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Composite beam identification using a variant of the inhomogeneous wave correlation method in presence of uncertainties

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Abstract

Purpose – This paper aims to propose numerical-based and experiment-based identification processes, accounting for uncertainties to identify structural parameters, in a wave propagation framework.

Design/methodology/approach – A variant of the inhomogeneous wave correlation (IWC) method is proposed. It consists on identifying the propagation parameters, such as the wavenumber and the wave attenuation, from the frequency response functions. The latter can be computed numerically or experimentally. The identification process is thus called numerical-based or experiment-based, respectively. The proposed variant of the IWC method is then combined with the Latin hypercube sampling method for uncertainty propagation. Stochastic processes are consequently proposed allowing more realistic identification.

Findings – The proposed variant of the IWC method permits to identify accurately the propagation parameters of isotropic and composite beams, whatever the type of the identification process in which it is included: numerical-based or experiment-based. Its efficiency is proved with respect to an analytical model and the Mc Daniel method, considered as reference. The application of the stochastic identification processes shows good agreement between simulation and experiment-based results and that all identified parameters are affected by uncertainties, except damping.

Originality/value – The proposed variant of the IWC method is an accurate alternative for structural identification on wide frequency ranges. Numerical-based identification process can reduce experiments' cost

without significant loss of accuracy. Statistical investigations of the randomness of identified parameters illustrate the robustness of identification against uncertainties.

Keywords Structural identification, Damping, Uncertainties, Honeycomb sandwich structure, Inhomogeneous wave correlation, Latin hypercube sampling

Paper type Research paper

1. Introduction

Structural identification forms an ever growing emphasis in engineering applications, such as vibroacoustics (Ablitzer *et al.*, 2014; Cherif *et al.*, 2015; Roozen *et al.*, 2017b). Special attention is paid by the scientific community to the damping phenomenon on which any vibration problem is directly dependent. Modeling and identifying damping is obviously necessary when designing and dimensioning structures. Numerical simulations become more and more inevitable to reduce the cost of experimental identification processes. In the literature, most works focus on the modal-based numerical approaches. Nevertheless, these approaches reach their limits in mid and high frequencies where great modal density exists. This disadvantage makes their use of limited interest. As alternatives, in a wave propagation framework, other methods based on the wavenumber space (k-space) analysis for parameter identification are introduced (Ichchou *et al.*, 2008a). The most frequently used methods are the Mc Daniel method (McDaniel *et al.*, 2000) and the inhomogeneous wave correlation (IWC) method (Berthaut *et al.*, 2005).

The Mc Daniel method consists on adjusting iteratively, for each frequency, the wavenumber and the damping. The wavenumbers for the neighboring frequencies are considered as initial estimate. The Mc Daniel method was used by McDaniel and Shepard (2000) to identify the damping of a freely suspended beam which is excited by an arbitrary transient load. Its efficiency was proved with respect to modal approaches. Indeed, modal descriptions of the structural response, such as half-power point method and many variations of modal analysis and testing, permit to estimate the damping loss factor only near resonance frequencies. However, the Mc Daniel method is capable to estimate the loss factor at any frequency, even if the latter varies significantly over the frequency range of interest, and the resonance frequencies are not close enough to track the variations. The extension of the method to two-dimensional applications was proposed by Ferguson *et al.* (2002). The authors combined a continuous Fourier transform with a least square minimization to identify a single dominant homogeneous wave when using a windowed field far away from the near-field sources which would otherwise create disturbances.

The principle of the IWC method is to correlate the vibratory field with an inhomogeneous wave. To extract the parameters in each direction of propagation and thus eliminate the near-field sources, a correlation index is introduced. It depends on propagation parameters and permits to build a frequency and direction-dependent dispersion equation from a space vibratory field. In the literature, Berthaut *et al.* (2005) and Ichchou *et al.*, (2008b) proved the ability of the IWC method to identify accurately structural parameters of isotropic and anisotropic ribbed panels and plates. Obtained results illustrated the dependence of the loss factor on the wave direction. However, it does not depend on boundary conditions, on geometry and on source location. The efficiency of the IWC method was also illustrated by Ichchou *et al.* (2008a) for honeycomb beams and panels identification with bending load at wide frequency band. Inqui  t   (2008) used the IWC method to identify the phase velocity of a quasi-isotropic laminated composite plate. Caillet *et al.* (2007) estimated the elasticity modulus of a plate using the IWC method with respect to a modal approach. Rak *et al.* (2008) applied both Mc Daniel and IWC methods to identify the

damping of a homogeneous beam covered with viscoelastic layers. The efficiency of these methods is proved with respect to standard Oberst tests. The use of an incorrect model of the wave field in the IWC method (Rak *et al.*, 2008) leads to unreliable identification of parameters, which is illustrated by negative values of the loss factor for lower frequencies. However, the Mc Daniel method enabled correct estimation of the loss factor within wide frequency range. Chronopoulos *et al.* (2013) identified the wave propagation characteristics of a composite panel for a wide frequency range. The IWC method identification was based on the vibratory data measured experimentally and was compared to the wave finite element method estimation. The IWC method has recently been used by Cherif *et al.* (2015) to identify the damping of orthotropic honeycomb panels, based on experimental and numerical (FEM) results. Its efficiency was proved with respect to other classical methodologies.

As mentioned in Berthaut (2004) and Inqui  t   (2008), the IWC method allows accurate parameter identification at mid and high frequencies when high modal overlap occurs. In fact, in this case, energy is distributed in all propagation directions. At low frequencies, when low modal overlaps occur, the IWC method reaches its limits. Some recent works focus on the improvement of the identification accuracy at low frequency ranges. VanBelle *et al.* (2017), for instance, proposed an extended form of the IWC method to validate, through experimental measurements of dispersion curves on a manufactured metamaterial sample, the dispersion curves calculated numerically with the unit cell approach. The principle of the extended IWC is to take into account the experimental excitation location when expressing the correlated inhomogeneous wave. This allows better estimates of the attenuation. Moreover, to improve the wavenumber estimates, Roozen *et al.*, (2017a) proposed to use only half of the measurement data in the IWC, called wave fitting approach, either to the left or to the right of the excitation position. This effect is due to the disturbing influence of the measurement data on the left of the excitation point when fitting the right running waves, and vice versa. Accurate identifications of the wavenumber were achieved using the wave fitting approach and the Prony method, compared to the spatial Fourier approach. Particularly, smoother estimate of the wavenumber was obtained when using the mean of left and right running waves, compared the use of only right or left running wave.

The main purpose of this work is to construct an identification model adapted to the complexity of dynamic systems for any considered frequency domain. The proposed identification model is a variant of the classical IWC method based on a summation of inhomogeneous waves, accounting for both forward and backward propagating waves. In the context of a numerical-based identification process, the proposed IWC-variant uses as inputs frequency response functions (FRFs), which are computed numerically, to identify propagation parameters such as damping, wavenumber, wave attenuation and phase velocity. Its efficiency is evaluated through its comparison to an analytical model, the classical IWC method and the Mc Daniel method, which is considered as reference.

Far from the idealization based on deterministic modeling, uncertainties can affect design parameters, loading, modeling processes, etc. To achieve realistic models, it is inevitable to account for uncertainties. Quantifying and propagating uncertainties permits to evaluate their impact and obtain an agreement between design models and experimental analyses. Two main approaches can be used to quantify uncertainties depending on their classification: parametric or non-parametric. A special emphasis is paid in this work to parametric uncertainties (e.g. geometrical parameters and loading forces). Parametric uncertainties can be quantified according to possibilistic or probabilistic frameworks. According to the former, uncertainties are, for instance, modeled by intervals, whereas according to the latter, random variables, for example, quantify the variability of uncertain

parameters. In this case, probabilistic methods, also called stochastic methods, allow propagating uncertainties to evaluate stochastic impact of uncertain input parameters on output responses if direct problems are considered and inversely if inverse problems are considered. Several stochastic methods are introduced in the literature. The most commonly used methods are the sample-based ones. The Monte Carlo simulations (MCS) (Fishman, 1996; Rubinstein and Kroese, 2008) and the Latin hypercube sampling (LHS) (McKay *et al.*, 1979; Helton and Davis, 2003) are frequently used. Both methods are based on a succession of deterministic evaluations corresponding to a set of realizations of random variables and allow obtaining accurate results through simple implementations. The LHS method permits to reduce the prohibitive computational time required by the MCS without a significant loss of accuracy. In fact, partitioning the variability space into regions of equal probability and selecting one sampling point in each region allow reducing the number of samples.

Combining the LHS method with the above-cited parameter identification methods (classical IWC, IWC-variant and Mc Daniel) allows constructing three stochastic identification processes. The main stochastic identification process, coupling the LHS method and the IWC-variant method, is compared to the two other processes to evaluate the efficiency of the proposed IWC-variant and the effect of uncertainties on its identification.

In the present paper, we focus on parameter identification of beam structures. A sandwich composite beam of honeycomb core is particularly considered. This choice is due to the growing industrial integration of composite materials, in particular for high-technology sectors, resulting from their interesting mechanical and material properties, high energy dissipation and resistance/weight ratios.

An experimental validation of the numerical-based identification results is then carried out through an experiment-based identification process using as inputs FRFs which are measured experimentally.

2. Theoretical backgrounds

2.1 Mc Daniel method

The main aim of the Mc Daniel method (McDaniel *et al.*, 2000; McDaniel and Shepard, 2000) is to estimate complex wavenumbers and amplitudes of waves which are propagated through damped structures, such as beams, plates and shells. The principle of the Mc Daniel method is to iteratively adjust, for each frequency, the wavenumber to approximate accurately the response. It considers the wavenumbers for neighboring frequencies as initial estimations. It uses the wave dispersion relations containing information about structure viscoelastic properties which are difficult to measure experimentally. The most important property is the loss factor which can only be estimated around resonant frequencies using the modal approach and at any frequency owing to the Mc Daniel method.

Mathematically, let us consider a harmonic displacement field which depends on space coordinates:

$$u = \Re\{Ue^{-i\omega t}\}, \quad (1)$$

where $\Re\{\cdot\}$ refers to the real part and U to the displacement amplitude.

The Mc Daniel method consists on solving the linear differential equation of motion of the neutral axis or surface of the structure which takes the form:

$$-\omega^2 U + \mathcal{L}\{U\} = 0, \quad (2)$$

where $\mathcal{L}\{U\}$ is a linear operator containing the displacement derivatives with respect to the space coordinate x .

Taking into account boundary conditions:

$$\mathcal{L}_b\{u\}|_{(x=x_b)} = \Re\{Be^{-i\omega t}\}, \quad (3)$$

where x_b represents boundaries and B is complex valued, the solution of equation (2) is expressed as:

$$U(x) = \sum_{n=1}^N \{F_n e^{ik_n x} + B_n e^{ik_n(L-x)}\}, \quad (4)$$

where N is the number of different waves. Each wave n is characterized by a wavenumber k_n of complex value, containing positive real and imaginary parts, and an amplitude F_n or B_n according to forward or backward propagation, respectively, computed using boundary conditions.

The wavenumber is computed according to the type of the propagated wave through the structure. In flexural, longitudinal and torsion wave case, respectively, the wavenumber is expressed as:

$$k_f = \sqrt[4]{\frac{\rho A \omega^2}{E(\omega)(1 - i\eta(\omega))I}}, k_l = \sqrt{\frac{\rho \omega^2}{E(\omega)(1 - i\eta(\omega))}}, k_t = \sqrt{\frac{\rho \omega^2}{G(\omega)(1 - i\eta(\omega))}}, \quad (5)$$

where $E(\omega)$ and $G(\omega) = E(\omega)/2(1 + \nu)$ are the real parts of the Young and shear modulus, respectively, ρ is the mass density, I the inertia moment, A the area of the transversal section and $\eta(\omega)$ the material loss factor.

The loss factor depends on the wavenumber and can consequently be identified, in the flexural wave case as:

$$\eta = \left| \frac{\Im\{k_f^4\}}{\Re\{k_f^4\}} \right|, \quad (6)$$

where $\Im\{\cdot\}$ and $\Re\{\cdot\}$ correspond, respectively, to the imaginary and real parts.

In longitudinal and torsion wave cases, the loss factor is expressed as:

$$\eta = \left| \frac{\Im\{k_i^2\}}{\Re\{k_i^2\}} \right|, \quad (7)$$

where the index i refers to indexes l or t according to longitudinal or torsion cases, respectively.

To verify the validity of the initially supposed wavenumber, it is compared to the obtained wavenumber through the error function defined as:

$$\varepsilon^2(k) = \sqrt{\frac{\sum_{m=1}^M \rho(x_m) |U^{mes}(x_m, \omega) - U(x_m, \omega)|^2}{\sum_{m=1}^M \rho(x_m) |U(x_m, \omega)|^2}}, \quad (8)$$

where M is the number of considered measurement points, $\rho(x_m)$ the coherence function and $U^{mes}(x_m, \omega)$ and $U(x_m, \omega)$ are the measured and real wave fields, respectively. To minimize this error, an optimization algorithm varying the wavenumber is applied.

2.2 Inhomogeneous wave correlation method

The principle of the IWC method is to project the vibratory field on inhomogeneous waves. In fact, damped propagating waves are fully identified from a spatial displacement field and by correlation with inhomogeneous waves (complex wavenumber and known direction). Consequently, the dispersion equation is completely reconstructed, and the wave attenuation is measured using the IWC method.

Mathematically, the IWC method uses a harmonic field $\hat{u}(x, y)$, calculated either from a harmonic excitation or from a temporal Fourier transform:

$$u(x, y, t) = \int_0^{+\infty} \hat{u}(x, y) e^{i\omega t} d\omega. \quad (9)$$

Propagation parameters are computed by correlating the vibratory field of the structure $u(x, y, \omega)$, denoted here $\hat{u}(x, y)$ where the ω -dependence comprises in the hat $\hat{\cdot}$, with an inhomogeneous wave:

$$u_{IWC} = A(\theta) e^{-ik(\theta)(1+i\gamma(\theta))(x\cos(\theta)+y\sin(\theta))}, \quad (10)$$

where θ is the wave direction, γ the wave attenuation and $A(\theta)$ the amplitude of the wave.

The correlation is performed through an *IWC* criterion defined, like a modal assurance criterion (Ewins, 1984), as:

$$IWC(k_{IWC}, \gamma_{IWC}, \theta) = \frac{\left| \iint_S \hat{u} \cdot u_{IWC}^* dx dy \right|}{\sqrt{\iint_S \hat{u} \cdot \hat{u}^* dx dy \times \iint_S u_{IWC} \cdot u_{IWC}^* dx dy}}, \quad (11)$$

where u_{IWC}^* is the complex conjugate of the wave u_{IWC} . This criterion represents the wave contribution in the field $\hat{u}(x, y)$ or also the ratio of the energy carried by the wave and the total energy contained in the field. Maximizing the *IWC* correlation criterion leads to optimizing the wavenumber and damping identification.

For numerical discrete analysis, integrals in equation (11) are replaced by weighted discrete sums:

$$IWC(k, \gamma, \theta) = \frac{\left| \sum_n \rho_i \hat{u} \cdot u_{IWC}^* S_i \right|}{\sqrt{\sum_n \rho_i |\hat{u}|^2 S_i \cdot \sum_n \rho_i |u_{IWC}|^2 S_i}}, \quad (12)$$

where S_i is the elementary surface of the structure around a measurement point M_i , ρ_i is the coherence of measurement data at point M_i ($\rho_i = 1$ (if the coherence is not available) and n is the number of measurement points.

In practice (Berthaut, 2004; Berthaut *et al.*, 2005; Ichchou *et al.*, 2008a, 2008b; Inqui  t  , 2008), the algorithm of application of the IWC method consists on putting the direction θ , for each frequency iteration, into a discrete set of values θ_j . For each direction θ_j , the maximum of the *IWC* criterion is located at a couple of values (k_j, γ_j) of the wavenumber and the wave attenuation, respectively. The triplet $(\theta_j, k_j, \gamma_j)$ corresponding to vanishing waves ($\gamma_j > 1$) is removed.

2.3 Proposed variant of the inhomogeneous wave correlation method

As found in literature, the classical form of the IWC method reaches its limits and thus gives inaccurate identification, especially for damping, at low frequencies. Modal overlap is, indeed, not sufficiently high to allow energy covering all propagation directions (Berthaut, 2004; Inqui  t  , 2008). To overcome such limits, some improved and extended forms of the IWC method were proposed in literature (Section 1). For instance, VanBelle *et al.* (2017) extended the IWC method with the experimental excitation location. The authors proposed to define an inhomogeneous wave including the excitation location to obtain a better estimate of the attenuation. Moreover, Roozen *et al.*, (2017a) proved that the identification accuracy is improved when using only half the space-time data set. This effect was verified in the context of both Prony method and wave fitting approach, which refers in literature to the IWC method, and is due to the disturbing influence of the measurement data on the left of the excitation point when fitting the right running waves, and vice versa. The authors compared both results of using right running wave, left running wave and the mean of these two wave types. The latter allowed more accurate identification of the wavenumber.

In the same context, a variant of the IWC method is proposed in this paper. It is based on a sum of inhomogeneous waves, of the form:

$$u_{IWCV} = e^{-ik(\theta)(1+i\gamma(\theta))(x\cos(\theta)+y\sin(\theta))} + e^{ik(\theta)(1+i\gamma(\theta))(x\cos(\theta)+y\sin(\theta))}, \quad (13)$$

which accounts for both right (e^{-ik}) and left (e^{ik}) running wave components.

The classical form of the IWC method considers only one term corresponding to forward propagating (incident) wave (e^{-ik}) [equation (10)] and neglects the term corresponding to backward propagating (reflected) wave (e^{ik}) and two other terms corresponding to evanescent waves (e^{-ik} and e^k). Note that the Mc Daniel method accounts for both incident and evanescent waves, thus for the four wave types mentioned above (e^{-ik} , e^{ik} , e^k and e^h). To improve the identification allowed by the IWC method, especially for damping, and to overcome the lack of terms in its classical form, the proposed variant accounts for both forward and backward traveling waves (e^{-ik} and e^{ik}).

The correlation between the vibratory field and the proposed wave is performed through the *IWC* criterion [equations (11) and (12)] which must be maximized to optimize the wavenumber and damping identification.

3. Analytical-numerical model of an isotropic beam

In the present study, only examples of beam structures are considered. The wave direction is, hence, fixed. Identification methods need FRFs at each point location, as primary input. The primary outputs are the wavenumber and the wave attenuation. These parameters being identified, the others can be calculated: phase velocity and damping loss factor. It is important to note that the input FRFs can be either experimental or numerical. Identification process is then called either experiment-based or numerical-based, respectively.

Let us consider an example of an isotropic beam which geometrical and mechanical properties are presented in Table I. Damping is introduced into the system using a complex Young modulus given by $E = E_0(1 + i\eta)$.

$b(\text{m})$	$h(\text{m})$	ν	$E_0(\text{GPa})$	$\rho(\text{kg.m}^{-3})$	$F_0(\text{N})$
0.029	0.015	0.29	210	7,800	10

Table I.
Geometrical and mechanical properties of the isotropic beam

The analysis is performed over a large frequency range. Two boundary condition configurations are considered: clamped-free beam and freely suspended beam. Two loading configurations are considered in this study: membrane and flexural loading. The harmonic displacement field $U(x, t)$ is computed, in each measurement point of the beam, using an analytical-numerical model. Identification is then performed using the aforementioned methods, in both deterministic and stochastic cases.

3.1 Deterministic identification

3.1.1 Membrane loading. If a membrane loading is considered, the equation of motion, also called the wave propagation equation, is expressed as:

$$-ES \frac{\partial^2 U(x, t)}{\partial x^2} + \rho S \frac{\partial^2 U(x, t)}{\partial t^2} = F(x, t), \quad (14)$$

where $F = F_0 e^{i\omega t}$. $U(x, t)$ refers to beam membrane deformation where x is the longitudinal direction. E is the Young modulus, ρ the density and S the cross section.

The solution of equation (14) takes the form:

$$U(x, t) = u(x) e^{i\omega t}, \quad (15)$$

where ω is the angular frequency and $u(x)$ is expressed as:

$$u(x) = A_1 e^{-ikx} + A_2 e^{ikx}. \quad (16)$$

The longitudinal wave equation (16) is a sum of traveling (incident) wave moving in the $-x$ sense and deflected wave moving in the $+x$ sense. A_1 represents thus the incident wave coefficient, whereas A_2 stands for the reflected wave coefficient. These unknown constants can be evaluated in terms of boundary conditions.

This longitudinal wave case implies defining analytically the wavenumber as:

$$k(\omega) = \frac{\omega}{c_\varphi}, \quad (17)$$

where $c_\varphi = \sqrt{\frac{E}{\rho}}$ is the wave (phase or propagation) velocity.

The wave attenuation γ , representing the spatial damping, is related to the structural damping loss factor η (Lyon and DeJong, 1995) by:

$$\eta = \frac{2c_g}{c_\varphi} \gamma, \quad (18)$$

where c_g is the group velocity defined as $c_g = \frac{\partial \omega}{\partial k}$. Consequently, $c_g = \frac{\omega}{k} = c_\varphi$ and $\eta = 2\gamma$.

Computing the constants A_1 and A_2 according to the clamped-free beam configuration (clamped boundary: $U(x=0, t) = 0$; free boundary: $ES \frac{\partial U(x=L, t)}{\partial x} = -F_0 e^{i\omega t}$) leads to the following expression of the harmonic displacement field:

$$u(x_j) = \frac{F_0}{iESk_l(e^{-ikL} + e^{ikL})} e^{-ikx_j} - \frac{F_0}{iESk_l(e^{-ikL} + e^{ikL})} e^{ikx_j}. \quad (19)$$

For a freely suspended beam configuration ($ES \frac{\partial U(x=0, t)}{\partial x} = 0$ and $ES \frac{\partial U(x=L, t)}{\partial x} = F_0 e^{i\omega t}$), the harmonic displacement field is expressed as:

$$u(x_j) = -\frac{F_0}{iESk_l(e^{-ikL} - e^{ikL})}e^{-ikx_j} - \frac{F_0}{iESk_l(e^{-ikL} - e^{ikL})}e^{ikx_j}. \quad (20)$$

The variant of the IWC method is applied to identify the wavenumber and the wave attenuation, compared to the analytical model, the Mc Daniel method and the classical IWC method. Then, equations (17) and (18) permit to identify the damping loss factor and the phase velocity.

The variation of the identified wavenumber and phase velocity according to frequency is illustrated by Figure 1. Identification is performed using the analytical model, the Mc Daniel method, the IWC method and the IWC-variant, for the clamped-free and freely suspended boundary conditions' configurations.

Good agreement is shown between the estimates of the IWC-variant, the analytical model and the Mc Daniel method. At low frequencies, the IWC-variant provides an alternative to the classical IWC method which estimates propagation parameters accurately only at mid and high frequencies. Varying boundary conditions does not affect the efficiency of the IWC variant. More detailed illustration is shown in Table II which lists phase velocities computed at some frequencies.

The group velocity, which determines the velocity of propagation of the energy transported by the wave, is then constant: $c_g = \frac{\partial \omega}{\partial k} = c_\varphi = \sqrt{E/\rho} = 5.1887 \cdot 10^3$. Hence, energy moves with the same velocity, independently of frequency: non-dispersive wave propagation occurs.

Figure 2 shows the variation of the wave attenuation and the damping loss factor according to frequency, for both clamped-free and freely suspended boundary conditions' configurations.

High oscillations affect the estimates of the classical IWC method, especially at low frequencies, around analytical and Mc Daniel estimates. This divergence results in an error exceeding 100 per cent, on the damping loss factor, as shown in Figure 4. The convergence of the method is faster at high frequencies. On the other hand, fluctuations are attenuated and more accurate results are obtained when applying the IWC-variant.

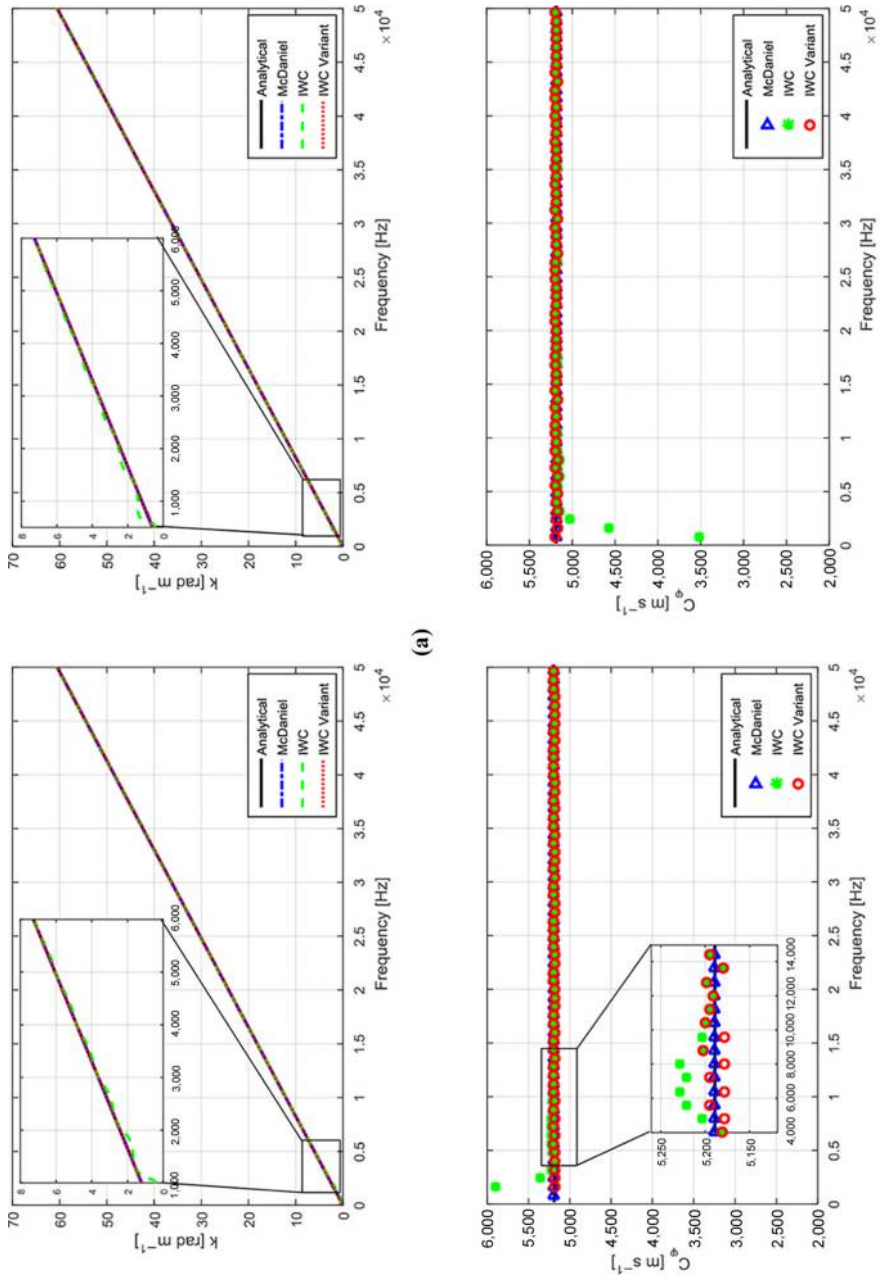
Errors on identified parameters are illustrated by Figures 3 and 4. The IWC-variant is at first compared to the Mc Daniel method, considering the analytical model as reference, and then to the classical IWC method, the Mc Daniel method being the reference.

As shown in Figure 4, the errors of the Mc Daniel method are very less. Its efficiency is thus verified, which allows considering its estimates as reference. The errors of the IWC-variant estimates oscillate around the Mc Daniel errors. Oscillations are small and strongly attenuated compared to those affecting the estimates of the classical IWC method. Varying boundary conditions does not affect the accuracy of the IWC-variant estimates.

As mentioned in theoretical part, more accurate identification is obtained for maximal *IWC* criterion. We propose thus to calculate an objective function, which consists on computing the maximal *IWC* criterion for each frequency step. The objective function of the classical IWC method is compared to the objective function of the IWC-variant (Figure 5). The maximal *IWC* criterion of the latter is greater than the maximal *IWC* criterion of the former and tends to the theoretical limit which is 1. Greatest difference between objective functions is obtained at the lowest frequencies. Difference is inversely proportional to frequency. These comparisons confirm that the IWC-variant is an interesting alternative to the classical IWC method, at low frequencies in particular.

In membrane loading case, the efficiency of the proposed IWC-variant is proved, compared to the classical IWC method, with respect to the Mc Daniel method and the

Figure 1. Variation of the wavenumber and the phase velocity of (a) clamped-free beam and (b) freely suspended beam, according to frequency, identified using the analytical model, the Mc Daniel method, the IWC method and the IWC-variant



analytical model, considered both as reference. This efficiency is independent of applied boundary conditions.

3.1.2 Flexural loading. If a flexural loading is considered, the wave propagation equation is expressed as:

$$EI \frac{\partial^4 U(x, t)}{\partial x^4} + \rho S \frac{\partial^2 U(x, t)}{\partial t^2} = F(x, t), \quad (21)$$

where $F = F_0 e^{i\omega t}$. $I = \frac{bh^3}{12}$ is the second moment of area and S the transverse cross section. Equation (21) is based on the Euler–Bernoulli beam theory which is based on the hypothesis: the shear deformations of the cross section and the rotational inertia effect are neglected.

The solution of equation (21) takes the form:

$$U(x, t) = u(x) e^{i\omega t}. \quad (22)$$

The deflection $u(x)$ of the beam is expressed as:

$$u(x) = A_1 e^{ikx} + A_2 e^{-ikx} + A_3 e^{-kx} + A_4 e^{kx}, \quad (23)$$

where A_1 is the coefficient of the wave propagating to the left, A_2 is the coefficient of the wave propagating to the right, A_3 is the coefficient of the evanescent wave decaying to the right and A_4 is the coefficient of the evanescent wave decaying to the left.

In flexural wave case, the analytical wavenumber is:

$$k = \sqrt[4]{\frac{\rho S \omega^2}{EI}}. \quad (24)$$

Here, the damping loss factor is expressed as Lyon and DeJong (1995):

$$\eta = \frac{2c_g}{c_\varphi} \gamma, \quad (25)$$

where c_g is the group velocity defined as $c_g = \frac{\partial \omega}{\partial k}$. Consequently, $c_g = 2 \frac{\omega}{k} = 2 \sqrt[4]{\frac{EI \omega^2}{\rho S}}$ and $\eta = 4 \gamma$.

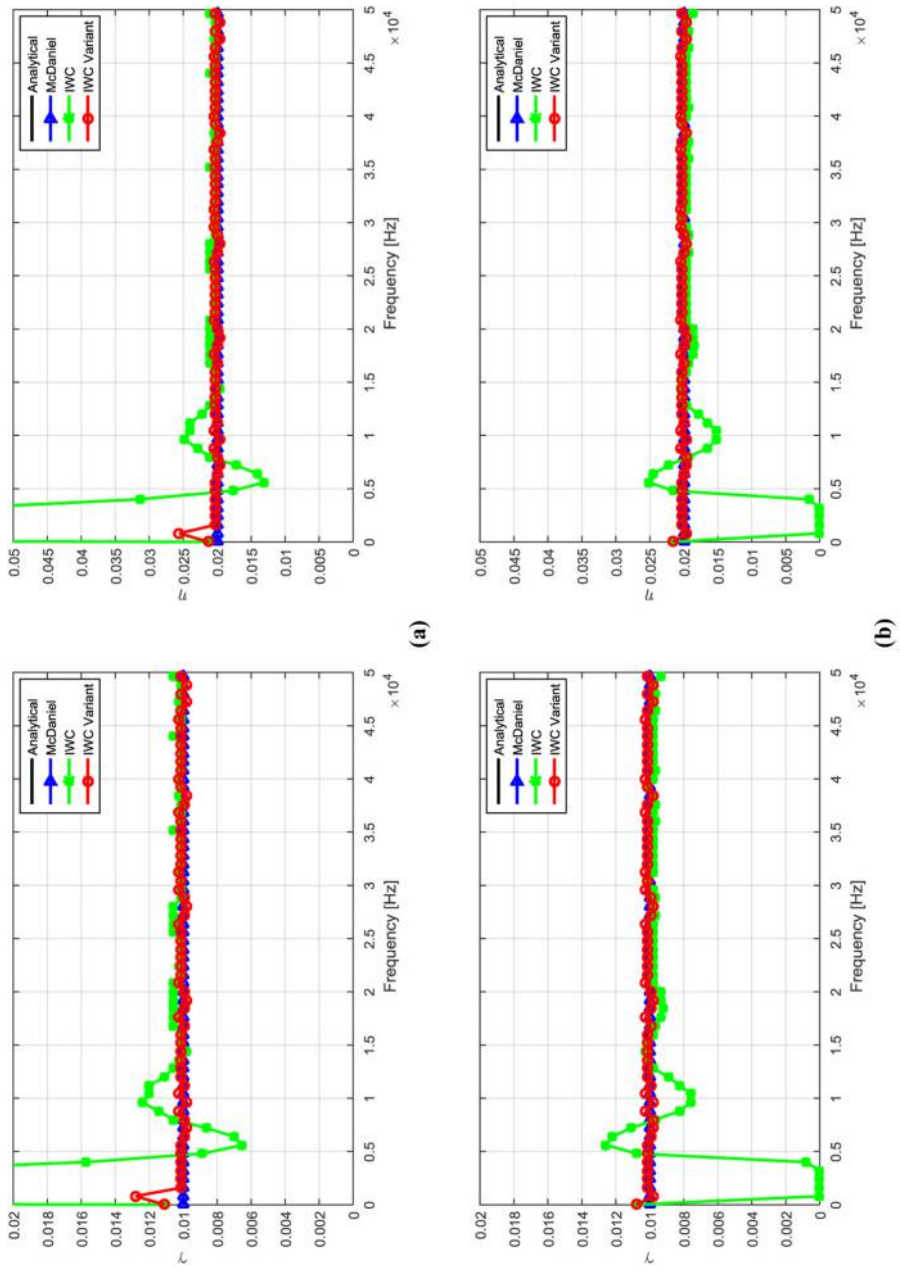
Similar to membrane loading case, two boundary conditions' configurations are considered here: clamped-free beam and freely suspended beam.

According to the clamped-free beam configuration [clamped boundary: $(x = 0, t) = 0$ and $\frac{\partial U(x=0, t)}{\partial x} = 0$; free boundary: $EI \frac{\partial^2 U(x=L, t)}{\partial x^2} = 0$ and $\frac{\partial^3 U(x=L, t)}{\partial x^3} = \frac{F}{EI}$], the constants $A_1 - A_4$ are solutions of the system $[D]\{A_j\} = \{f\}$, also expressed as:

Method	Frequencies (Hz) $\times 10^4$							
	0.16	0.24	0.56	0.8	1.6	2.88	3.6	4.4
Analytical				5.189				
Mc Daniel				5.190				
IWC	5.893	5.357	5.221	5.229	5.187	5.185	5.191	5.176
IWC-variant	5.187	5.189	5.195	5.178	5.187	5.195	5.191	5.202

Table II.
Phase velocity computed using the analytical model, the Mc Daniel method, the IWC method and the IWC-variant at some frequencies, for the clamped-free beam configuration

Figure 2. Variation of the wave attenuation and the damping loss factor of (a) clamped-free beam and (b) freely suspended beam, according to frequency, identified using the analytical model, the Mc Daniel method, the IWC method and the IWC-variant



$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ i & -i & -1 & 1 \\ -e^{ikL} & -e^{-ikL} & e^{-kL} & e^{kL} \\ -ie^{ikL} & ie^{-ikL} & e^{-kL} & e^{kL} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \frac{F_0}{EI k^3} \end{Bmatrix}. \quad (26)$$

The harmonic displacement field is consequently of the form:

$$\{u(x_j)\} = \{e^{ikx_j} \quad e^{-ikx_j} \quad e^{-kx_j} \quad e^{kx_j}\} \{A_j\}. \quad (27)$$

When freely suspended beam configuration is considered ($EI \frac{\partial^2 U(x=0,t)}{\partial x^2} = 0$, $\frac{\partial^3 U(x=0,t)}{\partial x^3} = 0$, $EI \frac{\partial^2 U(x=L,t)}{\partial x^2} = 0$ and $\frac{\partial^3 U(x=L,t)}{\partial x^3} = \frac{F}{EI}$), equation (26) takes the form:

$$\begin{bmatrix} -1 & -1 & 1 & 1 \\ -i & i & -1 & 1 \\ -e^{ikL} & -e^{-ikL} & e^{-kL} & e^{kL} \\ -ie^{ikL} & ie^{-ikL} & -e^{-kL} & e^{kL} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \frac{F_0}{EI k^3} \end{Bmatrix}. \quad (28)$$

Figure 6 illustrates the variation of the wavenumber and the phase velocity according to frequency computed using the analytical model, the Mc Daniel method, the classical IWC method and the IWC-variant, for the two boundary conditions' configurations.

In presence of membrane loading and flexural loading, very good agreement is found between the estimates of the IWC-variant, the analytical model and the Mc Daniel method. The shapes of the curves of the identified wavenumber and phase velocity agree very well. Less accurate results are obtained for the identified wave attenuation and damping loss factor, as shown in Figure 7. The IWC-variant results oscillate around those of the analytical model and the Mc Daniel method. These oscillations are strongly attenuated compared with the oscillations of the results of the classical IWC method. This is also shown through the comparison of the errors of the IWC-variant estimates and those resulting from the classical IWC method, with respect to the Mc Daniel estimates, as shown in Figure 8. Small errors are, in fact,

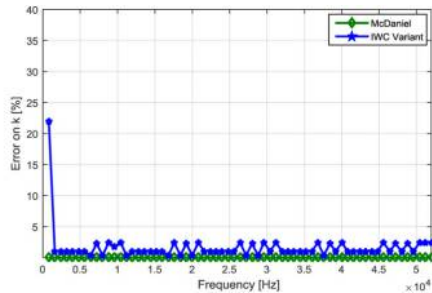
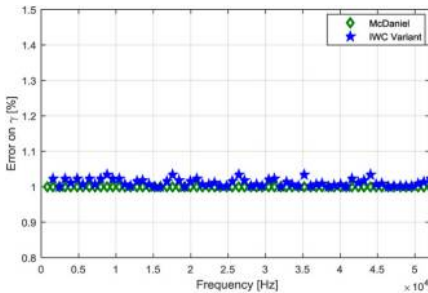
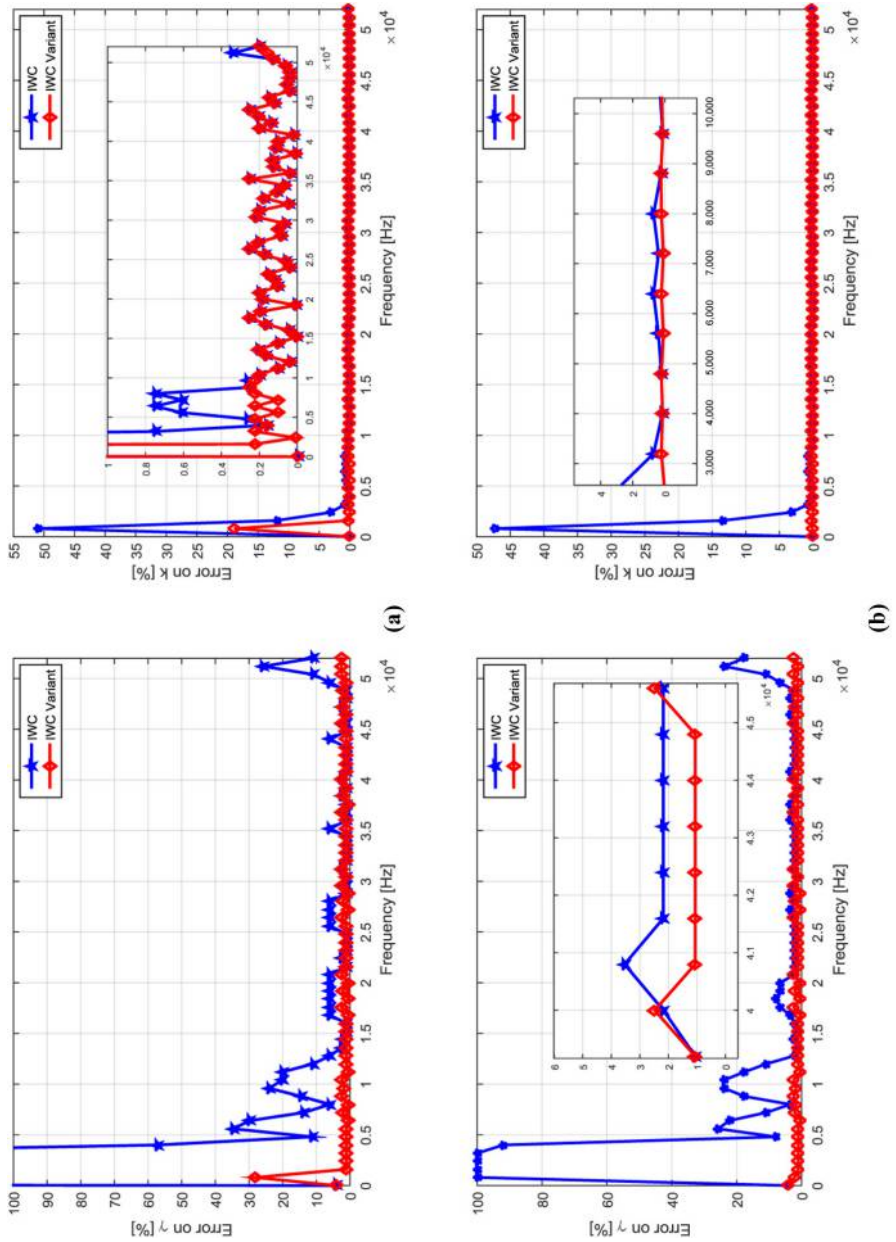
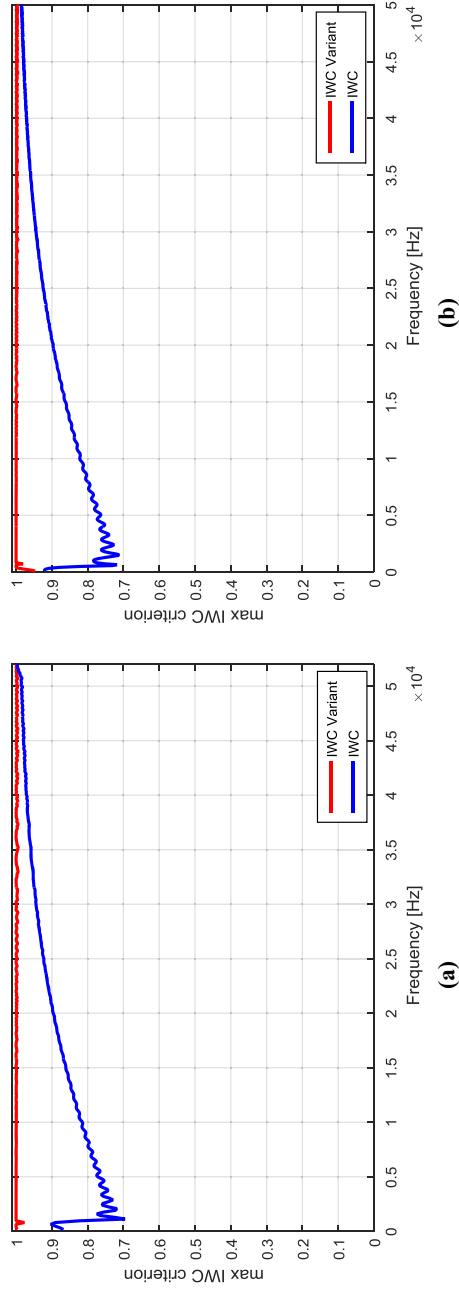


Figure 3.

Errors on the wave attenuation and the wavenumber identified using the Mc Daniel method and the IWC-variant, with respect to those identified analytically, for a clamped-free beam configuration

Figure 4. Errors on the wave attenuation and the wavenumber identified using the classical IWC method and the IWC-variant, with respect to those identified using the Mc Daniel method, for (a) clamped-free beam and (b) freely suspended beam configurations





Notes: (a) Clamped-free beam; (b) freely suspended beam

Figure 5.
Objective function variation according to frequency, associated to the IWC method and the IWC-variant, for configurations

obtained on damping loss factor using the IWC-variant. Regarding the wavenumber, the IWC-variant estimates are less accurate, especially at low frequencies, for both clamped-free and freely suspended boundary conditions' configurations.

In flexural loading case, dynamic flexural stiffness can also be identified. It is defined by $D = \frac{\sigma \omega^2}{k^4}$, where $\sigma = \frac{M}{S}$ is the mass density and $M = \rho Sh$ is the global mass of the structure. The identified dynamic flexural stiffness is illustrated in Figure 9 which compares the results of the different applied methods. The approximations of the IWC-variant agree very well with those of the analytical model and the Mc Daniel method. Small errors are detected at low frequencies.

The comparisons of the objective functions computed for the IWC-variant and the classical IWC method are illustrated in Figure 10. The maximal values of the *IWC* criterion achieved using the IWC-variant are always greater than those obtained by the classical IWC method, especially at low frequencies. The accuracy of the classical IWC method depends on the frequency range: it increases proportionally to frequency. However, nearly constant maximal values of the *IWC* criterion are kept, owing to the IWC-variant, along considered frequency range.

To recapitulate, the analytical model was considered as reference for this example. This permits to confirm the efficiency of the Mc Daniel method, already proved in the literature. The Mc Daniel method is hence considered as reference thereafter.

The efficiency of the proposed IWC-variant to identify accurately propagation parameters is proved. It neither depends on boundary conditions nor on loading type. The IWC-variant represents an interesting alternative to the classical IWC method, especially at low frequencies where the latter reaches its limits.

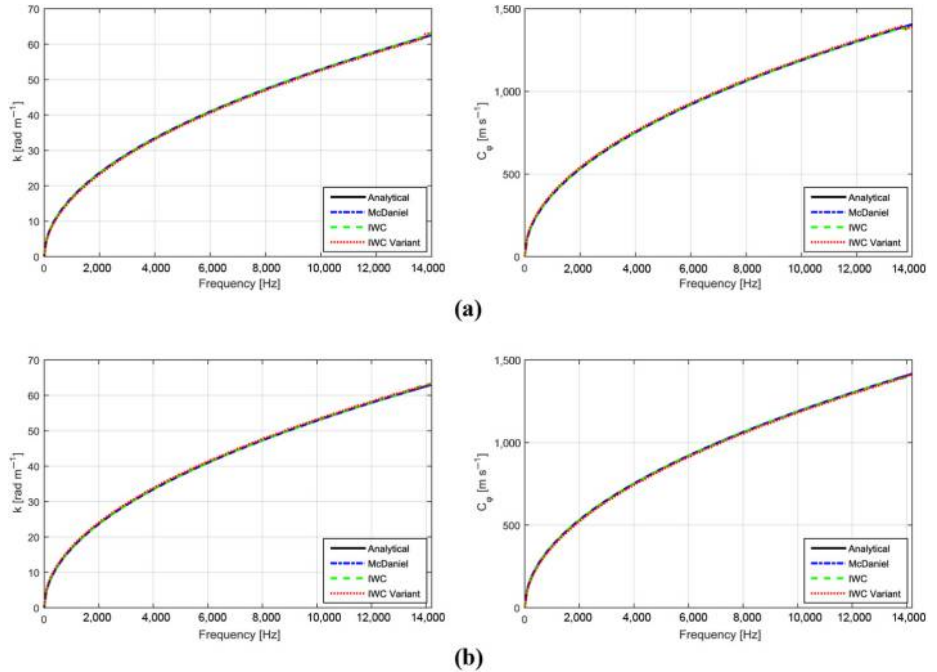


Figure 6. Variation of the wavenumber and the phase velocity of (a) clamped-free beam and (b) freely suspended beam, according to frequency, identified using the analytical model, the Mc Daniel method, the IWC method and the IWC-variant

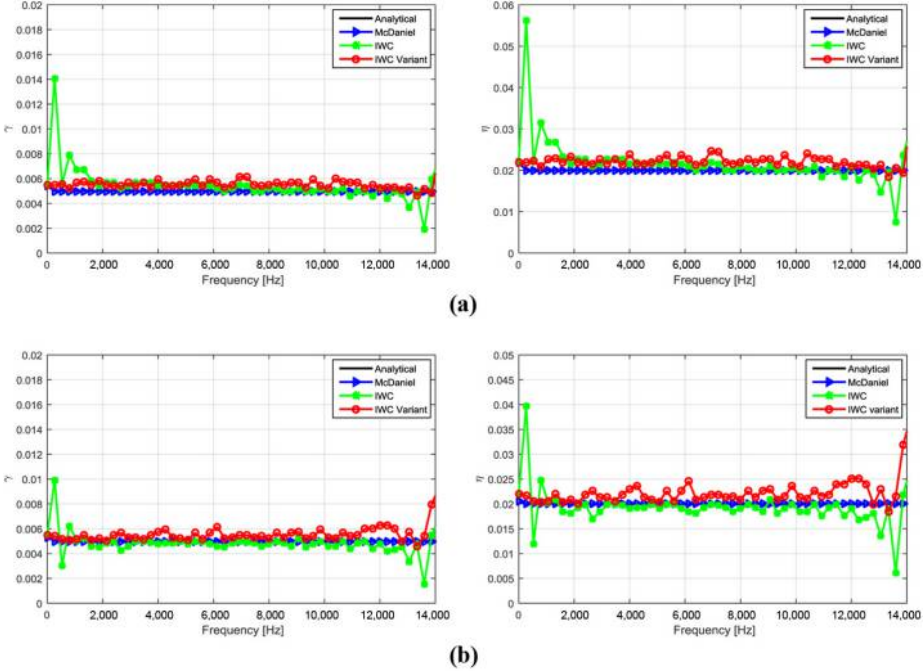


Figure 7. Variation of the wave attenuation and the damping loss factor of (a) clamped-free beam and (b) freely suspended beam, according to frequency, identified using the analytical model, the Mc Daniel method, the IWC method and the IWC-variant

To achieve more realistic structural identification, accounting for uncertainties is inevitable. The efficiency of the proposed IWC-variant being verified in deterministic case, its evaluation in presence of uncertainties is the purpose of the following section.

3.2 Identification in presence of uncertainties

Several types of uncertainties could be considered in this study. Only parametric uncertainties are considered here. Measurement points' coordinates are supposed to be uncertain. Actually, if some measurement points do not match the associated displacement fields, how would this affect the identified parameters?

The variability of the measurement points' coordinates is modeled, in a probabilistic framework, by random variables. The measurement points' vector is thus expressed as:

$$x_s = x(1 + \delta_x \xi), \quad (29)$$

where x is the vector containing the mean coordinates of the measurement points, δ_x is the dispersion value and ξ is a Gaussian random variable.

The effects of the randomness of the measurement points' coordinates on the identified parameters, which vary also randomly, are investigated through stochastic uncertainty propagation methods. In the present paper, the LHS method is used. In total, 1,000 samples of random variable are considered. Thus, 1,000 successive deterministic simulations are generated.

The combination of the LHS method with either Mc Daniel method or IWC-variant leads to two stochastic identification processes, the one including the Mc Daniel method being considered as reference.

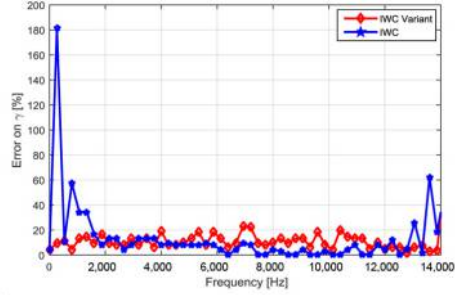
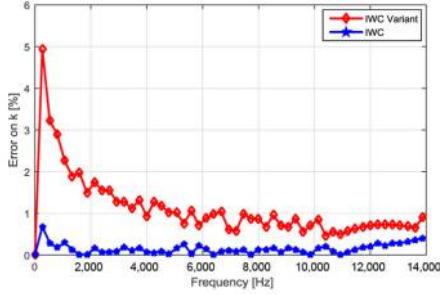


Figure 8.

Errors on the wavenumber and the wave attenuation identified using the classical IWC method and the IWC-variant, with respect to those computed using the Mc Daniel method, for (a) clamped-free beam and (b) freely suspended beam configurations

(a)

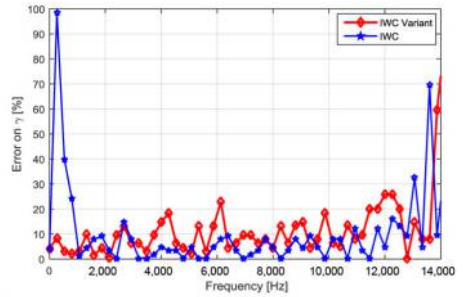
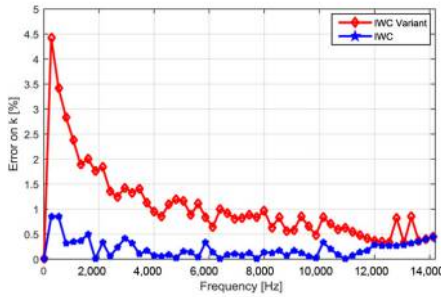
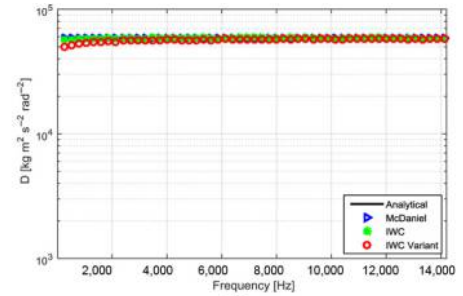
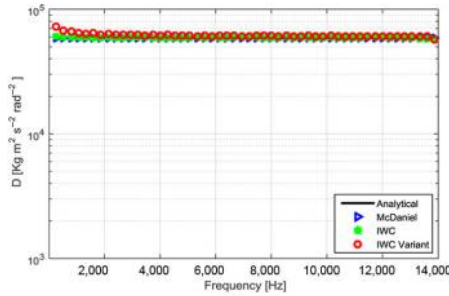


Figure 9.

Variation of the dynamic stiffness of (a) clamped-free beam and (b) freely suspended beam, according to frequency, identified using the analytical model, the Mc Daniel method, the IWC method and the IWC-variant



(a)

(b)

To quantify the variability of the identified parameters, several statistical post-processing evaluations are performed: means, envelopes, standard deviations and dispersions. An envelope refers here to extreme statistics, and dispersion is computed by the ratio between standard deviation and mean.

Note that only clamped-free boundary conditions' configuration is considered here. Flexural and membrane loading configurations are studied.

3.2.1 Flexural loading. Statistical evaluations quantifying the variability of the identified wavenumber and damping loss factor are illustrated in Figures 11 and 12, respectively.

Very good agreement is obtained between results of stochastic identification processes including either the Mc Daniel method or the IWC-variant. The efficiency of the proposed IWC-variant is not affected by uncertainties: the method is subsequently robust against uncertainties.

Regarding the variability of the wavenumber [Figure 11(b)], the envelope is larger when frequency increases. This is also illustrated by the increasing standard deviation in Figure 11(c). As the mean and the standard deviation increase similarly, a nearly constant dispersion is obtained [Figure 11(d)]. Note that a given dispersion value, 5 per cent, on measurement points' coordinates leads to a dispersion of nearly 5 per cent on the identified wavenumber.

Contrary to the wavenumber, no variability is found for the damping loss factor (Figure 12). This can be deduced from perfect superposition of the mean, maximal and minimal curves and the null standard deviation ($<10^{-16}$) and dispersion ($<10^{-14}$). The damping loss factor is, hence, not affected by uncertainties.

The results of the IWC-variant are compared to those obtained using the Mc Daniel method through errors' computation. Indeed, Figure 13 shows that the errors obtained on the means of the wavenumber and the damping loss factor, computed using the stochastic identification process including the IWC-variant are around those obtained in deterministic case, with respect to the results of the stochastic identification process including the Mc Daniel method. This comparison illustrates the efficiency of the proposed IWC-variant in both deterministic and stochastic case. The method allows robust identification against uncertainties.

3.2.2 Membrane loading. In this section, clamped-free boundary conditions' configuration and membrane loading are considered. Statistical evaluations of the wavenumber and the damping loss factor are illustrated in Figures 14 and 15, respectively.

Very good agreement is obtained between the estimates of the stochastic identification process using the Mc Daniel method and that including the proposed IWC-variant. The envelope width and the standard deviation of the wavenumber [Figure 14(b)-(c)] increase with frequency. When measurement points' coordinates are dispersed by 5 per cent, a nearly constant dispersion of about 5 per cent is obtained on the wavenumber [Figure 14(d)].

As obtained in flexural loading case, the damping is not affected by uncertainties (Figure 15). The mean, maximal and minimal curves are perfectly superposed, and null standard deviation and dispersion values are obtained.

As shown in Figure 16, the errors computed on the means of the wavenumber and the damping loss factor computed using the identification process including the IWC-variant,

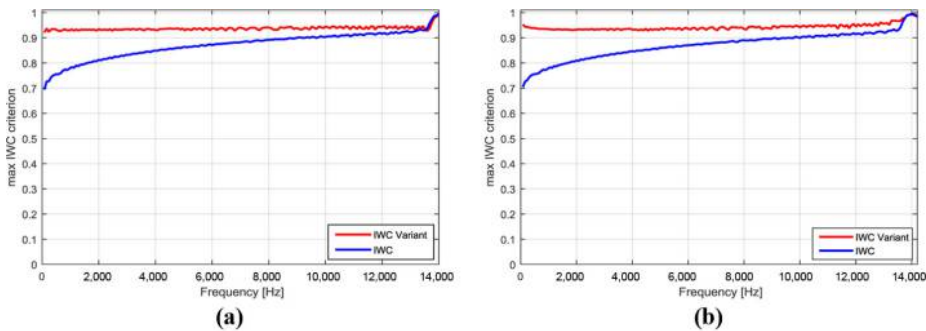


Figure 10. Objective function variation according to frequency, associated to the IWC method and the IWC-variant, for configurations

Notes: (a) Clamped-free beam; (b) freely suspended beam

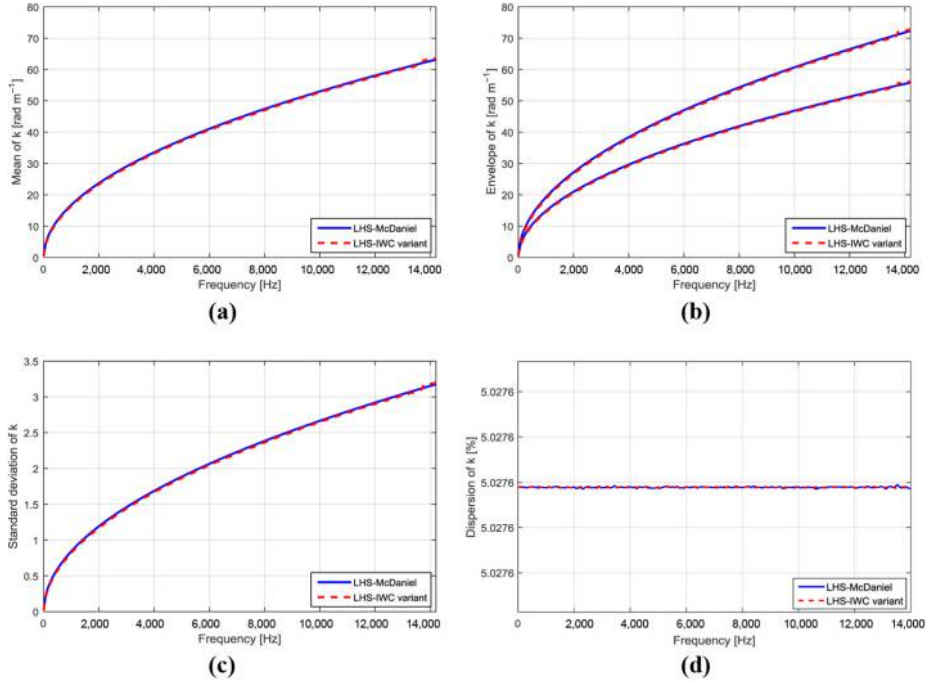
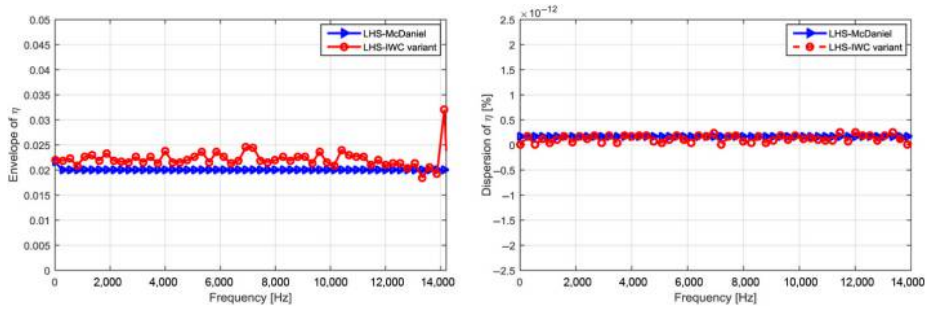


Figure 11.
(a) Mean,
(b) envelope, (c)
standard deviation
and (d) dispersion of
the wavenumber
computed using the
identification process
combining the LHS
method with the Mc
Daniel method and
the IWC variant for
 $\delta_x = 5\%$, in flexural
loading case

Figure 12.
Envelope and
dispersion of the
damping loss factor
computed using the
identification process
combining the LHS
method with the Mc
Daniel method and
the IWC variant for
 $\delta_x = 5\%$, in flexural
loading case



with respect to the errors resulting from the process based on the Mc Daniel method, are around those obtained in deterministic case. Subsequently, as deduced in the flexural loading case, the proposed IWC-variant is robust against uncertainties.

4. Composite beam model

Given the growing emphasis on industrial integration of composite materials in engineering applications, the proposed deterministic and stochastic identification processes are applied here to identify propagation parameters of a sandwich

composite beam with aluminum honeycomb core (Plate 1). The width of the beam is 0.029 m . The thickness of the core plate is 0.011 m . The face plates are made of 6-ply carbon-fiber composite oriented according to the directions $[45\ -45\ 45\ 45\ -45\ 45]$, respectively. Materials' properties of all components are listed in Table III. The beam is freely suspended and a flexural loading is applied.

To verify the efficiency of the proposed IWC-variant in estimating the propagation parameters of the sandwich beam structure, two types of identification processes are applied. The process using FRFs which are computed numerically is called numerical-based identification process. If FRFs are measured experimentally, the identification process is called experiment-based.

4.1 Numerical-based identification

The identification processes, applied in this section, are numerical based. The FRFs, computed numerically at each measurement point, are used as inputs. As was done in the isotropic beam case, the efficiency of the proposed IWC-variant is verified, first, in deterministic case. Stochastic properties and effects are then investigated, and the robustness of the IWC-variant identification against uncertainties is then evaluated.

4.1.1 Deterministic numerical-based identification. Figure 17 illustrates the displacement field of the sandwich beam computed numerically at several measurement points.

Following the identification of the wavenumber and the wave attenuation, the other propagation parameters can then be calculated, namely, the phase velocity and the damping loss factor. The comparisons of the IWC-variant estimates of the aforementioned parameters with those identified using the Mc Daniel method and the classical IWC method are illustrated in Figures 18 and 19.

As illustrated in Figures 18 and 19, small oscillations affect the curves of the IWC-variant estimates compared to those obtained by the classical IWC method. The curves follow the same trend and have nearly the same order of magnitude throughout the whole frequency band. Comparing these figures, it can be deduced that the wavenumber and phase velocity estimates are more accurate than those of the wave attenuation and the damping loss factor.

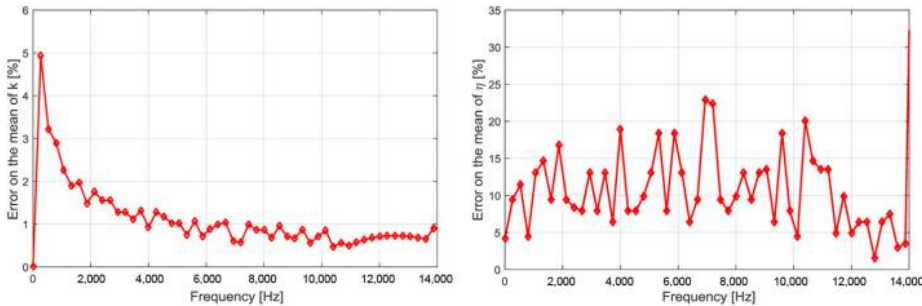


Figure 13. Errors on the means of the wavenumber and the damping loss factor computed using the identification process combining the LHS method with the IWC-variant, with respect to those computed using the identification process combining the LHS method with the Mc Daniel method for $\delta_x = 5\%$, in flexural loading case

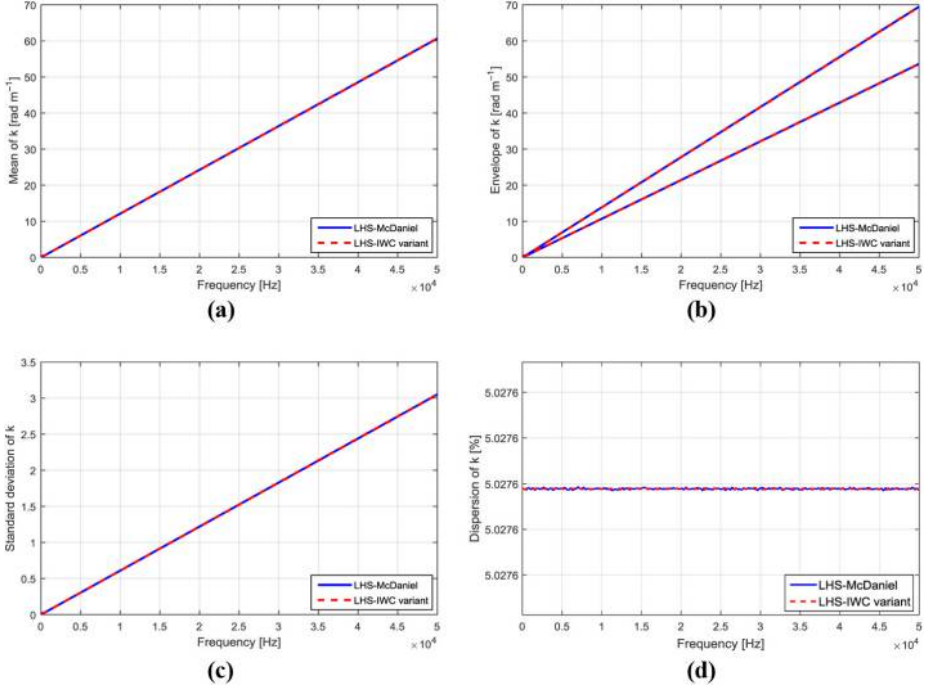


Figure 14. (a) Mean, (b) envelope, (c) standard deviation and (d) dispersion of the wavenumber computed using the identification process combining the LHS method with the Mc Daniel method and the IWC-variant for $\delta_x = 5\%$, in membrane loading case

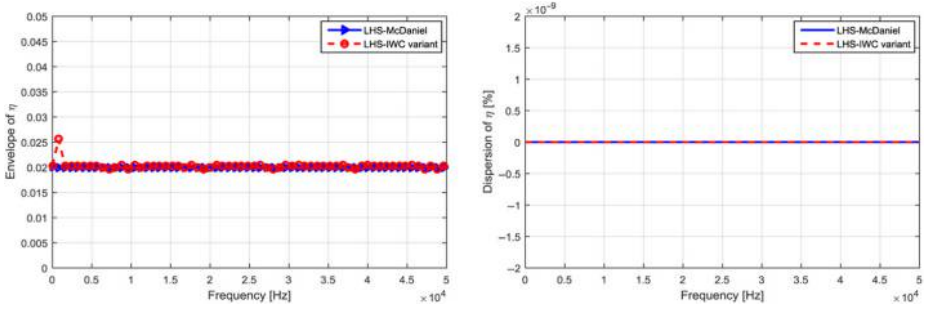


Figure 15. Envelope and dispersion of the damping loss factor computed using the identification process combining the LHS method with the Mc Daniel method and the IWC-variant for $\delta_x = 5\%$, in membrane loading case

This is also illustrated by the comparisons of the errors computed on the identified wavenumber and wave attenuation using the IWC-variant and the classical IWC method, with respect to the estimates of the Mc Daniel method (Figure 20). Errors' fluctuations are also attenuated when applying the IWC-variant, especially for higher frequencies where results converge faster.

The efficiency of the proposed IWC-variant can also be illustrated through the comparison of its objective function to that of the classical IWC method (Figure 21). The maximal *IWC* criterion values achieved by the IWC-variant are much more important than

those obtained by the classical IWC method. The difference is large at low frequencies but decreases at high frequencies. Note that the more elevated the *IWC* criterion is, the more accurate the identification is.

The variation of the identified dynamic flexural stiffness is shown in Figure 22. Oscillations are attenuated using the IWC-variant, and more accurate identification is obtained at higher frequencies.

As obtained for the isotropic beam case, the comparison of the IWC-variant estimates with those provided by the Mc Daniel method shows good agreement. The fluctuations affecting the shapes of the identified parameters' curves using the classical IWC method are

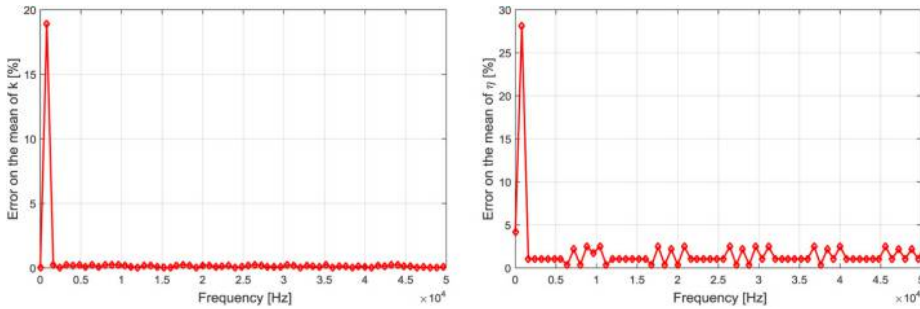


Figure 16. Errors on the means of the wavenumber and the damping loss factor computed using the identification process combining the LHS method with the IWC-variant, with respect to those computed using the identification process combining the LHS method with the Mc Daniel method for $\delta_x = 5\%$, in membrane loading case

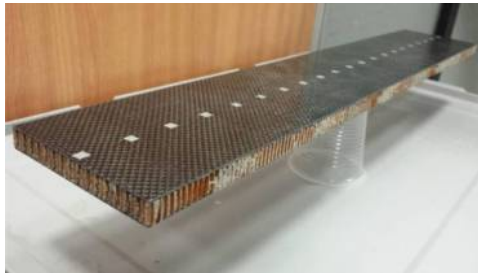


Plate 1. Honeycomb-core sandwich beam

Type	Material Reference	Young modulus (GPa)			Poisson's ratio			Shear modulus (GPa)			Density (Kg m ⁻³)
		E_1	E_2	E_3	ν_{12}	ν_{13}	ν_{23}	G_{12}	G_{13}	G_{23}	ρ
Face plates	Vicotex G803/914	60.27	60.27	5	0.029	0.35	0.35	5	5	5	1594
Core plate	5056 3.1 3/16.001	0.415	0.267	0.668	0.29	0.3	0.3	0.131	0.310	0.137	49.65

Table III. Material properties of the sandwich beam components

attenuated when the IWC-variant is used. More accurate identification and more smooth curves are obtained. The efficiency of the proposed IWC-variant identification being proved, it can thus be considered as an alternative to the classical IWC method.

4.1.2 Numerical-based identification in presence of uncertainties. Stochastic analysis allows evaluating the effects of the randomness of the measurement points' coordinates on the variability of the identified parameters of the composite beam. Statistical evaluations are, indeed, performed to quantify each parameter's variability. The identification methods are thus combined with the statistical LHS method. A dispersion level $\delta_x = 3$ per cent is considered here. Statistical investigations are, first, performed on the wavenumber and the phase velocity as shown in Figures 23 and 24, respectively.

As deduced in isotropic beam case, stochastic parameters' variability is proportional to frequency. As shown through Figures 23(b) and (c) and 25(b) and (c), most important variability corresponds to the highest frequencies. Envelopes, representing extreme statistics, are larger and standard deviations are greater. Furthermore, for a dispersion level

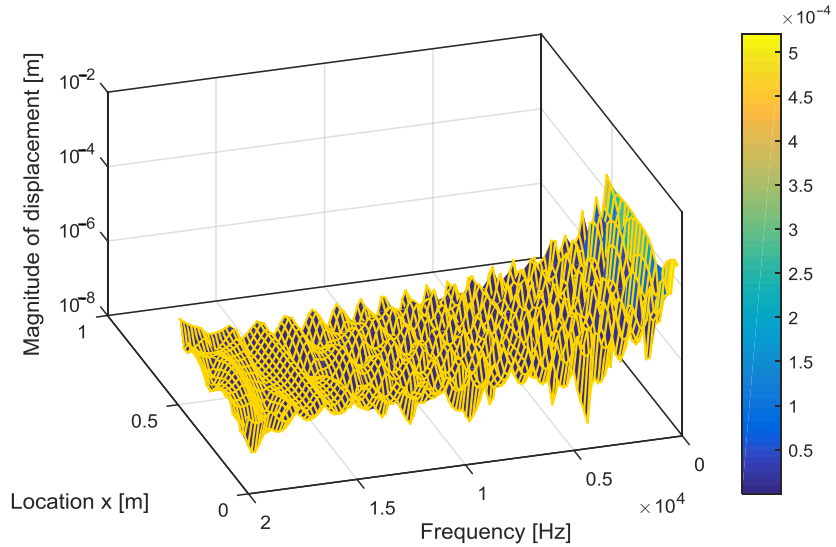
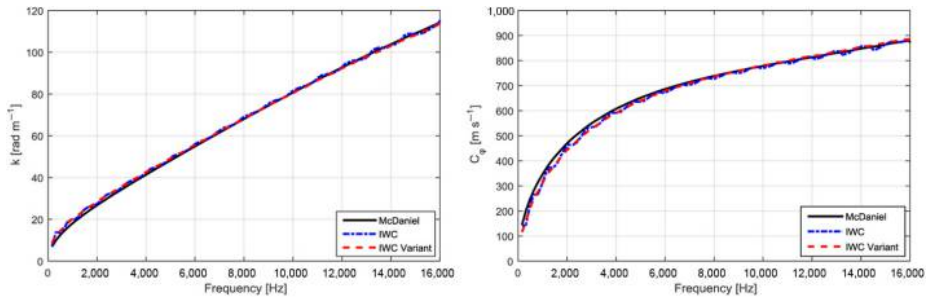


Figure 17. Variation of the sandwich beam displacement field according to variation of frequency and measurement points

Figure 18. Variation of the wavenumber and the phase velocity of the sandwich beam according to frequency, identified using the numerical-based Mc Daniel method and IWC-variant



of 3 per cent on the measurement points' coordinates, the identified wavenumber and phase velocity are dispersed by nearly 3 per cent. Comparing the IWC-variant estimates to those obtained by the Mc Daniel method, convergence is faster for the highest frequencies, as illustrated in deterministic case (Figures 23 to 26).

Effects of uncertainties on dynamic flexural stiffness are illustrated by statistical post-processing evaluations shown in Figure 27. Nearly constant variability is obtained. For a dispersion level of 3 per cent on the measurement points' coordinates, the dynamic flexural stiffness is dispersed by nearly 12 per cent. Output variability is here four times greater than that imposed on input. Hence, the dynamic flexural stiffness is the most affected by uncertainties.

Very good agreement is found between the estimates of the IWC-variant and the Mc Daniel method. Results are less accurate at low frequencies.

4.2 Experiment-based identification

Experiment-based identification processes are applied in this section. The FRFs are thus computed experimentally.

In practice, an electrodynamic shaker Bruel & Kjaer 4809 is used to excite mechanically the freely suspended beam. The vibratory response is measured using a Scanning Laser Vibrometer (Ometron VPI+), as shown in Figures 28 and Plate 2. The phase reference is obtained by a force transducer Bruel & Kjaer 8001. Both signals are sampled with a Hewlett-Packard Paragon 35654A.

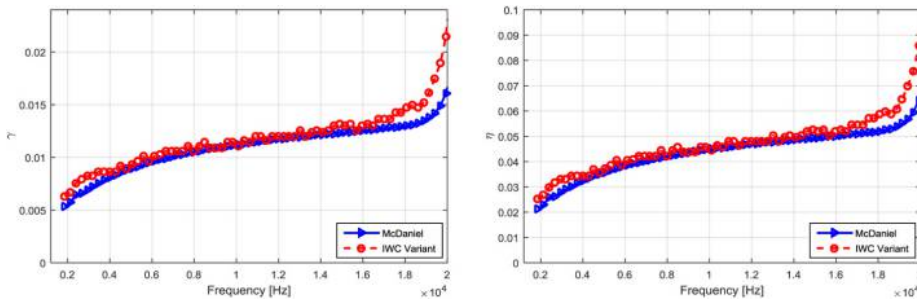


Figure 19. Variation of the wave attenuation and the damping loss factor of the sandwich beam according to frequency, identified using the numerical-based Mc Daniel method and IWC-variant

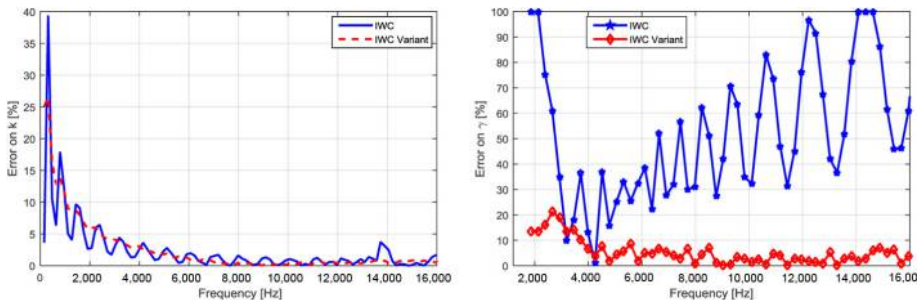


Figure 20. Errors on the wavenumber and the wave attenuation of the sandwich beam identified using the IWC method and the IWC-variant, with respect to those identified using the Mc Daniel method

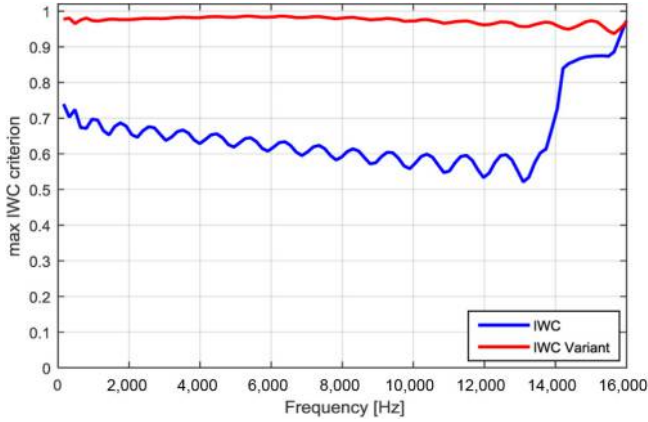


Figure 21.
Objective function variation according to frequency, associated to the IWC method and the IWC-variant

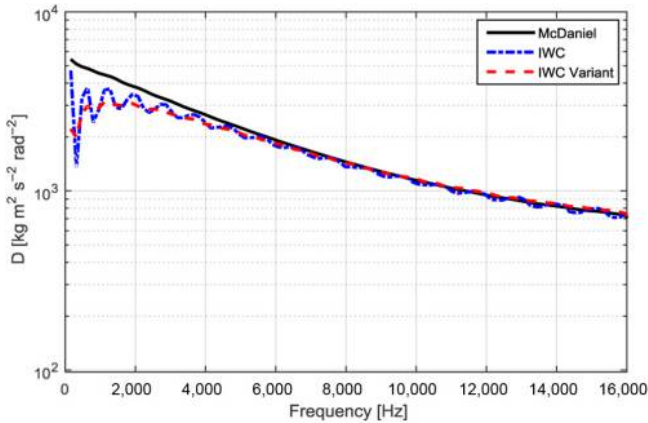


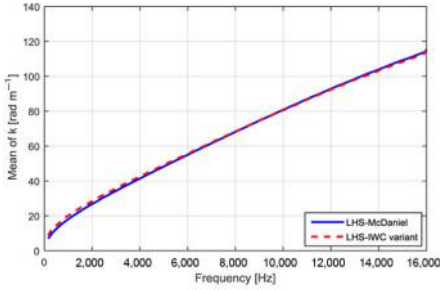
Figure 22.
Variation of the dynamic stiffness of the sandwich beam according to frequency, identified using the Mc Daniel method, the numerical-based IWC method and IWC-variant

4.2.1 Deterministic experiment-based identification. The wavenumber and the wave attenuation are here calculated based on the experimentally measured FRFs. The phase velocity and the damping loss factor are then deduced. Comparisons of the IWC-variant estimates of the aforementioned parameters and the dynamic stiffness with those identified using the Mc Daniel method and the classical IWC method are illustrated in Figures 29 and 30, respectively.

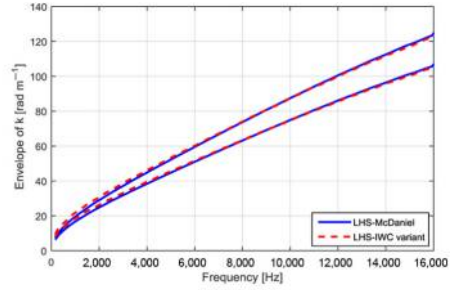
The results show the identification sensitivity to measurement disturbance and noise, especially at high frequencies, due to experimental conditions.

Regarding the variation of the wavenumber and the phase velocity, the oscillations of the classical IWC method estimates are strongly attenuated when applying the IWC variant, as shown in numerical simulations. The results of the Mc Daniel method and the IWC-variant follow the same trend throughout the whole frequency band. Inaccurate estimates are obtained by the classical IWC method, especially for wave attenuation and damping loss factor.

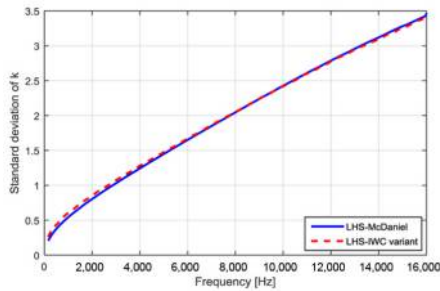
Moreover, comparing the objective functions shows that higher values of the maximal IWC criterion are achieved by the IWC-variant (Figure 31).



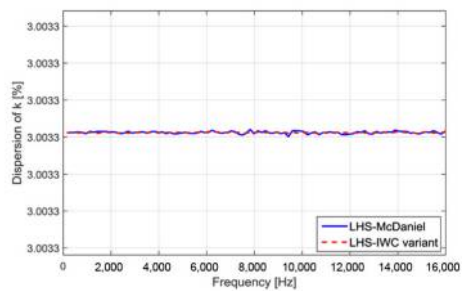
(a)



(b)

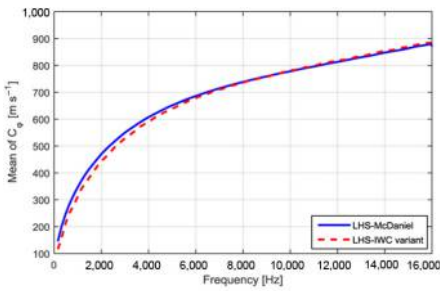


(c)

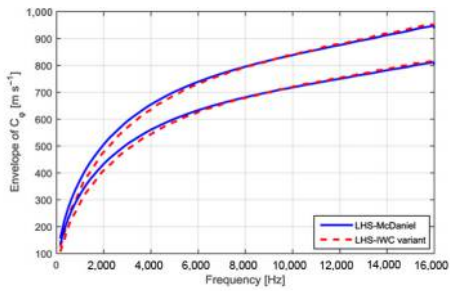


(d)

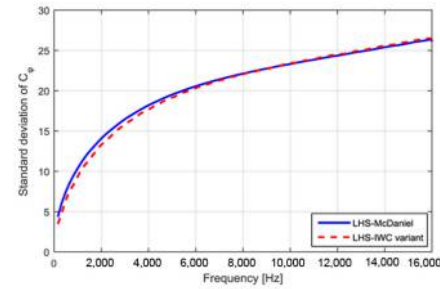
Figure 23. (a) Mean, (b) envelope, (c) standard deviation and (d) dispersion of the wavenumber of the sandwich beam computed using the identification process combining the LHS method with the McDaniel method and the IWC-variant for $\delta_x = 3\%$



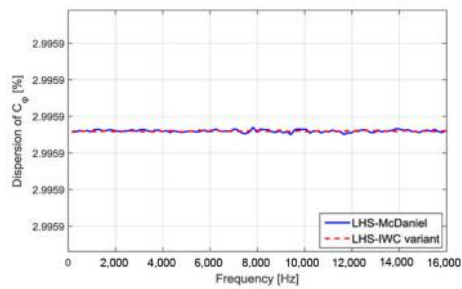
(a)



(b)



(c)



(d)

Figure 24. (a) Mean, (b) envelope, (c) standard deviation and (d) dispersion of the phase velocity of the sandwich beam computed using the identification process combining the LHS method with the McDaniel method and the IWC-variant for $\delta_x = 3\%$

The experiment-based results confirm the ability of the proposed IWC-variant to identify accurately the propagation parameters of the composite beam, which has been proven by numerical-based identification results. The comparison between the experiment-based and the numerical-based identification estimates is illustrated in Figure 32.

The estimates of the numerical-based identification process are in agreement with those of the experiment-based one. Note that fluctuations on experiment-based results are due to measurement disturbance and noise. The efficiency of the proposed IWC-variant is thus proved whatever the type of the identification process in which it is included: numerical-based or experiment-based process. Either method can then be considered as predictive tool for propagation parameters' identification of composite materials.

Regarding the computational time gain achieved by the numerical-based process, it can be considered as an alternative to the experiment-based one which leads to more expensive identification.

4.2.2 Experiment-based identification in presence of uncertainties. The study is extended in this section to presence of uncertainties. Measurement points' coordinates are supposed to vary randomly. Actually, this randomness can be due to measurement errors which can be committed by engineer and can also result from measuring tools and conditions. Hence, uncertainties on measurement points' coordinates could be one of the most influential parameters on experimental results' accuracy. The resulting randomness of the identified parameters is statistically investigated as illustrated in Figures 33-36.

Figure 25.

Envelope and dispersion of the damping loss factor of the sandwich beam computed using the identification process combining the LHS method with the Mc Daniel method and the IWC-variant for $\delta_x = 3\%$

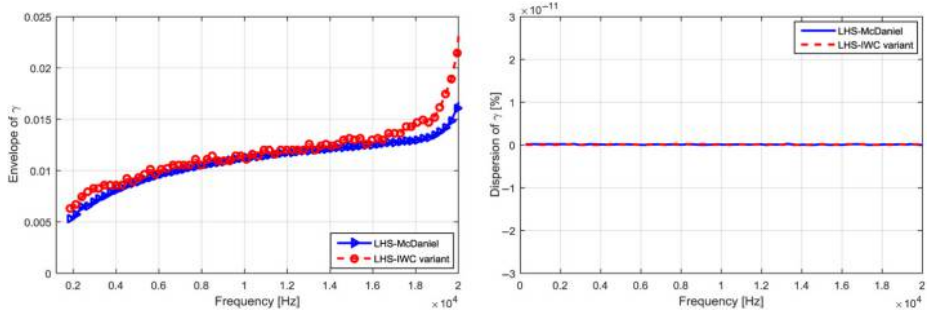
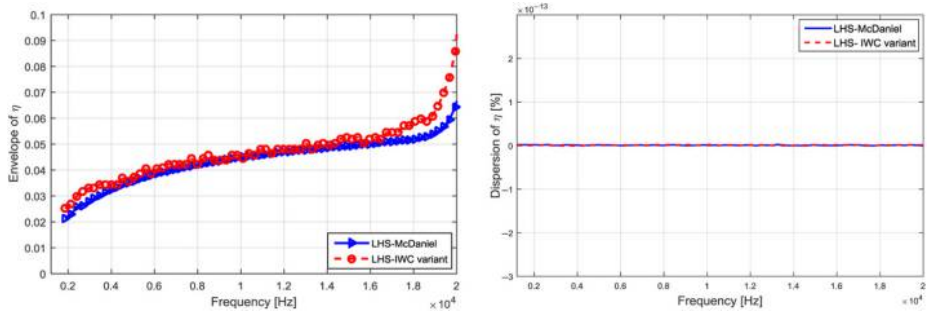


Figure 26.

Envelope and dispersion of the wave attenuation of the sandwich beam computed using the identification process combining the LHS method with the Mc Daniel method and the IWC-variant for $\delta_x = 3\%$



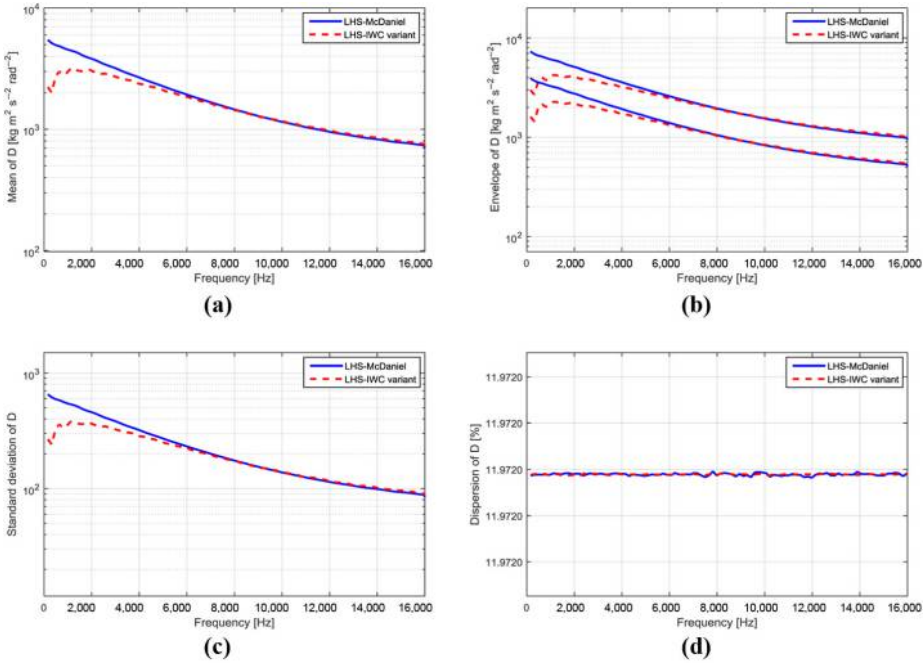


Figure 27. (a) Mean, (b) envelope, (c) standard deviation and (d) dispersion of the dynamic flexural stiffness of the sandwich beam computed using the identification process combining the LHS method with the McDaniel method and the IWC-variant for $\delta_x = 3\%$

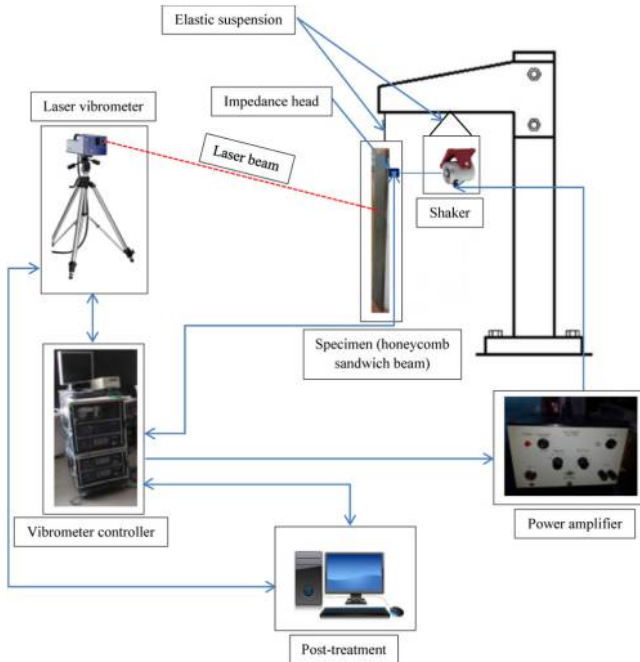


Figure 28. Experimental set-up for measuring FRFs

The IWC-variant estimates are in agreement with those of the Mc Daniel method. As shown in simulation results, some oscillations affect the IWC-variant estimates at low frequencies.

Furthermore, in agreement with simulation results, no variability is obtained on the wave attenuation and the damping loss factor (null dispersions and envelopes) (Figure 35). These parameters are thus not affected by uncertainties. Statistical quantifications of the randomness of the wavenumber, the phase velocity and the dynamic flexural stiffness are in very good agreement with those obtained with the numerical-based identification process. Indeed, comparing Figures 33, 34 and 36 with Figures 23, 24 and 27, one can deduce the conformity between dispersion values, in particular: for the aforementioned parameters, respectively, nearly 3, 3 and 12 per cent of dispersion values are obtained.

To recapitulate, the obtained results prove the efficiency of the proposed IWC-variant to identify accurately the propagation parameters, whatever the type of the identification process in which it is integrated: experiment-based or numerical-based process. The proposed IWC-variant can thus be considered as an alternative to the classical IWC method, especially at low frequencies.

Furthermore, in both deterministic and stochastic cases, the proposed method leads to accurate identification, which proves its robustness against parametric uncertainties.

Statistical investigations of the identified parameters' variability show that the randomness of the measurement points' coordinates does not affect the estimates of the wave attenuation and the damping loss factor. However, the wavenumber and the phase velocity vary randomly with dispersions nearly equal to that imposed at input. A four times greater dispersion is obtained on the dynamic flexural stiffness. One can deduce from these results that errors on measurement points' coordinates are very influential on identified parameters, except damping. Therefore, one must reduce these errors as possible and so address each point to its displacement field, to reduce the obtained variability on identified parameters.

5. Conclusion

The present paper proposed a variant of the IWC method for structural parameter identification. While the principle of the classical form of the IWC method is correlating the displacement field with an inhomogeneous wave, a projection of the displacement on a sum of inhomogeneous waves is ensured by the proposed variant of the method.

Varying boundary conditions and loadings proved the independence of the efficiency of the proposed approach of these aspects. Some limits of the classical IWC method are overcome by the proposed variant. The classical IWC method gives inaccurate identification at low frequencies, as the modal overlap is not sufficiently

Plate 2.

FRFs measurement setup of freely suspended composite honeycomb sandwich beam with electrodynamic shaker and scanning laser vibrometer



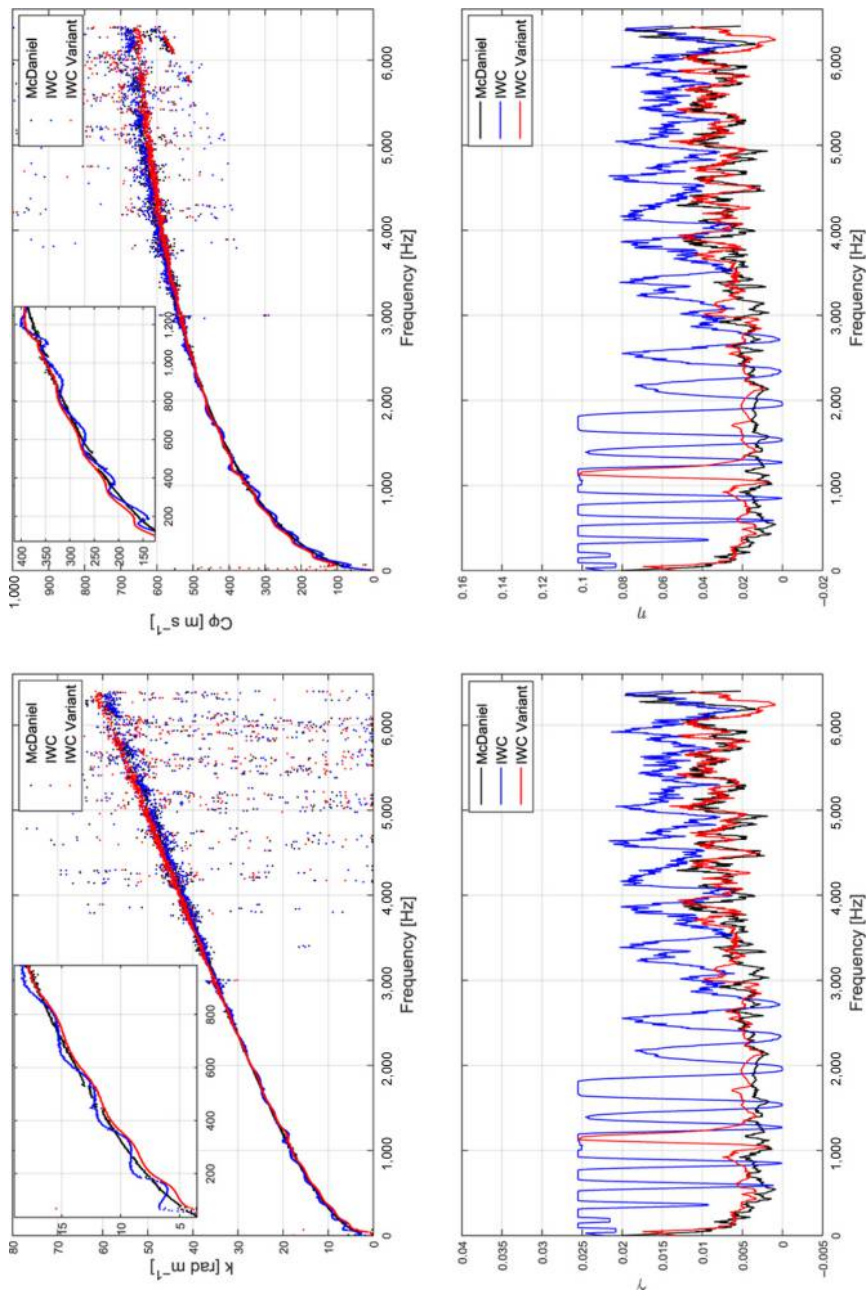


Figure 29. Variation of the wavenumber, the phase velocity, the wave attenuation and the damping loss factor of the sandwich beam according to frequency, identified using the experiment-based Mc Daniel method and IWC-variant

Figure 30. Variation of the dynamic stiffness of the sandwich beam according to frequency, identified using the experiment-based Mc Daniel method, IWC method and IWC-variant

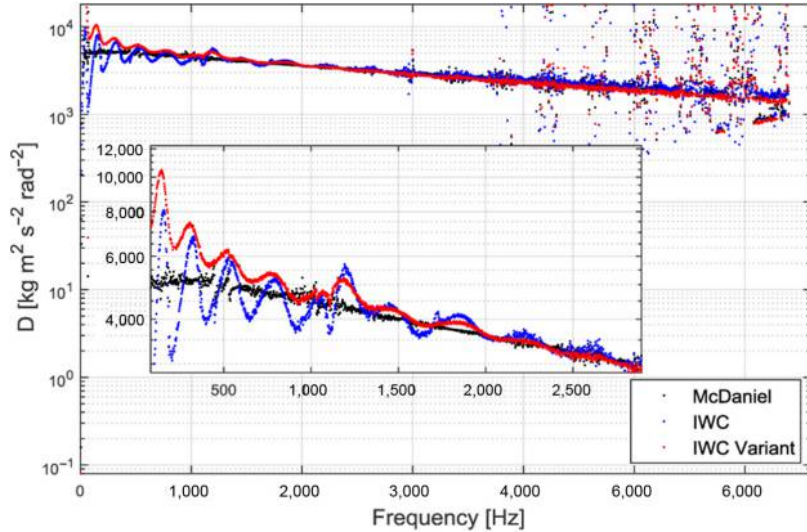
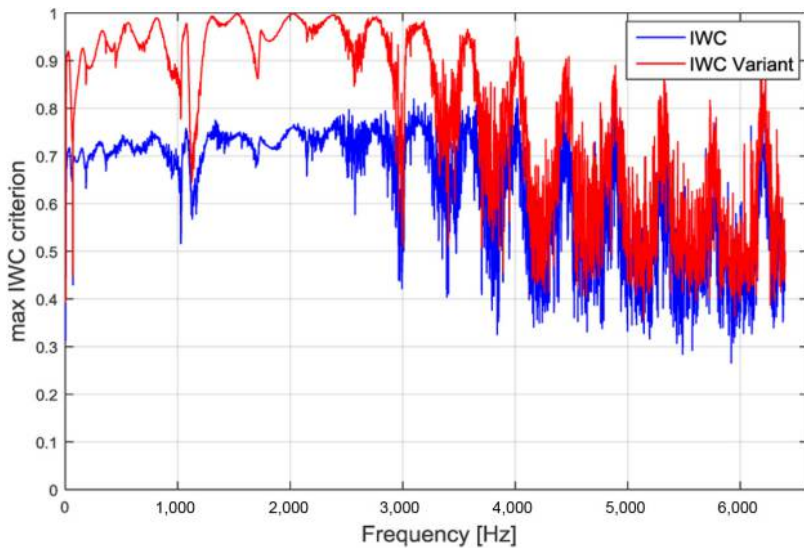


Figure 31. Objective function variation according to frequency, associated to the experiment-based IWC method and IWC-variant



high to allow energy covering all propagation directions. This identification's inaccuracy is improved when applying the proposed IWC-variant. Moreover, high oscillations of identification curves are obtained by the classical IWC method. The proposed variant permits to attenuate these oscillations and allows identifying structural parameters without significant loss of accuracy with respect to the analytical model and the Mc Daniel method considered, both, as reference. The efficiency of the proposed method was evaluated on both isotropic and sandwich beam structures.

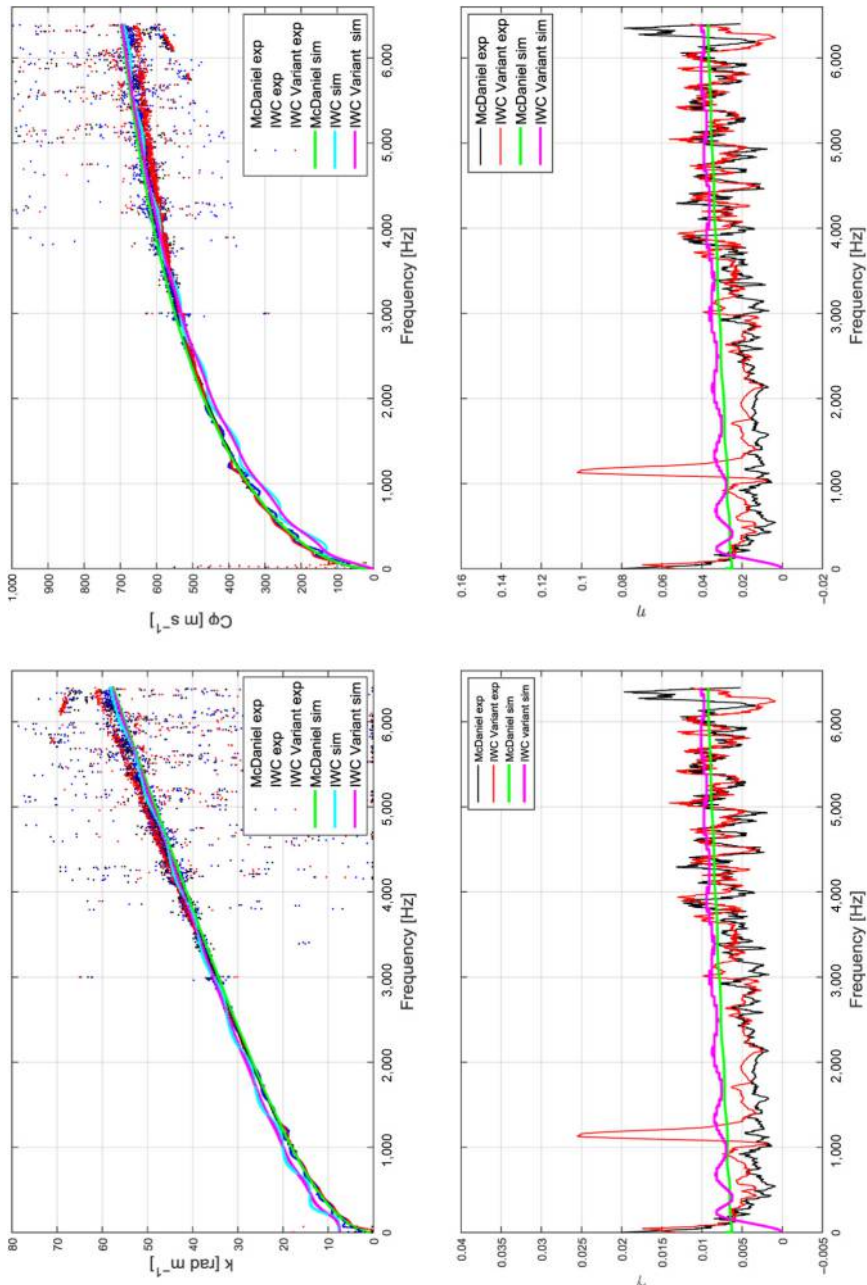
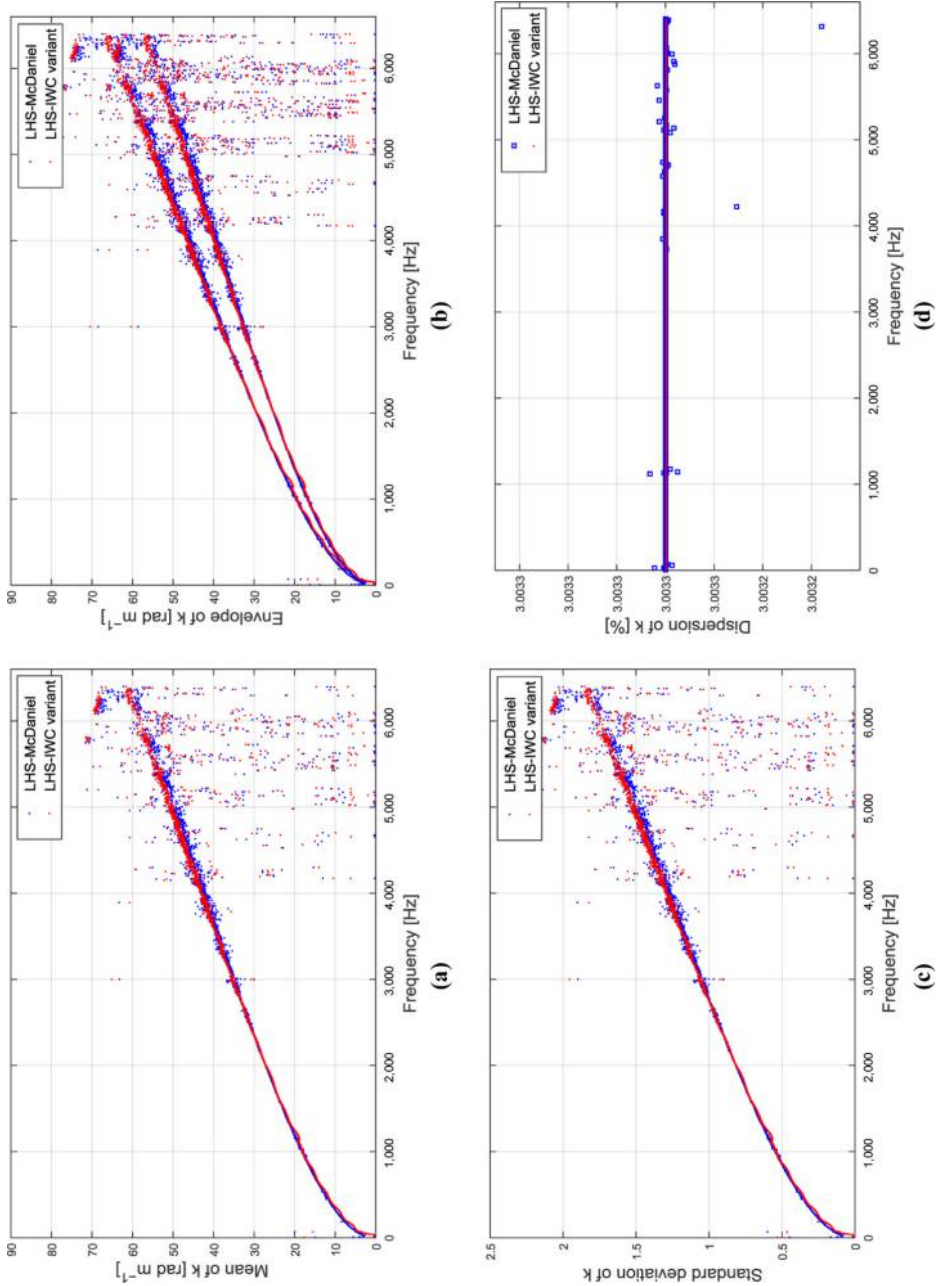


Figure 32. Comparison of the wavenumber, the phase velocity, the wave attenuation and the damping loss factor of the sandwich beam, identified using the Mc Daniel method and the IWC-variant, integrated into both experiment and numerical-based identification processes

Figure 33.
 (a) Mean, (b) envelope, (c) standard deviation and (d) dispersion of the wavenumber of the sandwich beam computed using the experiment-based identification process combining the LHS method with the McDaniel method and the IWC-variant for $\delta_x = 3\%$



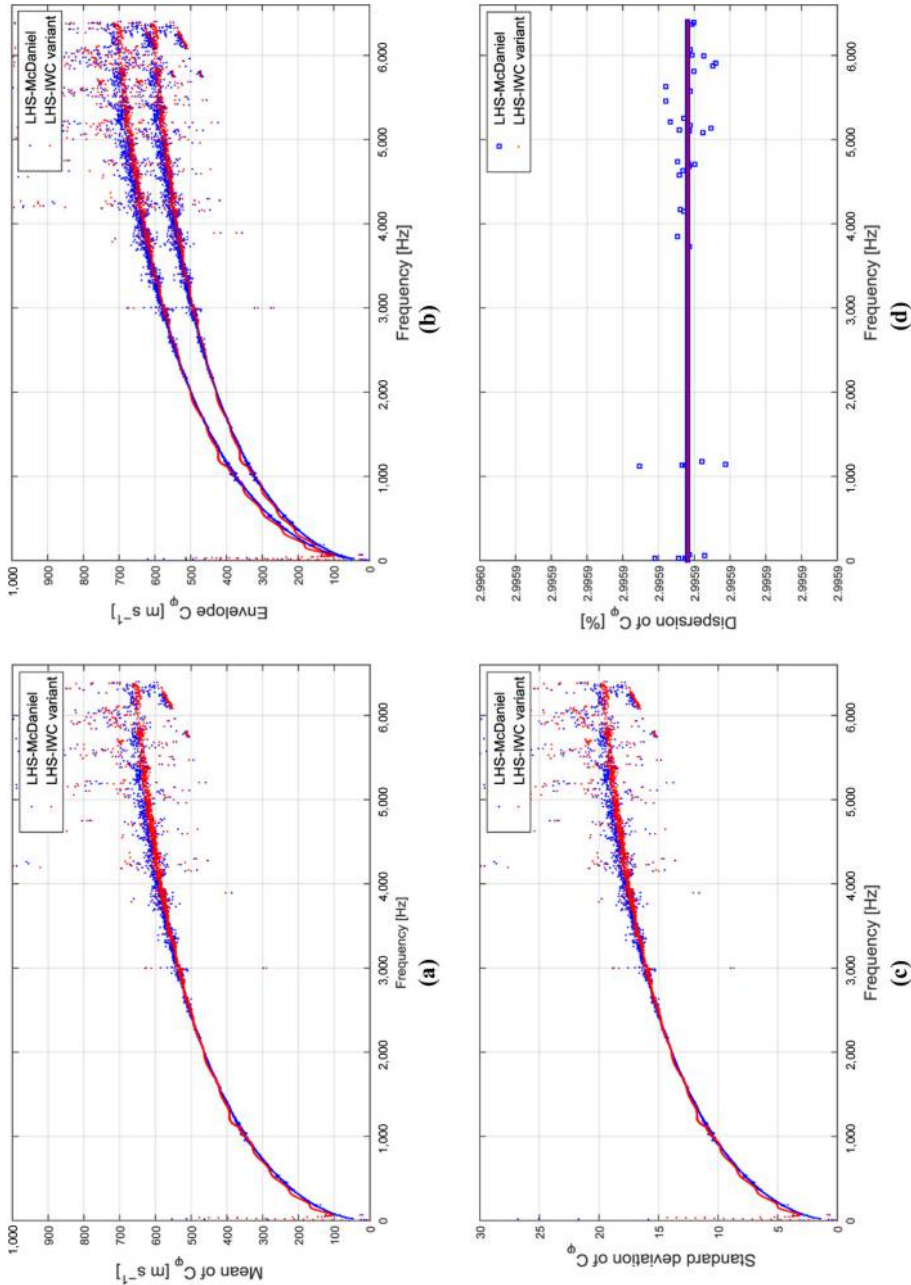
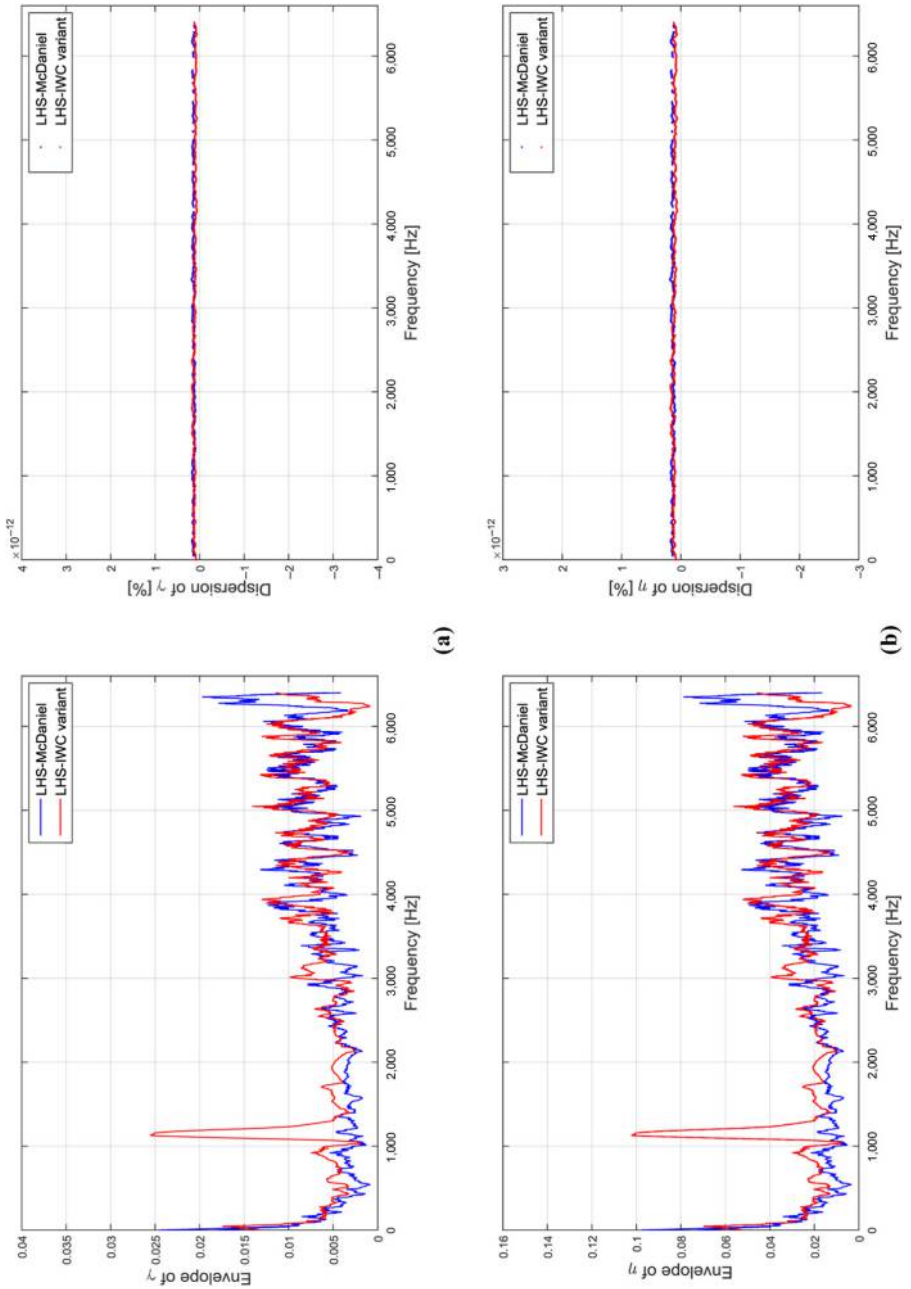


Figure 34.
 (a) Mean, (b) envelope, (c) standard deviation and (d) dispersion of the phase velocity of the sandwich beam computed using the experiment-based identification process combining the LHS method with the McDaniel method and the IWC-variant for $\delta_x = 3\%$

Figure 35. Envelopes and dispersions of (a) the wave attenuation and (b) the damping loss factor of the sandwich beam computed using the experiment-based identification process combining the LHS method with the McDaniel method and the IWC-variant for $\delta_x = 3\%$



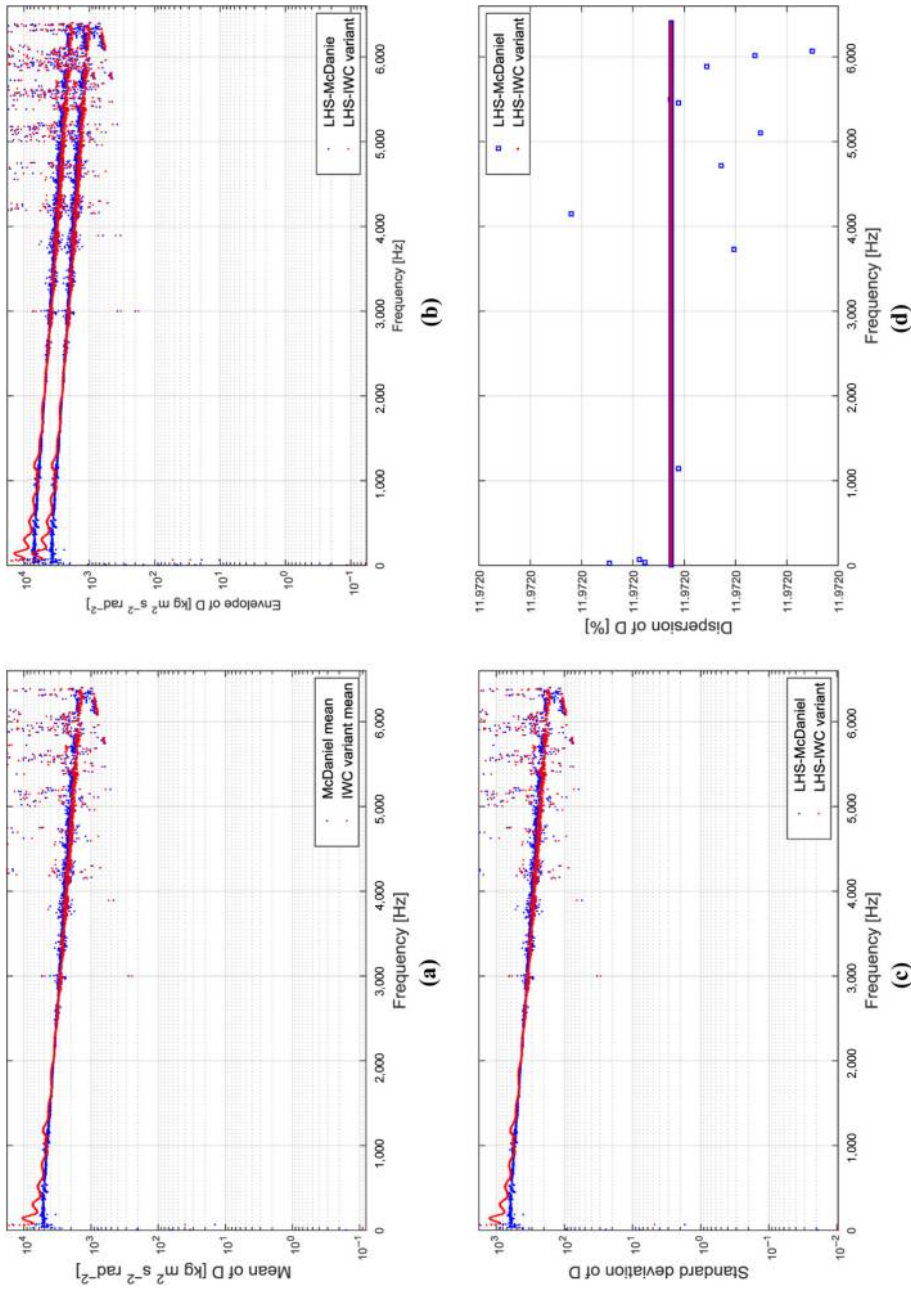


Figure 36. (a) Mean, (b) envelope, (c) standard deviation and (d) dispersion of the dynamic flexural stiffness of the sandwich beam computed using the experiment-based identification process combining the LHS method with the McDaniel method and the IWC-variant for $\delta_x = 3\%$

To achieve more realistic identification, parametric uncertainties were taken into account. Random variation was supposed on measurement points' coordinates, which can be due, actually, to measurement errors, committed by engineer or resulting from measuring tools and conditions. To analyze the uncertainty effect on identified parameters, a stochastic identification process combining the LHS uncertainty propagation method with the IWC-variant was proposed and compared to the identification process combining the LHS method with the Mc Daniel method. Statistical investigations of results illustrated the effect of uncertainties of measurement points' coordinates on all identified parameters except damping which is quantified through the wave attenuation and the loss factor.

The experimental validation of the obtained numerical results, in both deterministic and stochastic case, was carried out for the composite beam example. The identification methods, which were used initially in a numerical-based process, were then included into an experiment-based identification process, using as inputs experimentally measured responses.

The extension of the study to other types of uncertainties is an interesting perspective. In the context of parametric uncertainties, identification could be sensitive to some parameters more than others. Other uncertain parameters could also be considered with the aim of either sensitivity or uncertainty analysis.

Study is limited, in this work, to beam structure examples. Its extension to 2D composite structures is another interesting perspective, on which works in progress are focusing.

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