbf{m} + \mathbf{m}_1 \mathbf{e}_1 - \gamma \mathbf{m}_3 \mathbf{e}_3$$

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$r$  takes its values in  $\mathbb{R}^2$ ; Stability of the 0-solution ?

# Invariance by translation

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## Invariance by translation for Landau-Lifschitz

For  $\sigma \in \mathbb{R}$ ,  $(t, x, y) \mapsto M_0(x - \sigma)$  is also a solution for LL

One-parameter family of solutions for the  $r$ -equation :  $R(\sigma)(x) = \begin{pmatrix} M_0(x - \sigma) \cdot M_1(x) \\ M_0(x - \sigma) \cdot M_2 \end{pmatrix}$

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(cf. D. Henry (1d) and T. Kapitula (multi-d) for Cahn-Hilliard equation)

Equation for  $(\sigma, w)$

$$\partial_t w = \Lambda w + \mathcal{F}$$

$$\frac{d\sigma}{dt} = \mathcal{G}(\sigma, w)(w)$$

To be proved :

- $w \rightarrow 0$  in  $H^2$
- $\sigma \rightarrow \sigma_\infty$

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If  $\langle w_1 | \frac{1}{\cosh x} \rangle = 0$ ,

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The non symmetric part is compensated by the other terms

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The linear part is coercive so that  $\|(\|_{\mathbf{H}_p^2} w)$  is exponentially decreasing

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# Plan

- 1 Physical Context
- 2 Model for Ferromagnetic Nanotubes
- 3 Walls in Nanotubes
- 4 Stability Proof
- 5 Conclusion



## Conclusion

- First stability result for non static walls in multi-d
- The 2d-model is a bit more realistic than the 3d-model used for the stability of static walls in 3d
- multi-d model without boundary
- Large applied field ?

Stability threshold for  $|h_a| < \frac{1}{2}$  ?

Behavior for  $h_a > \frac{1}{2}$  ? Precessional Regime ?

André Thiaville and Yoshinobu Nakatani  
*Domain-Wall Dynamics in Nanowires and Nanostrips*, Spin Dynamics in Confined Magnetic Structures III, Topics in Applied Physics **101** (2006), 161–205

- Nanowire with elliptical section
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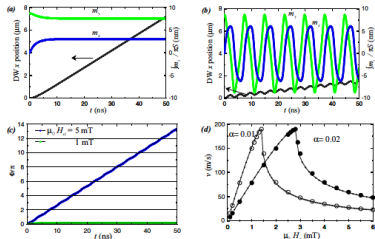


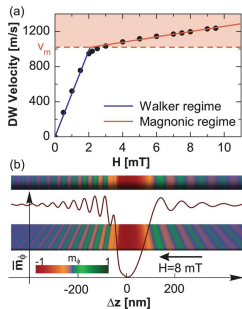
Fig. 11. Transverse-wall dynamics in a rectangular nanowire (edge sizes 10 nm and 5 nm, mesh size 5 nm). The field  $H_a$  applied along the wire axis is (a) 10 Oe ( $\mu_0 H_a = 1$  mT) and (b) 50 Oe ( $\mu_0 H_a = 5$  mT), the damping constant is  $\alpha = 0.01$ . The DW position is plotted as well as the integrated transverse magnetization components. The DW velocity is high at the low field (a), with no steady DW magnetization precession. At the higher field (b) this precession appears and the DW velocity drops and displays oscillations. The angle  $\phi$  of the DW magnetization orientation is displayed in (c) and the DW velocity in (d), versus applied field  $H_a$ , with the damping coefficient  $\alpha$  as a parameter. The analytical results (full thin lines and small dots) are drawn in (c) and (d), using the parameters of Table 1

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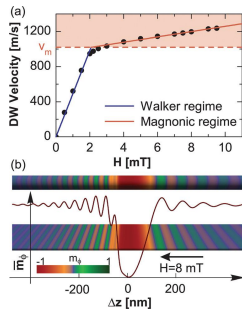
Better approximation for the demagnetizing field ?

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