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# **Infinite-length implementation of linear chip-level equalizer by blind recursive filtering for the DS-CDMA downlink**

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*Abstract* This paper deals with a receiver for the DS-CDMA downlink. User signals are transmitted synchronously using orthogonal spreading codes. Multipath propagation of the transmission channel generates inter-chip interference which deteriorates performance. Linear chip-level equalization followed by a filter matched to the code of the desired user allows performance to be improved. We show that an infinite-length implementation of the linear MMSE chip-level equalizer can be achieved using two recursive filters optimized from a blind criterion. Simulation results over a multipath Rayleigh channel show that the proposed receiver outperforms the finite-length implementation of the linear MMSE receiver and the ideal RAKE receiver.

## I. INTRODUCTION

In the downlink of direct spread code division multiple access (DS-CDMA) systems, the users within a same cell are transmitted synchronously using orthogonal spreading codes. In a single-cell system, interference is then exclusively created by multipath components, generating both multiple access interference between users and inter-chip interference for a single user. Linear receivers have shown their efficiency in restoring orthogonality among users by reducing the distortion induced by the multipath

channel. These linear receivers are built with a linear chip-level equalizer followed by a matched filter to the spreading code of the desired user [1-9], referred as code-matched filter in the following. When the number of users is large and/or the spreading factor is weak, performance of these linear receivers is better than the performance of the RAKE receiver.

In practice, the chip-level equalizers are implemented from finite-length discrete filters. So, the linear chip-level equalizer is often implemented by a transversal filter leading to a sub-optimum realization. In this paper, we show that linear receiver based on chip-level equalization can be optimally realized by using two recursive filters, a matched filter to the transmission channel and a matched filter to the code of the desired user. From this theoretical approach, we propose to implement the receiver with two recursive blind adaptive filters and a classical RAKE receiver.

Section II describes the transmission scheme for the DS-CDMA downlink. A structural decomposition of linear receivers for the downlink DS-CDMA is given in section III. Section IV proposes a low-complexity implementation for this receiver. Section V presents the performance of the proposed receiver and gives a comparison with the RAKE receiver performance over a multipath Rayleigh fading channel. Section VI gives our conclusions.

## II. SIMULATION SCHEME

The information data of user  $k$  are mapped on the symbol sequence  $a_k(nT_k) \{(\pm 1 \pm j)/\sqrt{2}\}$  with variance  $\sigma_a^2$  at rate  $1/T_k$ . Amplitude  $A_k$  and spreading code  $\mathbf{c}_k$  are assigned to user  $k$  of the system. The spreading codes of the  $K$  users of a same cell are assumed orthogonal. Spreading code  $\mathbf{c}_k$  is a vector of

$F_k$  chips  $c_k[pT_c]$  where  $T_c$  is the chip duration,  $F_k$  is the spreading factor of user  $k$ , and  $T_k = F_k T_c$ . After spreading, the contributions of the  $K$  users are summed and scrambled by a long pseudo-random complex sequence whose  $p$ th chip is  $s[pT_c]$ . Then, we define the chip-rate multi-user symbol as  $d[pT_c]$

$$d[pT_c] = s[pT_c] \sum_{k=1}^K A_k a_k[pT_c] c_k[pT_c] \quad (1)$$

where  $a_k[pT_c] = a_k(nT_k)$  when  $pT_c \in [nT_k; (n+1)T_k]$ .

Because of the scrambling sequence, the multi-user symbols can be modeled as stationary, white, circular and zero-mean with power  $\sigma_d^2$  given by

$$\sigma_d^2 = \sigma_a^2 \sum_{k=1}^K \frac{A_k^2}{F_k} \quad (2)$$

Multi-user symbols are transmitted through a time-invariant discrete equivalent transmission channel  $H(z) = \sum_{l=0}^L h_l z^{-l}$ , which encompasses the effects of pulse shaping, multipath channel and reception filtering. Both thermal noise and inter-cell interference are modeled by a complex additive white Gaussian noise  $w[pT_c]$ , stationary and circular, with zero mean and variance  $\sigma_w^2$ .

The observation at the receiver input has the following expression

$$r[pT_c] = \sum_{l=0}^L h_l d[(p-l)T_c] + w[pT_c] \quad (3)$$

Interference between multi-user symbols impairs the orthogonality of the spreading codes leading to poor performance when a code-matched filter is used to detect the user information. Improvements can be obtained by using a RAKE receiver or, even better, a linear chip-level equalizer followed by a filter matched to the code of the desired user. This latter receiver is depicted in figure 1 and detects the user

associated with spreading code  $m$ . The decision on the information symbol of user  $m$  at the output of the receiver is denoted  $\hat{a}_m(nT_m)$ .

Zero forcing (ZF) or minimum mean square error (MMSE) equalizers can be considered. In a DS-CDMA context, the power of the noise  $\sigma_w^2$  includes extra-cell interference and thermal noise, which are never equal to zero. Because a ZF equalizer tends to increase the power of noise at its output, it is usually more attractive to use an MMSE equalizer which makes a compromise between interference suppression and noise enhancement [3][5][8].

Then, the optimum linear filter  $F(z)$  can be optimized following the MSE criterion

$$\min_{F(z)} E\left\{\left|y[pT_c] - d[pT_c]\right|^2\right\} \quad (4)$$

where  $y[pT_c]$  denotes the output of the equalizer  $F(z)$ .

The linear MMSE chip-level equalizer  $F(z)$  is given by the Wiener formula

$$F(z) = \sigma_d^2 H^*(1/z^*) \left[ H(z) H^*(1/z^*) \sigma_d^2 + \sigma_w^2 \right]^{-1} \quad (5)$$

where the superscript  $*$  stands for complex conjugate.

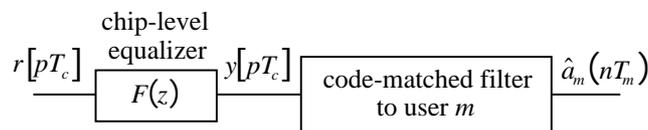


Fig 1. Linear MMSE receiver (LMMSE) for the DS-CDMA downlink

### III. STRUCTURAL ANALYSIS OF THE LINEAR MMSE RECEIVER

In practice, the linear MMSE receiver presented figure 1 has to be implemented by a finite-length chip-level transversal filter followed by a code-matched filter [3][6][7]. Because of the finite-length of the equalizer, the transversal linear MMSE receiver (T-LMMSE) is sub-optimum. Moreover, according to the transmission context, the transversal filter may need a very large number of coefficients to reach the performance of the infinite-length LMMSE receiver. Note that if the number of coefficients is too large, the complexity of the receiver can become prohibitive. In order to give a solution to this problem, we show that it is possible to implement an infinite-length version of the MMSE equalizer from recursive filters. The proposed receiver, called recursive linear MMSE (R-LMMSE) receiver, is realized from two recursive filters in front of a channel-matched filter followed by a filter matched to the code of the desired user, the whole structure being equivalent to the optimum linear MMSE receiver presented in section II.

In order to describe the R-LMMSE receiver structure, we use spectral factorization properties. There is a unique solution [10] to the following expression

$$H(z)H^*(1/z^*)\sigma_d^2 + \sigma_w^2 = SG(z)G^*(1/z^*), \quad (6)$$

when  $G(z) = 1 + \sum_{l=1}^L g_l z^{-l}$  is constrained to be a minimum phase filter. In this case,  $S$  is a positive value.

Then, the following relation gives the transfer function  $F(z)$  (4) of the infinite-length MMSE equalizer

$$F(z) = \frac{\sigma_d^2}{S} \frac{1}{G(z)} \frac{1}{G^*(1/z^*)} H^*(1/z^*) \quad (7)$$

As  $G(z)$  is a causal and minimum phase filter, i.e. with its roots inside the unit circle,  $1/G(z)$  can be implemented by a recursive stable filter. On the other hand, the transfer function  $G^*(1/z^*)$  is anticausal

and its roots are outside the unit circle, so it is impossible to implement directly  $1/G^*(1/z^*)$  by a recursive filter. Nevertheless, the filtering by  $1/G^*(1/z^*)$  of an input sequence can be decomposed into a time-reverse operation on the input sequence, a filtering by  $1/G(z)$  of the time-reversed input sequence and a new time-reverse of the filtered sequence [11][12]. Consequently, the infinite-length MMSE equalizer  $F(z)$  can be implemented as depicted in figure 2 using two recursive filters followed by a filter matched to the equivalent discrete channel. In figure 2,  $TR^*$  represents the time-reverse operation which takes a  $N$ -chips sequence at its input, and delivers the reverse and conjugate input sequence at its output.

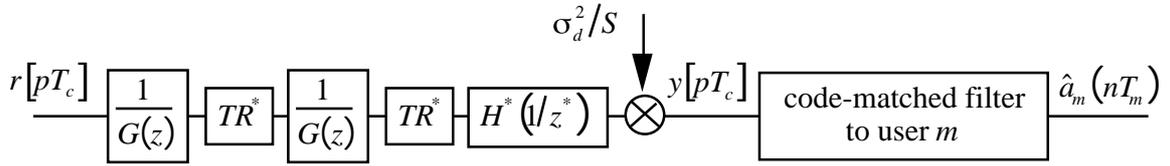


Fig 2. Recursive linear MMSE receiver (R-LMMSE)

#### IV. IMPLEMENTATION OF THE R-LMMSE RECEIVER

Firstly, let us discuss of the RAKE receiver which consists of “fingers”, each one corresponding to a delayed path of the received signal. In CDMA systems, the operator defines the number of fingers and the path searcher determines the delays. Since the chip period is very small, let us assume that delays are multiple of one chip period. Using this hypothesis, it can be demonstrate [1] that a RAKE receiver with  $L + 1$  fingers separated by a chip period is equivalent to the concatenation of the channel-matched filter with the filter matched to the code of the desired user. Moreover, if the  $L + 1$  coefficients of the discrete equivalent channel  $H(z)$  are known at the receiver side, the RAKE is ideal. Remarks that if delay between each path is lower than a chip period, it is possible to over-sample the input of the recursive filter at a rate higher than  $1/T_c$ . This modification can be realized from the results presented in [13].

Nevertheless, this latter modification increases the complexity of the receiver. For a practical point of view and to the detriment of optimality, the RAKE receiver can be implemented with a number of fingers lower than  $L + 1$ . According to the previous discussion, the R-LMMSE receiver depicted figure 3, can be seen as a pre-filtering in front of a RAKE receiver.

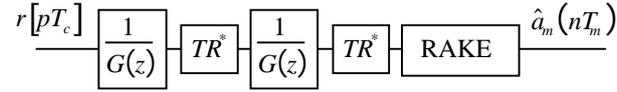


Fig 3. The recursive LMMSE receiver involves the RAKE receiver

Now, in order to compute the coefficients of filter  $G(z)$ , we propose a low-complexity blind method. Blind determination of the filter  $G(z)$  is interesting because none information on the system is needed. Let us denote  $e[pT_c]$  as the output of the recursive filter depicted in figure 4

$$e[pT_c] = r[pT_c] - \mathbf{G}^T \mathbf{E}_p \quad (8)$$

with  $\mathbf{G} = [g_1 \cdots g_L]^T$ ,  $\mathbf{E}_p = [e[(p-1)T_c] \cdots e[(p-L)T_c]]^T$ , and the superscript  $T$  stands for transposition.

We can show that the output of filter  $1/G(z)$  has a constant power spectral density [13][14]. Consequently, the recursive filter  $1/G(z)$  is the unique stable recursive filter whitener of its input  $r[pT_c]$ . This filter is obtained when vector  $\mathbf{G}$  minimizes the power of the predictive error at the output of the recursive filter

$$J(\mathbf{G}) = E\{[e[pT_c]]^2\} \quad (9)$$

There are a few algorithms available to minimize  $J(\mathbf{G})$ , e.g., the stochastic gradient algorithm. Because the filter is recursive, the gradient algorithm cannot be exactly computed, but the following relation gives an efficient approximate algorithm [15][16]

$$\mathbf{G}_p = \mathbf{G}_{p-1} + \mu_G e[pT_c] \mathbf{E}_p \quad (10)$$

where  $\mu_G$  is a step size and  $\mathbf{G}_0 = [0 \dots 0]^T$ .

This low-complexity solution has been often criticized [17][18] for its possible instability or wrong convergence. Effectively, if the zeros of  $G(z)$  are close to the unit circle, the step size is too large or the length of recursive filter is badly estimated, then the adaptive recursive filter can become unstable or converge to a local minimum. Nevertheless, in a DS-CDMA context, the power of the noise  $\sigma_w^2$  is never equal to zero and consequently the zeros of  $G(z)$  are rather distant from the unit circle, which confers a good robustness to the adaptive recursive filter. Besides, a step size lower than 0.003 and a number of coefficients greater than the channel length  $L$  for the recursive filter give excellent results. Since the maximum delay spread of the channel is roughly known for a given transmission environment, the latter constraint can be easily respected. In addition, this blind adaptive recursive filter was largely tested with success in real or simulated time-variant contexts without visible instability [13][14].

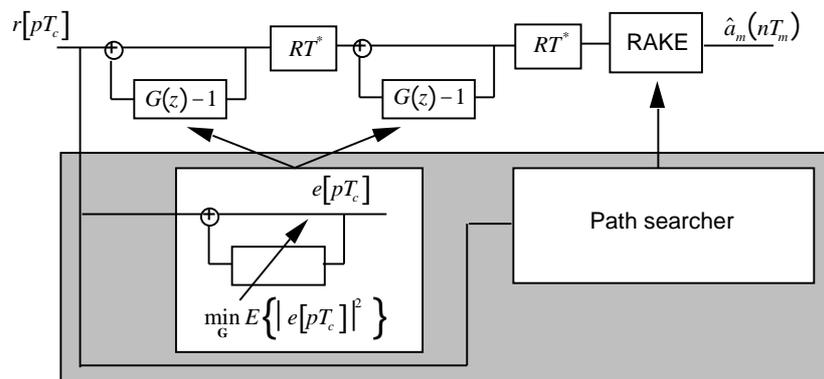


Fig 4. Implementation of the R- LMMSE receiver

Implementation of the R-LMMSE is depicted in figure 4. The time-reversed process constraints the R-LMMSE to work on a bufferized sequence of  $N$  chips. During this sequence, the channel must be invariant or slowly variant. Block computation is required which includes the following states:

- I. From a bufferized sequence of  $N$  samples  $r[pTc]$ , coefficients of filter  $\mathbf{G}$  are determined by the blind adaptive algorithm (10). After  $N$  iterations, we assume that vector  $\mathbf{G}$  is close to its optimum value and can be used in the R-LMMSE receiver.
- II. The bufferized sequence is filtered by the recursive filter. At its output, the  $N$ -length chip sequence is time-reversed and conjugated.
- III. The time-reversed sequence is filtered anew by the recursive filter, time-reversed, conjugate and presented at the input of the RAKE receiver.

## V. RESULTS

We plot the bit error rate (BER) versus the signal to noise ratio  $E_b/N_0$ , where  $E_b$  is the energy per bit of the desired user and  $N_0$  corresponds to the single-sided power spectral density of the additive noise. The MFB curve gives the performance of the matched filter bound.

Fig. 5 presents a performance comparison between the R-LMMSE and the T-LMMSE (see section III) over a time-invariant channel with 5 coefficients  $[2 - 0.4j; 1.5 + 1.8j; 1; 1.2 - 1.3j; 0.8 + 1.6j]$ . The T-LMMSE is realized with a 13 coefficients transversal equalizer followed by a code-matched filter. The R-LMMSE also has 13 coefficients, 4 coefficients in each recursive filter and 5 coefficients in the channel-

matched filter allowing an ideal RAKE receiver to be realized. The spreading factor is assumed the same for all users and equal to 32. As described in section IV, the coefficients of the equalizer are updated on a slot duration with  $N=2560$  chips.

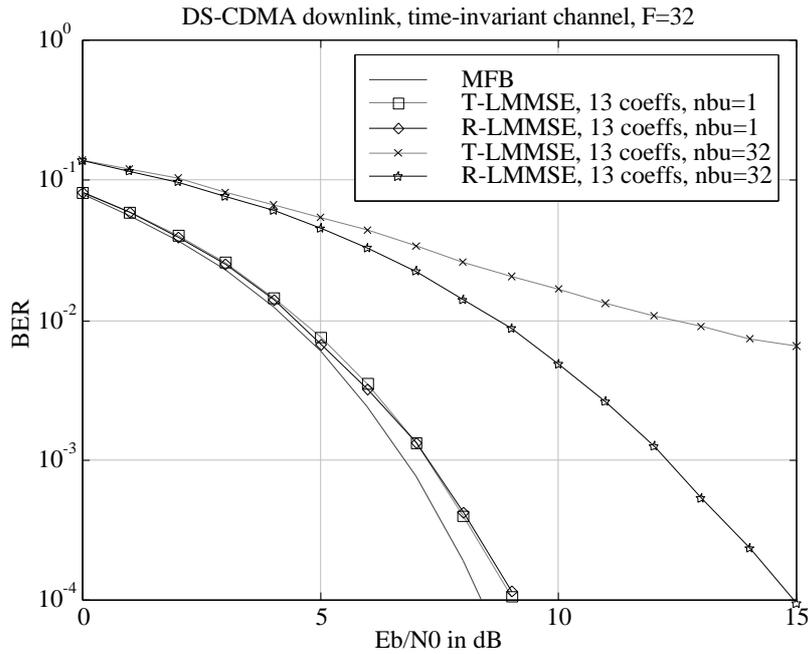


Figure 5. Performance comparison between the T-LMMSE and the R-LMMSE over a time-invariant channel.

For the simulation results shown in figure 5, the equivalent discrete channel is assumed known at the receiver side in order to have an optimal computation of the different filters. In the single user case, the R-LMMSE and the T-LMMSE receiver have the same performance. Nevertheless, when the cell is full-loaded, the R-LMMSE receiver allows better performance because it realizes an infinite-length chip equalizer whereas the T-LMMSE is based on a finite-length equalizer.

Figure 6 presents a performance comparison between the RAKE and the R-LMMSE receivers over a slowly Rayleigh fading channel. To simplify the simulation, the channel is assumed different over each slot and time-invariant over the slot duration. A slot includes  $N=2560$  chips which corresponds to the length of the UMTS slot. The ITU Vehicular A Rayleigh channel is considered, whose parameters are slightly modified in order to give path delay multiple of chip duration according to table 1.

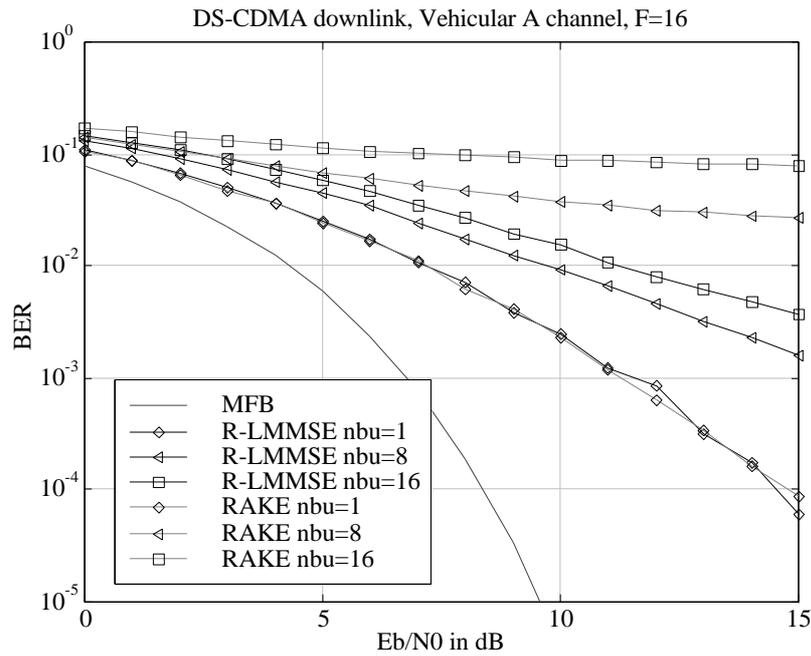


Fig. 6. Performance comparison between the R-LMMSE and the optimum RAKE over the Vehicular A channel.

Path	1	2	3	4	5	6
Delay ( $\times T_c$ )	0	1	3	4	7	10
Mean power ( $\times$ dB)	-3.14	-4.14	-12.14	-13.14	-18.14	-23.14

TABLE 1. Parameters of the multipath Rayleigh channel

The spreading factor is the same for all users and equal to 16. We consider three loads for the system: 1, 8 and 16 users. The recursive filter is computed from the adaptive blind algorithm proposed in section IV. There are 10 coefficients in each recursive filters and the step size of the adaptive algorithm (10) is equal to 0.003. The RAKE receiver is assumed ideal (section IV).

When there is a single user in the cell, performance of the RAKE and the R-LMMSE receivers is the same. In this case, the filter  $F(z)$  tends towards a channel-matched filter. When the number of users increases, a simple channel-matched filter for  $F(z)$  is insufficient. Since filter  $F(z)$  compensates the frequency selectivity of the transmission channel, the performance of the R-LMMSE is better than RAKE receiver performance.

## VI. CONCLUSIONS

This paper have presented an infinite-length implementation of the LMMSE receiver for the DS-CDMA downlink. From a structural analysis, the infinite-length LMMSE receiver has been decomposed as a RAKE receiver preceded by two recursive filters. A blind optimization of the recursive adaptive filters has been proposed which only needs the output of the discrete channel to work. No prior knowledge about the system such as the number of users in the cell, their spreading factors or the estimation of the noise variance is needed. Simulations results have shown that the proposed receiver, called R-LMMSE, is equivalent to the infinite-length LMMSE receiver. So, the R-LMMSE performance is always better than the performance of the RAKE receiver and especially when the system is strongly loaded. Note that in this paper, an ideal RAKE receiver has been considered which must be replaced in further work by a more classical implementation with only few fingers.

## REFERENCES

- [1] M. Bossert, T. Frey, "Interference Cancellation in the Synchronous Downlink of CDMA-Systems," *ITG-FACHTAGUNG: Mobile Kommunikation*, Sept. 1995.
- [2] A. Klein, "Data Detection Algorithms Specially Designed for Downlink of CDMA Mobile Radio Systems," *IEEE Vehic. Tech. Conf.*, vol. 1, pp.203-207, May 1997.
- [3] I. Ghauri and D.T.M. Slock, "Linear Receivers for the DS-CDMA Downlink Exploiting Orthogonality of Spreading Sequences," *32<sup>nd</sup> Asilomar Conf. On Signal, Systems and Computers*, vol. 1, pp.650-654, 1998.
- [4] C.D. Franck and E. Visotsky, "Adaptive Interference Suppression for Direct-Sequence CDMA Systems with Long Spreading Codes," *Proc. Annual Allerton Conf. On Communications, Control and Computing*, pp. 411-420, Sept 1998.
- [5] K. Hooli, M. Latva-aho, and M. Juntti, "Multiple access interference suppression with linear chip equalizers in WCDMA downlink receivers," in *Proc. IEEE Global Communications Conf*, Rio de Janeiro, Brazil, vol. 1, pp 467-471, Dec. 5-9 1999.
- [6] M. Heikkilä, P. Komulainen and J. Lilleberg, "Interference Suppression in CDMA Downlink through Adaptive Channel Equalization," *IEEE Vehic. Techn. Conf.*, VTC Fall, Amsterdam, Netherlands, pp. 978-982, Sept. 1999.
- [7] S. Werner and J. Lilleberg, "Downlink Channel Decorrelation in CDMA Systems with Long Codes," *IEEE Vehic. Techn. Conf.*, Houston, Texas, USA, pp 1614-1617, May 1999.
- [8] T.P. Krauss, M.D. Zoltowski, and G. Leus, "Simple MMSE equalizers for CDMA downlink to restore chip sequence: Comparison to Zero-Forcing and RAKE," in *ICASSP*, vol. 5, pp. 2865-2868, May 2000.

- [9] F. Petré, G. Leus, L. Deneire, M. Engels and M. Moonen, "Adaptive Space-Time Chip-Level Equalization for WCDMA Downlink with Code-Multiplexed Pilot and Soft Handover," *IEEE Int. Conf on Com.*, Vol. 3, pp. 1635-1639, 2002.
- [10] J.M. Cioffi, G.P. Dudevoir, M. Vedat Eyuboglu, G.D. Forney Jr., "MMSE Decision-Feedback Equalizers and Coding – Part I: Equalization Results," *IEEE Trans. on Commun.*, Vol 43, NO. 10, Oct. 1995.
- [11] A.V. Oppenheim and R.W. Schaffer, "Discrete-time signal processing," Prentice Hall Inc., 1989.
- [12] S. Ariyavisitakul, "Equalization of a Hard-Limited Slowly-Fading Multipath Signal Using a Phase Equalizer with a Time-Reversal Structure," in *IEEE Vehic. Techn. Conf.*, pp. 520-526, 6-9 May 1990.
- [13] J. Labat, C. Laot, "Blind Adaptive Multiple Input Decision Feedback Equalizer: a Self-Optimized Configuration," *IEEE Trans. on. Com.*, Vol. 49, No 4, pp. 646-654, April 2001.
- [14] J. Labat, O. Macchi and C. Laot, "Adaptive Decision Feedback Equalization: Can you Skip the Training Period? ", *IEEE Trans. on. Com.*, Vol. 46, No 7, pp. 921-930, July 98.
- [15] O. Macchi, "Adaptive Processing," in *The LMS Approach with Applications in Transmission*, New York, Wiley, 1995.
- [16] P.L. Feintuch, "An Adaptive Recursive LMS Filter," *IEEE Proc.*, vol. 64, pp.1622-1624, Sept. 1976.
- [17] C. R. Johnson, Jr, and M.G. Larimore, "Comments on and Additions to "An Adaptive Recursive LMS Filter," *IEEE Proc.*, Vol. 65, pp. 1399-1402, Sept. 1977.
- [18] R.A. Casas, C.R. Johnson, JR. ,J. Harp and S. Caffee, "On initialization Strategies for Blind Adaptive DFEs," *IEEE Wireless Communications and Networking Conference*, Vol. 2, pp. 792-796, 1999.