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PAPER

Free-fall of photons in a planar optical cavity

Maxime Richard

Univ. Grenoble Alpes, CNRS, Grenoble INP, Institut Néel, Fr-38000 Grenoble, France

E-mail: maxime.richard@neel.cnrs.fr

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Abstract

We consider a planar optical cavity with one of its in-plane axis aligned with the Earth's local gravity field. We examine the motion of a wavepacket of light trapped within the cavity spacer under the action of gravity. Using a general relativity framework, we find that the wavepacket initially at rest in terms of its in-plane group velocity, is subject to a non-relativistic free-fall motion of acceleration **g** in the Earth-bound reference frame.

1. Introduction

The effect of gravity on propagating electromagnetic waves leads to several fascinating phenomena such as gravitational lensing [1], redshift [2] and time dilation [3]. These effects are often associated with light propagating freely over macroscopic distances; and yet, they can also show up in photonic devices designed for this purpose such as interferometers. Thus for instance, the Sagnac interferometer is routinely used nowadays as an acceleration measurement device in inertial navigation [4]. A more fundamental example are the giant interferometers used in the LIGO-Virgo collaboration, capable of detecting gravitational wavetrains of astrophysical origin [5–7].

Another class of photonic device is less frequently considered and yet increasingly relevant in this context: that of optical resonators (i.e. Fabry–Perot optical cavities). Unlike interferometers, these devices are capable of storing a wavepacket of electromagnetic field as a standing wave, i.e. in a non-propagating state of light occupying a small volume within the cavity for a long time. In state-of-the-art cavities, this storage time has become strikingly large: in the near-visible domain, photons have been stored for several microseconds in whispering-gallery modes in solid-state cavities of large diameters [8–11], up to a record lifetime of 50 μs [12]. In the microwave domain, photons can be stored nowadays in a confocal superconducting cavity for as long as 0.13 s [13] which, once unfolded, correspond to a free space travel as large as 39000 km. Such space and time scales suggest that the field stored in these cavity might be influenced by earth's gravity in a measurable way.

With this idea in mind, a subclass of optical cavities is of particular conceptual interest: the large size ones that exhibit translational invariance along one or two dimension (2D). This is for instance the case of semiconductor planar optical cavities [14], or of optical-fiber-like cylindrical waveguides. We will focus in the next three sections on the two-dimensional (2D) case, without lack of generality. In a planar optical cavity, the kinetic properties of cavity photons maps accurately that of 2D massive particles freely moving in this 2D space [15]; Indeed, the textbook derivation of the cavity mode dispersion relation can be written as $\hbar\omega(\mathbf{p}_\parallel) = \sqrt{|\mathbf{p}_\parallel|^2c^2 + m_\parallel^2c^4}$, that involves a well-defined rest-energy and kinetic-energy terms associated to this in-plane motion, where $m_\parallel = \hbar\omega_0/c^2$ is the effective photon mass that, unlike in free space, correctly describes its rest energy, $\omega_0/2\pi$ is the cavity resonance at $k_\parallel = 0$, $\mathbf{k}_\parallel = \mathbf{p}_\parallel/\hbar$ is the in-plane wavenumber, and $\mathbf{p}_\parallel$ the in-plane momentum. Note that there are other longitudinal modes present in the cavity that potentially break up this description. However when the modes are well split from each others (i.e. large Finesse cavity), and for low wavenumber $k_\parallel$, photons in a given longitudinal mode number j cannot jump into the neighbouring modes $j \pm 1$ without breaking momentum and energy conservation.

In this work, in the first three sections, we examine theoretically the motion of a wavepacket of light living within the spacer of such a planar cavity, when at least one of the two free dimensions is aligned with the gravity

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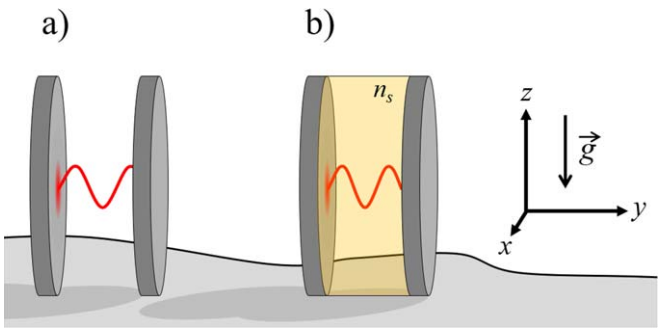

Figure 1. Illustration of the system considered in this work. The two mirrors of a planar Fabry–Perot cavity are shown standing vertically, i.e. parallel to the Earth's gravity field g, assuming an empty spacer in (a) as considered in section 2, and a spacer filled by a solid-state material of refractive index n_s in (b), as considered in sections 3 and 4. The axes convention and the direction of the gravity field are shown on the side. The size of the mirrors and the thickness of the spacer are not at scale (see section 4 for a realistic experimental implementation).

field vector **g**. In agreement with the equivalence principle, and upon taking care of applying it only along the translationally invariant directions, we show that the wavepacket of light undergoes free fall within the cavity spacer. Since photons are effectively stopped and put at rest with respect to the laboratory frame by the cavity, this free fall motion has a non-relativistic regime at low group velocity that matches our everyday life experience of the Newtonian free fall.

2. Gravitational free-fall in an empty cavity

The setup which is considered is that depicted in figure 1(a): a planar optical cavity with infinite size mirrors (i.e. much larger than the in-plane size of the trapped wavepacket of light. See section 4 for details) is lying at rest in the laboratory frame at the surface of the Earth, with its main axis (y) parallel with the horizon, and one of the in-plane axis (z) oriented parallel to **g**. The cavity embeds a long lived wavepacket of light which is initially at rest in terms of its in-plane group velocity $v_g^z(t=0) = \hbar k_\parallel/m_\parallel = 0$ in the laboratory frame.

Before going further, two properties that stem from the translational invariance of the cavity need to be underlined as they will prove crucial: (1) An observer situated outside the cavity can measure the energy and momentum of the photons inside the cavity, using the photons leaking throughout the imperfectly reflecting mirrors. Indeed, both the energy and the in-plane momentum of the light is conserved in this inside/outside weak coupling mechanism as a result of the cavity translational invariance [16]. (2) In the equivalence principle that can be formulated for this system, the motion along (z) or (x) of the two mirrors forming the cavity is irrelevant as, in a vacuum-filled cavity, it leaves the problem invariant. Note that in this idealized model, we disregard the light induced electronic excitations within the metallic material constituting the mirrors, otherwise statement (2) would not hold. We will see in the section 3 how having a light-excited (polarizable) material in the cavity indeed breaks down the equivalence principle.

With these precisions in mind, we can formulate the equivalence principle between two experimental situations (i) and (ii) as follows. (i) A wavepacket of light is excited inside the spacer of a cavity situated at the surface of the Earth. The wavepacket propagates freely inside the cavity along z under the influence of the gravitational field of acceleration $\mathbf{g} = -g\mathbf{u}_z$. An observer (that we identify with a detector) is at a fixed position in the Earth-bound reference frame (i.e. at the surface of the Earth) and measures the wavepacket motion. (ii) A wavepacket of light is excited inside the spacer of a cavity which is situated in a portion of space where the gravitational field is zero. An observer, and the cavity mirrors, are subject to an artificial acceleration of magnitude $-\mathbf{g} = g\mathbf{u}_z$, while the wavepacket is not (cf point (2) above). According to the equivalence principle, (i) and (ii) are strictly equivalent, and the observer will obtain the same results. (i) describes the experimental situation we are interested in in this work, and (ii) helps us understand that in (i), the observer should see the wavepacket move away, with an acceleration **g**.

In order to get a formal derivation of this gravity induced dynamics, we determine the intracavity dispersion relation in a general relativity framework. Since we restrict ourselves to a small intracavity volume $\mathcal{V}$, and elevation z above the surface of the Earth, as compared to the Schwarzschild metric curvature, we can use the Newtonian limit of the metric with the line element [17]:

$$ds_{\mathcal{V}}^2 = -[1 - r_s/(r_t + z)]c^2dt^2 + [1 - r_s/(r_t + z)]^{-1}dz^2 + dx^2 + dy^2, \quad (1)$$

where $r_s = 2GM_t/c^2 = 2gr_t^2/c^2 \ll r_t$ is the Earth Schwarzschild radius, r_t is the Earth radius, M_t is the Earth's mass, G is the gravitational constant, and $g = 9.81$ m.s^{-2} is the gravitational acceleration at the surface of the Earth. $z = 0$ is set at the surface of the Earth ($z \ll r_t$). We will see in the last section that the in-plane size of $\mathcal{V}$ that fixes the required mirror radius, is essentially fixed by the intracavity in-plane expansion of the wavepacket resulting from diffraction. $\mathcal{V}$ is indeed determined such that the fraction of the field reaching the mirror edges is negligible at any time during the cavity storage lifetime.

In order to derive the dispersion relation, we adopt a technique based on the Lagragian action principle developed in [18], which has the advantage of properly accounting for a possible polarizability of the medium, as will be done in section 3. In the present section, we consider an empty cavity. Using the aforementioned elements for the metric tensor, the transverse field dispersion relation reads

$$f_T = -g^{00}\omega^2 + c^2(k_z^2 g^{zz} + [k_y^j]^2 + k_x^2) = 0, \tag{2}$$

where

$$g^{00}(z) = g^{zz}(z)^{-1} = 1 - \frac{r_s}{r_t + z}, \tag{3}$$

and $\mathbf{k}_\parallel = (k_x, k_z)$. Since the metric is assumed flat along the cavity axis y, i.e. $g^{yy} = 1$, the cavity thickness L_c does not depend on the observer height z. The constructive interference along the cavity main axis thus leads to $k_y^j = j\pi/L_c$ where j is fixed the longitudinal mode number. Note that we have assumed a scalar field for the sake of simplicity, and thus neglect in this work a potentially rich, but much more complicated physics pertaining to the polarization degree of freedom. This approximation is well justified here, as the electromagnetic field oscillation plane tilt angle $\simeq c\sqrt{k_x^2 + k_z^2}/\omega_0$ with respect to the mirror plane remains very small at all time.

The dispersion relation can thus be rewritten as

$$f_T = -g^{00}(z)\omega^2 + c^2(k_z^2 g^{zz}(z) + m_\parallel^2 c^2/\hbar^2) = 0, \tag{4}$$

where $k_x = 0$ is assumed for simplicity and without loss of generality, and $m_\parallel$ is defined like in the previous section. This expression is the dispersion relation of light within the cavity, accounting for the gravitational correction. The group velocity of the wavepacket can be derived from equation (4) as [18]

$$v_g^z = \dot{z} = \frac{\partial f_T}{\partial k_z}\left[\frac{\partial f_T}{\partial \omega}\right]^{-1}. \tag{5}$$

A relation on the time derivative of the wavevector k_z can also be derived, resulting from the fact that the wavepacket is subject to gravitational acceleration:

$$\dot{k_z} = -\frac{\partial f_T}{\partial z}\left[\frac{\partial f_T}{\partial \omega}\right]^{-1}. \tag{6}$$

Equations (4), (5) and (6) provide a complete description of the wavepacket dynamics inside the cavity plane. We can for instance calculate the wavepacket acceleration $\ddot{z}$ within the cavity. To do so, we take the time derivative of equation (5), and express it as a function of v_g^z, and k_z as explained in detail in the appendix. We obtain a lengthy expression that simplifies into

$$a_g^z = \ddot{z} = \frac{-g}{[g^{00}(z)]^3} + \mathcal{O}\left(\frac{c^2 k_z^2}{\omega^2}\right), \tag{7}$$

where we have used the non-relativistic approximation $\epsilon = \dot{z}/c \ll 1$, which can be reformulated according to equation (5) as $ck_z/\omega \ll 1$. Then, in the limit of short distance free fall as compared to the Earth radius, it turns out that $g^{00}(z) \simeq 1$ and (cf appendix):

$$a_g^z = -g, \tag{8}$$

up to the second order in ϵ. We thus see that a wavepacket of light confined in a cavity plane with an initially vanishing in-plane group velocity, will exhibit first a non-relativistic motion, in which it will fall towards the ground with the same acceleration $\mathbf{g}$ as a any solid object, without qualitative nor quantitative distinction. This derivation also show that our 2D-restricted equivalence principle is well preserved, even in this limiting situation where the particle 2D effective inertial mass emerges from the confinement in the cavity plane. While we did not verify it, we expect this equivalence principle to hold in the relativistic regime as well.

3. Gravitational free-fall in a solid-state cavity

For practical purposes, we wish to examine how this free fall motion is modified when the cavity spacer is filled with a solid-state polarizable medium of refractive index $n_s > 1$ as depicted in figure 1(b), which is the case of most state-of-the-art optical cavities in the near-visible domain. To describe this situation, the perturbation of

the electrons rest position in the medium is added to the action to be minimized [18]. In our specific geometry, this method yields the following dispersion relation :

$$f_T^s = (1 - g^{00}(z) - n_s^2)\omega^2 + c^2(k_z^2 g^{zz}(z) + m_\parallel^2 c^2/\hbar) = 0. \tag{9}$$

Note that due to the prefactor in front of ω, the inertial mass is modified from its vacuum expression into $m_\parallel^s = n_s m_\parallel$, while the speed of light changes into $c_s = c/n_s$. We now carry out the same derivation as in section 2 to determine the dynamics of the wavepacket in this polarizable medium. Upon applying the non-relativistic approximation, we end up with the following gravity-induced acceleration of the wavepacket

$$a_g^z = \ddot{z} = \frac{-g}{g^{00}(z)[1 - n_s^2 - g^{00}(z)]^2} + \mathcal{O}\left(\frac{c^2 k_z^2}{\omega^2}\right), \tag{10}$$

that simplifies into

$$a_g^z = -g/n_s^4, \tag{11}$$

up to the second order in ϵ and $g^{00}(z)[1 - n_s^2 - g^{00}(z)]^2 \simeq n_s^4$. Thus, upon filling the cavity spacer with a polarizable material, our wavepacket of light still undergoes a Newtonian free fall but with a slowed down acceleration $g_s = g/n_s^4$. This feature can be understood by the fact that the electronic motion excited by the intracavity light and generating the polarization field are fixed in the cavity frame and thus are not in free fall, unlike the electromagnetic field. The earlier thus backacts on the field as a kind of drag force as pointed out already in the context of light propagation in transparent moving media [19].

Note that when n_s depends on the frequency (in a so-called dispersive medium), the notion of effective mass, or in-plane rest mass, associated with the in-plane wavepackets motion, also breaks down as this feature has the effect of altering the dispersion relation. A paradigmatic examples of this situation is the strong coupling regime in planar semiconductor microcavities [20], in which n_s modifies the dispersion relation so much that it opens a spectral gap in the vicinity of the bound electron-hole pair transition.

4. Experimental signature

In this section, we examine whether this non-relativistic free fall is actually observable in state-of-the-art optical cavities. To carry out this measurement, we can exploit the fact that, unlike classical solid bodies, we have access to the phase $\phi(z, t)$ of the wavepacket, of which k_z is the gradient along z. The principle of using interferometry to detect gravity induced dynamics on light is not new, it has been discussed and applied extensively already in different contexts [21–24]. Here we apply this principle as an experimental method to detect our cavity-induced non-relativistic dynamics. The expression for $\dot{k}_z$ is derived in the appendix and simplifies into

$$\dot{k}_z = -\frac{g\omega_0}{n_s^2 c^2} = -\frac{g_s\omega_0}{c_s^2}, \tag{12}$$

in the non-relativistic approximation. The dependence in n_s shows that the gravity-induced dynamics is slower in a dielectric than in vacuum. Yet, in both cases, the gravitational field builds-up a phase gradient over time that can be detected into an interferometric measurement. Assuming a quality factor $Q = 1 \times 10^{10}$, the phase gradient achieved within the intracavity lifetime is of the order of 10^{-6} rad.m^{-1}. This is a very small phase shift, that might prove challenging to detect, but not impossible. A phase resolution $< 10^{-4}$ rad is typically achievable in well-controlled but standard interferometric setup [25].

Let us examine a possible implementation of such a measurement: owing to their record performance in the visible domain, we choose to consider a solid-state cylindrical cavity as depicted in figure 2(a). Based on results from several groups [8, 9, 12], similar structures made up of ultra pure materials like CaF_2 or MgF_2 support whispering gallery mode (WGM) with a quality factor as large as $Q = 6.1 \times 10^{10}$ [12]. Note that the results we have derived so far can be transposed directly from planar into cylindrical geometry. In cylindrical cavity, the light is confined into whispering gallery modes in the transverse direction, and free to propagate along the main axis (with a scalar momentum k_z), which is thus oriented vertically along z.

The experiment would be carried out in the following way: at $t = 0$ a spatially Gaussian pulse of light (of initial waist σ_0 along z), is injected in the middle of the cylindrical cavity with an initial momentum $\hbar k_z = 0$, in the chosen whispering gallery mode by evanescent coupling. The free evolving wavepacket is then going to be subject to two different dynamics. The first one has nothing to do with gravity, and results from the laws of diffraction: the mode waist $\sigma(t)$ is going to expand in both directions of the WGM cavity as $\sigma^2(t) = \sigma_0^2(1 + [\lambda ct/(\pi\sigma_0^2 n_s^2)^2])$ [26]. We take advantage of this effect to couple the light out of the WGM only after a singificant delay fixed by this expansion, using grating-like diffracting elements placed at both ends of the waveguide (in z_0 and $-z_0$). Note that in absence of gravity this free expansion does not introduce any phase difference between (z) and $(-z)$ direction of the expansion.

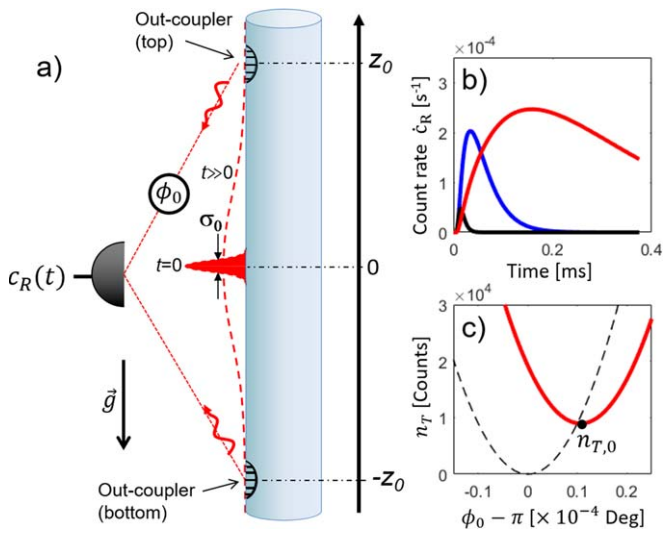

Figure 2. (a) Experimental setup for the measurement of Newtonian free fall in a cylindrical optical cavity. The WGM cavity has a diameter in the millimeter range in order to reach a giant quality factor [12]. (b) Calculated count rate $\dot{c}_R(t)$ as measured on the photodiode, following the injection of a short pulse of lightat $t = 0$, assuming $Q = 0.2 \times 10^{10}$ (black), $Q = 1 \times 10^{10}$ (blue), $Q = 6 \times 10^{10}$ (red). For this calculation we assumed a wavelength of $\lambda = 780$ nm, an initial waist of the injected WGM $\sigma_0 = 5$ mm, an out-coupling height $z_0 = 10$ cm, a refractive index $n_s = 1.43$ (CaF_2), and an outcoupler surface factor $s_{oc} = 5\%$. (c) Calculated time-integrated photons count number n_T versus initial phase difference ϕ_0 minus π, for $Q = 6 \times 10^{10}$ (red solid line). The non-zero minimum $n_{T,0}$ is shown as a black dot. The case $g = 0$ m.s^{-2} is shown for comparison as a black dashed line.

The second effect is the gravity-induced dynamics: the slowly increasing phase gradient builds up a phase difference between the two beams outcoupled at z_0 and $-z_0$. They are thus sent to interfere on a single photon detector with high quantum efficiency, such as an avalanche photodiode, in order to maximize the signal-to-noise ratio in this weak intensity regime. In absence of gravity, there is no time evolution of the phase difference. We thus insert a short tunable delay line in one of the arms of the interferometer, such that at $t = 0$, the initial phase difference between both paths is $\phi_0 = \pi$, and the interference is destructive. This tuning cancels any signal that is not due to the build-up of the gravity-induced phase difference.

The photon counts on the photodiode, can be simulated in this context within a simple rate equation model where

$$\dot{N} = -[\gamma_c + \gamma_{oc} s_{oc} \alpha(t)] N(t), \tag{13}$$

is the total loss rate of the photons outside the WGM cavity; $\gamma_c = \omega_0/Q$ is the nominal cavity loss rate, γ_{oc} is the outcoupling rate throughout the diffractive elements. $\alpha(t)$ is the overlap integral between the gaussian mode and the diffractive elements, and s_{oc} is a surface factor characterizing these elements size. The count rate on the photodiode then reads

$$\dot{c}_R = \gamma_{oc} s_{oc} \alpha(t) \{1 + \cos[\phi_0 + \Delta\phi(t)]\} N(t), \tag{14}$$

where $\Delta\phi(z_0, t) = 2 z_0 \omega_0 g_s n_s^2 t / c_s^2$ is the gravity induced time-dependent phase shift between points z_0 and $-z_0$ given by equation (12). We solved this model numerically using the realistic parameters given in the caption of figure 2. Examples of calculated $\dot{c}_R(t)$ are shown in figure 2(b) for $Q = \{0.2, 1, 6.0\} \times 10^{10}$. At early time, the plot shows a slow increase in count rate resulting from the gravity induced phase dynamics. In an experiment in which the aim is to demonstrate the free fall phenomenon, we do not need this time resolution, and just need to find a non-zero total photon counts n_T that exceed the experimental noise. It can be calculated as $n_T = n_p \int \mathrm{d}t \dot{c}_R(t)$, where $n_p = 2.7 \times 10^{11}$ is the number of pulses contributing during a 1h integration time. The results are summarized in the following table:

$Q(\times 10^{10})$	0.2	1.0	6.0
n_T	200	3750	19220
S_n	14	61	138

where $S_n = \sqrt{n_T}$ is the signal-to-noise ratio in a shot-noise-limited measurement. These number are highly encouraging even though it should be kept in mind that in realistic experiments, other source of noises like e.g. the acquisition electronics can spoil S_n significantly. A correct tuning of the initial phase difference to $\phi_0 = \pi$ at $t = 0$ could alos be hard to achieve experimentally. A simple way around this requirement, at the cost of a longer experiment, is to record n_T upon sweeping ϕ_0, in order to determine the time-integrated signal minimum $n_{T,0}$ of $n_T(\phi_0)$. Indeed in absence of gravity $n_{T,0} = 0$, while it takes a strictly positive value in presence of gravity as is

shown in the simulation in figure 2(c) realized for $Q = 6.0 \times 10^{10}$. Note that a continuous wave version of these measurements would give similar results. In fact, the major challenge to be overcome for a realistic implementation of these experiments is the fabrication of a WGM cavity with negligible fluctuations of the diameter along (z), that could otherwise cause the free fall to slow, stop, or even bounce back.

This proposed approach is of course not unique, superconducting cavities hold a record in terms of storage time, that could be exploited using observable that does not depend on the wavelength of light like in an interferometric measurement, like tracking the intensity maximum of the wavepacket over time. in another approach, we could also give up the need for translational invariance of the cavity. Indeed, while we have not considered the case in this work, we expect that gravity couples transverse modes in 3D confined cavity that feature a discrete photonic density of state along (z), like microdisks [8], microtorus [9] or microspheres [10], or any equivalent system in the microwave domain [11]. In this approach, we expect a slow transfer from one vertical transverse mode to the next as a signature of the free fall.

5. Conclusion

In summary, we have shown that when a planar or cylindrical optical cavity is oriented adequately in a gravitational field, a wavepacket of light initially at rest within the resonator undergoes a non-relativistic free-fall motion: its group velocity increase as gt like any solid object of finite mass. In order to examine the feasibility of detecting this motion experimentally, we envisaged a large diameter solid-state cylindrical cavity with state-of-the-art performances, and a detection method based on interferometry. Using realistic experimental parameters taken from the literature, we simulated this setup and found that the intracavity free fall motion is in principle observable in a shot-noise limited measurement.

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Appendix

A.1. Detailed derivation of a wavepacket group acceleration $\ddot{z}$ in a solid-state optical cavity of index n_s

We present here a detailed derivation of equation (10), that describe the in-plane gravitational acceleration of the wavepacket in a dielectric-filled cavity (of index $n_s > 1$) respectively. The results for a vacuum cavity can be determined from here by setting $n_s = 1$. We first expand explicit equation (5) using the expression of f_T^s in equation (9), which yields

$$\dot{z} = \frac{c^2 k_z}{\omega F H}, \tag{15}$$

where we define $F(z, t) \equiv g^{00}(z, t)$, and $H(z, t) \equiv 1 - F(z, t) - n_s^2$ for the sake of compactness. We also expand equation (6) into

$$\dot{k}_z = -g\frac{\omega^2/c^2 + k_z^2/F^2}{\omega H}. \tag{16}$$

Upon taking the time derivative of equation (15) we obtain three terms :

$$\ddot{z} = \frac{c^2}{\omega F H}\left\{\dot{k}_z - \frac{k_z}{\omega}\dot{\omega} - \frac{2gk_z(F-H)}{c^2 F H}\dot{z}\right\}, \tag{17}$$

where $\dot{k}_z(k_z, \omega)$ is given by equation (16), and $\dot{F} = 2g\dot{z}/c^2$, where $z \ll r_t$ has been assumed. $\dot{\omega}(,\dot{k}_z, \dot{z})$ can be be derived by taking the time derivative of equation (9), which results in

$$\dot{\omega} = \frac{-g}{c^2 H\omega}\left(\omega^2 + \frac{c^2 k_z^2}{F^2}\right)\dot{z} - \frac{c^2 k_z}{FH\omega}\dot{k}_z. \tag{18}$$

We can now inject equations (15), (16), and (18) into (17). After some algebra it simplifies into

$$\ddot{z} = \frac{-2g}{F}\left\{\frac{1}{2H^2} + \frac{c^2 k_z^2}{F^2 H^2 \omega^2} + \frac{(F-H)k_z^2 c^2}{F^2 H^3 \omega^2} - \frac{c^2 k_z^2}{2FH^3\omega^2}\left(1 + \frac{c^2 k_z^2}{\omega^2 F^2}\right)\right\}. \tag{19}$$

In this work, we are interested in the non-relativistic limit of the in-plane wavepacket motion, for which the group velocity $v_g = \dot{z} \ll c$. From equation (15), this limit yields imposes $ck_z/(\omega FG) \ll 1$. up to the second order in this ration, $\ddot{z}$ equation thus simplifies considerably into

$$\ddot{z} = \frac{-g}{FH^2} = \frac{-g}{F(1 - F - n_s^2)^2}. \tag{20}$$

We restrict ourselves to propagation free fall distance much shorter than the Earth radius $z \ll r_t$, $1 - F(z) \simeq 2gr_t/c^2 \simeq 1.4 \times 10^{-13} \ll 1$. Equation (20) thus simplifies into

$$\ddot{z} = -g/n_s^4. \tag{21}$$

In vacuum, in the same non-relativistic limit, the wavepacket of light undergoes a regular free fall of acceleration g.

ORCID iDs

Maxime Richard https://orcid.org/0000-0001-9596-2891

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