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# Modeling Selecting and Ranking of Alternatives Characterized by Multiple Attributes to Satisfy Multiple Objectives

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**Abstract.** A number of decision making problems consist in selecting and ranking alternatives (projects, candidates, policies, etc.) that are characterized by multiple attributes in order to satisfy multiple objectives. Furthermore, this process generally necessitate coping with many stakeholders opinion regarding the importance to assign to each attribute and/or each objective. Given an objective, there will be attributes that act in the sens of realization of this objective (supporting attributes), those working against the achievement of this objective (rejecting attributes) and finally some attributes may be neutral regarding the achievement of this objective. Building on such distinction of attributes, we propose in this paper an approach, based on satisficing game theory, that firstly determine satisficing alternatives, those alternatives for which the *selectability measure* (determined based on supporting attributes and stakeholders preferences) exceeds the *rejectability mesure* (computed from rejecting attributes and stakeholders preferences) and secondly assign priorities to those satisficing alternatives so that an overall selectability exceeds an overall rejectability. An interesting thing to be noticed about this approach compared to existing ones is that it allows non homogeneity of attributes (all the alternatives do not need be characterized by the same attributes).

**Keywords:** Selecting and Ranking, Multiiple Attributes, Multiple Objectives, Multiple Actors, Satisficing Games, AHP.

## 1. Introduction and statement of the problem

Selecting and/or ranking alternatives constitute the step in a decision making process where an algorithm or a procedure must be derived using information obtained in previous steps (alternatives, attributes, objectives) in order to recommend alternatives to be implemented. The context of any real world decision making (selecting, sorting, ranking alternatives) problem is characterized by at least one of the following features.

- Multiple attributes: alternatives to be ranked, sorted or chosen are characterized by many attributes.
- Multiple objectives: decisions are made when seeking to satisfy many objectives; the classical constrained optimization problems (see for instance [4, 7]) can be considered to be multiple objectives problems where some objectives are transformed into constraints.
- Multiple actors (stakeholders): for a number of practical decision making problems, the (antagonist) opinions regarding the importance of attributes as well as objectives of many actors have to be taken into account.
- Uncertainty: the realization of objectives or the attributes defining alternatives may be subjected to uncertainty.

We consider in this paper the problem of selecting a subset  $\Sigma$  of alternatives from an universe  $U$  and ranking them that is assigning a relative weight  $x_u$  to a selected alternative  $u \in \Sigma$  in the context defined by the following materials that corresponds to previous declined features except uncertainty:

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- each alternative of the universe  $U$  is characterized by a certain number of attributes that are not necessary the same for all of alternatives;
- there are  $m$  objectives functions  $f_j, j = 1, 2, \dots, m$  to be satisfied; these functions may be general statement such as enhance socioeconomic situation, respect environment, be competent, etc.;
- a number of stakeholders and/or experts intervene in the selecting and ranking process through their preferences regarding objectives and/or attributes.

The version of this problem with a single stakeholder known in general as decision maker is what is typically known in the literature as multi-attributes, multi-objectives, multi-criteria decision making or shortly decision analysis (see for instance [3, 5, 6, 10, 11, 18, 25, 26]) and have been used in economics and management science for years and has gradually crept in engineering. Many real-world problems are often formulated in terms of multiple objectives and/or multiple attributes optimization problems, see for instance [3, 5, 6, 10, 11, 15, 18, 25, 26] and references therein. For instance in a production planning problem one wants to maximize the output and minimize the resources utilized. In the domain of mechanical engineering, civil engineering, and material engineering, the design of a structure is a multiple objectives optimization problem in the sense that, it is required in many cases to minimize the mass or the volume of the material used and to maximize some index of safety, see for instance [3]. Software design and implementation require considering many conflicting objectives as minimization of the cost of development, maximization of the speed of the system, minimization of power consumption and the weight of the system mainly in what concern embedded systems design. Other objectives related to environment for instance can be considered, see [15].

In this paper we do not consider uncertainty and we consider that attributes are either numeric or have been assigned a numerical values by experts or stakeholders by applying the analytic hierarchy process (AHP), see [13, 14], to the hierarchy of Figure 1 for instance where one must answer a question of the form "given an attribute  $a$ , how well perform an alternative  $u$  compared to an alternative  $v$ ?" using a specific scale (see below) in order to derive a comparison matrix from which a value or a weight will be derived for each alternative. Because of the importance of the AHP approach through this paper (for weights derivation in the subsequent sections) its procedure is recalled below.

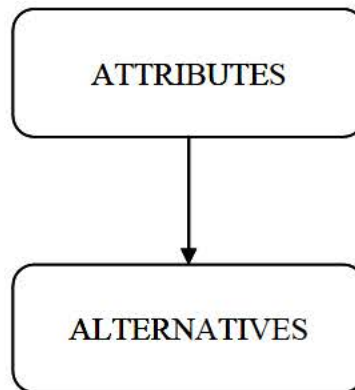


Figure 1: Attributes values elicitation hierarchy

### 1.1. Recall of AHP procedure

The analytic hierarchy process is a comprehensive, powerful and flexible decision making process to help people set priorities and make the best decision when both qualitative and quantitative aspects are used to evaluate alternatives, see [13, 14]. By reducing complex decisions to a series of one-on-one comparisons, then synthesizing the results, AHP not only helps decision makers arrive at the best decision, but also provides a clear rationale that it is the best. It is designed to reflect the way people actually think and is a widely used decision-making theory. The basic AHP decomposes a decision problems in different elements, grouped in clusters, that it arranges in a linear hierarchy form where the top element of the hierarchy is the overall goal of the decision making and is based on the following axioms (see [12]).

**Axiom 1 (reciprocity):** if element  $A$  is  $x$  times as important than element  $B$ , then element  $B$  is  $1/x$  times as important as element  $A$ .

**Axiom 1 (reciprocity):** if element  $A$  is  $x$  times as important than element  $B$ , then element  $B$  is  $1/x$  times as important as element  $A$ .

**Axiom 2 (homogeneity):** only comparable elements are compared. Homogeneity is essential for comparing similar things, as errors in judgement become larger when comparing widely disparate elements.

**Axiom 3 (independence):** the relative importance of elements at any level does not depend on what elements are included at a lower level.

**Axiom 4 (expectation):** the hierarchy must be complete and include all the criteria and alternatives in the subject being studied. No criteria and alternatives left out and no criteria and alternatives are included.

Given an hierarchy as that of Figure 1, the elements of cluster  $C_c$  in a top down hierarchy are pairwise compared with regard to each element of the cluster  $C_{c-1}$  to obtain a  $n_c \times n_{c-1}$  weighting matrix  $W_c$  where  $n_i$  is the number of elements in the cluster  $C_i$ . This matrix is given by equation (1)

$$W_c = \begin{bmatrix} w_c^1 & w_c^2 & \dots & w_c^{n_{c-1}} \end{bmatrix} \quad (1)$$

where  $w_c^i$  are  $n_c$  column vectors obtained as follows: for each element  $i$  of the cluster  $C_{c-1}$ , a pairwise comparison matrix  $W_c^i$  of elements of cluster  $C_c$  is constructed by answering questions of the form "how important is element  $X$  compared to the element  $Y$  of the cluster  $C_c$  with regard to upper level element  $Z$  of the cluster  $C_{c-1}$  ?" using the scales given by the following Table I ([13, 14])

Table I: scales for AHP comparison procedure.

Verbal scale	Numerical values
Equally important	1
Moderately more important	3
Strongly more important	5
Very strongly more important	7
Extremely more important	9
Intermediate scales (compromise)	2, 4, 6, 8

Once this matrix is constructed, the vector  $w_c^i$  is computed as the unique eigenvector of this matrix associated with eigenvalue  $n_c$ , that is the solution of the equation (2)

$$W_c^i w_c^i = n_c w_c^i \quad (2)$$

and a consistency<sup>1</sup> index is computed for possible modification of comparison weights (see [14]). The entries of vector  $w_c^i$  can be interpreted as the value or the weight of the element  $i$  (a given attribute on Figure 1) for the elements of the cluster  $c$  (alternatives on Figure 1). In the case of many stakeholders or experts, this process can be done separately by each stakeholders and then take the mean value.

The remainder of this paper is organized as follows: in the second section some classical approaches used to solve multiple objectives / multiple attributes decision making problems are reviewed; the third section recall the relevant materials of satisficing game theory that we need in this paper; in the fourth section we establish the satisficing game model for solving the selecting and ranking problems presented in the introduction section; the section five is devoted to the application of the approach established in the paper to a real world practical problems and concluding remarks are given in section six.

## 2. Classical approaches

Classical approaches for solving multiple objectives decision problems rely on the notion of the so-called Pareto dominance [9, 26] and Pareto-optimal set and the resolution is organized around two processes:

<sup>1</sup> A comparison matrix  $M$  is said to be consistent if it verifies:  $M_{ii} = 1$ ,  $M_{ji} = 1/M_{ij}$  and  $M_{ik} = M_{ij}M_{jk}$ .

search and decision making. Depending on how search (finding a sample of Pareto-optimal set) and decision process are combined, multiple objectives optimization methods can be classified in three categories [26].

- Decision making before search: the objective functions are aggregated into a single objective by using some preference of the decision maker.
- Search before decision making: here a sample (or totality) of Pareto-optimal set is obtained first and then a choice is made by a decision maker.
- Decision making during search: here an interactive sequential optimization is performed where after each search step, the decision maker is presented with a number of alternatives.

The first approach to deal with multiple objectives decision making problems has been the aggregation of objectives into a single objective in different ways leading to weighting methods, constraint methods and goal programming methods, see for instance [18]. The advantage of these methods is that efficient and broad algorithms developed for single objective optimization problems (see [4, 7, 8] and references therein) can be used to solve the resulting problems. The drawback of these techniques is that the subjective intervention of the user is needed to fix weighting factors and it is known [26] that these methods are most of the time not able to finding Pareto-optimal solutions in the case of non convex feasible space. To overcome these drawbacks, new methods have been designed based on evolutionary algorithms, mainly genetic algorithms that are able to generating efficiently Pareto-optimal solutions.

Other approaches that are considered in the multiple objectives /multiple attributes decision aid community are dominated by outranking approaches where a partial order of alternatives is derived by an interactive procedure between the analyst and the decision maker (see [1, 2, 11, 25]) and the evolutionary algorithms that are a class of stochastic optimization methods that attempt to simulate the process of natural evolution. Evolutionary algorithms have been proved useful in optimizing difficult functions that might mean: non-differentiable objective functions, many local optima, a large number of parameters, or a large number of configurations of parameters [26].

In this paper we consider a novel modeling approach that is based on the concept of supporting/rejecting attributes in the framework of satisficing game theory that is recalled in the following third section; then the approach considered in this paper is derived in the fourth section. Similar procedures have been derived by the author for decision making purposes including efficiency evaluation, object retrieving from database and priority setting, see [20, 21, 22, 23, 24].

### 3. Satisficing game theory

The underlying philosophy of most of the techniques used in the literature to construct selecting and ranking model is the superlative rationality, looking for the best, all the alternatives must be compared against each other. But the superlative rationality paradigm is not necessarily the way humans evaluate alternatives (and maybe not the best one). Most of the time humans content themselves with alternatives that are just "*good enough*" because their cognitive capacities are limited and information in their possession is almost always imperfect that is the fundamental idea behind the theory of bounded rationality that has its roots in the work by H. Simon [17]; the concept of being good enough allows a certain flexibility because one can always adjust its aspiration level. On the other hand, decision makers more probably tend to classify units as good enough or not good enough in terms of their positive attributes (benefit) and their negative attributes (cost) with regard to the decision goal instead of ranking units with regard to each other. For instance, to evaluate cars, we often make a list of positive attributes (driving comfort, speed, robustness, etc.) and a list of negative attributes (price, consumption per kilometer, maintainability, etc.) of each car and then make a list of cars for which positive attributes "exceed" negative attributes in some sense. This way of evaluation falls into the framework of praxeology or the study of theory of practical activity (the science of efficient actions). Here decision maker(s), instead of looking for the best options, look for satisficing alternatives. Satisficing is a term that refers to a decision making strategy where options, units or alternatives are selected which are "good enough" instead of being the best [19]. Let us consider a universe  $U$  of alternatives; then for each alternative  $u \in U$ , a *selectability function*  $\mu_s(u)$  and a *rejectability function*  $\mu_r(u)$  are defined to measure the degree to which  $u$  works towards success in achieving the decision maker's goal and costs associated with this alternative respectively. This pair of measures called *satisfiability functions or measures* are mass functions (they have the mathematical structure of the probabilities [19]): they are non negative and sum to one on  $U$ . The following definition then gives the set of options arguable to

be "good enough" because for these options, the "benefit" expressed by the function  $\mu_S$  exceeds the cost expressed by the function  $\mu_R$  with regard to the index of boldness  $q$ .

**Definition 1.** The satisficing set  $\Sigma_q \subseteq U$  with the index of boldness  $q$  is the set of alternatives defined by equation (3)

$$\Sigma_q = \{u \in U : \mu_S(u) \geq q\mu_R(u)\} \quad (3)$$

The boldness index  $q$  can be used to adjust the aspiration level: increase  $q$  if  $\Sigma_q$  is too large or on the contrary decrease  $q$  if  $\Sigma_q$  is empty for instance.

Applying the satisficing game theory to the selecting and ranking problem defined previously return then to determining satisfiability measures  $\mu_S(u)$  and  $\mu_R(u)$ ; the process of determining these measures will be considered in the following section.

#### 4. Satisficing selecting and ranking procedure

The approach considered in this paper is based on the idea that given an objective as defined in the introduction section, there are those attributes which variations are positively correlated to that objective (larger is better) and those for which variations are negatively correlated (smaller is better). The former are supporting attributes and the later rejecting ones for the considered objective. By so doing one can establish a selecting and ranking model based on two measures: selectability measure  $\mu_S(u)$  that aggregate supporting contributions and the rejectability measure  $\mu_R(u)$  that aggregate rejecting contributions in the framework of satisficing game theory [19] for the alternative  $u$ . The flow of information needed to establish this selecting and ranking procedure is given by the Figure 2.

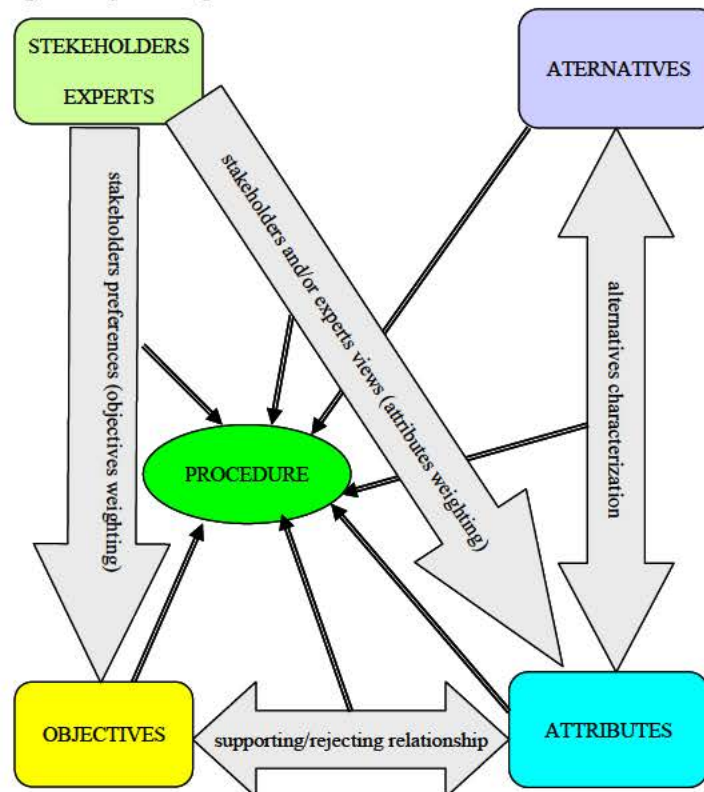


Figure 2: Flow of information needed by selecting and ranking procedure

In the following paragraph we will show how to compute these parameters from specification materials.

##### 4.1. Defining selectability and rejectability measures

The procedure for determining selectability and rejectability measures begins by the normalization of attributes to obtain a value like (see [16]) characterization as shown by equation (4)

$$a^n(u) = \frac{a(u) - a_{\min}}{a_{\max} - a_{\min}} \quad (4)$$

where  $[a_{\min} \ a_{\max}]$  is the range on which the attribute  $a$  is evaluated; when an attribute is common to all alternatives, this range can be determined by taking the corresponding *min/max* value over the universe  $U$ ; this normalization is necessary as the attributes do not have the same units (money, quantity, rank, etc.) nor evaluated on the same scale. As we stated in abstract, given an objective function  $f_j$  we divide the set of attributes of a given alternative  $u$  into two sets  $A_j^S(u)$  and  $A_j^R(u)$  containing supporting attributes and rejecting attributes respectively (see the following definition) with regard to that objective function.

**Definition 2.** An objective function  $f_j$  is said to be supported (respect. rejected) by an attribute  $a$  if and only if  $a(u) \geq a(v) \Rightarrow u$  is preferred to  $v$  for that objective (respect.  $a(u) \geq a(v) \Rightarrow v$  is preferred to  $u$  for that objective).

Once attributes are normalized, for each alternative  $u \in U$  and each objective function  $f_j$  we determine the measures  $\Psi_S^{f_j}(u)$  and  $\Psi_R^{f_j}(u)$  as given by equations (5)-(6)

$$\Psi_S^{f_j}(u) = \sum_{a \in A_j^S(u)} \alpha_a^{f_j} a^n(u) \quad (5)$$

$$\Psi_R^{f_j}(u) = \sum_{a \in A_j^R(u)} \beta_a^{f_j} a^n(u) \quad (6)$$

where  $\alpha_a^{f_j}$  and  $\beta_a^{f_j}$  are the relative supportability and rejectability importance assigned to attribute  $a$  (by stakeholders and/or experts) with regard to the objective function  $f_j$ . These measures represent supporting and rejecting weight of objective  $f_j$  for the alternative  $u$ . The aggregated selectability/rejectability measures for the alternative  $u$  are then given by equations (7)-(8)

$$\Psi_S(u) = \sum_{j=1}^m \omega_j \Psi_S^{f_j}(u) \quad (7)$$

$$\Psi_R(u) = \sum_{j=1}^m \omega_j \Psi_R^{f_j}(u) \quad (8)$$

where  $\omega_j$  is the relative importance of the objective function  $f_j$  with regard to selecting and ranking goal assigned by stakeholders.

Determination of weights  $\alpha_a^{f_j}$ ,  $\beta_a^{f_j}$  and  $\omega_j$  can be done using an AHP approach respectively on hierarchies (a), (b) and (c) of Figure 3 by each stakeholder and then taking the mean value, see for instance [20, 21] where a similar procedure for weights elicitation have been proposed. Experts and/or stakeholders that will determine this weights are not necessarily the same.

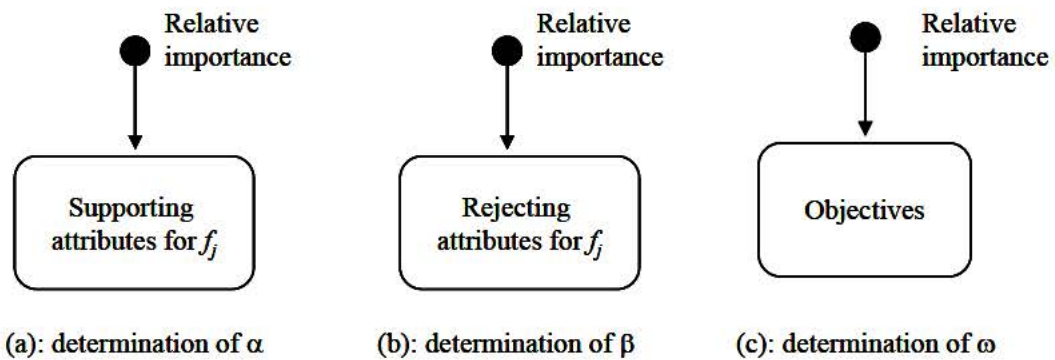


Figure 3: AHP hierarchy for determining weights  $\alpha_a^{f_j}$ ,  $\beta_a^{f_j}$  and  $\omega_j$

The selectability and rejectability measures  $\mu_S(u)$  and  $\mu_R(u)$  are then given by the following definition.



**Definition 3.** The selectability measure  $\mu_S(u)$  and the rejectability measure  $\mu_R(u)$  for the alternative  $u$  are given by equation (9)

$$\mu_S(u) = \frac{\Psi_S(u)}{\sum_{v \in U} \Psi_S(v)} \quad \text{and} \quad \mu_R(u) = \frac{\Psi_R(u)}{\sum_{v \in U} \Psi_R(v)} \quad (9)$$

Notice that these measures define probability tables over the set  $U$  and so fulfill the requirements of satisficing game theory. The following paragraph presents the procedures to select and rank alternatives arguable to be satisficing or good enough.

## 4.2. Satisficing selecting and ranking

### 4.2.1. Selecting

The selected subset is constituted by the alternatives for which the selectability measure exceeds the rejectability measure as given by the following definition.

**Definition 4.** The selected subset  $\Sigma_q$  at the index of boldness  $q$  is given by equation (10)

$$\Sigma_q = \{u \in U : \mu_S(u) \geq q\mu_R(u)\} \quad (10)$$

The caution index  $q$  can be used to adjust the number of alternatives one want to include in the selected subset  $\Sigma_q$ : small values of this index will lead to a lot of alternatives being declared satisficing whereas large values of  $q$  will reduce the number of satisficing alternatives. A sensitivity analysis can be carried up to determine the value  $q_{\min}$  below which all the alternatives of  $U$  will be declared satisficing and a value  $q_{\max}$  above which no alternative will be satisficing. For all alternatives of  $U$  to be declared satisficing the following inequality (11)

$$\mu_S(u) \geq q\mu_R(u) \quad \forall u \in U \Leftrightarrow q \leq q_{\min} = \min_{u \in U} \left( \frac{\mu_S(u)}{\mu_R(u)} \right) \quad (11)$$

must be verified so that for such indices of caution  $q$  we have equation (12)

$$\Sigma_q = U \quad (12)$$

On the contrary, there is no satisficing alternative, that is, equation (13)

$$\Sigma_q = \emptyset \quad (13)$$

if and only if the following inequality (14)

$$\mu_S(u) < q\mu_R(u) \quad \forall u \in U \Leftrightarrow q > q_{\max} = \max_{u \in U} \left( \frac{\mu_S(u)}{\mu_R(u)} \right) \quad (14)$$

is verified. Finally if the index of caution verifies  $q \in [q_{\min}, q_{\max}]$  then we have equation (15)

$$\Sigma_q \subseteq U \quad (15)$$

### 4.2.2. Ranking

Once the desired selected subset  $\Sigma_q$  is obtained, one will consider ranking its alternatives. The ranking process consist in assigning a weight  $x_u > 0$  to each alternative  $u \in \Sigma_q$  so that the overall satisficing condition of equation (16) is satisfied

$$\sum_{u \in \Sigma_q} \mu_S(u)x_u \geq q \sum_{u \in \Sigma_q} \mu_R(u)x_u \Leftrightarrow \sum_{u \in \Sigma_q} (\mu_S(u) - q\mu_R(u))x_u \geq 0 \quad (16)$$

subjected to conditions of equation (17)

$$\sum_{u \in \Sigma_q} x_u = 1, \quad x_u \geq \varepsilon \quad (17)$$

where  $\varepsilon$  is a very small real number to ensure that each alternative receives a non zero weight. These weights can be determined by solving the following linear programming problem (18)

$$\min_x 0 \text{ s.t. } \sum_{u \in \Sigma_q} (\mu_S(u) - q\mu_R(u))x_u \geq 0, \sum_{u \in \Sigma_q} x_u = 1, x_u \geq \varepsilon. \quad (18)$$

where *s.t.* stands for subjected to and  $x$  is a real vector of dimension  $|\Sigma_q|$ .

#### 4.2.3. Sensitivity analysis

Given a non satisfying alternative  $u$ , one may wonder how should variate its attributes values in order to render it satisfying when supposing that other alternatives remain unchanged; this process can be carried up hierarchically by determining first how much its aggregated supporting measures  $\Psi_S^{f_j}(u)$  and its aggregated rejecting measures  $\Psi_R^{f_j}(u)$  must variate and then inject these values into equations (5) and (6) to determine how much its attributes values must variate. To do so, let us derive how the variation  $d\mu_S(u)$  of the selectability measure does depend on the variations  $d\Psi_S^{f_j}(u)$ ,  $j=1, 2, \dots, m$  of the aggregated supporting measures values; this dependency derivation is given by equations (19) - (24)

$$d\mu_S(u) = \sum_{j=1}^m \left( \frac{\partial \mu_S(u)}{\partial \Psi_S^{f_j}(u)} \right) d\Psi_S^{f_j}(u) \quad (19)$$

$$= \sum_{j=1}^m \left( \frac{\frac{\partial \Psi_S(u)}{\partial \Psi_S^{f_j}(u)} \left( \sum_{v \in U} \Psi_S(v) \right) - \Psi_S(u) \frac{\partial}{\partial \Psi_S^{f_j}(u)} \left( \sum_{v \in U} \Psi_S(v) \right)}{\left( \sum_{v \in U} \Psi_S(v) \right)^2} \right) d\Psi_S^{f_j}(u) \quad (20)$$

$$= \sum_{j=1}^m \left( \frac{\omega_j \left( \sum_{v \in U} \Psi_S(v) \right) - \Psi_S(u) \omega_j}{\left( \sum_{v \in U} \Psi_S(v) \right)^2} \right) d\Psi_S^{f_j}(u) \quad (21)$$

$$= \sum_{j=1}^m \left( \frac{\omega_j \left( \frac{\Psi_S(u)}{\mu_S(u)} \right) - \Psi_S(u) \omega_j}{\left( \frac{\Psi_S(u)}{\mu_S(u)} \right)^2} \right) d\Psi_S^{f_j}(u) \quad (22)$$

$$= \sum_{j=1}^m \left( \frac{\omega_j \mu_S(u) (1 - \mu_S(u))}{\Psi_S(u)} \right) d\Psi_S^{f_j}(u) \quad (23)$$

$$= \left( \frac{\mu_S(u) (1 - \mu_S(u))}{\Psi_S(u)} \right) \sum_{j=1}^m \omega_j d\Psi_S^{f_j}(u) \quad (24)$$

In the same way we can show that the relation between the variation  $d\mu_R(u)$  of the rejectability measure and the variations  $d\Psi_R^{f_j}(u)$ ,  $j=1, 2, \dots, m$ , of the aggregated rejecting measures is given by equation (25)

$$d\mu_R(u) = \left( \frac{\mu_R(u) (1 - \mu_R(u))}{\Psi_R(u)} \right) \sum_{j=1}^m \omega_j d\Psi_R^{f_j}(u) \quad (25)$$

So that for a non satisfying alternative  $u$  to become a satisfying one when other alternatives remain unchanged the following inequality (26) must be satisfied

$$\mu_S(u) + d\mu_S(u) \geq q(\mu_R(u) + d\mu_R(u)) \quad (26)$$

which is equivalently to the linear inequality (27) in variation values  $d\Psi_S^{f_j}(u)$  and  $d\Psi_R^{f_j}(u)$ ,  $j=1, 2, \dots, m$  to be determined,

$$\left( \frac{\mu_S(u) (1 - \mu_S(u))}{\Psi_S(u)} \right) \sum_{j=1}^m \omega_j d\Psi_S^{f_j}(u) - q \left( \frac{\mu_R(u) (1 - \mu_R(u))}{\Psi_R(u)} \right) \sum_{j=1}^m \omega_j d\Psi_R^{f_j}(u) \geq q\mu_R(u) - \mu_S(u) \quad (27)$$

Furthermore for these variations to be feasible the following inequalities (28)-(29) must be verified<sup>2</sup>

$$d\Psi_S^{f_i}(u) \geq 0, 0 \leq \Psi_S^{f_i}(u) + d\Psi_S^{f_i}(u) \leq \sum_{a \in A_j^S(u)} \alpha_a^{f_j} \quad (28)$$

$$d\Psi_R^{f_i}(u) \geq 0, 0 \leq \Psi_R^{f_i}(u) + d\Psi_R^{f_i}(u) \leq \sum_{a \in A_j^R(u)} \beta_a^{f_j} \quad (29)$$

or equivalently inequalities (30)-(31)

$$0 \leq d\Psi_S^{f_i}(u) \leq \sum_{a \in A_j^S(u)} \alpha_a^{f_j} - \Psi_S^{f_i}(u) \quad (30)$$

$$-\Psi_R^{f_i}(u) \leq d\Psi_R^{f_i}(u) \leq \min\left(0, \sum_{a \in A_j^R(u)} \beta_a^{f_j}\right) \quad (31)$$

These variations can then be determined by solving a linear programming problem of the form (32)

$$\min_{d\Psi_S^{f_j}, d\Psi_R^{f_j}, j=1,2, \dots, m} \{0\} \quad s.t. \quad (27) - (30) - (31) \quad (32)$$

that is a mathematically ill-defined problem that can be rendered well defined by adding constraints and/or changing the function to be optimized in order to take into account practical concerns for instance. Once these values are determined, they will be injected into the equations (5)-(6) and one will solve linear programs of the form (33)

$$\min_{da^n(u)} \{0\} \quad s.t. \quad \begin{cases} d\Psi_S^{f_j}(u) = \sum_{a \in A_j^S(u)} \alpha_a^{f_j} da^n(u) \\ d\Psi_R^{f_j}(u) = \sum_{a \in A_j^R(u)} \beta_a^{f_j} da^n(u) \end{cases} \quad \forall f_j \text{ and } 0 \leq a^n(u) + da^n(u) \leq 1 \quad \forall a^n(u) \quad (33)$$

to finally determine the amount  $da^n(u)$  by which the attributes of the non satisficing alternative  $u$  must vary in order to become satisficing. Notice that for a practical case one may consider adding other constraints in (33); for instance if a given attributes participate only in supporting (respectively rejecting) some objectives it is obvious that one will constrain its variation to be non negative (respectively non positive).

**Remark 1.** A similar sensitivity analysis can be carried up with regard to almost all materials defining the parameters of the established model and mainly with regard to objectives weighting parameters  $\omega_j$ .

In the following section a real world application will be considered to show how the approach established in this paper does operate in practical situation.

## 5. Illustrative application

To illustrate the potentiality of this method let us consider a real-world example in the domain of waste management. This application is adapted from [15] where the objective was to find the most plausible solution to the municipal solid waste management problem in a region of Central Finland. Here, we are just interested in real data and testing how well our approach would have worked in real situation; we will modify the original formulation of this problem to fit our approach. A preliminary study has identified 11 alternatives (see [15] for the meaning of each alternative) and 8 attributes which meanings are described in the following points.

- **a<sub>1</sub>**: net cost per ton,
- **a<sub>2</sub>**: global effects,
- **a<sub>3</sub>**: local and regional health effects,
- **a<sub>4</sub>**: acidificative releases,
- **a<sub>5</sub>**: surface water dispersed releases,

<sup>2</sup> The right hand sides of equations (28)-(29) come from equations (5)-(6) as a result of the normalization of attributes.

- $a_6$ : technical reliability,
- $a_7$ : number of employees,
- $a_8$ : amount of recovered waste.

The evaluation of alternatives with regard to these attributes is well defined and row data (indicating units is not relevant here) are given on the Table II.

Table II: row data of the illustrative example.

Alternatives	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$
<b>IA</b>	787	155714560	148	364	505	9	20	4330
<b>IB1</b>	828	154887200	148	364	390	6	28	4080
<b>IB2</b>	837	154889339	148	364	390	6	24	5340
<b>IC1</b>	1062	139621200	201	377	370	7	35	11470
<b>IC2</b>	1050	139623330	201	377	370	7	28	12700
<b>IIA</b>	769	155061660	150	364	520	9	26	4330
<b>IIB</b>	861	154228170	138	364	310	6	32	5340
<b>IIC</b>	1048	138952170	203	377	300	7	36	12700
<b>IIIA</b>	894	154342000	137	364	470	5	25	3260
<b>IIIB</b>	997	153762000	137	364	300	5	32	4080
<b>IIIC</b>	1231	140035000	205	375	220	5	38	10600

To fit our approach we consider that two objectives functions  $f_1$  and  $f_2$  that are described below must be satisfied:

- $f_1$ : enhance the socioeconomic situation of the considered region;
- $f_2$ : respect the environment.

From the definition of attributes we consider supporting/rejecting attributes sets  $A_{f_1}^S / A_{f_1}^R$  and  $A_{f_2}^S / A_{f_2}^R$  (that are common to all alternatives) for these objectives to be given by equations (34)-(37).

$$A_{f_1}^S = \{a_6, a_7, a_8\} \quad (34)$$

$$A_{f_1}^R = \{a_1, a_2, a_3, a_4, a_5\} \quad (35)$$

$$A_{f_2}^S = \{a_6, a_8\} \quad (36)$$

$$A_{f_2}^R = \{a_2, a_3, a_4, a_5\} \quad (37)$$

These materials have been used by the procedure established in this paper to obtain the subsequent results; two cases, according to how objectives functions are weighted, are considered.

## 5.1. Results

### 5.1.1. Case 1: Equal importance

If we consider attributes as well as objectives to have the same importance, we obtain satisfiability results of the following Table III that are also depicted on Figure 4.

Table III: Results for equal importance case

Alternatives	$\Psi_S^{f_1}(u)$	$\Psi_R^{f_1}(u)$	$\Psi_S^{f_2}(u)$	$\Psi_R^{f_2}(u)$	$\mu_S(u)$	$\mu_R(u)$
<b>IA</b>	1.1133	2.1507	1.1133	2.1118	0.0958	0.0930
<b>IB1</b>	0.7813	1.8068	0.3369	1.6791	0.0481	0.0760
<b>IB2</b>	0.6926	1.8264	0.4703	1.6792	0.0500	0.0765
<b>IC1</b>	2.2030	3.1153	1.3697	2.4811	0.1536	0.1221
<b>IC2</b>	1.9444	3.0894	1.5000	2.4812	0.1481	0.1215
<b>IIA</b>	1.4467	2.1522	1.1133	2.1522	0.1101	0.0939
<b>IIB</b>	1.1370	1.4252	0.4703	1.2260	0.0691	0.0578
<b>IIC</b>	2.3889	2.8412	1.5000	2.2373	0.1672	0.1108
<b>IIIA</b>	0.2778	2.0220	0	1.7515	0.0119	0.0823
<b>IIIB</b>	0.7535	1.6437	0.0869	1.1502	0.0361	0.0609
<b>IIIC</b>	1.7775	2.9108	0.7775	1.9108	0.1099	0.1052

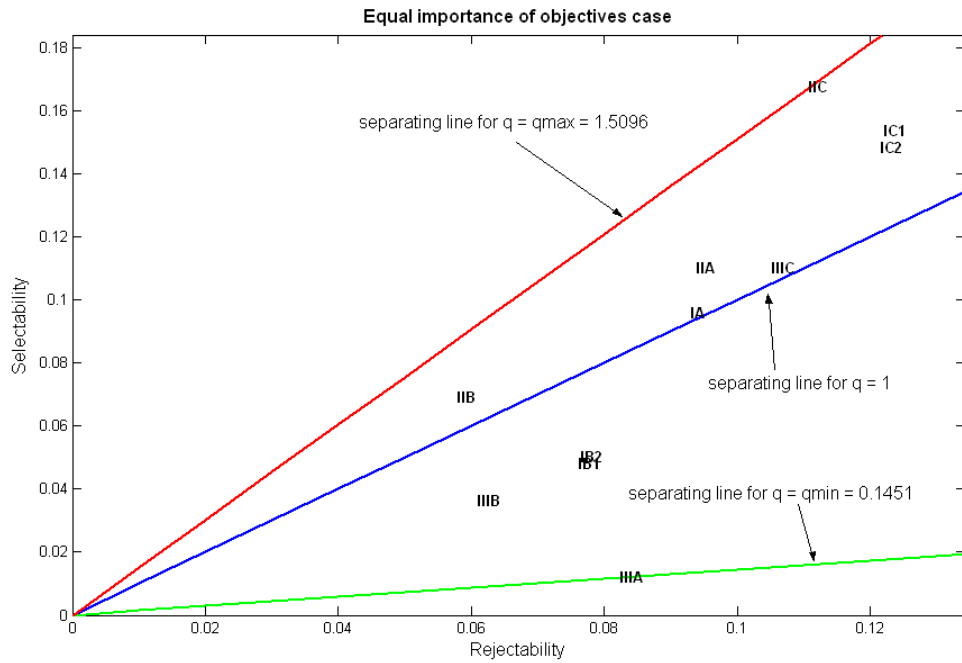


Figure 4: Results in the case of equal importance assumption: satisfying alternatives are those lying on or above the separating line for a give  $q$ .

So the satisfying alternatives subset  $\Sigma_1$ , with the index of boldness  $q=1$ , are given by equation (38)

$$\Sigma_1 = \{\mathbf{IA}, \mathbf{IC1}, \mathbf{IC2}, \mathbf{IIA}, \mathbf{IIB}, \mathbf{IIC}, \mathbf{IIIC}\} \quad (38)$$

and the solution of problem (18) is given by (39)

$$x = [0.1336 \quad 0.1479 \quad 0.1455 \quad 0.1403 \quad 0.1378 \quad 0.1603 \quad 0.1346] \quad (39)$$

that leads to the order of equation (40)

$$\mathbf{IIC} \succ \mathbf{IC1} \succ \mathbf{IC2} \succ \mathbf{IIA} \succ \mathbf{IIB} \succ \mathbf{IIIC} \succ \mathbf{IA} \quad (40)$$

Non satisfying alternatives set  $\bar{\Sigma}_1$  is given by (41)

$$\bar{\Sigma}_1 = U - \Sigma_1 = \{\mathbf{IB1}, \mathbf{IB2}, \mathbf{IIIA}, \mathbf{IIIB}\} \quad (41)$$

For these later alternatives a sensitivity analysis to determine how to render each one satisfying if other alternatives remain unchanged has been carried up and the results are summarized in the following Table IV

and on Figure 5 that shows how each of the 8 attributes values must variate in order to render the corresponding alternative satisfying. Notice that as the first 5 attributes contribute to the rejection of the two objectives and the 3 later ones to supporting them, we constrain the variations of the first ones to be non positive and the later to be non negative.

Table IV: Sensitivity analysis results

	<b>IB1</b>	<b>IB2</b>	<b>IIIA</b>	<b>IIIB</b>
$d\Psi_S^{f_1}(u)$	1.1536	1.1997	1.4181	1.1684
$d\Psi_R^{f_1}(u)$	-0.9210	-0.9311	-1.0309	-0.8380
$d\Psi_S^{f_2}(u)$	0.8645	0.7950	1.0417	0.9949
$d\Psi_R^{f_2}(u)$	-0.8559	-0.8560	-0.8929	-0.5862
$da_1^n$	-0.0651	-0.0751	-0.1380	-0.2518
$da_2^n$	-0.4808	-0.4808	-0.4676	-0.4473
$da_3^n$	-0.0863	-0.0863	0	0
$da_4^n$	0	0	0	0
$da_5^n$	-0.2888	-0.2888	-0.4253	-0.1389
$da_6^n$	0.3507	0.3827	0.5209	0.5409
$da_7^n$	0.2891	0.4047	0.3764	0.1736
$da_8^n$	0.5138	0.4123	0.5209	0.4540

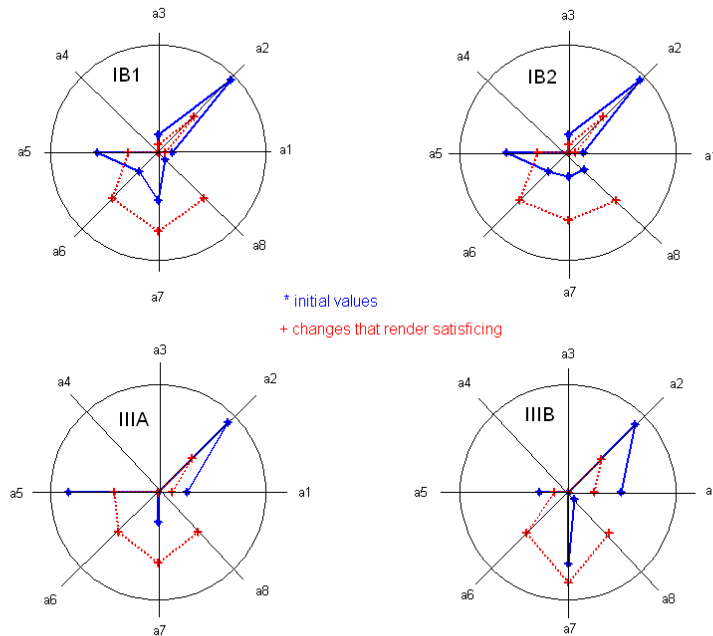


Figure 5: Initial values and changes of attributes that render each alternative **IB1**, **IB2**, **IIIA**, or **IIIB** satisfying considering that other alternatives remain unchanged.

### 5.1.2. Case 2: Weighting differently objectives

Let us consider now that the attributes have the same importance but the socioeconomic objective  $f_1$  is 80% more important than the environment objective  $f_2$ . The results in this case are given by the following

equations (42)-(44),

$$\Sigma_1 = \{IC1, IC2, IIA, IIB, IIC, IIIC\} \quad (42)$$

$$x = [0.1679 \quad 0.1635 \quad 0.1626 \quad 0.1643 \quad 0.1815 \quad 0.1585] \quad (43)$$

$$IIC \succ IC1 \succ IIB \succ IC2 \succ IIA \succ IIIC \quad (44)$$

We see that the alternative **IA** is no longer satisficing and the order is modified between alternatives **IIB**, **IC2** and **IIA**.

In contrary, if the environment objective  $f_2$  is considered to be 80% more important than the socioeconomic objective  $f_1$ , we obtain the following results (45)-(47) where the alternative **IA** becomes satisficing and the order between satisficing alternatives is significantly modified compared to previous results,

$$\Sigma_1 = \{IA, IC1, IC2, IIA, IIB, IIC, IIIC\} \quad (45)$$

$$x = [0.1378 \quad 0.1478 \quad 0.1506 \quad 0.1404 \quad 0.1317 \quad 0.1606 \quad 0.1311] \quad (46)$$

$$IIC \succ IC2 \succ IC1 \succ IIA \succ IA \succ IIB \succ IIIC \quad (47)$$

**Remark 2.** It is interesting to notice that the final accomplished alternative **IIC** in the original study [15] is the one that would have been selected by the approach presented in this paper although the formulation considered here is slightly different with some arbitrary considerations.

## 6. Conclusion

The problem of selecting and ranking alternatives characterized by multiple attributes to satisfy multiple objectives where some stakeholders opinions must be taken into account has been considered in this paper. The main idea of the selecting and ranking procedure established in this paper relies on first determining, for any objective, attributes that support it (larger is better) and attributes that reject it (smaller is better); then considering stakeholders preferences regarding the importance of objectives by weighting them as weights that stakeholders and/or experts may assign to each category of attributes, two measures, one known as selectability that act on supporting attributes and another known as rejectability are derived for each alternatives. Alternatives to be included in the selected subset are those for which the selectability measure exceeds the rejectability measure subjected to an index of caution that permits to adjust the size of this subset. A priority index is then determined to order the selected alternatives in order to optimize the difference between the aggregated selectability and rejectability measures. The sensitivity analysis proposed in this paper allows to quickly verify whether changes in attributes of a non satisficing alternatives will allow it to be come satisficing. Another interesting fact of the procedure established in this paper is that alternatives are not required to be characterized by the same attributes, the important thing is to be able to establish a supporting/rejecting relationship between these attributes and stakeholders objectives. The procedure is applied to a real world problem with interesting results that confirms the potentiality of the approach.

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