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1 **Geomagnetic jerks and rapid hydromagnetic waves focusing** 2 **at Earth's core surface**

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8 **Geomagnetic jerks are abrupt changes in the second time derivative - the secular acceleration-**
9 **of Earth's magnetic field that punctuate ground observatory records. They are presently the**
10 **major obstacle to the prediction of geomagnetic field behaviour years to decades ahead. Re-**
11 **cent jerks have been linked to short-lived, temporally alternating and equatorially localised**
12 **pulses of secular acceleration observed in satellite data, associated with rapidly alternating**
13 **flows at Earth's core surface. The dynamical origin of jerks has been unclear but can now be**
14 **investigated in numerical models of the geodynamo that realistically simulate the interaction**
15 **between slow core convection and rapid hydromagnetic waves. Using one such model, here**
16 **we show that the observed jerk patterns can be explained by the arrival of localised Alfvén**
17 **wave packets radiated from sudden buoyancy releases inside the core. As they reach the core**
18 **surface, the waves focus their energy towards the equatorial plane and along lines of strong**
19 **magnetic flux, creating sharp interannual changes in core flow and producing geomagnetic**
20 **jerks through the induced variations in field acceleration. The ability to numerically repro-**

21 **duce jerks offers a new way to probe the physical properties of Earth's deep interior.**

22 The geomagnetic field displays temporal variations on a broad range of time scales. Through
23 a self-sustained dynamo process (the geodynamo), slow convective motion in Earth's electrically
24 conducting and liquid core is believed to maintain the field and drive its changes over centuries
25 and longer. At the other end of the range, geomagnetic jerks with typical time scales a few years
26 or less¹ represent the fastest observed features of the internally-generated field. They were ini-
27 tially identified as 'V-shaped' patterns (see examples in Fig. 1a) in time series of the magnetic
28 field rate-of-change at ground observatories^{2,3} (the secular variation), indicating an abrupt change
29 in the field acceleration amidst periods where this acceleration is otherwise approximately con-
30 stant. Explaining the time scale disparity between rapid jerks and slow convection is a theoretical
31 challenge that has recently spurred significant progress, both in observational geomagnetism and
32 in numerical geodynamo simulations. In combination with an improving network of ground ob-
33 servatories, satellite magnetic field observations now provide a global and continuous view of the
34 geomagnetic secular acceleration over the past two decades^{4,5}, with horizontal spatial resolution of
35 approximately 2000 km at the core surface (spherical harmonic degree 9) and temporal resolution
36 down to about a year on the largest length scales. This has dramatically enhanced our empirical
37 knowledge of jerks, most notably by revealing⁶⁻⁹ their links to short-lived, temporally alternating
38 pulses of geomagnetic acceleration at the surface of the Earth (Fig. 1c,e), that at the core surface
39 are most prominent at low latitudes and localised in longitude (Fig. 2a,b). In the wake of these re-
40 sults, the earlier historical jerks of the twentieth century have also been linked to considerably less
41 resolved, but similarly alternating pulses with low-latitude foci¹⁰. It has long been suspected that

42 jerks could somehow represent the signature of hydromagnetic waves¹¹. This prompted an earlier
43 explanation¹² for jerks in terms of time-varying zonal flows that are kinematically consistent with
44 torsional Alfvén waves occurring between concentric, magnetically-coupled, axial cylinders in the
45 core. Though torsional waves have later been successfully identified in Earth’s core¹³ and in self-
46 consistent numerical simulations of the geodynamo^{14–17}, they have however been found to occur at
47 interannual periods shorter than earlier decadal estimates, and with an amplitude that is too weak
48 to account for the geomagnetic secular acceleration signal associated with jerks^{17,18}. Furthermore,
49 the complex patterns of magnetic acceleration found in satellite observations require localised (i.e.
50 non-axisymmetric), rapidly alternating flows beneath the core surface^{5,19–21}. These discount an ex-
51 planation in terms of torsional waves, but provide valuable new constraints on the rapid dynamics
52 taking place in Earth’s core.

53 **Rapid hydromagnetic waves in advanced numerical geodynamo simulations**

54 Numerical simulations of convective core magnetohydrodynamics have been successful at
55 describing the detailed morphology of the geomagnetic field^{22,23}, its temporal variations and the
56 underlying core flows²⁴. To achieve this, the magnetic Reynolds number comparing the magnetic
57 diffusion and convective core overturn time scales τ_η and τ_U (see definitions in Methods) needs to
58 be $Rm = \tau_\eta/\tau_U \approx 1000$, such that a realistic $\tau_U \approx 100$ yr is achieved when adopting an Earth-like
59 value $\tau_\eta \approx 10^5$ yr as the fundamental time scale for casting the dimensionless model results back
60 to the dimensional world (Methods). However, most existing simulations remain unrealistic when
61 considering time scales significantly shorter than τ_U . In Earth’s core, the dynamics of geomagnetic

62 jerks will involve strong rotational constraints from the Coriolis force, because the jerk's interan-
63 nual time scale is much longer than Earth's planetary rotation period $2\pi\tau_\Omega = 1$ day. Jerk dynamics
64 will also be affected by hydromagnetic waves given the proximity of the Alfvén time¹³ $\tau_A \approx 2$ yr.
65 However, because of computational limitations^{16,25}, and despite continuous advances^{14-16,26,27}, nei-
66 ther of these processes are correctly rendered in standard simulations where $2\pi\tau_\Omega$ and τ_A remain
67 much too long, and not sufficiently separated from τ_U . For instance, in our previous Coupled
68 Earth dynamo model²⁴ $2\pi\tau_\Omega \approx 10$ yr and $\tau_A \approx \tau_U \approx 100$ yr. In order to remedy these problems,
69 we have recently introduced a suite of numerical simulations^{17,25} following a well-defined path
70 through control parameter space that connects the original coupled Earth model to Earth's core
71 conditions. Our new simulations (Methods and Supplementary Table 1) involve a reasonably ac-
72 curate large-scale approximation that enables the exploration of parameters significantly beyond
73 current computational limits for direct numerical simulations. Along this path, the Earth-like field
74 morphology and kinematics of the coupled Earth model used as starting point are preserved, as are
75 the values of τ_U and τ_η , but the dynamics gradually evolves as $2\pi\tau_\Omega$ and τ_A decrease to become
76 realistic and increasingly separated from τ_U and τ_η . An asymptotic regime of strong rotational
77 and magnetic control pertaining to Earth's core conditions is reached¹⁷ at path positions beyond
78 30%. In addition to the slow background convection at time scale τ_U that is present throughout
79 the path, models in this regime additionally feature rapid magneto-inertial wave dynamics at time
80 scale τ_A . This dynamics includes geostrophic torsional Alfvén waves of weak amplitude, and also
81 non-axisymmetric, quasi-geostrophic Alfvén waves¹⁷. The relevance of the latter waves, that were
82 previously unexpected²⁸, has only recently been released^{17,29}. They offer a promising explana-

83 tion to jerks since they have been linked¹⁷ to the occurrence of intermittent magnetic acceleration
84 pulses occurring at low latitudes. In our most advanced Midpath model (50% of the path, Methods
85 and Supplementary Table 1), the Alfvén time scale value $\tau_A = 14.3$ yr implies that the waves have
86 interannual periods at wavelengths a fraction of the core size, well separated from convective pro-
87 cesses. Since $\tau_A \gg 2\pi\tau_\Omega = 0.19$ yr, such periods are also well within the rotationally-dominated
88 range where the Coriolis force plays a crucial role. Finally, a high ratio $\tau_\eta/\tau_A \approx 10^4$ of the mag-
89 netic diffusion and Alfvén time scales indicates that wave attenuation will be weak on large length
90 scales.

91 **Observed and simulated geomagnetic jerks**

92 Short-lived, intermittent pulses in the magnetic acceleration energy at Earth’s surface (Fig.
93 1d) are observed in numerical simulations throughout the parameter space path, but migrate to low
94 latitudes (Fig. 1f) only once the model conditions enter the rapid rotation regime¹⁷. In order to
95 highlight the link between such pulses and jerks, and to facilitate comparison with geomagnetic
96 field models of limited temporal resolution (Fig. 1c,e), we define jerk energy (Methods) as the
97 mean-squared difference between time averages of Earth’s surface magnetic acceleration taken
98 within two consecutive and non-overlapping 3-year time windows. With this definition, the tim-
99 ing of jerks in the simulation (Fig. 1d) can be properly defined from jerk energy pulses, and is
100 found to either shortly precede or follow that of magnetic acceleration pulses, as observed with
101 well-documented recent geomagnetic jerks⁶ (Fig. 1c). The intensity and duration of the events
102 also match rather well the observations. Abrupt slope changes in the magnetic variation time se-

103 ries are observed at specific locations (Fig. 1b), with approximately constant acceleration away
104 from the event, similar to the classic 'V-shaped' jerk signatures seen at ground observatories¹⁻³
105 (Fig. 1a). Maps of the radial magnetic acceleration before and after the events feature alternating
106 patterns (Fig. 1f) and indicate that simulated jerks are often visible over a large area (from Amer-
107 ica to Indonesia for the event shown here) at low and mid-latitudes, comparable to observations
108 of the well-characterised 2007.5 geomagnetic jerk (Fig. 1e) and to a number of earlier events^{30,31}.
109 Descending to the core surface (Fig. 2c,d), the corresponding structures are series of intense and
110 oppositely-signed patches of radial magnetic acceleration generated close to the equator and in a
111 narrow longitudinal band, beneath westward-drifting patches of intense radial magnetic flux (see
112 Fig. 3e) localised in the Atlantic hemisphere^{17,24}. The field acceleration patches alternate rapidly
113 in time for a few years (Supplementary Movie 1 from time -10 yr to 10 yr) before fading away.
114 The spatially localised morphology, interannual alternation time scale and the amplitude (approx-
115 imately 2,000 nT.yr⁻² up to degree 9) of the simulation output reproduce well the core surface
116 signature of geomagnetic jerks^{5,8} (Fig. 2a,b, see the events in 2007.5, 2011 and 2014.5).

117 **The origin of geomagnetic jerks and the role of hydromagnetic waves**

118 At the large scales accessible to observations (spherical harmonic degree up to 9), and in
119 the rapid rotation regime, the magnetic acceleration pulses in the simulations result from the ac-
120 tion of accelerating azimuthal core surface flows¹⁷ rather than from diffusive processes related to
121 flux expulsion that are common at the start of the parameter space path. In our Midpath model
122 sequence, a localised, intense and temporally alternating pulse of azimuthal flow acceleration is

indeed observed in the vicinity of the jerk time (Fig 2e,f, Supplementary Movie 2), resembling the localised alternating flows that have been inferred from geomagnetic variations⁵ associated with the 2007.5 geomagnetic jerk. The source of this perturbation can be traced back to a sudden buoyancy release from an isolated density anomaly at mid-depth in the core, 25 years before the event (Supplementary Movies 3,4). This release triggers strong azimuthal fluid flow accelerations that are entrained within the associated convective plume towards the core surface. The plume stalls at a cylindrical radius $s_c \approx 2950$ km (Supplementary Movie 3, Supplementary Fig. 1) where its decreasing radial velocity is overcome by the global westward drift. At cylindrical radii above s_c , we identify quasi-geostrophic Alfvén waves^{17,29} through the adherence of their trajectories to propagation at the locally variable, theoretical Alfvén speed and the deviation of their paths from that of material upwellings (Methods and Supplementary Fig. 1). In the upper outer core, material upwelling is indeed much slower than Alfvén waves, or even directed inwards. The perturbation energy however propagates further towards the core surface in well-defined, azimuthally extended, alternating wavefronts (Fig. 3a,b, Supplementary Movie 4) of columnar structure characteristic of rotationally-dominated dynamics¹⁷. The waves have a radial wavelength $d \approx D/4$ that is in line with the size of the density anomaly that initiated the event. Their energy becomes spatially concentrated as they approach the core-mantle boundary (Fig. 3c), yielding the intense, localised and temporally alternating surface flow acceleration signature (Fig. 2e,f) that causes the jerk, on a time scale comparable to the Alfvén wave period for these structures, $\sqrt{3}\tau_A d/D \approx 6$ yr. The energy concentration mechanism can be understood by noting that quasi-geostrophic Alfvén wavefronts are both guided along, and bounded by a strongly heterogeneous distribution of magnetic

144 field lines¹⁷. Beneath the jerk location, these field lines are arranged in an approximately axially-
145 invariant funnel-like structure (Fig. 3d) that is shaped by the slow convection and remains approxi-
146 mately static during the event. This causes the waves to be longitudinally focused towards a pair of
147 intense radial magnetic flux patches (see arrows in Fig. 3e) at the core surface. At the same time,
148 latitudinal focusing towards the equator occurs because of the effect of the spherical core-mantle
149 boundary on flow columns that tend to preserve their angular momentum as their height decreases
150 (Fig. 3b, Supplementary Video 4). Finally, the wave speed decreases close to the core-mantle
151 boundary (see curved green tracks in Supplementary Fig. 1) because the magnetic field is weaker
152 at the surface than at depth²⁵. To preserve the energy flux, the amplitude of wavefronts increases,
153 and preservation of the wave period also implies a reduction of the radial wavelength (Fig. 3c),
154 similarly to a shoaling process for water waves³². This three-dimensional energy focusing mecha-
155 nism is crucial in amplifying the weak quasi-geostrophic Alfvén waves so as to produce localised
156 and temporally alternating disturbances in the core surface flow acceleration that are significant
157 enough to cause jerks.

158 **Implications for geomagnetism and global geodynamics**

159 Since our models are in the dynamical regime of rapid rotation and strong magnetic control
160 relevant to Earth's core^{17,25}, their results can be extrapolated to natural conditions. From the mech-
161 anism described here, the duration and alternation time scale of jerk events are expected to scale
162 with the Alfvén time τ_A , which is about 7 times shorter in Earth's core¹³ than in our Midpath sim-
163 ulation (Supplementary Table 1). Yet the observed geomagnetic acceleration changes are only two

164 to three times faster than those simulated by the Midpath model (Fig. 2). This discrepancy is likely
165 related to the limited temporal resolution of present geomagnetic field models, which prevents the
166 true, potentially sub-annual³³ variations associated with jerks from being retrieved. In the upcom-
167 ing years, further insight will be obtained from jerk events that will be imaged with improved
168 resolution using data collected by the *Swarm* satellite mission. As we move along the parameter
169 space path, our models also indicate that energetic jerks occur more frequently (Fig. 4a). It is pos-
170 sible to construct statistical relationships between jerk energy and recurrence time (Fig. 4b), and
171 derive a scaling relationship for the evolution along the path of jerk energy at a given recurrence
172 time (Supplementary Fig. 2) in reasonable agreement with a theoretical prediction (Methods). The
173 extrapolation of this relationship to the end of the path (Fig. 4b) also agrees with the observed sub-
174 decadal to decadal jerk recurrence rates observed in the geomagnetic field^{1,5}. Jerk energy is also
175 found to decrease with increasing lower mantle conductance, because of the associated additional
176 Ohmic losses, and with increasing levels of upper outer core stratification (Supplementary Fig. 3).
177 This latter effect is due to changes in the geometry and amplitude of the background magnetic field
178 rather than to the wave mechanism itself, which is not sensitive to stratification. Finally, exam-
179 ining simulated records of the length of day (Methods, Supplementary Fig. 4) in the vicinity of
180 jerk events, we also observe signatures of the wave's arrival at the core surface. Rapid inflexions
181 in the rate of change of the length-of-day similar to those observed for Earth^{31,34} are caused by
182 pulses in the acceleration of the electromagnetic torque felt by the mantle. All these results high-
183 light the potential importance of the numerical reproduction of jerks, as it may lead to an improved
184 geomagnetic^{35,36} and geodetic³⁷ sounding of important, but poorly known physical properties such

185 as the lower mantle electrical conductivity and upper outer core thermal conductivity.

186 The integration of geomagnetic data into numerical geodynamo simulations through data
187 assimilation has significantly advanced in the recent past³⁸, leading to inferences of the dynamical
188 internal structure of the geodynamo and to predictions of the future geomagnetic field evolution³⁹
189 that have been integrated within the latest iteration^{40,41} of the International Geomagnetic Reference
190 Field (IGRF). At interannual to decadal time scales, the accuracy of such predictions is currently
191 hampered by the underlying dynamical model, which is located at the start of the parameter space
192 path and hence does not account for wave dynamics. The availability of advanced numerical
193 dynamo simulations that produce realistic rapid dynamics and jerks will significantly improve the
194 quality of the prior information on which the predictions are based (in particular regarding the
195 time-dependence of the field), with subsequent gains in their accuracy.

196 **Methods**

197 **Model description.** The full description of our numerical models can be found in refs. ^{17,25}. We
198 solve for Boussinesq convection, thermochemical density anomaly transport and magnetic induc-
199 tion in the magnetohydrodynamic approximation within an electrically conducting and rotating
200 spherical fluid shell of thickness $D = r_o - r_i$ representing the outer core, with $r_i/r_o = 0.35$ as in the
201 Earth. Our unknowns are the velocity field \mathbf{u} , magnetic field \mathbf{B} and density anomaly field C , and
202 we analyse the magnetic variation $\partial\mathbf{B}/\partial t$, magnetic acceleration $\partial^2\mathbf{B}/\partial t^2$ and the flow acceleration
203 $\partial\mathbf{u}/\partial t$. The fluid shell is electromagnetically coupled both to a solid inner core of radius r_i and to a
204 solid outer shell representing the mantle between radii r_o and $1.83r_o$. The inner core and mantle are
205 furthermore coupled together by a gravitational restoring torque. Both the inner core and mantle
206 feature a time-dependent axial differential rotation with respect to the outer core. The three regions
207 are assigned moments of inertia respecting the proportions²⁴ relevant to Earth’s mantle, inner and
208 outer core, and the ensemble has a constant angular momentum defining the planetary rotation rate
209 Ω .

210 The mechanical boundary conditions are of the stress-free type at both boundaries. In the low
211 viscosity regime where our models operate, these are undistinguishable from no-slip conditions²⁵
212 while alleviating the need to resolve the viscous boundary layers. Electrically conducting bound-
213 ary conditions are used at both boundaries. The electrical conductivity of the inner core is set
214 at the same value σ_c as that of the outer core. The mantle features an electrically conducting
215 region at its base, with thickness Δ and conductivity σ_m . In our four main model cases (Supple-
216 mentary Table 1) the dimensionless conductance has been set to a median geophysical estimate⁴²

217 $\Sigma = \Delta\sigma_m/D\sigma_c = 10^{-4}$. Two other models (Midpath-I and Midpath-H) explore the end-member
218 values $\Sigma = 0$ (insulating mantle) and $\Sigma = 10^{-3}$. The thermochemical boundary conditions are
219 of heterogeneous, fixed-flux type. The homogeneous part F of the density anomaly flux is pre-
220 scribed at the inner boundary. In our four main model cases the homogeneous density anomaly
221 flux vanishes at the outer boundary (neutral buoyancy). A volumetric sink term is then present
222 in the density anomaly transport equation to conserve mass. Within the Boussinesq approxima-
223 tion, this configuration models bottom-driven chemical convection originating from inner core
224 solidification, a fully convective outer core and an exactly adiabatic heat flow at the core-mantle
225 boundary. An additional model (Midpath-S) explores the effect of a possible stratification of the
226 upper outer core⁴³ by prescribing a negative (adverse buoyancy) density anomaly flux at the core-
227 mantle boundary (see Stratified Core section below). Spatial modulations of the density anomaly
228 fluxes are prescribed at both boundaries¹⁷, with the same geometry as in the coupled Earth model²⁴.
229 These are meant to model a spatially heterogeneous growth of the inner core, and thermal control
230 from the heterogeneous lower mantle.

231 **Model parameters, parameter space path and time scales.** The four main control parameters
232 of the model are the flux-based Rayleigh, Ekman, Prandtl and magnetic Prandtl numbers

$$Ra_F = \frac{g_o F}{4\pi\rho\Omega^3 D^4}, \quad (1)$$

$$E = \frac{\nu}{\Omega D^2}, \quad (2)$$

$$Pr = \frac{\nu}{\kappa}, \quad (3)$$

$$Pm = \frac{\nu}{\eta}. \quad (4)$$

233 Here g_o , ρ , ν , κ and η are respectively the gravity at the outer boundary of the model, the fluid den-
 234 sity, viscosity, thermo-chemical and magnetic diffusivities ($\eta = 1/\mu\sigma_c$, with μ the fluid magnetic
 235 permeability). We have recently introduced²⁵ the concept of a unidimensional path in parameter
 236 space, by showing that the variations in these control parameters that are necessary to bridge the
 237 gap between our previous coupled Earth model²⁴ and Earth's core conditions can be represented as
 238 power laws of a single variable ϵ . Any model along the path is defined using the following rules:

$$Ra_F = \epsilon Ra_F(\text{CE}), \quad (5)$$

$$E = \epsilon E(\text{CE}), \quad (6)$$

$$Pr = 1, \quad (7)$$

$$Pm = \sqrt{\epsilon} Pm(\text{CE}). \quad (8)$$

239 Here $Ra_F(\text{CE}) = 2.7 \cdot 10^{-5}$, $E(\text{CE}) = 3 \cdot 10^{-5}$ and $Pm(\text{CE}) = 2.5$ are the control parameters of the
 240 coupled Earth dynamo model defining the start of the path ($\epsilon = 1$), and we have shown²⁵ that
 241 conditions relevant to Earth's core are reached at the end of path defined by $\epsilon = 10^{-7}$. Our models
 242 are defined in refs. ^{17,25} and in Supplementary Table 1 by the values $\epsilon = 10^{-2}$, $3.33 \cdot 10^{-3}$, 10^{-3}
 243 and $3.33 \cdot 10^{-4}$, respectively corresponding to 29%, 35%, 43%, and 50% of the path (the Midpath
 244 model). The parameters of the Midpath model are the closest to Earth's core conditions employed
 245 to date in a numerical dynamo simulation, at the expense of a large scale approximation (see
 246 Numerical Implementation section below).

247 The model outputs follow scaling laws²⁵ depending on ϵ that also closely approach the con-
 248 ditions expected in Earth's core as we progress along the path (Supplementary Table 1). Once the

249 magnetic diffusion time $\tau_\eta = D^2/\eta$ is set to an Earth-like value (see Rescaling section below), the
 250 end of path simultaneously matches the Earth’s core rotational time $\tau_\Omega = 1/\Omega$, convective overturn
 251 time $\tau_U = D/U$, and Alfvén time $\tau_A = \sqrt{\rho\mu}D/B$ (here U and B are respectively the root-mean-
 252 squared velocity and dynamo-generated magnetic field in the fluid shell). Numerical models taken
 253 along this path can therefore be understood as continuously progressing from imperfect towards
 254 geophysically appropriate conditions in all relevant aspects of their inputs and outputs. The dimen-
 255 sional values of τ_Ω , τ_U , and τ_A reached in our models and at the end of path are listed together with
 256 Earth’s core estimates in Supplementary Table 1 (see ref. ¹⁷ for a complete list of dimensionless
 257 time scale ratios achieved in the models).

258 **Stratified core case.** The Midpath-S model (Supplementary Table 1) explores the effects of a pos-
 259 sible upper outer core stratification⁴³ on the occurrence of simulated jerks. Within the Boussinesq
 260 approximation, stratification is modelled by adding an adverse density anomaly gradient⁴⁴ to the
 261 background gradient prescribed by the neutral buoyancy conditions described above:

$$\frac{dC}{dr} = \frac{-N^2\rho}{2g_o} (1 + \tanh((r - r_s)/\delta)). \quad (9)$$

262 Here N is the Brunt-Väisälä frequency pertaining to the stratification level at the core surface, r
 263 is radius, $r_s = 3340$ km is the radius at which stratification sets in, and $\delta = 10^{-2}D = 22.6$ km is
 264 the thickness of the stratified layer front. The thickness of the stratified layer is $r_o - r_s = 140$ km,
 265 as proposed in ref. ⁴³. In the Midpath-S model we set $N = 1/\tau_\Omega$, as also proposed in ref. ⁴³.
 266 The output of the Midpath-S model demonstrates the preservation of simulated jerks against core
 267 stratification, albeit at a reduced energy level given the modifications of the background magnetic
 268 field that guides the waves.

269 **Dimensional rescaling of dimensionless model output.** The dimensionless model length unit is
270 adjusted to the thickness $D = 2260$ km of Earth’s core. Time is rescaled by adjusting the magnetic
271 diffusion time scale $\tau_\eta = D^2/\eta$ to the value $\tau_\eta = 135\,000$ yr, corresponding to a value $\eta = 1.2$ m²/s
272 at the midpoint of current estimates²⁵. Given the invariance of the magnetic Reynolds number
273 $Rm = \tau_\eta/\tau_U \approx 1\,000$ along the parameter space path, this rescaling choice ensures $\tau_U \approx 130$ yr and
274 Earth-like convective geomagnetic variations¹⁷. The fluid and Alfvén wave velocities are rescaled
275 by using these length and time units. The magnetic field amplitude is presented by setting the
276 Elsasser magnetic field unit $\sqrt{\rho\mu\eta\Omega}$ to the value 0.9 mT. Given the approximate invariance of the
277 Elsasser number $B^2/\rho\mu\eta\Omega \approx 20$ along the path²⁵, this amounts to setting the root-mean-squared
278 field amplitude within the core to a value of about 4 mT, in agreement with Earth’s core current
279 estimate¹³. Note that concerning the time and magnetic field units our choices slightly differ (by
280 less than 5%) from ref. ¹⁷, as we adopt here the same units across all simulations. This change is
281 done in order to obtain a consistent comparison between the original path models and those with
282 a modified setup (Midpath-S,I,H, Supplementary Table 1) introduced in this study. Finally, the
283 density anomaly rescaling used in Supplementary Movie 3 follows from the velocity rescaling and
284 from adjustment of the dimensionless, time average convective power in the shell to an estimate²⁵
285 $P = 3$ TW of the geodynamo power.

286 **Jerk energy definition and scaling.** In Fig. 1c,d, we present the energy E_{SA} of the magnetic
287 acceleration, defined as a mean-squared average over Earth’s surface S_E :

$$E_{SA} = \left\langle (\partial_t^2 \mathbf{B})^2 \right\rangle = \frac{1}{S_E} \int_{S_E} \left(\frac{\partial^2 \mathbf{B}}{\partial t^2} \right)^2 dS. \quad (10)$$

288 To facilitate comparison of the model output with geomagnetic field models of limited temporal
 289 resolution, the jerk energy E_J is defined as a sliding finite difference between consecutive 3-yr time
 290 windows rather than an instantaneous rate-of-change:

$$E_J(t) = \left\langle \left(\left[\partial_t^2 \mathbf{B} \right]_t^{t+3 \text{ yr}} - \left[\partial_t^2 \mathbf{B} \right]_{t-3 \text{ yr}}^t \right)^2 \right\rangle. \quad (11)$$

291 As introduced above, the angle brackets denote the average over Earth's surface, and the square
 292 brackets denote a time average. Jerk recurrence statistics in Fig. 4b are obtained from time series
 293 (Fig. 4a) of E_J , by dividing the duration of the model run with the number of samples reaching
 294 or exceeding a given jerk energy. In Supplementary Fig. 2, jerk energies at 10, 30 and 100 yr
 295 recurrence times are extracted from Fig. 4b and scaled with the path parameter ϵ , revealing a
 296 common dependency in $\epsilon^{-0.19 \pm 0.01}$. The end-of-path prediction in Fig. 4b is obtained by collapsing
 297 the jerk statistics onto a single master curve according to this scaling, and extrapolating the master
 298 curve to the end-of-path conditions corresponding to $\epsilon = 10^{-7}$.

299 The amplitude of a secular acceleration pulse scales with the magnetic field amplitude B
 300 times the wave-induced flow acceleration U/τ_A . Given that the pulse duration should also scale
 301 with τ_A , jerk energy then scales with

$$E_J \sim \frac{\tau_A}{3 \text{ yr}} (BU/\tau_A)^2 \sim \frac{(BU)^2}{\tau_A} \quad (12)$$

302 Along the parameter space path, the dimensional values of U and B are approximately preserved,
 303 and the above scaling suggests that E_J should be inversely proportional to τ_A . The Alfvén number
 304 $A = \tau_A/\tau_U$ has been shown²⁵ to scale like $\epsilon^{0.25}$ along the path. The invariance of τ_U along the path
 305 (Supplementary Table 1) then leads to $\tau_A \sim \epsilon^{0.25}$ and $E_J \sim \epsilon^{-0.25}$, close to the numerical result

306 $E_J \sim \epsilon^{-0.19}$. The residual discrepancy mainly stems from the value of B which slightly decreases
 307 along the path (see ref. ²⁵).

308 **Identification of quasi-geostrophic Alfvén waves.** In Supplementary Fig. 1 we repeat the analy-
 309 sis carried out in ref. ¹⁷ to identify hydromagnetic wave propagation. The flow acceleration patterns
 310 that we analyse have a columnar structure that derives from the dominant rotational constraint of
 311 the Coriolis force. At any given time t , cylindrical radius s and at a fixed analysis longitude φ_0 , we
 312 therefore first compute the columnar average $\partial u_c / \partial t$ of azimuthal flow acceleration:

$$\frac{\partial u_c}{\partial t}(s, \varphi_0, t) = \frac{1}{z_+ - z_-} \int_{z_-}^{z_+} \frac{\partial(\mathbf{u} \cdot \mathbf{e}_\varphi)}{\partial t}(s, \varphi_0, z, t) dz. \quad (13)$$

313 Here s, φ, z are cylindrical coordinates, \mathbf{e}_φ is the unit vector in the azimuthal direction, and the
 314 vertical integral is evaluated between the lower and upper heights $z_{-,+}$ of an axial column parallel
 315 to the rotation vector $\mathbf{\Omega}$ at cylindrical radius s . We then represent time-cylindrical radius maps of
 316 $\partial u_c / \partial t$ and overplot ray-tracing theoretical propagation tracks obtained by integrating in time the
 317 column-averaged Alfvén velocity c_A and column-averaged cylindrical radial fluid velocity V_s :

$$c_A(s, \varphi_0, t) = \sqrt{\frac{1}{z_+ - z_-} \int_{z_-}^{z_+} \frac{(\mathbf{B} \cdot \mathbf{e}_s)^2}{\rho\mu}(s, \varphi_0, z, t) dz}, \quad (14)$$

$$V_s(s, \varphi_0, t) = \frac{1}{z_+ - z_-} \int_{z_-}^{z_+} \mathbf{u}(s, \varphi_0, z, t) \cdot \mathbf{e}_s dz. \quad (15)$$

318 Here \mathbf{e}_s is the unit vector in the cylindrical radial direction. The adherence of $\partial u_c / \partial t$ to the Alfvén
 319 tracks and deviation from material upwelling tracks demonstrates Alfvén wave propagation.

320 **Length-of-day variations.** The numerical simulation solves for the deviations Ω_M of the mantle
 321 angular velocity from the background planetary rotation rate Ω (see ref. ⁴² for details):

$$I_M \frac{d\Omega_M}{dt} = \Gamma_M + \Gamma_G. \quad (16)$$

322 Here I_M is the Earth’s mantle moment of inertia, and $\Gamma_{M,G}$ are respectively the magnetic and grav-
 323 itational torques felt by the mantle. The corresponding rate of change $d(\text{LOD})/dt$ in the length of
 324 the day is then

$$\frac{d(\text{LOD})}{dt} = -\frac{2\pi}{\Omega^2} \frac{d\Omega_M}{dt}, \quad (17)$$

325 where we have used $\Omega_M \ll \Omega$. Time series of $d(\text{LOD})/dt$ in the vicinity of jerk events are presented
 326 in Supplementary Fig. 4. The magnetic acceleration pulses cause pulses in $d^2\Gamma_M/dt^2$, and hence
 327 rapid inflexions in $d(\text{LOD})/dt$ with shape similar to that observed in geodetic time series^{31,34}. Note
 328 though that the amplitude of the inflexions is significantly weaker in the numerical simulations than
 329 in Earth’s core, because the inverse squared Alfvén number $1/A^2$ measuring the relative importance
 330 of magnetic forces and inertia is about 50 times weaker²⁵ in the Midpath model than in the core.

331 **Numerical Implementation.** Our numerical implementation involves a decomposition of the fields
 332 in spherical harmonics up to degree and order 133, and a discretisation in the radial direction on
 333 a second-order finite-differencing scheme (see ref. ²⁵ for numerical resolution details). We use
 334 the spherical harmonics transform library⁴⁵ SHTns freely available at [https://bitbucket.org](https://bitbucket.org/nschaeff/shtns)
 335 [/nschaeff/shtns](https://bitbucket.org/nschaeff/shtns). Time stepping is of second-order, semi-implicit type. Angular momentum
 336 conservation is controlled at each time step. To handle an increasing hydrodynamic turbulence
 337 along the path that does however only weakly affect the large-scale solution²⁵, hyperdiffusion is
 338 implemented on the velocity and density anomaly fields, but not on the magnetic field which re-
 339 mains fully resolved. The details, physical justification and validation of this approximation are
 340 presented in ref. ²⁵. Each model on the path is initialised using the output of the previous step.
 341 Integration times after statistical equilibration are listed in Supplementary Table 1. In our main

342 models these represent at least 18% of a magnetic diffusion time and 75% of a dipole decay time
343 $r_o^2/\pi^2\eta$. Within this time all model outputs are in a statistically-steady state¹⁷ demonstrating self-
344 sustained dynamo action. In particular, all models produced an axial dipole-dominated magnetic
345 field that did not reverse polarity.

346 **Data availability.** The numerical code and the simulation datasets analysed during the current
347 study are available from the corresponding author on reasonable request.

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450 **Author contributions** J.A. designed the project, designed and carried out the numerical experiments, and
451 wrote the manuscript. C.C.F. processed the geomagnetic data, constructed the CHAOS-6x5 geomagnetic
452 field model and led its comparison with the simulation results. J.A. and C.C.F. processed the results and
453 discussed the manuscript.

454 **Author Information** Correspondence and requests for materials should be addressed to J. Aubert (email:
455 aubert@ipgp.fr). The authors declare no competing financial interests.

456 **Figure 1 Signature of geomagnetic jerks at Earth's surface.** **a,c,e**, Observatory geo-
457 magnetic data and output from the CHAOS-6x5 geomagnetic field model⁵ based on satellite
458 and ground observations. **b,d,f**, Output from the Midpath numerical model (Methods and Sup-
459 plementary Table 1). **a**, Time series of the downward geomagnetic secular variation (rate of
460 change of the geomagnetic field, blue lines and crosses) from annual differences of revised
461 monthly means at two selected observatories, Kourou and Tamanrasset (marked as green
462 dots in **e**), plotted together with the CHAOS-6x5 output (black). **b**, Downward vertical secular
463 variation time series in the Midpath numerical model, at two locations marked with green dots
464 in **f**. **c,d**, Time series of the mean-squared secular acceleration E_{SA} (grey) and of jerk energy
465 E_J (black, see Methods for definitions and Fig. 4a for a longer time series of E_J), showing how
466 secular acceleration pulses relate to the strong acceleration changes that characterise jerks.
467 As identified locally from secular variation time series and globally by peaks in E_J , vertical
468 dashed lines in **a,c** mark geomagnetic jerks⁶⁻⁹ occurring near epochs 2007.5, 2011, 2014 and
469 in **b,d** the synthetic jerk event used to define the simulation time origin. **e,f**, Hammer pro-
470 jections of the radial secular acceleration (orange is outwards) before and after a jerk event,
471 showing patterns alternating in time over a large portion of Earth's surface.

472 **Figure 2 Signature of geomagnetic jerks at Earth's core surface.** **a,b**, Output from
473 the CHAOS-6x5 geomagnetic field model⁵. **c-f**, Output from the Midpath numerical dynamo
474 model. **a,c**, Time-longitude plots of the radial secular acceleration at the equator (orange is
475 outwards), filtered at spherical harmonic degree 9. The horizontal dashed lines locate geo-
476 magnetic jerks epochs⁶⁻⁹ in **a** and the synthetic jerk time in **c**. The vertical lines respectively

477 locate the longitudes selected for analysis in ref. ⁵ for the real events, and in **f** for the synthetic
478 event. **b,d**, Hammer projections of the radial secular acceleration (same spatial filtering as
479 in **a,c**) before and after jerk events, showing localised patterns alternating in time (see also a
480 numerical model temporal sequence in Supplementary Movie 1). Black curves again locate
481 the longitudes selected for analysis. **e**, Miller map showing details of the azimuthal flow ac-
482 celeration at the core surface (native model spatial resolution, blue is westwards) during the
483 simulated jerk event (see also global map and temporal sequence in Supplementary Movie
484 2). The black vertical lines locate the analysis longitude used in **f**. **f**, Temporal evolution of
485 the core surface, equatorial azimuthal flow acceleration at the analysis longitude, showing the
486 structure of the wave packet that causes the jerk.

487 **Figure 3 Hydromagnetic waves inside the core and magnetic field structure from the**
488 **Midpath model.** **a**, Planform of the azimuthal flow acceleration (blue is westward) at time
489 -3.22 yr before the jerk event, in a quarter of the equatorial plane between longitudes $0^\circ E$
490 and $90^\circ E$. Also shown are the directions of the rotation vector Ω and the wave vector **k**. **b**,
491 Meridional planform of azimuthal flow acceleration outside the axial cylinder tangent to Earth's
492 core and at the analysis longitude located by a black line in **a**. See supplementary Movie 4 for
493 the corresponding temporal sequences. **c**, closeup of equatorial azimuthal flow acceleration
494 corresponding to the dashed box in **a**, showing the concentrated wave structures below the
495 core-mantle boundary. **d**, Semi-transparent detail of **a** (see second dashed box), with a volu-
496 metric rendering of the magnetic field lines (grey, thickness proportional to local magnetic field
497 amplitude) that channel and focus the waves towards the core surface. **e**, Hammer projection

498 of the radial magnetic field at the core surface (native model spatial resolution, orange is out-
499 wards) at time -3.22 yr. Arrows in **d,e** locate the core surface magnetic flux patches where the
500 focused waves emerge.

501 **Figure 4 Statistics of jerk recurrence time. a,** Time series of jerk energy E_J (see Methods)
502 in the 29% of path and Midpath models (Supplementary Table 1). The arrow locates the event
503 at time 0 yr analysed in Figs. 1-3. **b,** Distribution of the average recurrence time of jerks
504 reaching or exceeding a given energy, for models within the rapid rotation regime¹⁷ (solid lines).
505 Also shown is an extrapolation (see Methods and Supplementary Fig. 2) of the recurrence time
506 distribution for the Earth's core conditions (dashed line and light grey shaded uncertainty area).
507 The dark grey line segment locates the output of CHAOS-6x5 as estimated from Fig. 1c (three
508 jerks with $E_J \geq 80 \text{ nT}^2 \cdot \text{yr}^{-4}$ and two with $E_J \geq 90 \text{ nT}^2 \cdot \text{yr}^{-4}$ within 19 years).

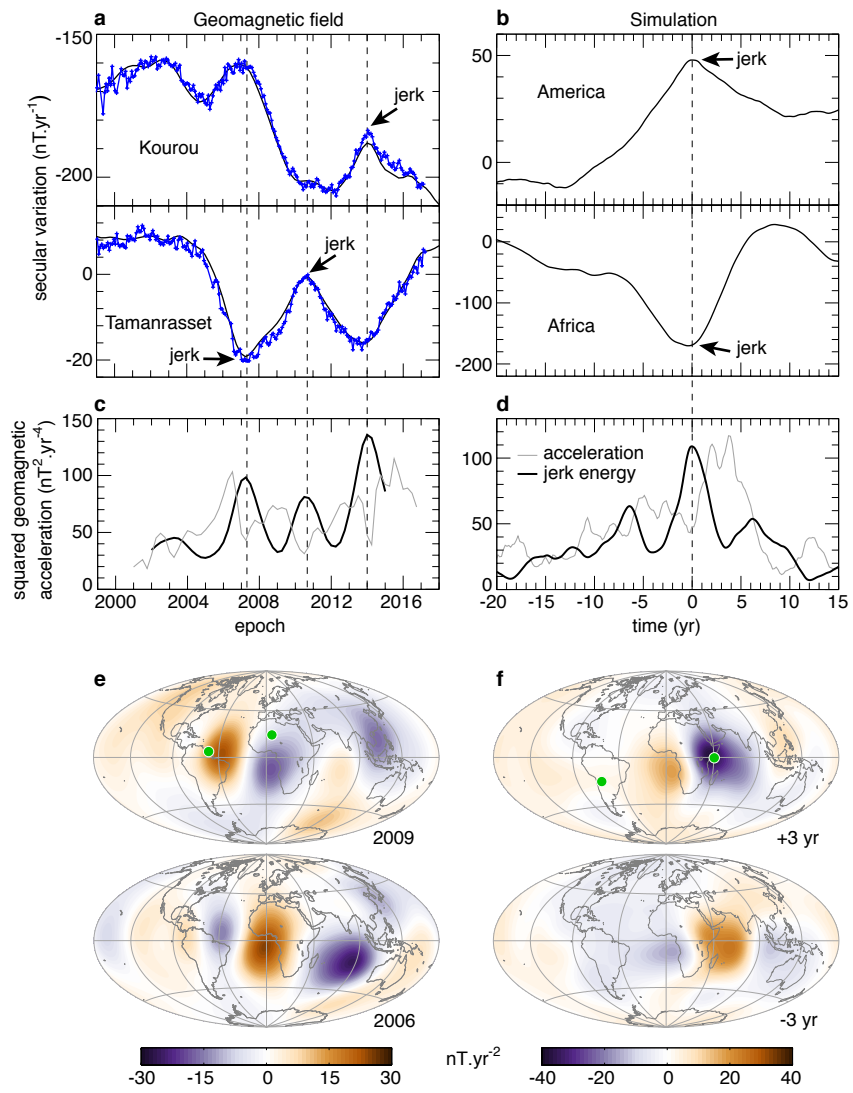


Figure 1

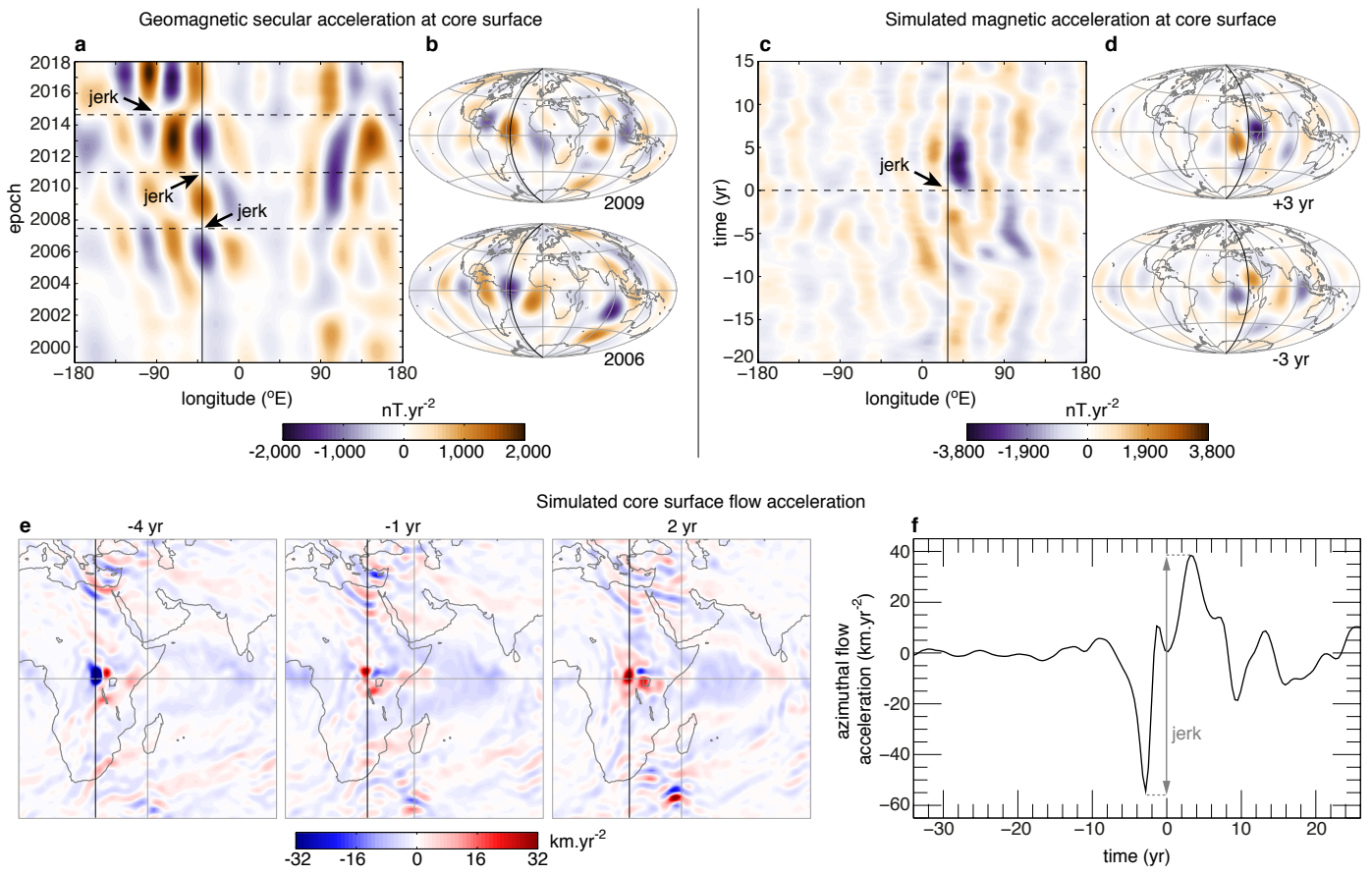


Figure 2

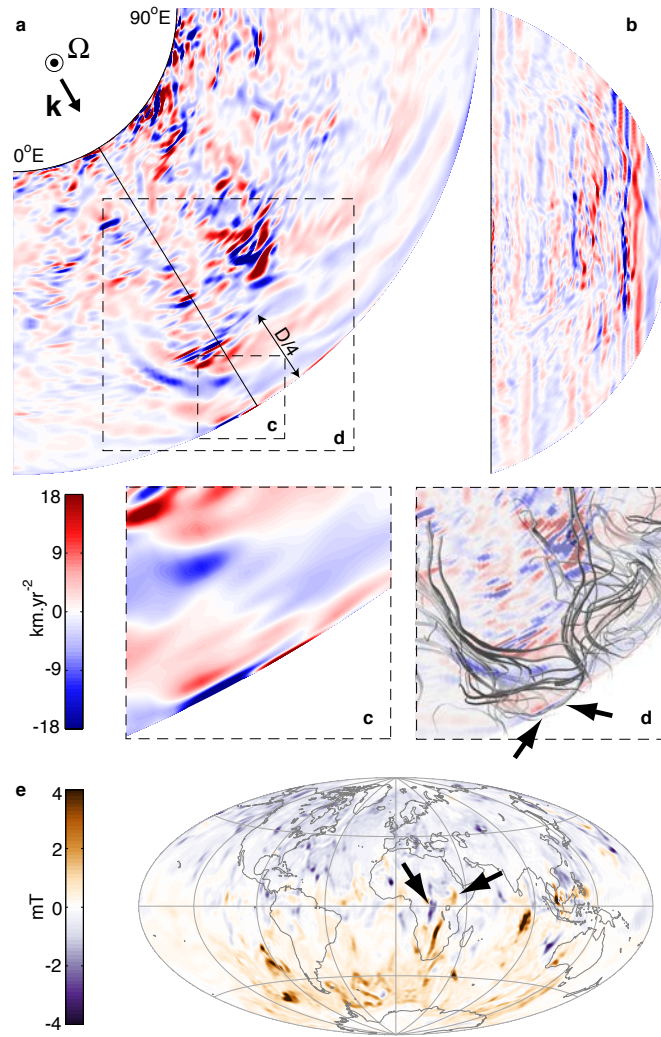


Figure 3

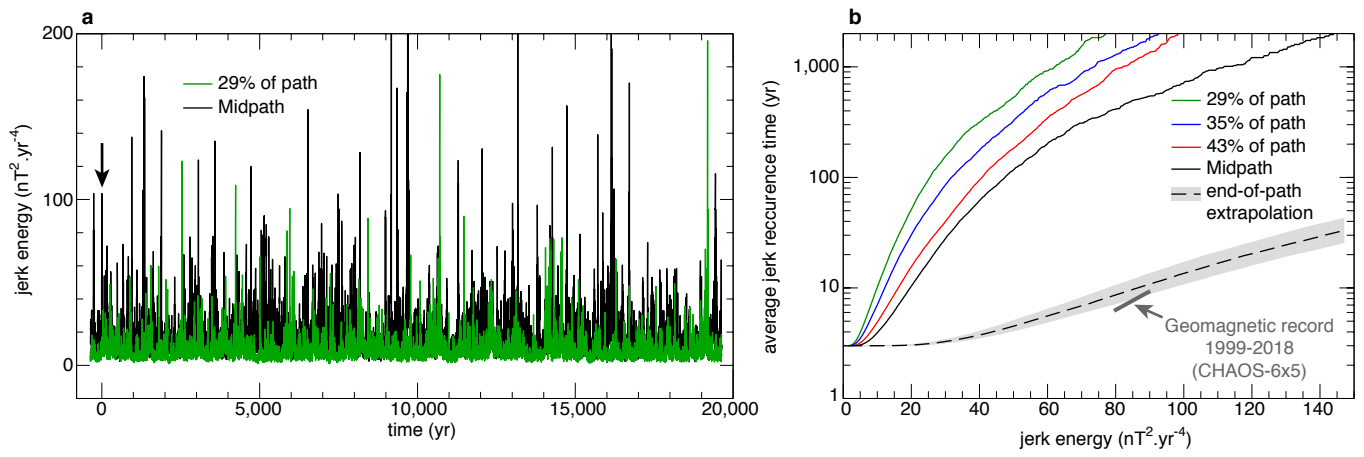


Figure 4

Supplementary information for: Geomagnetic jerks and rapid hydromagnetic waves focusing at Earth's core surface

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Contents:

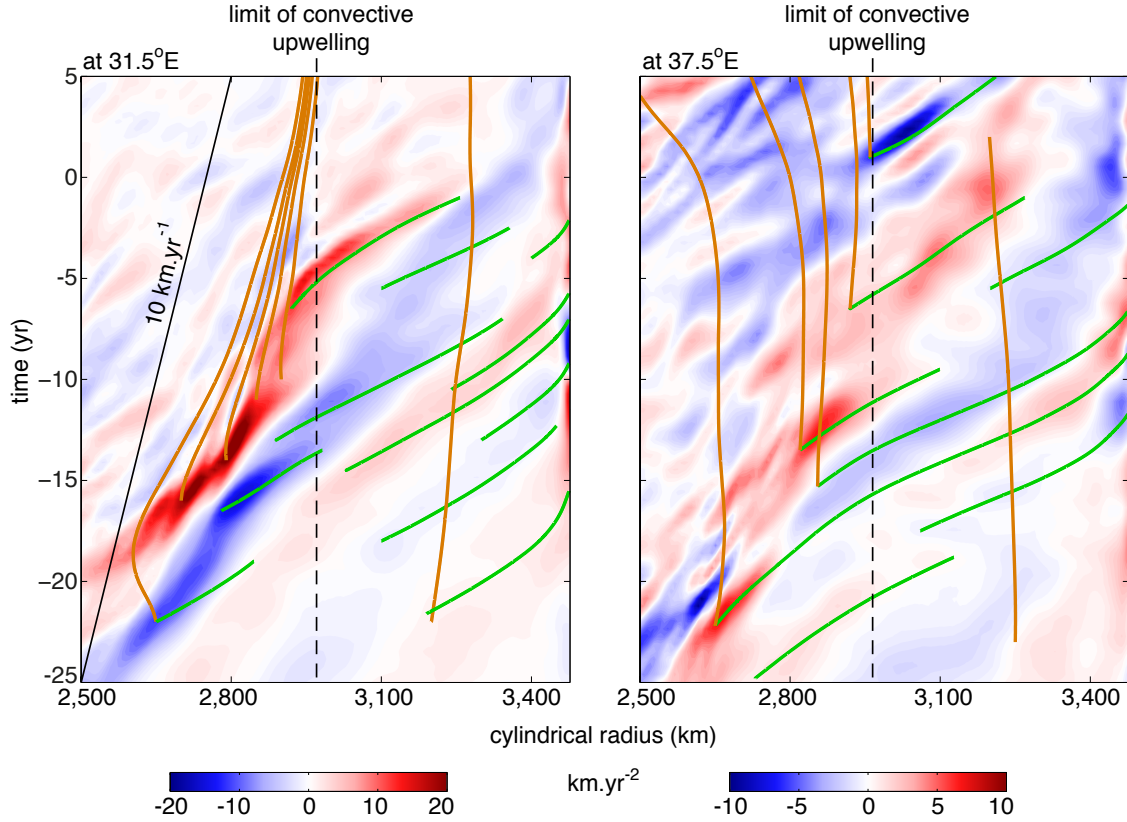
Supplementary Table 1

Supplementary Figs. 1-4

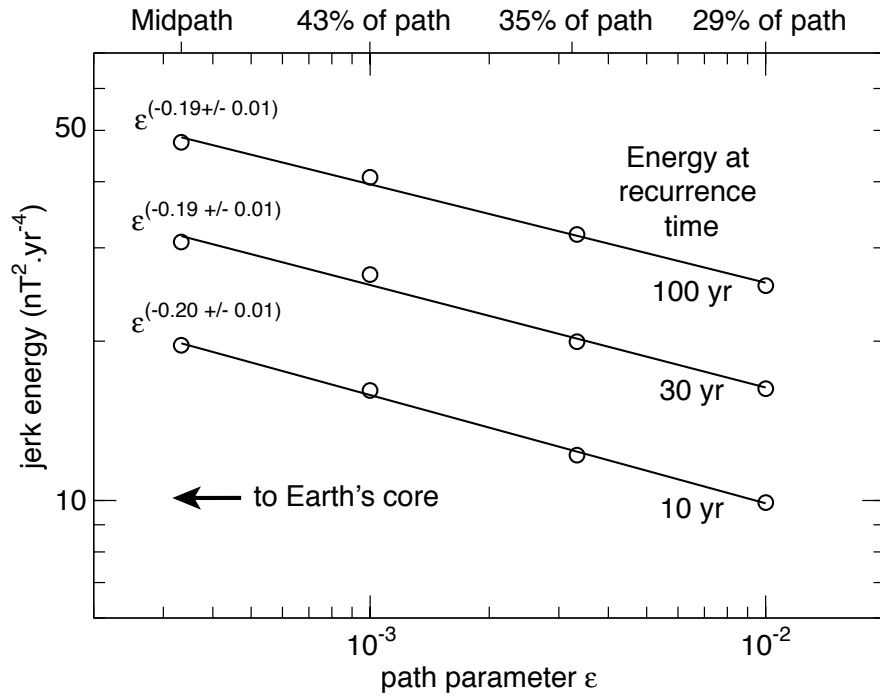
Captions for Supplementary Movies 1-4

Label	Path parameter ϵ	Path position	$\frac{\Delta\sigma_m}{D\sigma_c}$	core surface buoyancy	τ_U (yr)	τ_A (yr)	$2\pi\tau_\Omega$ (yr)	Integration time (yr)
	10^{-2}	29%	10^{-4}	neutral	129	31.5	1.0	42 900
	$3.33 \cdot 10^{-3}$	36%	10^{-4}	neutral	126	24.0	0.6	34 600
	10^{-3}	43%	10^{-4}	neutral	123	18.2	0.3	24 900
Midpath	$3.33 \cdot 10^{-4}$	50%	10^{-4}	neutral	125	14.3	0.2	24 400
Midpath-I	$3.33 \cdot 10^{-4}$	50%	0	neutral	120	14.2	0.2	11 400
Midpath-H	$3.33 \cdot 10^{-4}$	50%	10^{-3}	neutral	128	14.6	0.2	11 300
Midpath-S	$3.33 \cdot 10^{-4}$	50%	10^{-4}	adverse	121	14.5	0.2	10 100
End of path	10^{-7}	100%			130	1.9	$3.2 \cdot 10^{-3}$	
Earth					≈ 140	≈ 2	$2.7 \cdot 10^{-3}$	

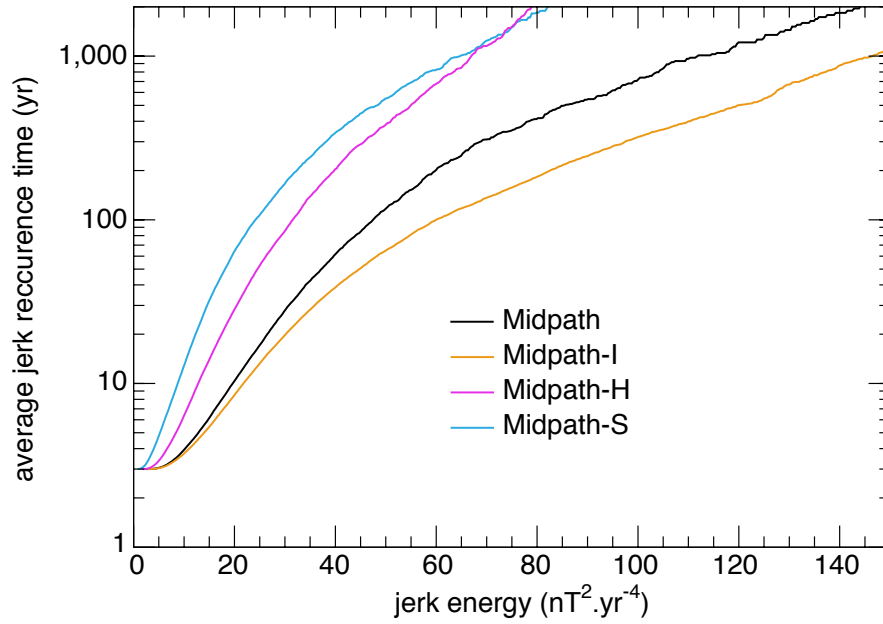
Supplementary Table 1: **Models along a parameter space path to Earth’s core.** Key parameters and corresponding dimensional time scale values for numerical models located along a parameter space path²⁵ towards Earth’s core conditions. See Methods for definitions and ref. ¹⁷ for complete parameter data. Dimensional time scales values are obtained from the dimensionless time scale ratios reported in ref. ¹⁷ and the magnetic diffusion time scale set to $\tau_\eta = 135\,000$ yr in this study. Also shown are the values, closely approaching Earth’s core estimates, obtained by extrapolating scaling laws determined along the path²⁵ to its end point.



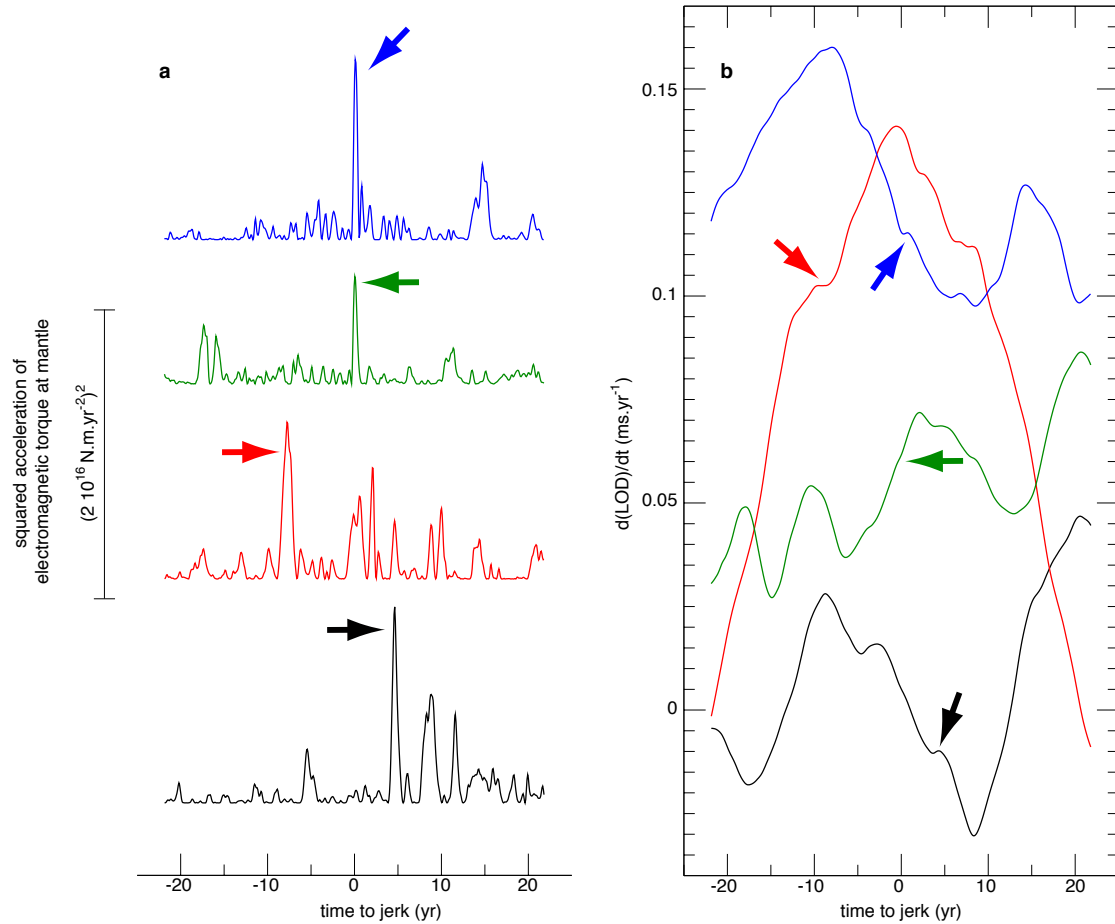
Supplementary Figure 1: **Alfvén wave propagation in the upper outer core.** Time-cylindrical radius diagrams of the column-averaged azimuthal flow acceleration $\partial u_c / \partial t$ (blue is westwards, see Methods for definitions) evaluated at two analysis longitudes, 31.5°E (as in Fig. 3b) and 37.5°E. Similar to ref. ¹⁷, green and brown curves respectively represent the ray-tracing theoretical propagation tracks of hydromagnetic waves at the column-averaged Alfvén speed c_A , and of material advection at the column-averaged cylindrical radial fluid velocity V_s . The slanted black line on the left panel denotes upward propagation at a speed 10 km/yr.



Supplementary Figure 2: **Scaling of jerk energy along the parameter space path.** Evolution of jerk energy at recurrence times 10, 30 and 100 years, as extracted from Fig. 4b, with the path parameter ϵ (Methods and Supplementary Table 1).



Supplementary Figure 3: **Sensitivity of the jerk recurrence time distribution to physical conditions in the lower mantle and upper outer core.** Distribution of the average recurrence time of jerks reaching or exceeding a given energy (same as Fig. 4b), for the models Midpath-I and Midpath-H with variable lower mantle electrical conductance, and model Midpath-S with a stratified region in the upper outer core (Methods). The Midpath model result from Fig. 4b is also reproduced for reference.



Supplementary Figure 4: **Signature of simulated jerks in the length of the day.** **a**, Squared acceleration $(d^2\Gamma_M/dt^2)^2$ (see Methods for definitions) of the electromagnetic torque exerted on the mantle by the outer core, as a function of time in the vicinity of four jerk events of model Midpath-H. **b**, first time derivative $d(\text{LOD})/dt$ of the simulated length of the day, as a function of time during the same jerk events. Arrows locate the pulses in the torque accelerations, that correspond to rapid inflexions in the rate of change of the length-of-day.

Supplementary Movie 1: Hammer projection of the core surface radial secular geomagnetic acceleration (orange is outwards) from the Midpath model, filtered at spherical harmonic degree 9, in the vicinity of the jerk event occurring at time 0 yr.

Supplementary Movie 2: Hammer projection of the core surface azimuthal flow acceleration (blue is westwards) from the Midpath model, in the vicinity of the jerk event occurring at time 0 yr.

Supplementary Movie 3: Partial equatorial cut (left) and meridional cut outside the tangent cylinder (right) of the convective density anomaly (orange denotes lighter fluid) from the Midpath model in the vicinity of the jerk event occurring at time 0 yr. The meridional cut in the right panel is taken at the analysis longitude marked by a black line in the left panel.

Supplementary Movie 4: Partial equatorial cut (left) and meridional cut outside the tangent cylinder (right) of azimuthal flow acceleration (blue is westwards) from the Midpath model in the vicinity of the jerk event occurring at time 0 yr. The meridional cut in the right panel is taken at the analysis longitude marked by a black line in the left panel.