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SANPEV: a Satisficing Analytic Network Process framework for Efficiency eValuation of alternatives

Ayeley P. TCHANGANI * †

Abstract. The purpose of this paper is to derive a procedure to evaluate a group of alternatives or units considered as systems where a certain transformation process consumes heterogeneous input attributes or items (to be understood in a broad sense including effort that must be done, a negative impact, consumable resources, ...) to produce or deliver heterogeneous output items (including subjective satisfaction, tangible products, beneficial impact, ...). In general, many actors with different points of view as well as different information sources more or less reliable will be involved in the evaluation process. The evaluation context we consider here consists in *stakeholders* (decision makers, experts, users, ...) that give their opinion regarding the impact of each item with regard to the evaluation goal ; the information or data (values of items for different alternatives) about items are collected from or supplied by different more or less reliable *information or data sources* (news papers, magazines, web, agencies and consulting cabinets, experts, ...). The established model aims to integrate the interactions between these different components (stakeholders, items, information sources and alternatives) and consists basically for each alternative or unit in computing two measures: an aggregated measure known as the *selectability* at the output of the system and an aggregated measure at the input known as the *rejectability* in the framework of *satisficing game theory*. The derivation of these measures is carried up by a pairwise comparison process using the *analytic network process* (ANP) approach, an extension of the well known *analytic hierarchy process* (AHP), that allows to take into account complex interactions of evaluation process components such as dependency and feedback.

Keywords: Evaluation Model, Multi Attributes Decision Making, Information Fusion, Satisficing Game Theory, Analytic Network Process, AHP, DEA.

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1 Introduction

The problem of evaluating the performance of an alternative or a unit within a group of alternatives with regard to a goal is of great importance for efficient decision making (choosing, sorting or ranking alternatives) regarding this group. Here, we understand by alternative a place where a certain transformation process (in a broad sense) consumes some input items (resources) to produce some output items (deliveries) as shown by Figure 1; consumption and production must be understood in a very broad way and can be represented respectively by the effort one must furnish, negative impact one must tolerate, prize one must pay, ... to obtain some satisfaction, some goods, ... A typical alternative or unit could be a hospital, a manufacturing plant, a

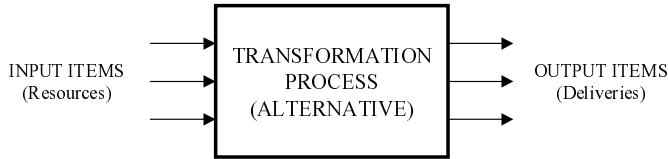


Figure 1: A typical transformation process or system

university, a police department, a restaurant, a call center, a physical object (a car, a computer, a house, ...), or a non physical thing (job, project, ..), etc.. Evaluating an alternative is always a relative process, that is, its resource utilization versus its output performance will be compared to other alternatives of the same kind. Classically, the evaluation process is carried by computing an absolute efficient index e_u for each alternative or unit u , from a set or universe \mathcal{U} , using equation (1)

$$e_u = \frac{\sum_{j=1}^{m_o} O(u, j)}{\sum_{i=1}^{m_i} I(u, i)}; \quad (1)$$

where $I(u, i)$ is the value of the input item i ($i = 1, 2, \dots, m_i$) used by the unit u to deliver m_o output items valued by $O(u, j)$ (the value of the output item j produced by the unit u). This approach has some serious drawbacks such as the fact that, in general, all input items or output items do not have the same importance with regard to the evaluation goal and even by trying to weight them to obtain an efficiency index as given by equation (2),

$$e_u = \frac{\sum_{j=1}^{m_o} \alpha_u^j O(u, j)}{\sum_{i=1}^{m_i} \beta_u^i I(u, i)}, \quad \alpha_u^j, \beta_u^i > 0; \quad (2)$$

there still exists the problem of determining relative weights α_u^j and β_u^i . A serious attempt to overcome these drawbacks comes from data envelopment analysis (DEA) established by [1]. It is an extreme point method that determines weights α_u^j and β_u^i in order to define a relative efficiency of each alternative compared to the best one (possibly virtual) by solving linear programs (see for instance [3] for definition

of linear programming concepts). Data envelopment analysis (DEA) is a technique for assessing and ranking the performance of corporations, research projects or other entities where an entire array of indicators of performance are to be evaluated. It was invented by [1] and is a linear programming based technique for measuring the relative performance of systems where the presence of multiple inputs and outputs makes comparisons difficult. It is an extreme point method and compares each system (also known in the DEA literature as producer or decision making unit (DMU)) with only the “best” one (possibly virtual). A fundamental assumption behind the DEA method is that if a given DMU, A , is capable of delivering $Y(A)$ in the output with $X(A)$ as inputs, then other systems should also be able to do the same if they were to operate efficiently. The procedure of finding the best virtual producer can be formulated as a linear program. Analyzing the efficiency of n producers then requires solving n linear programming problems. The following formulation is one of the standard forms for the DEA. We consider that there are n producers, m_i input items and m_o output items for each producer; the value of input item i and the value of output item j for producer u are $I(u, i)$ and $O(u, j)$ respectively. The n linear programming problems to be solved are defined by equation (3)

$$\begin{aligned}
& \max_{\alpha_u^j, \beta_u^i} \sum_{j=1}^{m_o} \alpha_u^j O(u, j) \\
& \sum_{j=1}^{m_o} \alpha_v^j O(v, j) - \sum_{i=1}^{m_i} \beta_v^i I(v, i) \leq 0, \quad 1 \leq v \leq n, \quad v \neq u \\
& \sum_{i=1}^{m_i} \beta_u^i I(u, i) = 1 \\
& \alpha_u^j, \beta_u^i \geq \varepsilon, \quad 1 \leq u \leq n, \quad 1 \leq j \leq m_o, \quad 1 \leq i \leq m_i \\
& 1 \leq u \leq n
\end{aligned} \tag{3}$$

where ε denotes a small strictly positive real number. The DEA method has been applied in many situations such as: health care (hospitals, doctors), education (schools, universities), banks, manufacturing, benchmarking, management evaluation, fast food restaurants, retail stores, police departments (see for instance [9], [22], [23], [25]). The power of the DEA as a tool for performance evaluation is due to the following characteristics (see for instance [7] and references therein for applications of this method).

- DEA can handle multiple input and multiple output models.
- It doesn't require an assumption of a functional form relating inputs to outputs.
- DMUs are directly compared against the best (possibly virtual) DMU.
- Inputs and outputs can have very different units.

But the DEA method does have a number of drawbacks reported in the literature among which are the following concerns.

- A judicious choice of weights will lead to a high proportion of units being efficient rendering the method less discriminatory. For instance a unit which has the highest ratio of one of the outputs to one of the inputs will be efficient, or have an efficiency which is very close to one by putting as much weight as possible on that ratio and the minimum weight (ε) on the other inputs and outputs. As

it is probable to find such ratio for many units, the efficient units set will be large. So a unit can appear efficient simply because of its pattern of inputs and outputs and not because of any inherent efficiency. To overcome this problem, some authors propose to apply weights restriction (see [6]) or to incorporate maximum weights on outputs (see [10]) approaches that are difficult to justify in practice.

- Since DEA is an extreme point technique, noise (even symmetrical noise with zero mean) such as measurement error can cause significant deviations that make the method not robust.
- Since a standard formulation of the DEA creates a separate linear program for each DMU, large problems will be obviously computationally intensive.

Besides the technical drawbacks raised concerning DEA approach, there is an important issue that was not raised yet: to apply DEA approach or the classical absolute index (1) approach to which it succeeds, items (input items as well as output ones) are required to take numerical values whereas in practice decision makers face heterogeneous (quantitative and/or qualitative) characterization of items. Furthermore, in general, decision makers or the analyst (one who is in charge of building the evaluation model) need experts, that do not have the same sensitivity regarding each item nor are competent to evaluate them, to give an opinion about the importance of each item with regard to the evaluation goal. Finally information that characterizes alternatives with regard to items will be collected from different sources that are more or less reliable and the established model must take into account all these concerns.

The approaches used to build evaluation models that take into account some of concerns that are not considered by DEA approach are methods from multicriteria decision making (MCDM) literature, see [2], [11], [12], [14], [17], [24] and references therein. These methods can be regrouped into two main categories.

- Evaluation models based on *value function(s)*: roughly speaking, these techniques consider a numerical function π defined on the universe \mathcal{U} such that

$$\pi(u) \geq \pi(v) \Leftrightarrow u \succsim v \quad (4)$$

where “ $u \succsim v$ ” stands for “ u is at least as good, with regard to the evaluation goal, as v or u dominates v ” leading to a weak order. The evaluation modeling process then consists in building such a function based on the interactions between different components of decision process; there are many techniques employed in the literature for constructing such a function where a number of them suppose a particular form for π such as expected utility or additive value function (see [2], [12], [17], [11], [24] and references therein).

- Evaluation models allowing incomparability and/or intransitivity known in the literature as outranking methods such as the family of ELECTRE procedures and PROMETHEE techniques (see [2] [11] [24]); these techniques necessitate in general iterative interaction between decision makers and the analyst.

Let us notify before continuing that the methods evoked here correspond to the discrete universe \mathcal{U} case; the classical methods for continuous universe case include for instance goal and compromise programming and a recent study by the author (see [21]) applies the satisficing game approach for this later case.

Another issue raised by DEA approach is the spirit of comparing each unit to the “best one” and our main objection with regard to the majority of techniques from multicriteria decision making literature is that they do not make difference in the comparison procedure between negative criteria (input items) and positive criteria (output items). We do think that, to evaluate units, humans often evaluate each unit firstly with regard to how efficiently it uses its resources to produce its output and secondly they look at how good this unit is compared to its counterparts. If we think of how students are evaluated, we see that each student is first evaluated individually (their marks reflecting the effort they have made) and then compared to the best student of the same class. Of course if there are intangible aspects in the characterization of units, they will introduce some preemptivity that must be given priority in the evaluation process so that the approach we are considering here concerns only input and output items that are pairwise comparable respectively. This observation suggests that an alternative should be evaluated by comparing its deliveries (output performance) to its resources consumption at first and with regard to the other alternatives in a second stage. A framework that seems interesting to tackle this problem with is the recently developed satisficing game theory [18] that, basically for our problem, will consist in defining *selectability* (with regard to output items) and *rejectability* (with regard to input items) functions or measures known as *satisfiability* functions. An efficient unit will be that for which the selectability is at least equal to the rejectability. This approach has been developed and applied with success to a real world problem in [20] for the case of numerical valuation of input/output items. The satisfiability functions here will depend on the interactions between stakeholders, items, information sources and alternatives so that the *analytic network process* (ANP), see [13], a generalization of the *analytic hierarchy process* (AHP, see [12]), will be used to derive such functions that capture the complex interdependencies between elements of the evaluation context.

The remainder of this paper is organized as follows: in the second section the elements of the satisficing game theory that are relevant to our problem are presented; the third section is devoted to a recall of analytic network process method; the fourth section shows how the performance evaluation problems described in the introduction section can be formulated using satisficing game approach and the analytic network process; the section five presents potential applications of the approach considered so far and finally a conclusion is presented in the sixth section.

2 Satisficing game theory

The underlying philosophy of the DEA approach for performance evaluation is substantive rationality i.e., looking for the best. But the substantive rationality paradigm is not necessarily the way humans evaluate options (and maybe not the best one).

Most of the time humans content themselves with options that are just “good enough”; the concept of being good enough allows a certain flexibility because one can always adjust one’s aspiration level. On the other hand, decision makers more probably tend to classify units as good enough or not good enough in terms of their positive attributes (benefit) and their negative attributes (cost) with regard to the evaluation goal instead of ranking units with regard to each other. For instance, to evaluate cars, we often make a list of positive attributes (driving comfort, speed, robustness, etc.) and a list of negative attributes (price, petrol consumption, maintainability, etc.) of each car and then make a list of cars for which positive attributes “exceed” negative attributes in some sense. This way of evaluation falls into the framework of praxeology or the study of the theory of practical activity (the science of efficient action) derived from epistemic logic (the branch of philosophy that classifies propositions on the basis of knowledge and belief regarding their content; for a proposition to be admissible it must be both believable and informative) and developed by [18]. Here decision maker(s), instead of looking for the best options, look for the satisficing options. This approach for decision making is rather close to Simon’s theory of bounded rationality (see for instance [16]) who suggests to change the optimization paradigm by the satisfaction one as one will never get all the necessary information regarding relationships of different components of a decision problem nor have enough computing power to search for the optimal solution so that one can content himself with options that satisfy its aspiration’s level.

Satisficing is a term that refers to a decision making strategy where options, units or alternatives are selected which are “good enough” instead of being the best [18]. Let us consider a universe \mathcal{U} of options, alternatives or units; then for each unit $u \in \mathcal{U}$, a *selectability function* $p_s(u)$ and a *rejectability function* $p_r(u)$ are defined so that $p_s(u)$ measures the degree to which u works towards success in achieving the decision maker’s goal and $p_r(u)$ is the cost associated with this unit. This pair of measures called *satisfiability functions* are in general normalized on \mathcal{U} . The following definition then gives the set of options which can be considered to be “good enough” because, for these options, the “benefit” expressed by the function p_s exceeds the “cost” expressed by the function p_r with regard to an index of caution q .

Definition 1 *The satisficing set $\Sigma_q \subseteq \mathcal{U}$ is the set of units defined by equation (5)*

$$\Sigma_q = \{u \in \mathcal{U} : p_s(u) \geq qp_r(u)\}. \quad (5)$$

Small values of the index of caution q will lead to lot of units being declared satisficing whereas large values of q will reduce the number of satisficing units. A sensitivity analysis can be carried up to determine the value q_{\min} below which all the units of \mathcal{U} will be declared satisficing and a value q_{\max} above which non unit will be satisficing. For all units of \mathcal{U} to be declared satisficing the following inequality (6)

$$p_s(u) \geq qp_r(u) \forall u \in \mathcal{U} \Leftrightarrow q \leq q_{\min} = \min_{u \in \mathcal{U}} \left(\frac{p_s(u)}{p_r(u)} \right) \quad (6)$$

must be verified so that for such an index of caution q we have

$$\Sigma_q = \mathcal{U}. \quad (7)$$

On the contrary, there is no satisficing unit, that is

$$\Sigma_q = \emptyset \quad (8)$$

if and only if the following inequality (9)

$$p_s(u) < qp_r(u) \quad \forall u \in \mathcal{U} \Leftrightarrow q > q_{\max} = \max_{u \in \mathcal{U}} \left(\frac{p_s(u)}{p_r(u)} \right) \quad (9)$$

is verified. Finally if the index of caution verifies $q \in [q_{\min}, q_{\max}]$ then

$$\Sigma_q \subseteq \mathcal{U}. \quad (10)$$

But for a satisficing unit u there can exist other satisficing units that are better (having more selectability and at most the same rejectability or having less rejectability and at least the same selectability) than u ; it is obvious that in this case any rational decision maker will prefer the latter units. So the interesting set is that containing satisficing units for which there are no better units: this is called the *satisficing equilibrium* set \mathcal{E}_q^S . To define this set, let us define first, for any unit $u \in \mathcal{U}$, the set $\mathcal{B}(u)$ of units that are strictly better than u (or strictly dominate u)

$$\mathcal{B}(u) = \mathcal{B}_s(u) \cup \mathcal{B}_r(u), \quad (11)$$

where $\mathcal{B}_s(u)$ and $\mathcal{B}_r(u)$ are defined as:

$$\begin{aligned} \mathcal{B}_s(u) &= \{v \in \mathcal{U} : p_r(v) < p_r(u) \text{ and } p_s(v) \geq p_s(u)\}, \\ \mathcal{B}_r(u) &= \{v \in \mathcal{U} : p_r(v) \leq p_r(u) \text{ and } p_s(v) > p_s(u)\}. \end{aligned}$$

Concretely if $v \in \mathcal{B}(u)$, it means that either “ $p_r(v) < p_r(u)$ and $p_s(v) \geq p_s(u)$ ” or “ $p_r(v) \leq p_r(u)$ and $p_s(v) > p_s(u)$ ” that is v does better in terms of rejectability and in terms of selectability than u . The equilibrium set \mathcal{E} (units for which there are no strictly better units) is defined by equation (12)

$$\mathcal{E} = \{u \in \mathcal{U} : \mathcal{B}(u) = \emptyset\} \quad (12)$$

and then the satisficing equilibrium set, \mathcal{E}_q^S , is given by equation (13)

$$\mathcal{E}_q^S = \mathcal{E} \cap \Sigma_q. \quad (13)$$

In the next section we will present elements of the analytic network process approach that are relevant to compute, given an evaluation problem, the satisfiability measures $p_s(u)$ and $p_r(u)$ for each alternative u that will be used to construct the value function π on the universe \mathcal{U} .

3 Analytic Network Process

The Analytic Network Process (ANP) is an extension of the Analytic Hierarchy Process (AHP) which allows the incorporation of the interdependency between elements being compared. The analytic hierarchy process is a comprehensive, powerful

and flexible decision making process to help people set priorities and make the best decision when both qualitative and quantitative aspects are used to evaluate alternatives, see [12]. By reducing complex decisions to a series of one-on-one comparisons, then synthesizing the results, AHP not only helps decision makers arrive at the best decision, but also provides a clear rationale that it is the best. It is designed to reflect the way people actually think and is a widely used decision-making theory. The basic AHP decomposes a decision problems in different elements, grouped in clusters, that it arranges in a linear hierarchy form where the top element of the hierarchy is the overall goal of the decision making. The hierarchy goes from the general to more particular until a level of operational criteria against which the decision alternatives can be evaluated is reached. The elements of cluster C_c in a top down hierarchy are pairwise compared with regard to each element of the cluster C_{c-1} to obtain the weighting $n_c \times n_{c-1}$ matrix \mathbf{W}_c where n_i is the number of elements in the cluster C_i . This matrix is given by equation (14)

$$\mathbf{W}_c = [w_c^1 \quad w_c^2 \quad \dots \quad w_c^{n_{c-1}}] \quad (14)$$

where w_c^i are n_c column vectors obtained as follows: for each element i of the cluster C_{c-1} , a pairwise comparison matrix \mathbf{W}_c^i of elements of cluster C_c is constructed by answering questions of the form “how important is element X compared to the element Y of the cluster C_c with regard to upper level element Z of the cluster C_{c-1} ?” using the scales given by the following Table I (see [12], [13])

Verbal scale	Numerical values
Equally important	1
Moderately more important	3
Strongly more important	5
Very strongly more important	7
Extremely more important	9
Intermediate scales (compromise)	2, 4, 6, 8

Table I: scales for AHP comparison procedure.

Once this matrix is constructed, the vector w_c^i is computed as the unique eigenvector of this matrix associated with eigenvalue $\lambda_{\max}(\mathbf{W}_c^i)$ (the maximum eigenvalue of the corresponding matrix), that is the solution of the equation (15)

$$\mathbf{W}_c^i w_c^i = \lambda_{\max}(\mathbf{W}_c^i) w_c^i \quad (15)$$

and an inconsistency¹ measure is calculated to evaluate the consistency of the decision maker when eliciting the judgements (see [13]). The overall weights vector ω of the bottom cluster, that is the alternatives cluster, with regard to the decision goal is then given by the following equation (16)

$$\omega = \mathbf{W}_N \mathbf{W}_{N-1} \dots \mathbf{W}_1 \quad (16)$$

¹A comparison matrix M is said to be consistent if it verifies: $M_{ii} = 1$, $M_{ji} = \frac{1}{M_{ij}}$ and $M_{ik} = M_{ij}M_{jk}$.

where \mathbf{W}_1 is a column vector representing the comparison weights of the first cluster with regard to the overall decision goal and \mathbf{W}_N (N is the number of clusters) is the comparison matrix of alternatives with regard to the direct upper level cluster elements (measurable criteria). The methodology of AHP (see [12]) assumes the inner and outer independence between elements, thus the hierarchy that represents the problems corresponds to directed graph (for instance Figure 2), where the arrows mean dependency; elements that are pointed by an arrow are evaluated against the elements from which the arrow emanate.

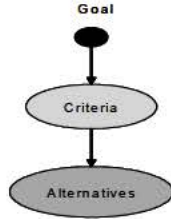


Figure 2: A typical AHP organization of a decision problem

By so doing, AHP approach eliminates the possibilities of feedback and inter-dependency. The Analytic Network Process (see [13]) abandons the requirement of linear dependence; here loops and feedback are allowed. The Analytic Network Process allows both interaction and feedback within clusters of elements (inner dependence) and between clusters (outer dependence). Such feedback best captures the complex effects of interplay in human society, especially when risk and uncertainty are involved. To determine the overall weights of a ANP decision making problem, one form first the supermatrix \mathbf{W} of the problem as equation (17)

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} & \cdot & \mathbf{W}_{1N} \\ \mathbf{W}_{21} & \mathbf{W}_{22} & \cdot & \mathbf{W}_{2N} \\ \cdot & \cdot & \cdot & \cdot \\ \mathbf{W}_{N1} & \mathbf{W}_{N2} & \cdot & \mathbf{W}_{NN} \end{bmatrix} \quad (17)$$

where \mathbf{W}_{ij} is the weighting matrix of cluster i with regard to cluster j obtained as in the case of analytic hierarchy process. The blocks of the supermatrix \mathbf{W} are then weighted by the corresponding cluster importance with regard to the decision goal to obtain a weighted supermatrix \mathbf{W}_w that is a column stochastic matrix. The global weighting matrix $\mathbf{\Omega}$ is obtained as the limiting power of \mathbf{W}_w as shown by equation (18)

$$\mathbf{\Omega} = \lim_{k \rightarrow \infty} (\mathbf{W}_w)^k \quad (18)$$

if \mathbf{W}_w is irreducible and acyclic. In the case of cyclicity and/or reducibility, there will exist many limiting supermatrices \mathbf{W}_w^l , let say L , and the Cesario sum (see [13])

is considered, equation (19)

$$\Omega = \lim_{k \rightarrow \infty} \frac{1}{L} \sum_{l=1}^L (\mathbf{W}_w^l)^k. \quad (19)$$

The bloc Ω_{ij} of the limiting supermatrix gives the overall priority of the elements of cluster C_i with regard to the elements of cluster C_j . A typical ANP is given by Figure 3 and, let us denote by \mathbf{w} , the weight vector of criteria with regard to the overall goal, \mathbf{W}_c the iner-weight matrix of criteria cluster, \mathbf{W}_{ca} the weight matrix of criteria with regard to alternatives, \mathbf{W}_{ac} the weight matrix of alternatives with regard to criteria and finally \mathbf{W}_a the iner-weight matrix of alternatives, then the supermatrix of this problem is given by equation (20)

$$\mathbf{W} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{w} & \mathbf{W}_c & \mathbf{W}_{ca} \\ \mathbf{0} & \mathbf{W}_{ac} & \mathbf{W}_a \end{bmatrix}. \quad (20)$$

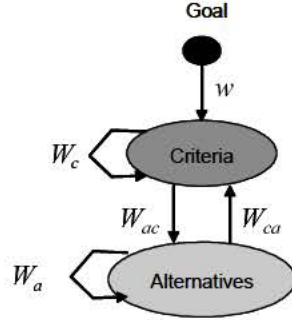


Figure 3: A typical ANP organization of a decision problem

For a large problem, computing the limiting supermatrix may be time consuming, fortunately some software such as Super Decisions (see [19]) can be used to support a decision making problem in analytic network process framework so that one can concentrate on modeling effort. We are now ready to develop the SANPEV framework in the next section.

4 SANPEV Framework

4.1 Evaluation process

Before establishing the overall performance model to compute the satisfiability measures in the next paragraph, let us first establish how these measures will be used

depending on the evaluation goal; this goal may be to choose the most efficient unit, to rank units or to sort (clustering, categorization, ...) them. Choosing and ranking are relative evaluation operations whereas sorting is an absolute operation (see [2]) that requires defining norms and categories. From the rejectability measure $p_r(u)$ and the selectability measure $p_s(u)$ for any unit $u \in \mathcal{U}$, an evaluation or value function $\pi(u)$ is defined as (21)

$$\pi(u) = \pi(p_s(u), p_r(u)) \quad (21)$$

which will take particular form depending on the criteria that guide the evaluation process and then, in the case of selection, the selected subset \mathcal{U}_s is defined by equation (22)

$$\mathcal{U}_s = \left\{ u : u = \arg \max_{v \in \mathcal{E}_q^S} \pi(v) \right\}. \quad (22)$$

If the evaluation goal is to rank units then one can define the ranking relation by equation (23)

$$u \succeq v \Leftrightarrow \pi(u) \geq \pi(v) \quad (23)$$

where “ $u \succeq v$ ” means that the unit u is at least as good as the unit v with regard to the evaluation goal and finally different norms can be defined using this function for sorting purpose; for instance, in the case $\pi(u) = p_s(u) - qp_r(u)$, two natural partitions of \mathcal{U} are given by equation (24)

$$\mathcal{C}_1 = \Sigma_q = \{u \in \mathcal{U} : \pi(u) \geq 0\} \text{ and } \mathcal{C}_2 = \mathcal{U} - \mathcal{C}_1. \quad (24)$$

Besides this possibility of sorting, the satisficing game approach leads to a natural categorization of the alternatives set \mathcal{U} into four subsets, namely \mathcal{E}_q^S , $\mathcal{E} - \mathcal{E}_q^S$, $\Sigma_q - \mathcal{E}_q^S$, and $\mathcal{U} - \Sigma_q \cup \mathcal{E}$. In terms of preference the subset \mathcal{E}_q^S is obviously preferred to the rest; it contains alternatives arguable to be “good enough” (their selectability exceeds their rejectability and there are no alternatives that are better than them) and the set $\mathcal{U} - \Sigma_q \cup \mathcal{E}$ contains completely inefficient alternatives (they are not satisficing alternatives nor equilibrium); there is no obvious conclusion for the subsets $\mathcal{E} - \mathcal{E}_q^S$ and $\Sigma_q - \mathcal{E}_q^S$ and a sensitivity analysis can be done for these alternatives (see for instance [20]). This dominance scheme is shown on the following Figure 4. Some

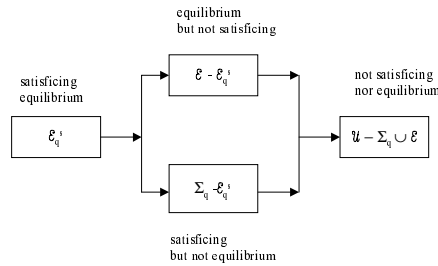


Figure 4: Dominance scheme of satisficing categorization process

of the criteria that can guide the final ordering of each category of alternatives are discussed in the following.

4.1.1 Input/output trade-off criterion

This is certainly the most intuitive criterion, that is one will compare the selectability measure and the rejectability measure, the evaluation function can then be defined by equation (25)

$$\pi(u) = p_s(u) - qp_r(u) \text{ or } \pi(u) = \frac{p_s(u)}{p_r(u)} \quad (25)$$

that gives the priority to alternatives with large difference between the selectability measure and the rejectability measure given the index of caution or that considers alternatives with the largest index of caution.

4.1.2 Input or output insensitive criterion

The evaluation goal may be defined with regard to the output performance only (input insensitive); this can happen if all the attributes available are related (or considered as such) to the deliveries of the systems; in this case the evaluation function is given by equation (26)

$$\pi(u) = p_s(u), \quad (26)$$

because the input insensitivity means that one can consider the rejectability measure $p_r(u)$ to be the same for any alternative $u \in \mathcal{U}$ so that the evaluation function $\pi(u)$ in equation (25) depends only on the selectability measure $p_s(u)$; that is the most selectable units are privileged. On the contrary, the evaluation process may be output insensitive, only resources consumption is considered for the evaluation purpose; in this case the evaluation function will privilege the least rejectable units, that is $\pi(u)$ is given by equation (27)

$$\pi(u) = \frac{1}{p_r(u)} \quad (27)$$

as the selectability measure $p_s(u)$ can be considered constant (output insensitivity) and as for a positive measure $p_r(u)$, maximizing $-p_r(u)$ or $\frac{1}{p_r(u)}$ is equivalent.

In the following paragraph we will establish the model that will be used to compute the satisfiability measures using the materials and relationships (stakeholders, items, information sources and alternatives) that define the evaluation problem using the analytic hierarchy/network process approach.

4.2 Modeling

The modeling framework consists in n alternatives or units to be evaluated, m_i items at the input and m_o items at the output, h stakeholders and s data or information

sources. We will begin by establishing the model in the case of pure hierarchy where independent stakeholders express their opinions or preferences about the items and perfect data concerning these items for each alternative are available. In the second case we will consider more complex relationships (mainly dependence and feedback) among evaluation process elements.

4.2.1 Pure hierarchy: linearity

In this paragraph we consider the case of pure hierarchy where stakeholders² (experts, decision makers, users, ...) express their concerns or visions about items that characterize the alternatives or units to be evaluated without feedback (stakeholders are not evaluated with regard to items). Furthermore, we consider that the input/output items are either numerically valued (strictly positive values or rendered strictly positive by some transformation) and data are supplied by a reliable source (this case, without considering stakeholders, corresponds to the case where DEA approach can be applied) or alternatives can be compared with regard to items by reliable experts. The transformation that renders the valuation of input/output items positive numeric is considered in the framework of utility theory where it is known that utilities are unique only up to a positive affine transformation (see for instance [15]), so that this transformation will be given by the following equation (28)

$$\times(u, i) \leftarrow \frac{\times(u, i) - \min_{v \in \mathcal{U}} \{\times(v, i)\}}{\max_{v \in \mathcal{U}} \{\times(v, i)\} - \min_{v \in \mathcal{U}} \{\times(v, i)\}} \quad (28)$$

where \times stands for I or O and $I(u, i)$ (respectively $O(u, i)$) is the value (measured or assigned in ordinal form by an authorized expert) of input item i (respectively output item i) for the alternative u . It is then straightforward to derive an algorithm to compute the rejectability measure p_r related to what happen at the input and the selectability measure p_s that corresponds to the output performance using the analytic hierarchy process as shown by Figure 5. Let us denote by $\mathbf{W}_{IS,I}^k$ (respect. $\mathbf{W}_{IS,O}^k$), for $k = 1, 2, \dots, h$, the pairwise comparison matrix of input (respect. output) items according to the stakeholders k that supplies weights w_{ij}^k (how important is item i compared to j according to stakeholder k ?). One rapid way³ to obtain this matrix is to ask the stakeholder to choose a pivot item p and compare other items to it using the standard AHP scales by supplying weights w_{ip}^k (how important is item i compared to the pivot item p according to stakeholder k ?). Then one construct a consistent comparison matrix $\mathbf{W}_{IS,\times}^k$ using the relations defined by equations (29) - (30)

$$\mathbf{W}_{IS,\times}^k(i, i) = 1, \quad \mathbf{W}_{IS,\times}^k(i, p) = w_{ip}^k, \quad \mathbf{W}_{IS,\times}^k(p, i) = \frac{1}{w_{ip}^k}, \quad (29)$$

²One can imagine that the stakeholders who judge the input items are not necessarily those who judge the output items.

³This rapid way of constructing the pairwise comparison matrices $\mathbf{W}_{IS,\times}^k$ ensures to obtain consistent matrices. Of course nothing prevents stakeholders of using the general way of pairwise comparison matrix construction (with inconsistency checking) to obtain these matrices and use the equation (31) to compute the weight vectors $\omega_{IS,\times}^k$.

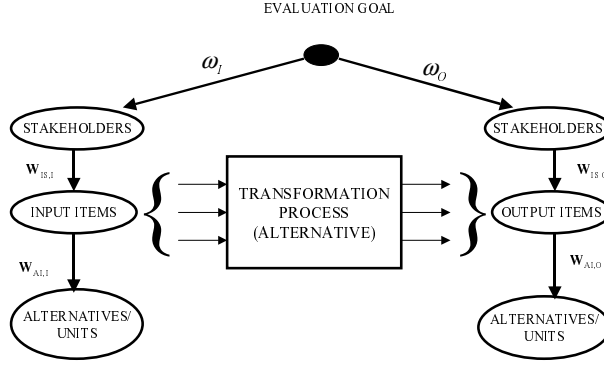


Figure 5: SANPEV architecture in the case of pur hierarchy

$$\mathbf{W}_{IS,\times}^k(i, j) = \mathbf{W}_{IS,\times}^k(i, l) \cdot \mathbf{W}_{IS,\times}^k(l, j). \quad (30)$$

Once these matrices are obtained, the column weight vectors $\omega_{SI,I}^k$ and $\omega_{SI,O}^k$ for items according to each stakeholder k are computed as shown by equation (31)

$$\omega_{IS,\times}^k(i) = \frac{1}{m_{\times}} \sum_{j=1}^{m_{\times}} \left(\frac{\mathbf{W}_{IS,\times}^k(i, j)}{\sum_{l=1}^{m_{\times}} \mathbf{W}_{IS,\times}^k(l, j)} \right), \quad i = 1, \dots, m_{\times}, \quad k = 1, \dots, h, \quad \times = I, O \quad (31)$$

and the $m_i \times h$ (respect. $m_o \times h$) weighting matrix $\mathbf{W}_{IS,I}$ (respect. $\mathbf{W}_{IS,O}$) is given by equation (32)

$$\mathbf{W}_{IS,I} = [\omega_{IS,I}^1 \quad \omega_{IS,I}^2 \quad \dots \quad \omega_{IS,I}^h] \quad \text{and} \quad \mathbf{W}_{IS,O} = [\omega_{IS,O}^1 \quad \omega_{IS,O}^2 \quad \dots \quad \omega_{IS,O}^h]. \quad (32)$$

Now we must compute the weighting matrix $\mathbf{W}_{AI,\times}$ of alternatives with regard to items. To do so, we suppose that either reliable experts are able to supply matrices $\mathbf{W}_{AI,\times}^i(u, v)$ that measure the relative importance of alternatives with regard to items or one dispose of positive values $I(u, i)$ (respectively $O(u, j)$) the value of input item i (respectively output item j) for the alternative u so that the pairwise comparison matrices $\mathbf{W}_{AI,I}^i$ and $\mathbf{W}_{AI,O}^j$ for the input item i and output item j are given by the equation (33)

$$\mathbf{W}_{AI,I}^i(u, v) = \frac{I(u, i)}{I(v, i)} \quad \text{and} \quad \mathbf{W}_{AI,O}^j(u, v) = \frac{O(u, j)}{O(v, j)}. \quad (33)$$

The weight vectors $\omega_{AI,I}^i$ (for the input item i) and $\omega_{AI,O}^j$ (for the output item j) are given by equation (34)

$$\omega_{AI,\times}^i(u) = \frac{1}{n} \sum_{v=1}^n \left(\frac{\mathbf{W}_{AI,\times}^i(u, v)}{\sum_{w=1}^n \mathbf{W}_{AI,\times}^i(w, v)} \right), \quad i = 1, \dots, n, \quad \times = I, O \quad (34)$$

and finally the $n \times m_i$ input weighting matrix $\mathbf{W}_{AI,I}$ and $n \times m_o$ output weighting matrix $\mathbf{W}_{AI,O}$ are given by the following equation (35)

$$\mathbf{W}_{AI,I} = [\omega_{AI,I}^1 \quad \omega_{AI,I}^2 \quad \dots \quad \omega_{AI,I}^{m_i}] \text{ and } \mathbf{W}_{AI,O} = [\omega_{AI,O}^1 \quad \omega_{AI,O}^2 \quad \dots \quad \omega_{AI,O}^{m_o}] . \quad (35)$$

Let us denote by ω_I and ω_O , h dimension column vectors representing the relative importance of stakeholders at the system input/output respectively with regard to evaluation goal, then the following proposition gives the rejectability measure $p_r(u)$ and the selectability measure $p_s(u)$.

Proposition 2 *The rejectability measure $p_r(u)$ and the selectability measure $p_s(u)$ for the alternative u are obtained by the following equation (36)*

$$p_r(u) = [\mathbf{W}_{AI,I} \cdot \mathbf{W}_{IS,I} \cdot \omega_I]_u \text{ and } p_s(u) = [\mathbf{W}_{AI,O} \cdot \mathbf{W}_{IS,O} \cdot \omega_O]_u . \quad (36)$$

Application To show the potentiality of the model established previously, let us consider a real world problem from DEA literature. The purpose of this problem is to measure the efficiency of 20 hospitals (see [4]) that are characterized by the number of minor and acute treated cases as the output items and the full time equivalent nursing staff and the number of beds as the resources (input items). Data collected are shown on the following table (Table II).

Hospital	Output items		Input items	
	Minor cases	Acute cases	Nurses	Beds
01	150	50	200	600
02	225	75	600	1200
03	90	10	200	200
04	160	40	600	300
05	50	50	500	200
06	75	75	320	150
07	200	50	375	450
08	350	100	400	320
09	400	90	550	480
10	250	300	900	660
11	350	350	850	720
12	350	400	720	940
13	275	375	900	850
14	220	40	250	370
15	300	10	115	250
16	320	275	600	590
17	375	230	550	710
18	230	50	200	280
19	290	90	450	410
20	360	70	415	575

Table II: input/output data for hospitals example

Applying the model established here when considering all items from each category (input or output) to be with equal importance leads to satisfiability measures shown on the Table III where we show also the efficiency index obtained by DEA approach in the original study [4].

Hospital number	Efficiency (DEA)	\mathbf{p}_s	\mathbf{p}_r
01	0.63	0.0241	0.0396
02	0.31	0.0361	0.0895
03	0.39	0.0108	0.0201
04	0.48	0.0232	0.0456
05	0.50	0.0141	0.0355
06	1.00	0.0212	0.0238
07	0.46	0.0291	0.0413
08	1.00	0.0531	0.0362
09	0.75	0.0563	0.0518
10	0.93	0.0797	0.0786
11	1.00	0.0988	0.0789
12	1.00	0.1080	0.0830
13	0.94	0.0959	0.0879
14	0.59	0.0301	0.0309
15	1.00	0.0317	0.0181
16	1.00	0.0821	0.0597
17	0.90	0.0794	0.0630
18	0.85	0.0320	0.0240
19	0.71	0.0453	0.0432
20	0.62	0.0487	0.0494

Table III: DEA efficiency, selectability measure and rejectability measure calculated using pure hierarchy SANPEV model for hospitals example

Applying (6) and (9) we find that $q_{\min} = 0.3973$ and $q_{\max} = 1.7499$. With a caution index of 1, we obtain the following sets

$$\Sigma_1 = \{08, 09, 10, 11, 12, 13, 15, 16, 17, 18, 19\}, \quad (37)$$

$$\mathcal{E}_1^s = \mathcal{E} = \{08, 09, 11, 12, 15, 16, 18\}. \quad (38)$$

The hospitals 10, 13, 17 and 19 are satisficing but not equilibria meaning that there are some hospitals that dominate them.

- $\mathcal{B}(10) = \{16\}$, the hospital number 10 is dominated by the hospital number 16 ($p_s(16) = 0.0821 > p_s(10) = 0.0797$ and $p_r(16) = 0.0597 < p_r(10) = 0.0786$); looking at original data, we see that the hospital 16 performs more efficiently on all items than the hospital 10 except for acute cases where it does 5 points less than the hospital 10.
- $\mathcal{B}(13) = \{11, 12\}$, the original data show a coherency with this domination scheme.

- $\mathcal{B}(17) = \{16\}$, the hospital 17 is mainly penalized by its beds number, a difference of 120 with regard to hospital 16.
- $\mathcal{B}(19) = \{08\}$, the hospital 19 is completely dominated by the hospital 08 as one can see from the original data.

One can see that all the hospitals that are considered as efficient by DEA approach are satisficing equilibria by our approach except the hospital 06 that is neither satisficing nor an equilibrium; it is dominated by the hospital 15 and one can see that it performs among the bottom hospitals in terms of delivery whereas its resources consumption is comparable to some of efficient hospitals; we do think that it is declared efficient by DEA approach because of technical drawbacks of this method raised previously. We obtain the same results as those obtained in [20] with the same problem; this approach has been applied also on many other problems, mainly from the DEA database (see [5]) with success. Furthermore, the intervention of stakeholders can be easily integrated. In the following paragraph we will consider the case of more complex relationships among evaluation process components.

4.2.2 Dependence and feedback: network

For the majority of real world evaluation problems, the interaction between the elements in presence, namely, stakeholders, input/output criteria or items, alternatives, data or information sources, may be very complex with possible dependency and feedbacks. In this case one cannot use the linear hierarchy approach as in the previous paragraph but rather an analytic network process approach. As stated in the introduction section, the considered evaluation problem consists in 4 main clusters: STAKEHOLDERS: experts, decision makers, users, ...; ITEMS: input (resources) as well as output (deliveries) items; DATA or INFORMATION SOURCES: news papers, magazines, web, specialized journals, experts, agencies, ...; and ALTERNATIVES or UNITS. There may exist a complex interaction between these clusters: stakeholders can influence each other regarding the importance of items for the achievement of the overall goal; the importance of items as well as the performance of alternatives regarding each item may be viewed differently by each stakeholder or a group of stakeholders. We consider the point of view of the analyst that must construct a model to support performance evaluation by using existing alternatives and their evaluation regarding the items and the opinions of stakeholders. The interaction between stakeholders is considered with regard to their opinions regarding the importance of each item; this interaction is resumed in the $h \times m_{\times}$ weighting matrix $\mathbf{W}_{SI,\times}$ obtained using procedure similar to pure hierarchy case. Conversely items will be compared according to the stakeholders opinions to obtain the $m_{\times} \times h$ weighting matrix $\mathbf{W}_{IS,\times}$. The architecture of the overall model is shown on Figure 6 where a fifth cluster named ITEMS_SOURCES is added for modeling convenience. The $(m_{\times}d_{\times}^4) \times m_{\times}$ matrix $\mathbf{W}_{ISI,\times}$ and $(m_{\times}d_{\times}) \times d_{\times}$ matrix $\mathbf{W}_{ISD,\times}$ on Figure 6 that traduce the neutral effect

⁴ d_{\times} is the number of data sources (input or output)

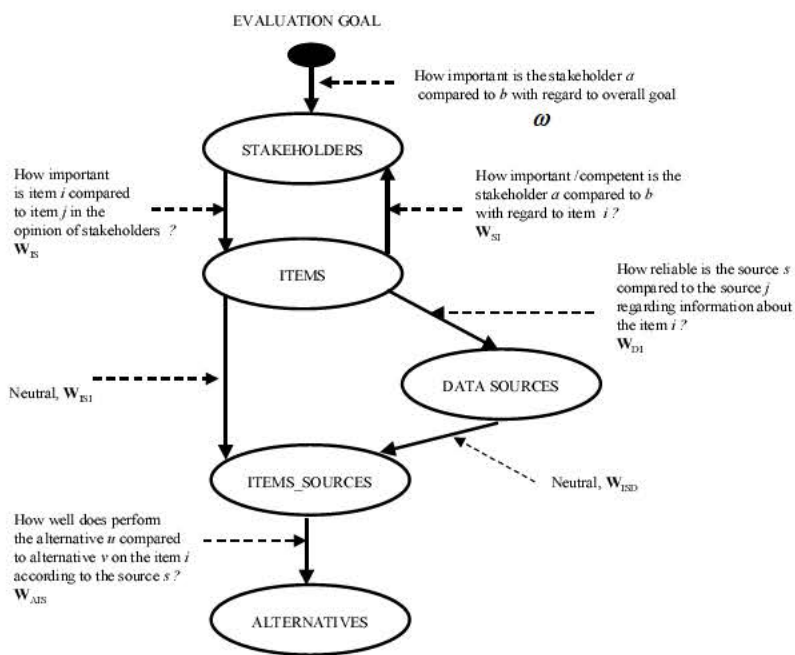


Figure 6: SANPEV Model in the case of dependency and feedback

of items cluster and data sources cluster respectively on the fictive items_sources cluster will be formed by the same column vectors to mean equal importance. Finally the alternatives are compared with regard to items_sources in terms of “how well does perform the alternative u compared to the alternative v with regard to item i according to data source s ?” and the results stocked in the $n \times (m \times d)$ weighting matrix $\mathbf{W}_{AIS,\times}$ and the column vector ω_\times indicates the relative importance of stakeholders with regard to the evaluation goal. The overall supermatrix \mathbf{W}_\times of the established model is then given by equation (39)

$$\mathbf{W}_\times = \begin{array}{c|cccccc} & \mathbf{EG} & \mathbf{ST} & \mathbf{IT} & \mathbf{DS} & \mathbf{IS} & \mathbf{AL} \\ \hline \mathbf{EG} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{ST} & \omega_\times & \mathbf{0} & \mathbf{W}_{SI,\times} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{IT} & \mathbf{0} & \mathbf{W}_{IS,\times} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{DS} & \mathbf{0} & \mathbf{0} & \mathbf{W}_{DI,\times} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{IS} & \mathbf{0} & \mathbf{0} & \mathbf{W}_{ISI,\times} & \mathbf{W}_{ISD,\times} & \mathbf{0} & \mathbf{0} \\ \mathbf{AL} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{W}_{AIS,\times} & \mathbf{0} \end{array} \quad (39)$$

where **EG**, **ST**, **IT**, **DS**, **IS** and **AL** mean evaluation goal, stakeholders, items, data (or information) sources, items_sources and alternatives respectively. The weighted supermatrix and finally the limiting supermatrix Ω_\times are obtained as presented in the third section, possibly using a software such as Super Decisions. Notice that in fact two such architectures will be constructed: one at the input of the system to measure the effect of resources consumption for each alternative that lead to the rejectability mass function $p_r(u)$ (for this input architecture the comparison of items with regard to stakeholders opinion is judged negatively) and a second architecture at the output of the system to measure the delivery performance where the items will be judged positively and the result will lead to the selectability mass function $p_s(u)$. The following proposition gives the satisfiability measures.

Proposition 3 *The rejectability measure vector p_r (respect. the selectability measure vector p_s) is constituted by the normalized n last values of the first column of limiting supermatrix Ω_r (respect. Ω_s).*

In the following section, we will present potential applications, in different engineering domains, of the method developed in this paper.

5 Applications

5.1 Potential application domains

The approach presented so far can be applied in different domains such as the following.

- Environment and geography: sites selection for locating infrastructures (roads, airports, power plants, waste management facility, ...) where *stakeholders* may

consist in politicians, public administration, local community, economic associations, environmental associations, ...; *items* can be considered to reflect environmental impact, socioeconomic development, the realization cost, technical feasibility and reliability, ...; *information sources* may consist in domain experts opinion, economic and/or environment agencies or cabinets, ...

- Manufacturing and service: evaluation of production/service units (manufacturing plants, banks, hospitals, call centers, ...) for their performance and/or activity restructuring purpose for instance. Here input/output *items* will be naturally defined by the resources consumed and the throughput whereas *stakeholders* may consist in group of actors reflecting the concerns and hopes of production team, maintenance team, sales team, managers,
- Management (strategy and marketing): the problem may be to select new projects or products for a company, to reorganize or to restructure the activities of a company, investing in researches and development, ... Here *stakeholders* will be the board of governors, the managers, the unions and personnels, ... and *items* may be investment cost, short/long term return, notoriety impact, impact on the environment, advertising cost, benefice, and the *information sources* will be specialized magazines, stock exchanges behavior over a certain horizon, other companies,
- Human resources management: choosing appropriate candidates for appropriate jobs is a crucial objective for human resources managers. The *stakeholders* here will be the managers, targeted team manager, future colleagues, ...; *items* may be qualifications, communication skills, experience, ... and *information sources* can be the opinion of past employers, past colleagues, attended education institutions, ...
- Governments: any government in the world and mainly governments of developing countries face the problem of prioritizing its investment projects in different domains: health, education, infrastructures, military, researches and development, ... in order to ensure economic growth, wealthy, human development, ... for the population. The method developed here can be used in this prioritization process where *stakeholders* will be government agencies, unions, non governmental organizations, parliament, international financial institutions such as world bank or IMF, ...; the items will be naturally defined by needed resources for each project and the expected results; the *information sources* can be consulting groups, international experts, benchmark projects from other countries, ...
- ...

5.2 Illustrative application

To illustrate the potentiality of this method let us consider a real-world example in the domain of waste management. This application is adapted from [14] where

the objective was to find the most plausible solution to the municipal solid waste management problem in a region of Central Finland. Here, we are just interested in real data and testing how well our approach would have worked in real situation; we will modify the original formulation of this problem to fit our approach. A preliminary study has identified 11 alternatives (see [14] for the meaning of each alternative) and 8 items that we regroup in 5 input items (items that correspond to an effort, a negative impact on the environment or a cost) and 3 output items (beneficial items); the meaning of these items are described in the following.

- Input items
 - I_1 : net cost per ton,
 - I_2 : global effects,
 - I_3 : local and regional health effects,
 - I_4 : acidificative releases,
 - I_5 : surface water dispersed releases,
- Output items
 - O_1 : technical reliability,
 - O_2 : number of employees,
 - O_3 : amount of recovered waste,

The evaluation of alternatives with regard to items is well defined and row data (indicating units is not relevant here) are given on the Table IV; in terms of SANPEV structure of Figure 6, the nodes DATA SOURCES and ITEMS_SOURCES do not exist.

AL	I.I.					O.I.		
	I_1	I_2	I_3	I_4	I_5	O_1	O_2	O_3
IA	787	155714560	148	364	505	9	20	4330
IB1	828	154887200	148	364	390	6	28	4080
IB2	837	154889339	148	364	390	6	24	5340
IC1	1062	139621200	201	377	370	7	35	11470
IC2	1050	139623330	201	377	370	7	28	12700
IIA	769	155061660	150	364	520	9	26	4330
IIB	861	154228170	138	364	310	6	32	5340
IIC	1048	138952170	203	377	300	7	36	12700
IIIA	894	154342000	137	364	470	5	25	3260
IIIB	997	153762000	137	364	300	5	32	4080
IIIC	1231	140035000	205	375	220	5	38	10600

Table IV: row data for different alternatives of waste management facility example; AL stands for Alternatives, I.I. for Input Items and O.I. for Output Items.

From data of Table IV, we deduce the weighting matrices $\mathbf{W}_{AI,I}$ and $\mathbf{W}_{AI,O}$ that are given on the Table V.

AL	$\mathbf{W}_{AI,I}$					$\mathbf{W}_{AI,O}$		
	I_1	I_2	I_3	I_4	I_5	O_1	O_2	O_3
IA	0.0759	0.0949	0.0815	0.0898	0.1218	0.1250	0.0617	0.0553
IB1	0.0799	0.0944	0.0815	0.0898	0.0941	0.0833	0.0864	0.0522
IB2	0.0808	0.0944	0.0898	0.0898	0.0941	0.0833	0.0741	0.0683
IC1	0.1025	0.0851	0.1107	0.0930	0.0893	0.0972	0.1080	0.1466
IC2	0.1013	0.0851	0.1107	0.0930	0.0893	0.0972	0.0864	0.1623
IIA	0.0742	0.0945	0.0826	0.0898	0.1255	0.1250	0.0802	0.0553
IIB	0.0831	0.0940	0.0760	0.0898	0.0748	0.0833	0.0988	0.0683
IIC	0.1011	0.0847	0.1118	0.0930	0.0724	0.0972	0.1111	0.1623
IIIA	0.0863	0.0940	0.0754	0.0898	0.1134	0.0694	0.0772	0.0417
IIIB	0.0962	0.0937	0.0754	0.0898	0.0724	0.0694	0.0988	0.0522
IIIC	0.1188	0.0853	0.1129	0.0925	0.0531	0.0694	0.1173	0.1355

Table V: weighting matrices of alternatives with regard to items for the waste management example.

The main modification with regard to the original paper [14] is that we consider that some stakeholders opinion must be taken into account. In [14] a committee of 45 members were asked to express their opinion by assigning weights (from 1 to 7, 7 being the most important) to criteria and these weights were aggregated for use by the ELECTRE III approach (see for instance [11], [24] for ELECTRE approaches) to rank the alternatives. The final accomplished alternative was the alternative **IIC**. Here we consider that the selection process must integrate the opinions of different groups of actors that can be splitted up as in the following.

- Political group (**PG**). This group is the group that will ultimately decide and will be concerned mostly by financial issues, jobs creation issues and the health of their administrees.
- Technical group (**TG**). This is the group of actors that will supervise the realization of the project; it is concerned mainly by the items regarding the technical issues such as the reliability of the project, the efficiency (the amount of treated waste), ...
- Environmental group (**EG**). This group may comprise different ecological associations and will be more concerned by items impacting on the environment such as the majority of input items.
- Beneficiary group (**BG**). We include here, the population whose waste will be treated and that will benefit of the jobs creation.

For output items, let us suppose that the political group considers the item O_2 (number of employees) and item O_3 (amount of recovered waste) to be very strongly

more important and strongly more important than item O_1 (technical reliability) respectively and considering similar suppositions for other actors (see Table A1 at appendix section for complete comparison), we obtain the weighting matrix of output items with regard to stakeholders $\mathbf{W}_{IS,O}$ given below (equation (40))

$$\mathbf{W}_{IS,O} = \begin{array}{c|cccc} & \mathbf{PG} & \mathbf{TG} & \mathbf{EG} & \mathbf{BG} \\ \hline O_1 & 0.0769 & 0.6522 & 0.1176 & 0.0909 \\ O_2 & 0.5385 & 0.1304 & 0.2941 & 0.4545 \\ O_3 & 0.3846 & 0.2174 & 0.5882 & 0.4545 \end{array} \quad (40)$$

A similar consideration for input items (see Table A2 at the appendix section) lead to the following input items weighting matrix $\mathbf{W}_{IS,I}$ (equation 41) with regard to stakeholders

$$\mathbf{W}_{IS,I} = \begin{array}{c|cccc} & \mathbf{PG} & \mathbf{TG} & \mathbf{EG} & \mathbf{BG} \\ \hline I_1 & 0.3488 & 0.4478 & 0.0370 & 0.0588 \\ I_2 & 0.1163 & 0.2239 & 0.1852 & 0.2353 \\ I_3 & 0.3488 & 0.0896 & 0.3333 & 0.2941 \\ I_4 & 0.1163 & 0.1493 & 0.2593 & 0.2353 \\ I_5 & 0.0698 & 0.0896 & 0.1852 & 0.1765 \end{array} \quad (41)$$

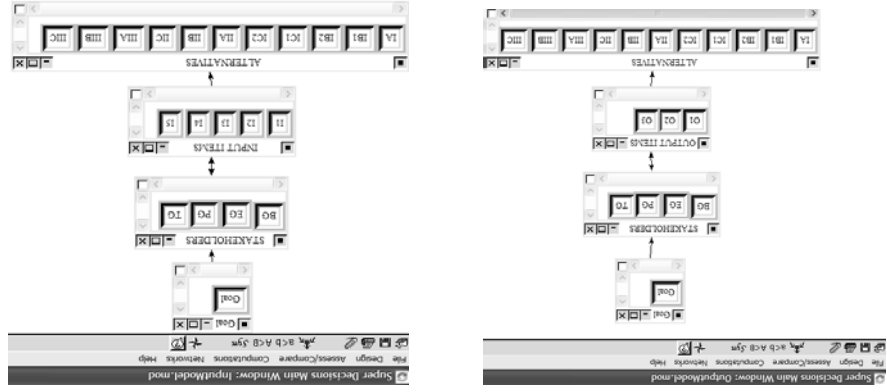
The groups of actors are also compared with regard to input/output items in terms of which group opinion is more important with regard to each item and that leads to the following weighting matrices $\mathbf{W}_{SI,O}$ (equation (42)) and $\mathbf{W}_{SI,I}$ (equation (43))

$$\mathbf{W}_{SI,O} = \begin{array}{c|ccc} & O_1 & O_2 & O_3 \\ \hline \mathbf{PG} & 0.1029 & 0.5607 & 0.0781 \\ \mathbf{TG} & 0.6564 & 0.1121 & 0.6245 \\ \mathbf{EG} & 0.1313 & 0.1402 & 0.2082 \\ \mathbf{BG} & 0.1094 & 0.1869 & 0.0892 \end{array} \quad (42)$$

$$\mathbf{W}_{SI,I} = \begin{array}{c|ccccc} & I_1 & I_2 & I_3 & I_4 & I_5 \\ \hline \mathbf{PG} & 0.1193 & 0.0828 & 0.1212 & 0.0962 & 0.0962 \\ \mathbf{TG} & 0.5966 & 0.1931 & 0.1515 & 0.6731 & 0.6731 \\ \mathbf{EG} & 0.1989 & 0.5793 & 0.6061 & 0.1346 & 0.1346 \\ \mathbf{BG} & 0.0852 & 0.1448 & 0.1212 & 0.0962 & 0.0962 \end{array} \quad (43)$$

These weighting data are then grouped to form the output supermatrix \mathbf{W}_O equation (44) and the input supermatrix \mathbf{W}_I equation (45)

Figure 7: Output and input analytic network of waste management facility problem constructed using *Super Decisions* software. (the selectability measure p_s and the rejectability measure p_r) obtained from Super Decision is given by the **case 2** columns of the Table VI; the satisfiability measures given by the **case 1** columns of this table are obtained when considering the pure hierarchy without intervention of stakeholders.



where $\omega = \omega_I = \omega_T = [0.25 \ 0.25 \ 0.25 \ 0.25]^T$ (stakeholders are considered to be with equal importance with regard to evaluation goal). To compute the limiting supermatrices Ω_s and Ω_r and extract the selectability measure p_s and the rejectability measure p_r , we use *Super Decisions* (see [19]), a software that implement the analytic network process approach to construct two networks as shown by Figure 7. The result

$$W_I = \begin{array}{|c|c|c|c|c|} \hline & EG & ST & IT & AT \\ \hline EG & 0 & 0 & 0 & 0 \\ ST & \omega_I & 0 & 0 & 0 \\ IT & 0 & W_{SI,I} & 0 & 0 \\ AT & 0 & 0 & W_{SI,I} & 0 \\ \hline \end{array} \quad (45)$$

$$W_O = \begin{array}{|c|c|c|c|c|} \hline & EG & ST & IT & AT \\ \hline EG & 0 & 0 & 0 & 0 \\ ST & \omega_O & 0 & 0 & 0 \\ IT & 0 & W_{SI,O} & 0 & 0 \\ AT & 0 & 0 & W_{SI,O} & 0 \\ \hline \end{array} \quad (44)$$

Alternatives	Case 1		Case 2	
	P_s	P_r	P_s	P_r
IA	0.0807	0.0926	0.0826	0.0890
IB1	0.0740	0.0878	0.0714	0.0862
IB2	0.0752	0.0896	0.0754	0.0909
IC1	0.1173	0.0960	0.1173	0.0972
IC2	0.1153	0.0957	0.1164	0.0969
IIA	0.0868	0.0932	0.0880	0.0902
IIB	0.0835	0.0834	0.0826	0.0832
IIC	0.1235	0.0924	0.1236	0.0945
IIIA	0.0628	0.0916	0.0628	0.0891
IIIB	0.0735	0.0853	0.0729	0.0862
IIIC	0.1074	0.0924	0.1071	0.0967

Table VI: selectability and rejectability measures in the case of pure hierarchy (case 1) and when stakeholders and items interaction is considered (case 2) for the waste management example.

Different categorizations of the alternatives set using the measures of Table VI are given by the following.

- **Case 1:** pure hierarchy. Here applying (6) and (9) we find that $q_{\min} = 0.6856$ and $q_{\max} = 1.3366$ and the following sets are obtained for $q = 1$.

$$\Sigma_1 = \{\mathbf{IC1}, \mathbf{IC2}, \mathbf{IIB}, \mathbf{IIC}, \mathbf{IIIC}\}, \quad (46)$$

$$\mathcal{E}_1^S = \mathcal{E} = \{\mathbf{IIB}, \mathbf{IIC}, \mathbf{IIIC}\}, \quad (47)$$

$$\mathcal{B}(\mathbf{IC1}) = \mathcal{B}(\mathbf{IC2}) = \{\mathbf{IIC}\}. \quad (48)$$

$\mathcal{B}(\mathbf{IC1}) = \{\mathbf{IIC}\}$ because from Table VI we have

$$p_r(\mathbf{IIC}) = 0.0924 < p_r(\mathbf{IC1}) = 0.0960 \text{ and } p_s(\mathbf{IIC}) = 0.1235 > p_s(\mathbf{IC1}) = 0.1173 \quad (49)$$

and a similar inequalities hold between $\mathbf{IC2}$ and \mathbf{IIC} . Though alternatives $\mathbf{IC1}$ and $\mathbf{IC2}$ are satisficing, they are dominated by the alternative \mathbf{IIC} ; the rest of alternatives are neither satisficing nor equilibrium.

- **Case 2:** network with equal importance of stakeholders. The application of (6) and (9) leads to $q_{\min} = 0.7048$ and $q_{\max} = 1.3079$ and for $q = 1$ we obtain the following sets

$$\Sigma_1 = \{\mathbf{IC1}, \mathbf{IC2}, \mathbf{IIC}, \mathbf{IIIC}\}, \quad (50)$$

$$\mathcal{E} = \{\mathbf{IIA}, \mathbf{IIB}, \mathbf{IIC}\}, \quad (51)$$

$$\mathcal{E}_1^S = \{\mathbf{IIC}\}, \quad (52)$$

$$\mathcal{B}(\mathbf{IC1}) = \mathcal{B}(\mathbf{IC2}) = \mathcal{B}(\mathbf{IIIC}) = \{\mathbf{IIC}\}. \quad (53)$$

One can easily verify from the Table VI that inequalities similar to (49) hold between the alternatives $\mathbf{IC1}$, $\mathbf{IC2}$, \mathbf{IIIC} on one hand and the alternative

IIC on the other hand that justify $\mathcal{B}(\mathbf{IC1})$, $\mathcal{B}(\mathbf{IC2})$ and $\mathcal{B}(\mathbf{IIC})$. Here, the only satisficing equilibrium alternative is **IIC** that dominates other satisficing alternatives, namely **IC1**, **IC2** and **IIC**; the alternatives **IIA** and **IIB** are not dominated but do have unfavorable input/output trade-off (they are not satisficing).

It is interesting to notice that the final accomplished alternative **IIC** in the original study [14] is the one that would have been selected by the approach presented in this paper although the formulation considered here is slightly different with some arbitrary considerations.

6 Conclusion

The problem of constructing a model to evaluate alternatives characterized by multiple heterogenous negative attributes (considered as the input items that must be consumed by the alternative) and multiple heterogenous positive attributes that are considered as the deliveries of the alternative where multiple stakeholders must express their opinions and information is collected from multiple sources has been considered in this paper. The underlying paradigm for this purpose is the satisficing game theory associated with analytic network process; a systematic method has been established that allows an analyst to calculate the satisfiability measures (the selectability measure at the output and the rejectability measure at the input of the alternative) using attributes values collected from information sources or attributes comparison weights obtained from experts and other interactions influence expressed by comparison weights. The categorization of items in terms of input (negative) and output (positive) and the decomposition scheme of analytic network process make it easy to carry up the one-to-one comparison. A practical example has been considered that shows the applicability of this approach. The method established so far in this paper is easy to understand and to apply with potentially many domains of applications (the generic architecture of the model in terms of Figure 6 is easily adaptable to new situations) but the necessity to divide attributes into positive attributes and negative attributes may raise some problems in practice; we do think that this later point may benefit by works in psychology and other social sciences.

7 Appendix

	PG			TG			EG			BG		
	O_1	O_2	O_3	O_1	O_2	O_3	O_1	O_2	O_3	O_1	O_2	O_3
O_1	1	1/7	1/5	1	5	3	1	2/5	1/5	1	1/5	1/5
O_2	7	1	7/5	1/5	1	3/5	5/2	1	1/2	5	1	1
O_3	5	5/7	1	1/3	5/3	1	5	2	1	5	1	1

Table A1: comparison of output items with regard to stakeholders for waste

management example.

	PG					TG				
	I_1	I_2	I_3	I_4	I_5	I_1	I_2	I_3	I_4	I_5
I_1	1	3	1	3	5	1	2	5	3	5
I_2	1/3	1	1/3	1	5/3	1/2	1	5/2	3/2	5/2
I_3	1	3	1	3	5	1/5	2/5	1	3/5	1
I_4	1/3	1	1/3	1	5/3	1/3	2/3	5/3	1	5/3
I_5	1/5	3/5	1/5	3/5	1	1/5	2/5	1	3/5	1

	EG					BG				
	I_1	I_2	I_3	I_4	I_5	I_1	I_2	I_3	I_4	I_5
I_1	1	1/5	1/9	1/7	1/5	1	1/4	1/5	1/4	1/3
I_2	5	1	5/9	5/7	1	4	1	4/5	1	4/3
I_3	9	9/5	1	9/7	9/5	5	5/4	1	5/4	5/3
I_4	7	7/5	7/9	1	7/5	4	1	4/5	1	4/3
I_5	5	1	5/9	5/7	1	3	3/4	3/5	3/4	1

Table A2: comparison of input items with regard to stakeholders for waste management example.

	O_1				O_2				O_3			
	PG	TG	EG	BG	PG	TG	EG	BG	PG	TG	EG	BG
PG	1	1/7	1	6/7	1	5	4	3	1	1/8	3/8	7/8
TG	7	1	5	6	1/5	1	4/5	3/5	8	1	3	7
EG	7/5	1/5	1	6/5	1/4	5/4	1	3/4	8/3	1/3	1	7/3
BG	7/6	1/6	5/6	1	1/3	5/3	4/3	1	8/7	1/7	3/7	1

Table A3: comparison of stakeholders with regard to output items for waste management example.

	I_1				I_2				I_3			
	PG	TG	EG	BG	PG	TG	EG	BG	PG	TG	EG	BG
PG	1	1/5	3/5	7/5	1	3/7	1/7	4/7	1	4/5	1/5	1
TG	5	1	3	7	7/3	1	1/3	4/3	5/4	1	1/4	5/4
EG	5/3	1/3	1	7/3	7	3	1	4	5	4	1	5
BG	5/7	1/7	3/7	1	7/4	3/4	1/4	1	1	4/5	1/5	1

	I_4				I_5			
	PG	TG	EG	BG	PG	TG	EG	BG
PG	1	1/7	5/7	1	1	1/7	5/7	1
TG	7	1	5	7	7	1	5	7
EG	7/5	1/5	1	7/5	7/5	1/5	1	7/5
BG	1	1/7	5/7	1	1	1/7	5/7	1

Table A4: comparison of stakeholders with regard to input items for waste management example.

References

- [1] Charnes A., Cooper W. W., Rhodes E., Measuring the efficiency of decision making units, *European Journal of Operational Research*, **2**, 1978, 429-444.
- [2] Bouyssou D., Marchant T., Perny P., Pirlot M., Tsoukiàs A., Vincke P., *Evaluation and Decision Models: A critical perspective*, Kluwer Academic Publishers, Dordrecht, 2001.
- [3] Dantzig G.B., *Linear programming and extensions*, Princeton University Press, Princeton, NJ, 1963.
- [4] Steering Committee, *Data Envelopment Analysis. A technique for measuring the efficiency of government delivery service*, Commonwealth of Australia, ISBN: 0-646-33533-2, 1997.
- [5] DEA Database, <http://www.deazone.com/datasets/FILE1/index.asp>
- [6] Dyson R. G., Thanassoulis E., Reducing weight flexibility in data envelopment analysis, *J. Op. Res. Soc.*, **39**, 1988, 563-576.
- [7] Emrouznejad A., Podinovski V., *Data Envelopment Analysis and Performance Management*, 2004.
- [8] Jacquet-Lagrèze, Sisko J., Assessing a set of additive utility functions for multicriteria decision making: The UTA method, *European Journal of Operational Research*, **10**, 2, 1982, 151-164.
- [9] Land K. C., Lovell C. A. K., Thore S., Productive Efficiency under Capitalism and State Socialism: An Empirical Inquiry Using Chance-Constrained Data Envelopment Analysis, *Technological Forecasting and Social Change* **46**, 1994, 139-152.
- [10] Roll Y., GOLANY B., Alternative Methods of Treating Factor Weights in DEA, *Omega*, **21**, 1, 1993, 99-109.
- [11] Roy B., Bouyssou D., *Aide Multicritère a la Decision: Methodes et Cas*, Edition Economica, Paris, 1993.
- [12] Saaty T., *The Analytic Hierarchical Process: Planning, Priority, Resource Allocation*, McGraw Hill, New York, 1980.
- [13] Saaty T., *The Analytic Network Process: Decision Making with Dependence and Feedback*, RWS Publications, Pittsburgh, 2005.
- [14] Salminen P., Hokkanen J., Lahdelma R., *Multicriteria Decision Analysis Project on Environmental Problems*, Report 5/1996, Department of Mathematics, Laboratory of Scientific Computing, University of Jyväskylä, 1996.

- [15] Salo A.A. and Hämäläinen R.P., On the Measurement of Preferences in the Analytic Hierarchy Process, *Journal of Multicriteria Decision Analysis*, **6**, 1997, 309-319.
- [16] Simon H. A., *Administrative Behavior: A study of decision-making processes in administrative organizations*, Fourth Edition, The Free Press, 1997.
- [17] Steuer R. E., *Muticriteria Optimization: Theory, Computation, and Application*, Wiley, New York, 1986.
- [18] Stirling W. C., *Satisficing Games and Decision Making: With Applications to Engineering and Computer Science*, Cambridge University Press, 2003.
- [19] Super Decisions, <http://www.superdecisions.com/>
- [20] Tchangani A. P., A Satisficing Game Approach for Group Evaluation of Production Units, *Decision Support Systems*, **42**, 2, 2006, 778-788.
- [21] Tchangani A.P., Multiple Objectives and Multiple Actors Load/Resource Dispatching or Priority Setting: Satisficing Game Approach, *Advanced Modeling and Optimization: An Electronic International Journal*, **8**, 2, 2006, 111 - 134.
- [22] Thore S. A., *Technology Commercialization: DEA and Related Analytical Methods for Evaluating the Use and Implementation of Technical Innovation*, Kluwer Academic Publishers, Boston/Dordrecht/London, 2002.
- [23] Thore S., Kozmetsky G., Phillips F., DEA of Financial Statements Data: the U.S. Computer Industry, *Journal of Productivity Analysis*, **2**, 1994, 229-248.
- [24] Vincke P., *L'aide multicritère à la décision*, Editions de l'Université Libre de Bruxelles, Bruxelles, 1989.
- [25] Winston W. L., *Operations Research: Applications and Algorithms*, Third Edition, Duxbury Press, 1994.