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# Satisfaction Indicators Taking into Account the Measurement and Computation Uncertainties for the Comparison of Data in Electromagnetics: Motivations and Scheduled Tasks of the French National Working Group CDIIS

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**Abstract**—A national study of criteria able to provide a satisfaction indicator about the comparison of data from electromagnetic measurement and computation, taking into account their associated uncertainties, has been organized in the framework of a French Working Group of the GdR Ondes, called CDIIS (Comparaison de Données entachées d’Incertitudes: Indicateurs de Satisfaction). This Working Group involves several industrial or academic research laboratories, including laboratories depending on governmental organisms. Four tasks have been defined: 1/ Identification of a set of satisfaction indicators. 2/ Identification of pertinent test cases in different application domains of electromagnetics. 3/ Application of the above criteria on the test cases. 4/ Conclusions: which criterion is best adapted to a given electromagnetism problem. This paper deals with Task 1 and discusses the results of various indicators applied to a canonical RCS (Radar Cross Section) test case.

## I. INTRODUCTION

When designing a new product, an important step is to manufacture a prototype and measure it as a proof of concept. It is not unusual to observe some discrepancies between the actual measurements and the expected outputs. Several errors or uncertainties may explain these differences including technological defaults due to the manufacturing process, measurement inaccuracies and much more. To conduct a rigorous analysis of the differences, the elementary error sources should be identified, for the computations (considering the best knowledge of the input data and the potential model errors) as well as for the measurements. The next step is to propagate these uncertainties through the model of the measurement procedure (based on the Guide to the Uncertainties Measurement -GUM- recommendations) and that of the computation procedure. Measured and computed data are therefore available, along with their confidence interval. The next ques-

tion is: how do these measured and computed data compare, and which comparison criterion can be established to quantifiably express the level of satisfaction or dissatisfaction.

This is the topic of a new Working Group called CDIIS (Comparaison de données entachées d’incertitudes: Indicateurs de Satisfaction), initiated by the CNRS (French National Center for Scientific Research) and the GdR Ondes (French research network about waves) piloted by the CEA (French Atomic Energy Commission), the IETR (Institut d’Electronique et de Telecommunications de Rennes) and Institut Fresnel. The CDIIS is in direct continuation with the studies previously performed during the Working Group GTi (Groupe de Travail sur les incertitudes de mesure en chambre anéchoïque) on the measurement of uncertainties in anechoic chambers [1]. In the framework of this former GTi, nine French laboratories (including a simulation laboratory providing computational reference data) participated to a national comparison of RCS measurement [2]. This study naturally gave rise to the question: how to compare a set of data with uncertainties.

The first discussions of the CDIIS enabled to point out that the criterion should be adapted to each specific field of the electromagnetic problem at hand (RCS, antenna, electromagnetic compatibility, materials characterization, radar dosimetry, etc...), each field exhibiting a specific variation of signals (e.g. resonances or low varying signals). Hence, the necessity appears that various test cases in various fields should be defined, on which a set of criteria should be applied. The first aim was consequently to define and select a set of indicators, and next to apply them on a given set of test cases.

In the following, only a RCS test case will be considered, in order to concentrate on the information that can be derived from these criteria.

## II. EXAMPLES OF INDICATORS

In RCS, the quantity of interest is the electric field. This quantity denoted as  $\mathbf{E}$  varies along the angular direction and/or the frequency. For a given angular direction and a given frequency, usually  $\mathbf{E}$  is nothing but a deterministic signal to which a random variable is added.

As we want to compare two distributions of data, we will therefore consider two independent multivariate random vectors  $\mathbf{E}_1$  and  $\mathbf{E}_2$ , with respective means  $\boldsymbol{\mu}_1$  and  $\boldsymbol{\mu}_2$ , and respective covariance matrices  $\boldsymbol{\Sigma}_1$  and  $\boldsymbol{\Sigma}_2$ . Their probability density functions will be denoted as  $p_1$  and  $p_2$ . To further simplify, we will herein assume that they are both normally distributed.

A first criterion may be directly defined as the likelihood of the (unknown) underlying true value of the collected data. The problem may be stated as follows. Let a couple of collected data be compared to one another, namely  $y_1$  (within  $E_1$ ) and  $y_2$  (within  $E_2$ ). Assuming  $p_1$  and  $p_2$ , is it likely to find an expected (true) value  $y_t$  in the vicinity of  $y_1$  and  $y_2$ . The maximum likelihood (ML) [3]  $p_t(y_t)$  of  $y_t$  is sought in a subspace around  $y_1$  and  $y_2$  such that:

$$ML = \max_{y_t} p_t(y_t) \quad (1)$$

with

$$p_t(y_t) = \frac{p_1(y_1/y_t - 1) \times p_2(y_2/y_t - 1)}{\int p_1(y_1/y_t - 1) \times p_2(y_2/y_t - 1) dy_t} \quad (2)$$

Another way to compare the data could be the ‘‘consistence criterion’’: a statistical test that quantifies the overlap of two distributions. First the confidence interval  $D_s$ , at a given probability  $\alpha$ , of one of the distributions has to be computed:

$$P(Y \in D_s | sim) = 1 - \alpha$$

with  $\alpha = 5\%$  and considering the data corresponding to the simulation as an example. Thus the ‘‘consistence criterion’’  $C$  is defined as the probability that the quantity of interest, according to the measurement, be in the confidence interval of the simulation:

$$C(Y | sim, Y | meas) = P(Y \in D_s | meas)$$

Note that this criterion is not symmetrical, i.e., it gives different results if the simulation or the measurement (as in this example) is considered as the reference. The value of this criterion is held between 0 (no overlap of the 2 distributions) and 1 (measurement distribution included in the simulation distribution).

The Mahalanobis distance has also been suggested as a comparison criterion. This distance is classically used to quantify the similarity or overlap between probability distributions or sample groups, in many fields such as classification or statis-

tical pattern recognition [4]. The Mahalanobis distance  $D_M$  is given by:

$$D_M^2(p_1, p_2) = (\boldsymbol{\mu}_2 - \boldsymbol{\mu}_1)^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu}_2 - \boldsymbol{\mu}_1) \quad \text{with} \quad \boldsymbol{\Sigma} = \frac{\boldsymbol{\Sigma}_1 + \boldsymbol{\Sigma}_2}{2}$$

It can be estimated from random samples, with empirical means and empirical covariance matrices. In a previous study, this criterion was applied to the RCS of a cone-sphere and showed that it could express one’s satisfaction or dissatisfaction when comparing measurement and computation data, both associated to their uncertainty [5].

The Mahalanobis distance is in fact a particular case of the Bhattacharyya distance which also measures the degree of similarity between two probability distributions [6]. When the multivariate distributions are normally distributed, this Bhattacharyya distance  $D_B$  reads as

$$D_B(p_1, p_2) = \frac{1}{8} (\boldsymbol{\mu}_2 - \boldsymbol{\mu}_1)^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu}_2 - \boldsymbol{\mu}_1) + \frac{1}{2} \ln \left( \frac{\det \boldsymbol{\Sigma}}{\sqrt{\det \boldsymbol{\Sigma}_1 \det \boldsymbol{\Sigma}_2}} \right)$$

This distance can take values from 0 to infinity. A derived version, bounded this time between 0 and 1, has also been proposed. It is called the Hellinger distance  $D_H$  and is defined as

$$D_H(p_1, p_2) = \sqrt{1 - \exp[-D_B(p_1, p_2)]}$$

In the following,  $1 - D_H$  will be considered.

Another criterion for comparing probability density distributions, which is well-spread in the information theory community, is the Kullback-Leibler divergence  $D_{KL}$  [6]. When the multivariate distributions are normally distributed, this Kullback-Leibler distance  $D_{KL}$  reads as

$$D_{KL}(p_1, p_2) = \frac{1}{2} (\boldsymbol{\mu}_2 - \boldsymbol{\mu}_1)^T \boldsymbol{\Sigma}_2^{-1} (\boldsymbol{\mu}_2 - \boldsymbol{\mu}_1) + \frac{1}{2} \ln \left( \frac{\det \boldsymbol{\Sigma}_2}{\det \boldsymbol{\Sigma}_1} \right) + \frac{1}{2} \text{tr}(\boldsymbol{\Sigma}_2^{-1} \boldsymbol{\Sigma}_1) - \frac{k}{2}$$

where  $k$  is the dimensionality of the multivariate distribution (here  $k=1$ ). Such a divergence is not symmetrical and is in fact not a real distance per se, as it does not verify the Minkowski inequality. To symmetrize the results, two divergences have been proposed, the Jensen-Shannon divergence  $D_{JS}$  and the Resistor divergence  $D_R$  [7],

$$D_{JS}(p_1, p_2) = \frac{D_{KL}(p_1, p_2) + D_{KL}(p_2, p_1)}{2}$$

$$\frac{1}{D_R(p_1, p_2)} = \frac{1}{D_{KL}(p_1, p_2)} + \frac{1}{D_{KL}(p_2, p_1)}$$

## III. EXAMPLE OF APPLICATION

In the following, the above functions are applied to a RCS (Radar Cross Section) case. The configuration corresponds to the bistatic diffraction values in VV polarization of a 38.1 mm diameter sphere over the range  $[-130^\circ, 130^\circ]$ , illuminated at the incidence of  $180^\circ$  and at the frequency of 9.6 GHz. The permittivity of the dielectric sphere, made of PMMA, has been experimentally estimated to be equal to  $\epsilon_r^{exp} = 2.6 + j0$  with the methodology described in [8].

Two sets of data are available, with their associated uncertainties. The first one corresponds to RCS measured in an anechoic chamber at the Centre Commun de Ressources en Mi-

croondes (CCRM) in Marseille, France. The mean and variances have been determined as explained in [9]. The simulation has been performed using Mie formalism [10][11], combined with a Monte-Carlo method [12] in order to derive the mean and variance of the scattered field, assuming that the real and imaginary parts of the sphere permittivity follow a uni

form distribution in the range  $[2.52, 2.65] + j [0.00, 0.06]$ , while the radius uniformly varies within  $38.1 \pm 0.2$  mm. The real part and the imaginary part of the simulated scattered field and the measured scattered field are both assumed to be normally distributed.

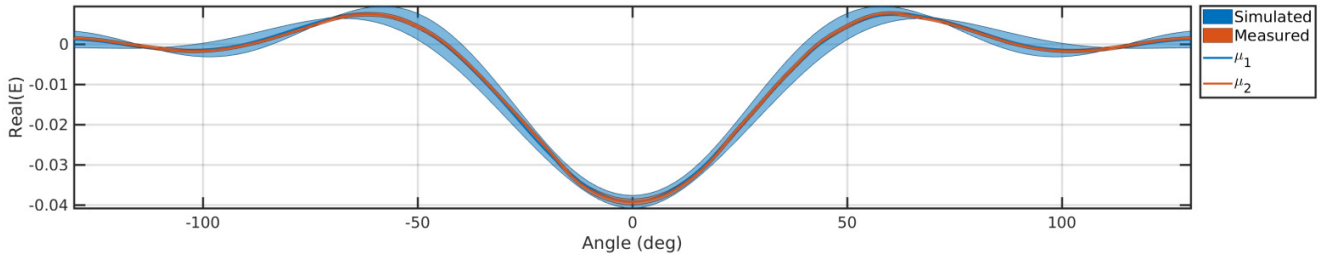


Figure 1: Angular evolution of the real part of the RCS of a dielectric sphere, in a bistatic configuration. The dashed areas represent a  $\pm 3\sigma$  variation around the mean value  $\mu$ . Experimental and simulated results are presented.

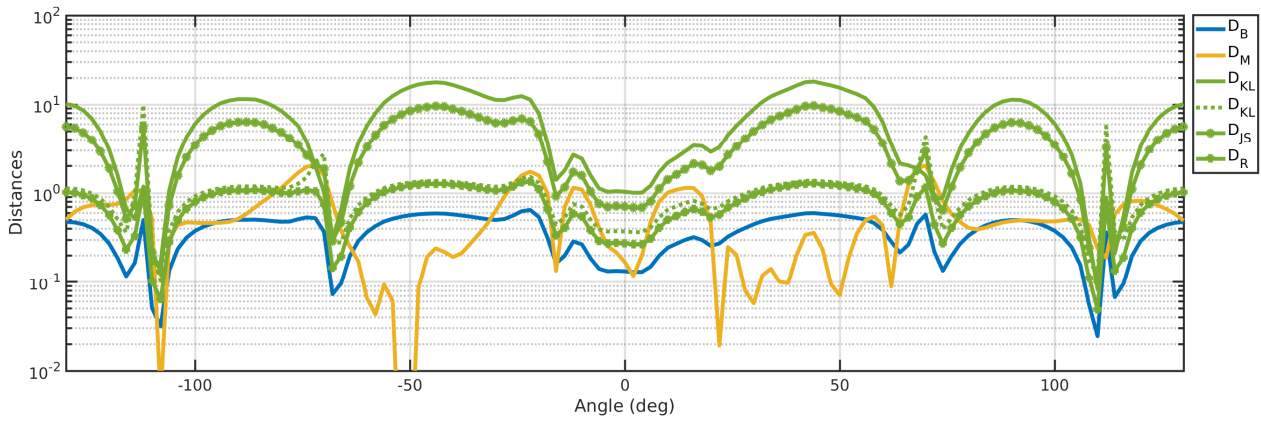


Figure 2: Comparison of the criteria applied on the real parts of the simulated and measured RCS. They are plotted in logarithmic scale as they are ranging from 0 to  $\infty$ .

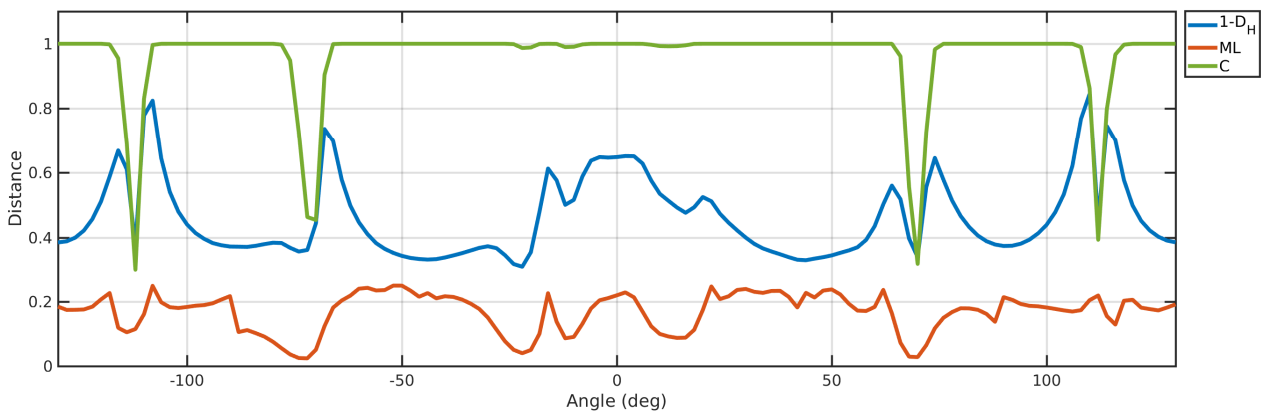


Figure 3: Comparison of the criteria applied on the real parts of the simulated and measured RCS. They are plotted in linear scale as they are ranging from 0 to 1.

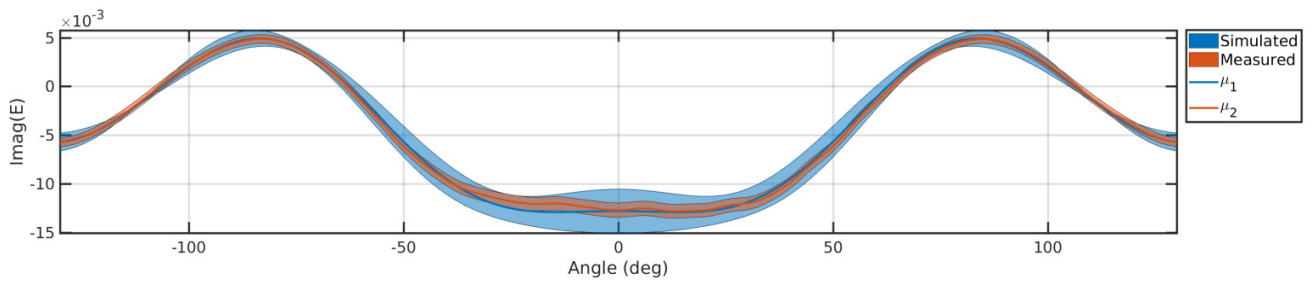


Figure 4: Angular evolution of the imaginary part of the RCS of a dielectric sphere, in a bistatic configuration. The dashed areas represent a  $\pm 3\sigma$  variation around the mean value  $\mu$ . Experimental and simulated results are presented.

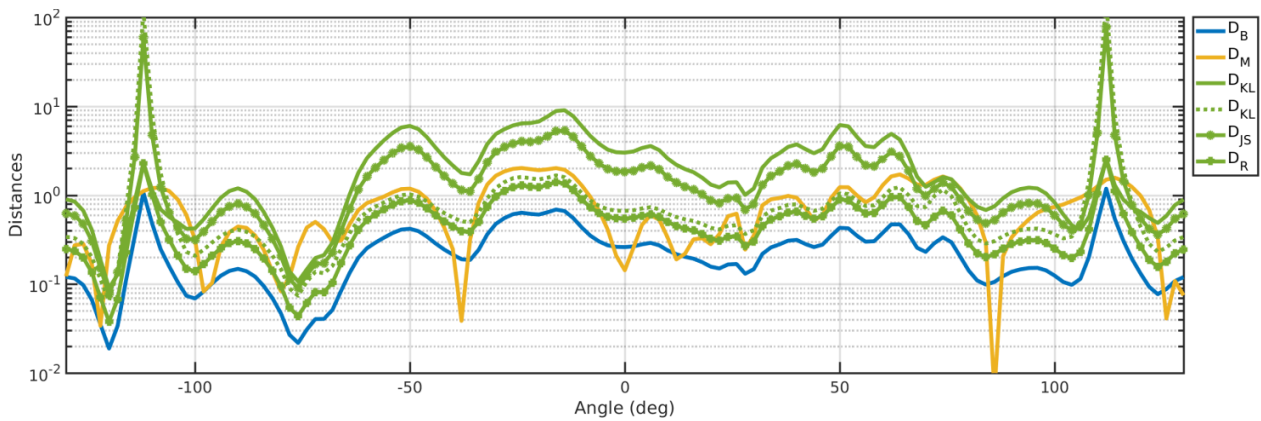


Figure 5: Comparison of the criteria applied on the imaginary parts of the simulated and measured RCS. They are plotted in logarithmic scale as they are ranging from 0 to  $\infty$ .

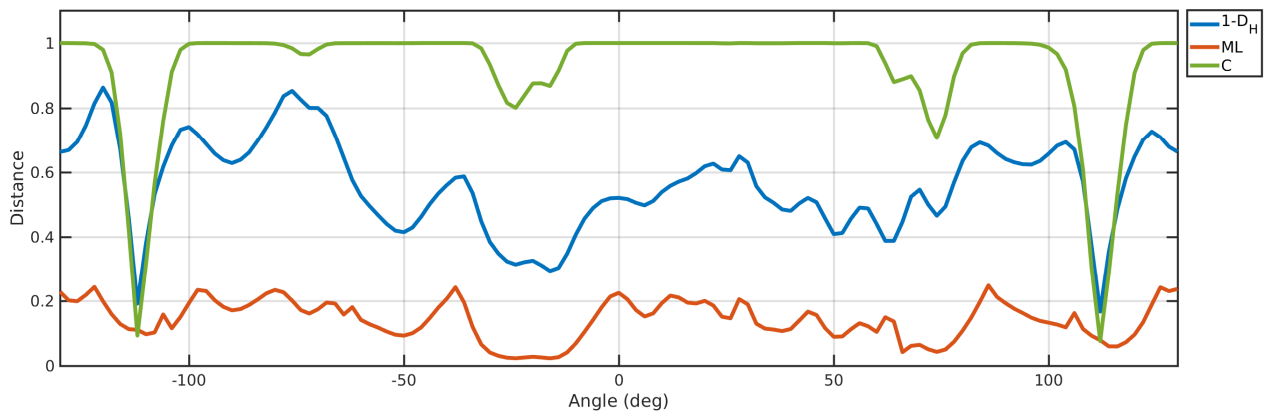


Figure 6: Comparison of the criteria applied on the real parts of the simulated and measured RCS. They are plotted in linear scale as they are ranging from 0 to 1.

In Figure 1 (resp. Figure 4), the angular evolution of the real (resp. imaginary) part of the measured and simulated scattered field is shown. As the mean values and uncertainties vary in the two sets of data, there are some areas where the error bars (corresponding to a  $\pm 3\sigma$  variation) do overlap and some others where the overlapping is reduced. It is of interest to note tha

the areas of overlapping may differ from the real part and the imaginary part.

The various criteria are plotted in Figure 2 and Figure 3 (resp. Figure 5 and Figure 6) for the real part (resp. imaginary part). The Kullback-Leibler divergence, due to its non-symmetrical property, has been plotted for  $D_{KL}(p_1, p_2)$  and  $D_{KL}(p_2, p_1)$ .

Nevertheless, in the current case, there is not a large difference between these two divergences.

Having a closer look at these criteria, it can be noticed that some of these criteria do follow the same trends. This is the case for example for the Bhattacharyya distance  $D_B$ , the Kullback-Leibler divergence, the Jensen-Shannon divergence  $D_{JS}$  and the Resistor divergence  $D_R$ . One of the reasons of this similar behavior might be due to the strong assumption that the random signals are normally distributed. The very good agreement between the measured and simulated field might also contribute to the fact that the criteria behave in a similar fashion.

Even if the Hellinger distance is only limited to the [0,1] range, it also follows the same behavior as the Bhattacharyya distance  $D_B$ . This is the reason why we advise to focus on this particular criterion, as it is a bounded criterion.

Another interesting aspect is that the Likelihood criterion  $ML$  seems to be the mirror image of the Mahalanobis criterion  $D_M$  along an horizontal line. The  $ML$  criterion is indeed built up as an inverse function of the distance between curves according to  $p_1$  and  $p_2$ .

Some of the criteria seem to be more influenced by the bias, as for the example the Mahalanobis distance, while some others such as the Hellinger distance or the consistency criterion combine bias and variance information and provide an indication on how well the two  $\pm 3\sigma$  areas overlap. The areas where a good fit is visible are detected by all of these latter criteria. Nevertheless, the consistency criterion is the one which provides the most clear-cut result, even if this criterion is non-symmetrical. Of course, such conclusions need to be consolidated by examining more data sets, in particular ones where the fitting between the two datasets is not that good.

#### IV. CONCLUSIONS AND FUTURE WORKS

In the frame of the GdR Ondes and the CDIIS Working Group, the definition of a relevant satisfaction indicator for comparing uncertain data in electromagnetic problems has been planned. A set of indicators has been investigated. The next step is to continue prospecting for interesting criteria. It is also planned to apply the whole set of criteria to various electromagnetic test cases such as: characterization of a sample of material in a TEM cell, current induced along a shielded cable, coupling in a cavity, radiation pattern of a log-periodic antenna, RCS of a disk with a slot.

At the time of the publication, almost 20 French laboratories are involved in this project.

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