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Direct Synthesis of Quad-Band Band-Pass Filter by Frequency Transformation Methods

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***Abstract*— This article presents a method to design a quad-band band-pass filter. It is based on a frequency transformation function allowing the direct transformation of a low-pass prototype in a quad-band band-pass filter. By defining the low and high frequencies of each pass-band, all the parameters of the frequency transformation (resonant angular frequencies and susceptance slope parameters) can be retrieved analytically. To validate the method, a third-order Chebyshev filter is designed and fabricated using microstrip technologies. Very good agreements are achieved between theoretical responses and measurements.**

***Keywords*— Quad-band band-pass filter, frequency transformation function, microstrip technologies**

I. Introduction

Multi-band microwave filters are now an attractive solution in modern wireless communication systems for reducing their volume and mass. Many recent works have proposed synthesis methods for dual- or triple-band [1]–[8] or, less often, quad-band or multi-band band-pass filters [9]–[11]. Most of them imposed the topology based on either stubs [5], [9] or step-impedance resonators [1], [7], [11]. Nevertheless, these latter for instance may require an important optimization process to reach the required filter performance. Coupling matrices are now also used to synthesize multi-band filters [3], [8], but it may also require a lot of time to optimize the matrix parameters and this is especially true with the increase of the number of bands.

Another strategy consists in using frequency transformation techniques to switch from a low-pass prototype to a multi-band band-pass filter. To do so, transmission zeros are introduced in a band-pass filter to create the required bands. This method was first presented in [2] for a dual-band case and then in [4] for a triple-band one. A generalization of this procedure for an arbitrary number of bands was proposed in [10]. Nevertheless, the proposed procedure imposes the use of frequency-invariant susceptances.

In this paper, we propose to extend the method presented in [2] to the case of quad-band band-pass filters keeping the synthesis fully analytic. Our work differs from [10] by the use of the slope parameters instead of coupling matrix and by the fact that no frequency-invariant susceptance is imposed. The paper is organized as follows. The first part introduces the transform function. All equations and expressions for constructing the filter on a lumped-elements configuration are given. The method is then applied to the design of a third-order Chebyshev quad-band band-pass filter. An example of implementation of this filter in microstrip technology is provided in the second part which includes simulated and measured results. Finally, we will give a summary of the whole paper.

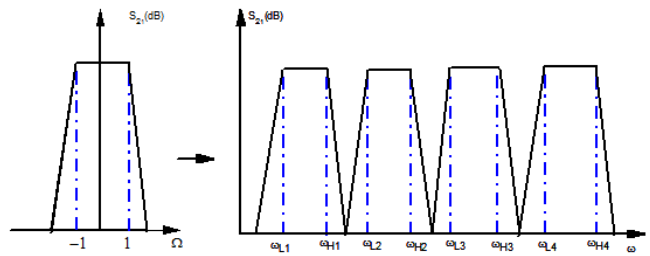

Fig. 1. From a classical low-pass prototype to a quad-band band-pass filter

II. Quad-band band-pass filter synthesis

The quad-band band-pass filter's response is shown in Fig.1. The frequency transformation from a classical low-pass filter prototype to the quad-band band-pass filter can mathematically expressed by

$$\Omega = T(\omega) = b_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) - \sum_{k=1}^{3} \frac{1}{b_k\left(\frac{\omega}{\omega_k} - \frac{\omega_k}{\omega}\right)}, \quad (1)$$

where Ω is the normalized angular frequency (associated to the normalized low-pass filter), $T(\omega)$ is the transformation function, ω is the de-normalized angular frequency (associated to the quad-band band-pass filter), ω_0 and b_0 are respectively the resonant angular frequency and the susceptance slope parameter of the band-pass resonators, ω_k and b_k ($k \in [1;3]$) are the ones of the stop-band resonators. These 8 parameters define the transformation and the objective of the synthesis process is to link them to the input parameters that are the low and high cut-off angular frequencies of the four bands ω_{L_i} and ω_{H_i} ($i \in [1;4]$) defined in Fig.1.

To do that, it can be assumed that the lower cut-off angular frequencies for the 4 pass-bands ω_{L_i} ($i \in [1;4]$) map to -1 in the normalized domain Ω, while the upper ones, ω_{H_i} ($i \in [1;4]$), map to +1. Because of the odd symmetry of $T(\omega)$, we can obtain:

$$T(-\omega_{L_i}) = T(\omega_{H_i}) = 1 \quad (2)$$

The four high cut-off angular frequencies and the opposite of the four low cut-off angular ones are therefore the zeros of the function $U(\omega)$ defined as:

$$U(\omega) = T(\omega) - 1 \tag{3}$$

In addition, using (1), $U(\omega)$ may be written as the ratio of two polynomials :

$$U(\omega) = \frac{\omega^8 + \sum_{p=0}^{7} n_p \omega^p}{-(n_7\omega^7 + n_5\omega^5 + n_3\omega^3 + n_1\omega)} \tag{4}$$

There are two ways to determine the value of n_p ($p \in [0;7]$). The first one uses the fact that the high cut-off angular frequencies and the opposite of the low cut-off ones are the zeros of $U(\omega)$. The second one consists in using directly the expression of $T(\omega)$ from (1) in (3). The expression of all the n_p coefficients as a function of the cut-off angular frequencies on the one hand and of the resonant angular frequencies and the slope parameters on the other hand are given in appendix. These 8 equations allow us to express analytically the 4 resonant angular frequencies ω_k and the 4 slope parameters b_k ($k \in [0;3]$) as a function of the input parameters:

$$\omega_0 = \sqrt{\frac{-n_0 n_7}{n_1}} \tag{5}$$

$$\omega_1 = \sqrt{\frac{-n_5 + \sqrt{A}(\cos\frac{\theta}{3} - \sqrt{3}\cos\frac{\theta}{3})}{3n_7}} \tag{6}$$

$$\omega_2 = \sqrt{\frac{-n_5 + \sqrt{A}(\cos\frac{\theta}{3} + \sqrt{3}\cos\frac{\theta}{3})}{3n_7}} \tag{7}$$

$$\omega_3 = \sqrt{\frac{-n_5 - 2\sqrt{A}\cos\frac{\theta}{3}}{3n_7}} \tag{8}$$

$$b_0 = \sqrt{\frac{-n_0}{n_1 n_7}} \tag{9}$$

$$b_1 = \frac{\omega_1(\omega_1^2 - \omega_2^2)(\omega_1^2 - \omega_3^2)}{X\omega_1^4 + Y\omega_1^2 + Z} \tag{10}$$

$$b_2 = \frac{\omega_2(\omega_2^2 - \omega_1^2)(\omega_2^2 - \omega_3^2)}{X\omega_2^4 + Y\omega_2^2 + Z} \tag{11}$$

$$b_3 = \frac{\omega_3(\omega_3^2 - \omega_1^2)(\omega_3^2 - \omega_2^2)}{X\omega_3^4 + Y\omega_3^2 + Z} \tag{12}$$

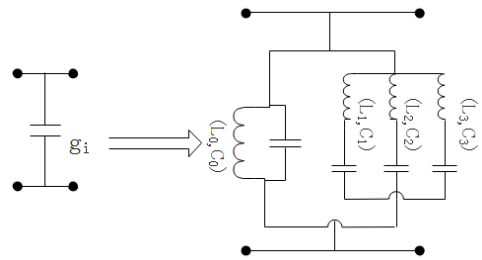

Fig. 2. Low-pass capacitance transformed into quad-band band-pass resonators

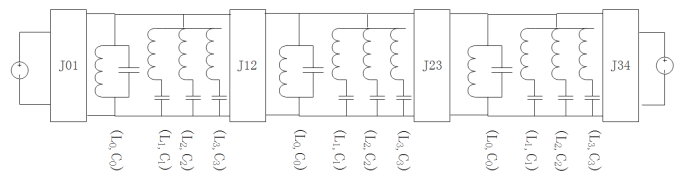

Fig. 3. Ideal lumped-elements quad-band band-pass filter.

where:

$$A = n_5^2 - 3n_7n_3 \tag{13}$$

$$B = n_5n_3 - 9n_7n_1 \tag{14}$$

$$C = n_3^2 - 3n_5n_1 \tag{15}$$

$$\theta = \arccos\frac{2An_5 - 3Bn_7}{2A^{3/2}} \tag{16}$$

$$X = \frac{n_6 - \frac{n_0n_7}{n_1} - \frac{n_5}{n_7}}{n_7} \tag{17}$$

$$Y = \frac{n_4 - \frac{n_0n_5}{n_1} - \frac{n_3}{n_7}}{n_7} \tag{18}$$

$$Z = \frac{n_2 - \frac{n_0n_3}{n_1} - \frac{n_1}{n_7}}{n_7} \tag{19}$$

Since all the parameters for building the quad-band band-pass filter are available, a unit capacitance in a classical low-pass prototype can be transformed into a quad-band resonator according to the structure presented in Fig.2.

An example of third-order Chebyshev filter using such quad-band resonators and ideal J-inverters is presented in Fig.3. The synthesis technique used here implies that the three quad-band resonators are identical. Table 1 presents the specification values (i.e the cut-off frequencies, $f_{Li} = \omega_{Li}/2\pi$ and $f_{Hi} = \omega_{Hi}/2\pi$, $i \in [1;4]$), the resonant frequencies ($f_k = \omega_k/2\pi$, $k \in [0;3]$), the slope parameters obtained from the synthesis and finally, the components' value of the third-order lumped-elements Chebyshev filter. The admittance inverters are $J_{01} = J_{34} = 0.01287$ S and $J_{12} = J_{23} = 0.007271$ S. The frequency response of this filter is reported in Fig.4.

Table 1. Application of the synthesis to a third-order quad-band filter: specifications and output values.

Cut-off frequencies				
Pass-band n°	1	2	3	4
f_{Li} (GHz)	1.1	1.3	1.5	1.7
f_{Hi} (GHz)	1.15	1.35	1.55	1.75
Frequency transformation parameters				
Resonator	Band-pass	Band-stop		
n°	0	1	2	3
f_k (GHz)	1.371	1.209	1.431	1.652
b_k	6.857	3.763	4.270	5.980
Lumped-elements value				
Resonator	Band-pass	Band-stop		
n°	0	1	2	3
C_k (pF)	5.610	2.035	1.557	0.904
L_k (pH)	2.402	8.516	7.945	10.27

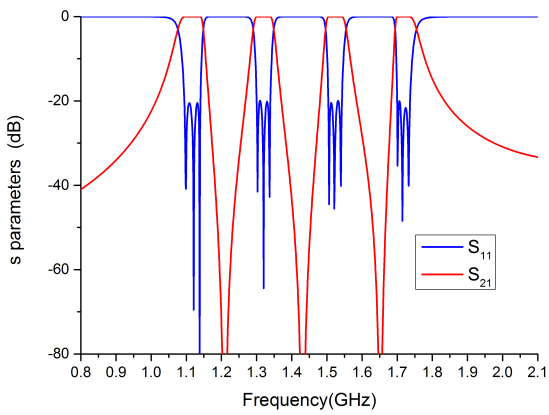

Fig. 4. Simulated frequency response of the lumped-elements quad-band band-pass filter (cf. Fig.3).

III. Example of implementation in microstrip technology

An example of implementation of this quad-band band-pass filter in microstrip technology is presented in this section. The filter was fabricated on Rogers RO4033 substrate of height $h = 0.508$mm, relative permittivity $\varepsilon_r = 3.55$ and $\tan\delta = 0.0027$, associated with a double-sided copper layer with thickness 17.5μm and conductivity 5.8×10^7S.m^{-1}. For each quad-band resonator, the band-pass part is implemented using a quarter-wavelength short-circuited stub and ideally the three stop-band ones using three quarter-wavelength open-circuited stubs. Nevertheless, the line impedance of these latter are too high ($> 500\Omega$) to be made in the chosen technology. So, the series resonators have been replaced by the association of a admittance inverter and a shunt resonator implemented as a quarter-wavelength line and quarter-wavelength short-circuited stub as also proposed in [2].

The quad-band band-pass filter was designed using ADS 2017 from Keysight Technologies ©. A picture of the realized quad-band band-pass filter is shown in Fig.5. Fig.6 presents the EM simulated and measured frequency responses of the designed quad-band band-pass filter. A very good correlation is obtained, except in term of losses since a difference of 1.5dB can be observed in each band. This is mainly due to an underestimate of the $\tan\delta$ value of the substrate in simulation, the latter being increased after laser engraving.

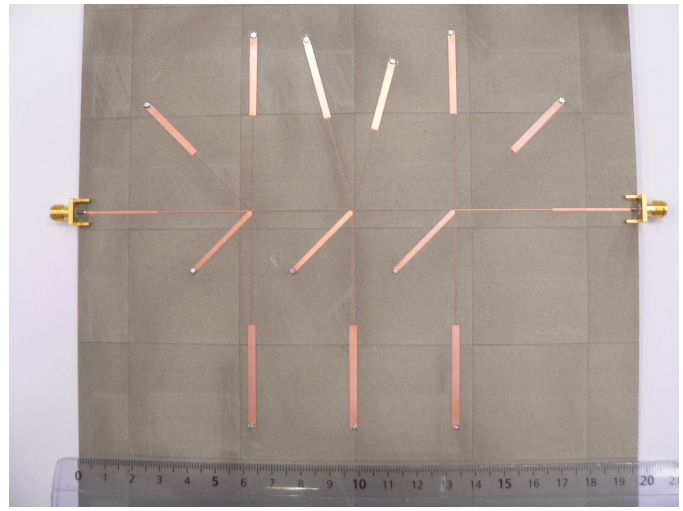

Fig. 5. Photograph of the completed quad-band band-pass filter (Size: 150×139mm^2 without 50Ω input and output lines).

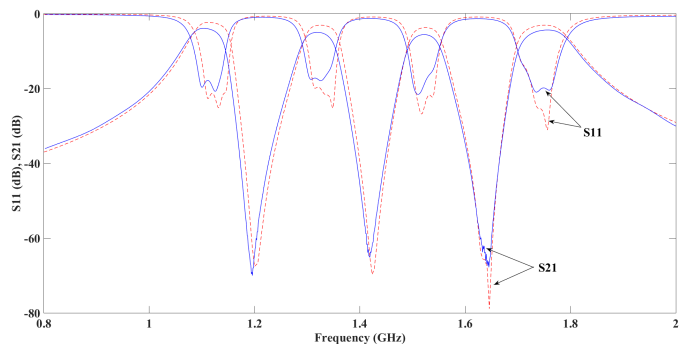

Fig. 6. EM simulated (red dashed lines) and measured (blue continuous line) frequency responses of the fabricated quad-band band-pass filter

IV. Conclusion

A novel synthesis method based on frequency transformation has been presented in this paper. This transformation based on the parallel association of one band-pass resonator and three band-stop ones allows to obtain analytically all the parameters of quad-band band-pass filter in a lumped-element configuration. A 3-order Chebyshev quad-band band-pass filter has been synthesized using this technique and a prototype in microstrip technology has been designed, fabricated and tested. The filter's response agrees well with the theory and full wave simulation, which validate our synthesis technique.

References

[1] Kuo, Jen-Tsai, Tsung-Hsun Yeh, and Chun-Cheng Yeh, "Design of microstrip bandpass filters with a dual-passband response." *IEEE Trans.*

on Microwave Theory and Techniques, vol. 53, n°.4, pp. 1331-1337, Apr. 2005.

[2] Macchiarella, Giuseppe, and Stefano Tamiazzo, "Design techniques for dual-passband filters." *IEEE Trans. on Microwave Theory and Techniques*, vol. 53, n°.11, pp. 3265-3271, Nov. 2005.

[3] Mokhtaari, Marjan, et al., "Coupling-matrix design of dual and triple passband filters." *IEEE Trans. on Microwave Theory and Techniques*, vol. 54, n°.11, pp. 3940-3946, Nov. 2006.

[4] Lee, Juseop, and Kamal Sarabandi, "Design of triple-passband microwave filters using frequency transformations." *IEEE Trans. on Microwave Theory and Techniques*, vol. 56, n°.1, pp. 187-193, Jan. 2008.

[5] Zhang, Xiu Yin et Xue, Quan, "Novel centrally loaded resonators and their applications to bandpass filters." *IEEE Trans. on Microwave Theory and Techniques*, vol. 56, n°.4, pp. 913-921, Apr. 2008.

[6] Chin, K. S., et Yeh, J. H., "Dual-wideband bandpass filter using short-circuited stepped-impedance resonators." *IEEE Microwave and Wireless Components Lett.*, vol. 19, n°.3, pp. 155-157, 2009.

[7] Mo, Yuxia, Kaijun Song, and Yong Fan, "Miniaturized triple-band bandpass filter using coupled lines and grounded stepped impedance resonators." *IEEE Microwave and Wireless Components Lett.* vol. 24, n°.5, pp. 333-335, 2014.

[8] Chu, Hui, and Jian-Xin Chen. "Dual-band substrate integrated waveguide balun bandpass filter with high selectivity." *IEEE Microwave and Wireless Components Lett.*, vol. 24, n°.6, pp. 379-381, 2014.

[9] C. Quendo, A. Manchec, Y. Clavet, E. Rius, J.-F. Favennec and C. Person, "General synthesis of N-band resonator based on N-order Dual-Behavior Resonator." *IEEE Microwave and Wireless Components Lett.*, vol. 17, n5, pp. 337-339, May 2007.

[10] A. Garcia-Lamparez and M. Salazar-Palma, "Single-band to multiband frequency transformation for multiband filters." *IEEE Trans. on Microwave Theory and Techniques*, vol. 59, n12, pp. 3048-3058, Dec. 2011.

[11] Liu, Haiwen, et al., "Quad-band high-temperature superconducting bandpass filter using quadruple-mode square ring loaded resonator." *IEEE Trans. on Microwave Theory and Techniques*, vol. 62, n12, pp. 2931-2941, Dec. 2014.

APPENDIX

This appendix presents the expressions of all the n_p coefficients ($n_p \in [0;7]$) as a function of the cut-off angular frequencies firstly and then as a function of the resonant angular frequencies and the susceptance slope parameters.

$$n_7 = \sum_{i=1}^{4} \omega_{Li} - \sum_{i=1}^{4} \omega_{Hi} = -\frac{\omega_0}{b_0} \tag{20}$$

$$\begin{aligned} n_6 &= \sum_{\substack{i,j=1\\ i\neq j}}^{4} \omega_{Hi}\omega_{Hj} + \sum_{\substack{i=j=1\\ i\neq j}}^{4} \omega_{Li}\omega_{Lj} - \sum_{i=1}^{4}\omega_{Li} \cdot \sum_{i=1}^{4}\omega_{Hi} \\ &= -\sum_{s=0}^{3}\omega_{Ls}^2 - \frac{\omega_0}{b_0}\cdot\sum_{s=1}^{3}\frac{\omega_s}{b_s} \end{aligned} \tag{21}$$

$$\begin{aligned} n_5 &= \sum_{\substack{i,j,k=1\\ i\neq j\neq k}}^{4} \omega_{Li}\omega_{Lj}\omega_{Lk} - \sum_{\substack{i,j,k=1\\ i\neq j\neq k}}^{4} \omega_{Hi}\omega_{Hj}\omega_{Hk} \\ &+ \sum_{\substack{i,j=1\\ i\neq j}}^{4} \omega_{Hi}\omega_{Hj} \cdot \sum_{i=1}^{4}\omega_{Li} - \sum_{\substack{i,j=1\\ i\neq j}}^{4} \omega_{Li}\omega_{Lj} \cdot \sum_{i=1}^{4}\omega_{Hi} \\ &= \frac{\omega_0}{b_0}\cdot\sum_{s=1}^{3}\omega_s^2 \end{aligned} \tag{22}$$

$$\begin{aligned} n_4 &= -\sum_{\substack{i,j,k=1\\ i\neq j\neq k}}^{4} \omega_{Hi}\cdot\omega_{Hj}\cdot\omega_{Hk} \cdot \sum_{i=1}^{4}\omega_{Li} \\ &- \sum_{\substack{i,j,k=1\\ i\neq j\neq k}}^{4} \omega_{Li}\cdot\omega_{Lj}\cdot\omega_{Lk} \cdot \sum_{i=1}^{4}\omega_{Hi} \\ &+ \sum_{\substack{i,j=1\\ i\neq j}}^{4} \omega_{Li}\cdot\omega_{Lj} \cdot \sum_{\substack{i,j=1\\ i\neq j}}^{4}\omega_{Hi}\cdot\omega_{Hj} + \prod_{i=1}^{4}\omega_{Li} + \prod_{i=1}^{4}\omega_{Hi} \\ &= \sum_{\substack{s,p=0\\ s\neq p}}^{3}\omega_s^2\cdot\omega_p^2 + \frac{\omega_0}{b_0}\cdot\sum_{s=1}^{3}\left(\frac{\omega_s}{b_s}\cdot\sum_{\substack{p,q=1\\ p\neq q\neq s}}^{3}\omega_p^2\omega_q^2\right) \end{aligned} \tag{23}$$

$$\begin{aligned} n_3 &= \prod_{i=1}^{4}\omega_{Hi}\cdot\sum_{i=1}^{4}\omega_{Li} - \prod_{i=1}^{4}\omega_{Li}\cdot\sum_{i=1}^{4}\omega_{Hi} \\ &+ \sum_{\substack{i,j,k=1\\ i\neq j\neq k}}^{4}\omega_{Li}\omega_{Lj}\omega_{Lk}\cdot\sum_{\substack{i,j=1\\ i\neq j}}^{4}\omega_{Hi}\omega_{Hj} \\ &- \sum_{\substack{i,j,k=1\\ i\neq j\neq k}}^{4}\omega_{Hi}\omega_{Hj}\omega_{Hk}\cdot\sum_{\substack{i,j=1\\ i\neq j}}^{4}\omega_{Li}\omega_{Lj} \\ &= -\frac{\omega_0}{b_0}\cdot\sum_{\substack{s,p=1\\ s\neq p}}^{3}\omega_s^2\omega_p^2 \end{aligned} \tag{24}$$

$$\begin{aligned} n_2 &= \prod_{i=1}^{4}\omega_{Hi}\cdot\sum_{\substack{i,j=1\\ i\neq j}}^{4}\omega_{Li}\omega_{Lj} + \prod_{i=1}^{4}\omega_{Li}\cdot\sum_{\substack{i,j=1\\ i\neq j}}^{4}\omega_{Hi}\omega_{Hj} \\ &- \sum_{\substack{i,j,k=1\\ i\neq j\neq k}}^{4}\omega_{Li}\omega_{Lj}\omega_{Lk}\cdot\sum_{\substack{i,j,k=1\\ i\neq j\neq k}}^{4}\omega_{Hi}\omega_{Hj}\omega_{Hk} \\ &= -\sum_{\substack{s,p,q=0\\ s\neq p\neq q}}^{3}\omega_s^2\omega_p^2\omega_q^2 - \frac{\omega_0}{b_0}\cdot\sum_{s=1}^{3}\left(\frac{\omega_s}{b_s}\cdot\prod_{\substack{p,q=1\\ p\neq q\neq s}}^{3}\omega_p^2\omega_q^2\right) \end{aligned} \tag{25}$$

$$\begin{aligned} n_1 &= \prod_{i=1}^{4}\omega_{Hi}\cdot\sum_{\substack{i,j,k=1\\ i\neq j\neq k}}^{4}\omega_{Li}\omega_{Lj}\omega_{Lk} \\ &- \prod_{i=1}^{4}\omega_{Li}\cdot\sum_{\substack{i,j,k=1\\ i\neq j\neq k}}^{4}\omega_{Hi}\omega_{Hj}\omega_{Hk} \\ &= \frac{\omega_0}{b_0}\cdot\prod_{s=1}^{3}\omega_s^2 \end{aligned} \tag{26}$$

$$n_0 = \prod_{i=1}^{4}\omega_{Li}\omega_{Hi} = \prod_{s=0}^{3}\omega_s^2 \tag{27}$$