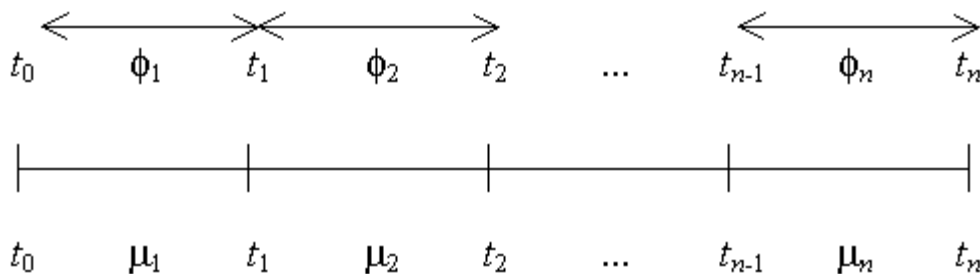


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Michael Schaub, Roger Pradel, Lukas Jenni, and Jean-Dominique Lebreton. 2001. Migrating birds stop over longer than usually thought: an improved capture-recapture analysis. *Ecology* 82: 852-859.

Appendix. Derivation of formula 2.

When analyzing capture-recapture data, one gets estimates of probabilities of departure over discrete periods of time as represented on the diagram below.



Here we derive a formula for the stopover duration after time t_0 . It depends not only on the discrete probabilities of departure f_i , but also on the pattern of the instantaneous rate of departure within each interval. Without any additional information, a reasonable assumption is that the instantaneous rate of departure m_i is constant within each interval. Then, departures during some interval follows a Poisson process and $m_i = -\ln(f_i)$ (see for instance, Seber, 1982 p.3). To calculate the stopover duration after time t_0 of a bird present at time t_0 , let us denote y the actual time of departure and Y the associated random variable. Let i be the integer verifying $t_{i-1} \leq y < t_i$; this is always possible if t_i can take the value $+\infty$. Then

$$\Pr[\text{stays to } y] = \Pr[\text{stays to } t_1] \times \Pr[\text{stays from } t_1 \text{ to } t_2] \dots \times \Pr[\text{stays from } t_{i-1} \text{ to } y] = f_1 f_2 \dots f_{i-1} \exp(-m_i (y-t_{i-1})).$$

The cumulative distribution function of Y being $F(y) = \Pr[Y < y] = 1 - \Pr[\text{stays to } y]$, its probability density function is

$$f(y) = F'(y) = m_i f_1 f_2 \dots f_{i-1} \exp(-m_i (y-t_{i-1}))$$

and the mean time of residency is

$$Sa = E[Y] = \int_0^{\infty} y f(y) dy$$

The following two lemmas, which are easily established, will be useful to calculate the integral above:

lemma 1: $\left(-\frac{1}{\mu} - y\right)e^{-\mu y}$ is a primitive of $\mu y e^{-\mu y}$

lemma 2: $\int_a^b \mu y e^{-\mu y} dy = e^{-\mu a} \left(\frac{1}{\mu} + a\right) - e^{-\mu b} \left(\frac{1}{\mu} + b\right)$

If y is any real and $n(y)$ the integer verifying $t_{n(y)} < y < t_{n(y)+1}$, then

$$\int_0^y z f(z) dz = \sum_{i=1}^{n(y)} \int_{t_{i-1}}^{t_i} z f(z) dz + \int_{t_{n(y)}}^y z f(z) dz =$$

$$\sum_{i=1}^{n(y)} e^{\mu_i t_{i-1}} \left(\prod_{j=1}^{i-1} \phi_j \right) \int_{t_{i-1}}^{t_i} \mu_i z e^{-\mu_i z} dz + e^{\mu_{n(y)+1} t_{n(y)}} \left(\prod_{j=1}^{n(y)} \phi_j \right) \int_{t_{n(y)}}^y \mu_{n(y)+1} z e^{-\mu_{n(y)+1} z} dz =$$

$$\sum_{i=1}^{n(y)} \left(\prod_{j=1}^{i-1} \phi_j \right) \left[\left(\frac{1}{\mu_i} + t_{i-1} \right) - \phi_i \left(\frac{1}{\mu_i} + t_i \right) \right] + \left(\prod_{j=1}^{n(y)} \phi_j \right) \left[\left(\frac{1}{\mu_{n(y)+1}} + t_{n(y)} \right) - e^{-\mu_{n(y)+1}(y-t_{n(y)})} \left(\frac{1}{\mu_{n(y)+1}} + y \right) \right] =$$

$$\sum_{i=1}^{n(y)} \left(\prod_{j=1}^{i-1} \phi_j \right) (1 - \phi_i) \left(\frac{1}{\mu_i} \right) - \left(\prod_{j=1}^{n(y)} \phi_j \right) t_{n(y)} + \left(\prod_{j=1}^{n(y)} \phi_j \right) \left[\left(t_{n(y)} + \frac{1}{\mu_{n(y)+1}} \right) - e^{-\mu_{n(y)+1}(y-t_{n(y)})} \left(y + \frac{1}{\mu_{n(y)+1}} \right) \right] =$$

$$\sum_{i=1}^{n(y)} \left(\prod_{j=1}^{i-1} \phi_j \right) (1 - \phi_i) \left(\frac{1}{\mu_i} \right) + \left(\prod_{j=1}^{n(y)} \phi_j \right) \left[\left(\frac{1}{\mu_{n(y)+1}} \right) (1 - e^{-\mu_{n(y)+1}(y-t_{n(y)})}) - y e^{-\mu_{n(y)+1}(y-t_{n(y)})} \right]$$

To go further, an assumption is required about the pattern of departure beyond the study period. We opted for a constant instantaneous rate of departure m_{n+1} and derived it from a weighted gliding average of the last three estimable f 's according to the formula

$$\phi_{n+1} = \frac{\phi_{n-2} + 2\phi_{n-1} + 4\phi_n}{7}$$

With $m_{n+1} = -\ln(f_{n+1})$, the stopover duration after time t_0 can now be calculated. Finally, we have

$$Sa = \sum_{i=1}^n \left(\prod_{j=1}^{i-1} \phi_j \right) (1 - \phi_i) \left(\frac{1}{\mu_i} \right) + \left(\prod_{j=1}^n \phi_j \right) \frac{1}{\mu_{n+1}}.$$

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