

Stabilization of walls in pinched ferromagnetic nanowires

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International IFCAM conference on nonlinear PDEs

TIFR CAM Bangalore, 28th and 29th March 2017

1 Physical Context

2 1d Model for ferromagnetic nanowires

3 Walls in constant-section wire

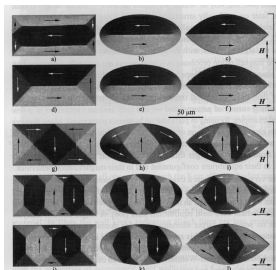
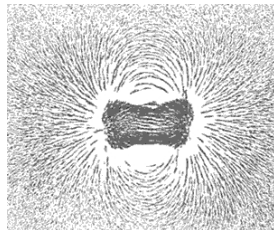
4 One wall configurations for pinched nanowires

5 Wire with several notches

Ferromagnetic materials

Properties

- Spontaneous magnetization
- Magnetic field generated by the magnetization
- Formation of domains and domain walls



A. Aharoni, Introduction to the Theory of Ferromagnetism, Oxford University Press, 1996

Industrial Applications

- Electromagnets
- Transformers
- Radar absorbing paints
- Mobile phones
- Modern Recording Devices

Magnetic Moment

- Magnetic domain : $\Omega \subset \mathbb{R}^3$
- Magnetization described by the magnetic moment $m : \mathbb{R}_t^+ \times \Omega_x \rightarrow \mathbb{R}^3$

$$|m(t, x)| = 1 \text{ p. p.}$$

- Constitutive Relation : $B = H + m$

Landau-Lifschitz Equation

$$\frac{\partial m}{\partial t} = -m \times \mathcal{H} - m \times (m \times \mathcal{H})$$

$$\mathcal{H} = -\partial_m \mathcal{E}(m) = A\Delta m + h_d(m)$$

$$\mathcal{E}(m) = \frac{A}{2} \int_{\Omega} |\nabla m|^2 + \frac{1}{2} \int_{\mathbb{R}^3} |h_d(m)|^2$$

$$\text{curl } h_d(m) = 0, \quad \text{div } (h_d(m) + \bar{m}) = 0$$

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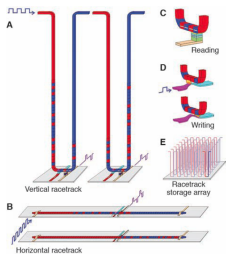
Landau-Lifschitz-Gilberg Equation

$$\frac{\partial m}{\partial t} - m \times \frac{\partial m}{\partial t} = -2m \times (A\Delta m + H_d(m))$$

$$\mathcal{H} = -\partial_m \mathcal{E}(m) = A\Delta m + h_d(m)$$

$$\mathcal{E}(m) = \frac{A}{2} \int_{\Omega} |\nabla m|^2 + \frac{1}{2} \int_{\mathbb{R}^3} |h_d(m)|^2$$

Data storage : ferromagnetic nanowires

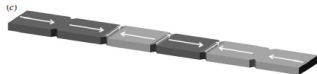


S. Parkin, M. Hayashi, L. Thomas, *Magnetic Domain-Wall Racetrack Memory*, *Science* **320** (2008)

- Bits encoded by the direction of the magnetization
- Stabilization of walls' position by notches (or pinchings)

Stuart S. P. Parkin, Masamitsu Hayashi, Luc Thomas, *Magnetic Domain-Wall Racetrack Memory*, *Science* **320** (2008)

- 3d devices for the storage of numerical data
- Fast access

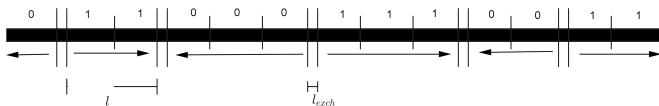


G. Hrkac, J. Dean and D. A. Allwood, *Nanowire spintronics for storage class memories and logic*, *Phil. Trans. R. Soc. A* (2011) **369**

Easy axis in the direction of the wire \Rightarrow Formation of domains

For data storage :

- Length of the domains \sim length of the sequences of zeros or ones in the data



- Walls positions must be unchanged (reliability of the storage)
- Information is transported to the reading head by application of an electric current

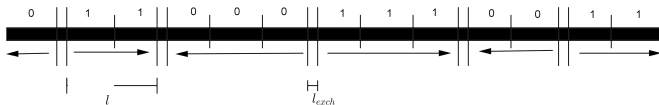
Objectives

- Obtention and justification of equivalent simpler 1d-models for nanowires
- Description of walls configurations and walls dynamics
- Effects of pinching on walls' stability

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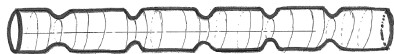
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1d model of wire with non constant cross section



$$\Omega_\eta = \{(x_1, x_2, x_3), x_1 \in \mathbf{I}, (x_2)^2 + (x_3)^2 < \eta^2(\rho(x_1))^2\}$$

Radius of the cross section : ρ

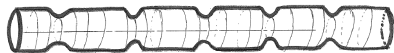
- $\rho : \mathbf{I} \rightarrow \mathbb{R}^{*+}, \mathcal{C}^\infty,$
- $0 < \rho_{\min} \leq \rho(x_1) \leq 1$
- $\mathbf{s}(x_1) = \pi(\rho(x_1))^2$

Weak Formulation :

$$\int_{[0, T] \times \Omega_\eta} \left(\frac{\partial m}{\partial t} - m \times \frac{\partial m}{\partial t} \right) \cdot \Psi = 2 \int_{[0, T] \times \Omega_\eta} \sum_{i=1}^3 m \times A \frac{\partial m}{\partial x_i} \cdot \frac{\partial \Psi}{\partial x_i} - 2 \int_{[0, T] \times \Omega_\eta} m \times H_d(m) \cdot \Psi.$$

- rescaling : fix domaine $\Omega_1 = \{(x_1, \tilde{x}_2, \tilde{x}_3), x \in \mathbf{I}, (\tilde{x}_2)^2 + (\tilde{x}_3)^2 < (\rho(x_1))^2\}$
- $\eta \rightarrow 0$ in the weak formulation with $\Psi(t, x_1, \tilde{x}_2, \tilde{x}_3) = \varphi(t, x_1)$

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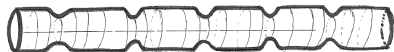
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$$m : \mathbb{R}_t^+ \times \mathbf{I}_x \rightarrow \mathbb{S}^2,$$

$$\int_{[0, T] \times \mathbf{I}} \mathbf{s}(x) \left(\frac{\partial m}{\partial t} - m \times \frac{\partial m}{\partial t} \right) \varphi = 2 \int_{[0, T] \times \mathbf{I}} \mathbf{s}(x) m \times A \frac{\partial m}{\partial x} \cdot \frac{\partial \varphi}{\partial x} - 2 \int_{[0, T] \times \Omega_\eta} \mathbf{s}(x) m \times \frac{1}{2} (m_2 \mathbf{e}_2 + m_3 \mathbf{e}_3) \cdot \varphi.$$

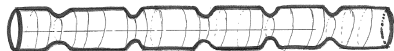
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Model of pinched wire

$$\frac{\partial m}{\partial t} = -m \times \mathcal{H}_{eff} - m \times (m \times \mathcal{H}_{eff})$$
$$\mathcal{H}_{eff} = A \partial_{xx} m + A \frac{\mathbf{s}'}{\mathbf{s}} \partial_x m - \frac{1}{2} (m_2 \vec{e}_2 + m_3 \vec{e}_3)$$

1d Ferromagnetism Energy

$$E(m) = \frac{A}{2} \int_{\mathbf{I}} \mathbf{s}(x) |\partial_x m|^2 dx + \frac{1}{4} \int_{\mathbf{I}} \mathbf{s}(x) (|m_2|^2 + |m_3|^2) dx$$

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Model of Nanowire

$$m : \mathbb{R}_t^+ \times I_x \rightarrow \mathcal{S}^2, \quad I = \mathbb{R} \text{ or } I = [-L, L]$$

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Static Solutions : Domains

$$\text{Domains : } m = \pm \vec{e}_1.$$

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- φ is constant
- $\theta'' + \cos \theta \sin \theta = 0$
- $\theta \rightarrow \pm\pi/2$ when $x \rightarrow \pm\infty$ or $\theta'(-L) = \theta'(L) = 0$

Static Solutions : Walls

- $M(x) = R_{\varphi(x)} \begin{pmatrix} \sin \theta(x) \\ \cos \theta(x) \\ 0 \end{pmatrix}$

$$R_{\varphi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & -\sin \varphi \\ 0 & \sin \varphi & \cos \varphi \end{pmatrix}$$

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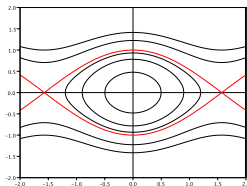
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G. Carbou, S. Labbé, *Stability for Static wall in a ferromagnetic Nanowire*, Discrete Contin. Dyn. Syst. Ser. B 6 (2006)

Stability of the constant profiles but not asymptotic stability

$$m(t, x) \rightarrow R_{\varphi_\infty} M_0(x - \sigma_\infty) \text{ when } t \rightarrow \infty$$

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Controlability of the wall position by the mean of the applied field

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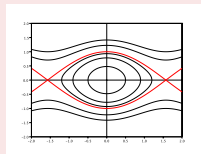
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$$M_0(x) = \begin{pmatrix} \sin \theta(x) \\ \cos \theta(x) \\ 0 \end{pmatrix}, \quad \begin{aligned} \theta'' + \cos \theta \sin \theta &= 0, \\ \theta'(-L) = \theta'(L) &= 0 \end{aligned}$$



G. Carbou, S. Labbé, *Stabilization of walls for nano-wires of finite length*, ESAIM Control Optim. Calc. Var. 18 (2012)

- One wall in a finite wire : centered in the middle of the wire
- This solution is unstable

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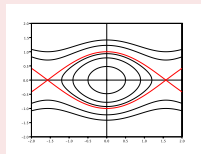
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The position of walls is unstable or non asymptotically stable



The data storage is not reliable

Stabilization by Pinching

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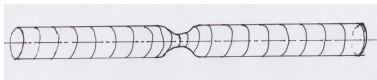


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Stabilization by Pinching

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Pinched Wire with one notch



$$\Omega_\eta = \{(x, y, z), x \in I, y^2 + z^2 < \eta^2(\rho(x))^2\}$$

Radius of the section : ρ

- $\rho : \mathbb{R} \rightarrow \mathbb{R}^+, C^\infty$,
- $\rho = 1$ outside $[-a, a]$,
- $\rho(-x) = \rho(x)$,
- ρ non increasing on $[0, a]$
- $\mathbf{s}(x) = \pi(\rho(x))^2$

Model of pinched Nanowire

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$$\mathcal{H}_{\text{eff}} = A \partial_{xx} m + A \frac{\mathbf{s}'}{\mathbf{s}} \partial_x m - \frac{1}{2} (m_2 \vec{e}_2 + m_3 \vec{e}_3)$$

$$E(m) = \frac{A}{2} \int_I \mathbf{s}(x) |\partial_x m|^2 dx + \frac{1}{4} \int_I \mathbf{s}(x) (|m_2|^2 + |m_3|^2) dx$$

Model of Nanowire

$$m : \mathbb{R}_t^+ \times I \rightarrow S^2,$$

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Theorem (with D. Sanchez)

- Existence of a profile for one wall
- Asymptotically stable for the position

Model of infinite Nanowire

$$m : \mathbb{R}_t^+ \times \mathbb{R}_x \rightarrow \mathcal{S}^2,$$

$$\frac{\partial m}{\partial t} = -m \times \mathcal{H}_{\text{eff}} - m \times (m \times \mathcal{H}_{\text{eff}})$$

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Theorem (with D. Sanchez)

- Existence of a profile for one wall
- **Asymptotically stable for the position**

Model of finite Nanowire

$$m : \mathbb{R}_t^+ \times [-L, L] \rightarrow \mathbb{S}^2,$$

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$$\mathcal{H}_{\text{eff}} = \partial_{xx} m + \frac{\mathbf{s}'}{\mathbf{s}} \partial_x m - (m_2 \vec{e}_2 + m_3 \vec{e}_3)$$

Theorem (with D. Sanchez)

- Existence of a profile for one wall for $L \geq \pi/2$
- **Asymptotically stable for the position** if L is large enough

- $M(x) = R_{\varphi(x)} \begin{pmatrix} \sin \theta(x) \\ \cos \theta(x) \\ 0 \end{pmatrix}$

$$R_{\varphi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & -\sin \varphi \\ 0 & \sin \varphi & \cos \varphi \end{pmatrix}$$

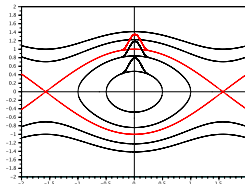
- BC : $\theta \rightarrow \pm\pi/2$ when $x \rightarrow \pm\infty$

$$\theta'(-L) = \theta'(L) = 0$$

- φ is constant

- $\theta'' + \frac{\mathbf{s}'}{\mathbf{s}} \partial_x \theta + \cos \theta \sin \theta = 0$

- $\left((\theta')^2 - \cos^2 \theta \right)' = -2 \frac{\mathbf{s}'}{\mathbf{s}} (\theta')^2$



Pinched Nanowire

$$m : \mathbb{R}_t^+ \times I_x \rightarrow \mathcal{S}^2$$

$$\frac{\partial m}{\partial t} = -m \times H_{\text{eff}} - m \times (m \times H_{\text{eff}})$$

$$\mathcal{H}_{\text{eff}} = m_{xx} + \frac{\mathbf{s}'}{\mathbf{s}} m_x - m_2 \vec{e}_2 - m_3 \vec{e}_3$$

Boundary Conditions

Wall Profile

$$M_0(x) = \begin{pmatrix} \sin \theta(x) \\ \cos \theta(x) \\ 0 \end{pmatrix}$$

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$$\theta'(-L) = \theta'(L) = 0$$

Stability of M_0 , asymptotic stability for the position of the wall ?

If $\|m(t=0) - M_0\|_{H^1(\mathbb{R})}$ is small, then

- for all time, $\|m(t) - M_0\|_{H^1(\mathbb{R})}$ remains small,
- $m(t)$ tends to $R_{\theta_\infty}(M_0)$ when t tends to $+\infty$.

Geometrical constraint $|m| = 1$

We describe the perturbations in the mobile frame

$$m(t, x) = \sqrt{1 - |r(t, x)|^2} M_0(x) + r_1(t, x) M_1(x) + r_2(t, x) M_2$$

$$M_0(x) = \begin{pmatrix} \sin \theta(x) \\ \cos \theta(x) \\ 0 \end{pmatrix} \quad M_1(x) = \begin{pmatrix} -\cos \theta(x) \\ \sin \theta(x) \\ 0 \end{pmatrix} \quad M_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

New Equations for r

$$r : \mathbb{R}_t^+ \times I_x \rightarrow \mathbb{R}^2$$

$$\partial_t r_1 = -L_1(r_1) - L_2(r_2) + \text{non linear}$$

$$\partial_t r_2 = L_1(r_1) - L_2(r_2) + \text{non linear}$$

$$L_1(r) = -\partial_{xx} r - \frac{s'}{s} \partial_x r + (\sin^2 \theta - \cos^2 \theta) r$$

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Exact Solution

M_0 for LL ~ 0 for the new system

Stability of 0 ?

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Exact Solution

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Stability of 0 ?

On $L^2(\mathbb{R})$, weighted scalar product :

$$\langle u | v \rangle = \int_{\mathbb{R}} \mathbf{s}(x) u(x) v(x) dx,$$

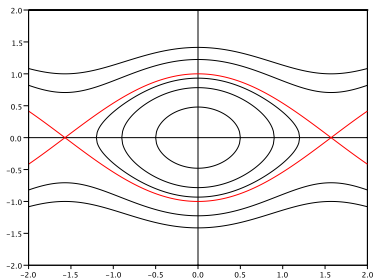
$$L_2 = -\frac{1}{\mathbf{s}} \partial_x (\mathbf{s} \partial_x) + (\sin^2 \theta_0 - (\theta'_0)^2) \implies L_2 \text{ is self-adjoint for } \langle \langle \rangle \rangle.$$

$$\langle L_2 u | v \rangle = \langle \ell u | \ell v \rangle, \quad \text{with } \ell = \partial_x + \theta' \tan \theta$$

$$L_2 \geq 0, \quad \text{Ker } L_2 = \mathbb{R}(\cos \theta)$$

- Valid whatever the configurations of the wire (several pinched zones)
- for one wall, on $(\cos \theta)^\perp$, $L_2 \geq 1$
- **Invariance by rotation** $\implies 0$ is an eigenvalue of the linearized

$$L_1 = L_2 + (\theta')^2 - \cos^2 \theta$$

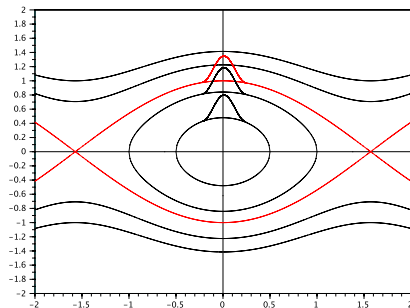


Key point : $L_1 \geq 0$?

- Infinite nanowire without pinching : $L_1 = L_2$, $L_1 \geq 0$. **Stability**
- Finite nanowire without pinching : $L_1 = L_2 - c$. **Bad sign, unstability for the position**

Properties of the linearized : L_1

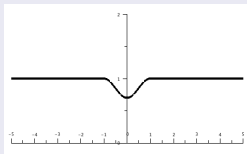
$$L_1 = L_2 + (\theta')^2 - \cos^2 \theta$$



- Infinite wire : $(\theta')^2 - \cos^2 \theta > 0$ on $\text{Ker } L_2 \implies L_1 > 0$
- Short finite wire : $(\theta')^2 - \cos^2 \theta \leq 0 \implies \text{Unstability}$
- Long finite wire : $(\theta')^2 - \cos^2 \theta > 0$ on a sufficiently large zone $\implies L_1 > 0$

- 1 Physical Context
- 2 1d Model for ferromagnetic nanowires
- 3 Walls in constant-section wire
- 4 One wall configurations for pinched nanowires
- 5 Wire with several notches**

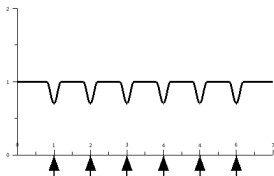
Elementary cell of length L



radius of the cross section : $\bar{\rho}$

- $\bar{\rho} = 1$ outside $[-a, a]$,
- $\bar{\rho}(-x) = \bar{\rho}(x)$,
- $\bar{\rho}$ non decreasing on $[0, a]$
- $\bar{\mathbf{s}}(x) = \pi(\bar{\rho}(x))^2$

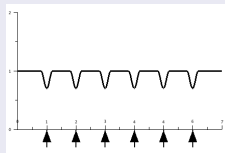
Wire with $N - 1$ notches, L : distance between 2 consecutive notches



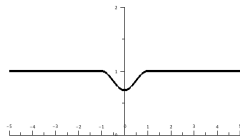
Model of wire

$$\frac{\partial m}{\partial t} = -m \times \mathcal{H}_{eff} - m \times (m \times \mathcal{H}_{eff})$$

$$\mathcal{H}_{eff} = \partial_{xx} m + \frac{\mathbf{s}'}{\mathbf{s}} \partial_x m - (m_2 \vec{e}_2 + m_3 \vec{e}_3)$$



$$\mathbf{s}(x) = \begin{cases} \pi & \text{if } 0 \leq x \leq \frac{L}{2} \\ \bar{\mathbf{s}}(x - kL) & \text{if } kL - \frac{L}{2} \leq x \leq kL + \frac{L}{2} \\ \pi & \text{if } (N-1)L + \frac{L}{2} \leq x \leq NL \end{cases}$$



$$\in \{0, 1\}^N$$

Example 00111010

bit 0 : domain where $m \sim -e_1$

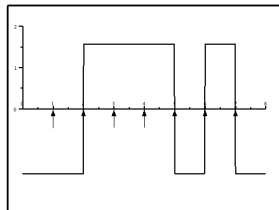
bit 1 : domain where $m \sim e_1$

Stationary solution for the
pinched wire

$$\mathcal{F}_L(\theta) = -\partial_{xx}\theta - \frac{s'}{s}\partial_x\theta - \cos\theta\sin\theta$$

$$\mathcal{F}_L(\theta^L) = 0$$

$$M = \begin{pmatrix} \sin\theta \\ \cos\theta \\ 0 \end{pmatrix}$$



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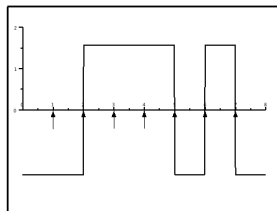
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Theorem (with D. Sanchez)

If L is great enough, for all N , for all data in $\{0, 1\}^N$,

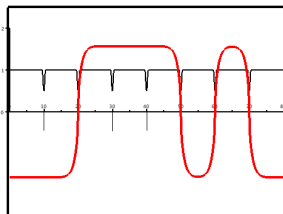
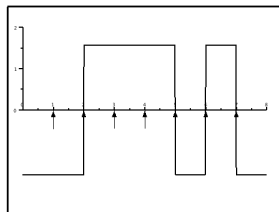
- there exists a static solution encoding this data,
- this solution is asymptotically stable modulo rotation

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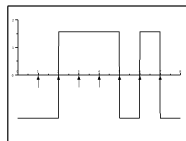


Stationary solution for the pinched wire

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00111010



1 Construction of an approximate solution

- In a domain : $\Theta_a^L = \pm\pi/2$
- In a wall : $\Theta_a^L \sim \bar{\Theta}$, the solution for one notch
- For L great enough, $\mathcal{F}_L(\Theta_a^L)$ exponentially small

2 Coercivity of Λ_a^L , the linearization around Θ_a^L

- One domain : OK
- One wall : OK
- Several domains and walls : IMS formula

3 Fixed Point Theorem : $\Theta_L = \Theta_a^L + v^L$

$$0 = \mathcal{F}_L(\Theta_a^L + v^L) = \mathcal{F}_L(\Theta_a^L) + \Lambda_a^L(v^L) + C(\Theta_a^L, v^L)(v^L, v^L), \quad \text{i.e. } v^L = \Phi_L(v^L)$$

$$\Phi_L(v) := -(\Lambda_a^L)^{-1} \left(\mathcal{F}(\Theta_a^L) + C(\Theta_a^L, v)(v, v) \right).$$

We proved the reliability of this kind of storage devices if L great enough

- Stability threshold for L ?
- Dynamics of the walls in pinched wires ?
- Recording (Creation of walls) ?

Thank you for your attention

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