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# FREQUENCY DOMAIN TURBO EQUALIZATION FOR SINGLE CARRIER SPACE-TIME BLOCK CODE PER BLOCK

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## ABSTRACT

This paper deals with multiple-input multiple-output communication system with single-carrier modulation and frequency domain equalization. Space-time coding schemes adapted to frequency-selective channels namely space-time block codes per block are used. An efficient generalized receiver based on space-time frequency domain detector is investigated. We propose the frequency domain derivation of a generalized minimum mean square error turbo equalizer for orthogonal space-time block code per block. The main advantage of our proposed scheme is that it can be applied to any orthogonal space-time block code per block. Simulation results show that the scheme performs efficiently over frequency-selective block fading channels.

## I. INTRODUCTION

In the last few years we have seen an impressive development of wireless communication systems. The volume of information sent through a band-limited transmission medium is growing up. At the same time, other requirements are to be considered in indoor environments with non-line of sight (NLOS) propagation to improve as far as possible the performance of the receiver.

The standard IEEE 802.16 introduces the use of techniques to avoid the inter-symbol interference (ISI) due to multipath propagation like orthogonal frequency division multiplexing modulation (OFDM) or single-carrier modulation with frequency domain equalization [1] (SC-FDE). In addition, the standard IEEE 802.16 suggests orthogonal space-time block codes (OSTBC) like the Alamouti code [2] and its generalization proposed by Tarokh *et al* [3]. The main advantage of OSTBC is their linear maximum likelihood (ML) detection and thus low receiver complexity.

These codes have become a subject of interest in the research field. An important contribution made by Tirkkonen *et al*, generalizes the construction of complex OSTBC [4] based on the Clifford algebra. Moreover, Tirkkonen *et al* propose the optimization of one or more code features such as power allocation based on unitary transformations on elementary OSTBC.

To adapt the Alamouti scheme to frequency-selective fading channels, the standard IEEE 802.16 proposes to use reversal techniques and symbol packet encoding instead of unique symbol encoding. In [5], Amis *et al* generalizes the construction of OSTBC per block (OSTBCB) and derive an efficient generalized decision feedback equalizer adapted to them. On the other hand, turbo equalization [6, 7] has been proved to achieve near optimal performance on ISI channels.

In this paper, we use the generalization of OSTBCB proposed in [5], and we derive the frequency domain turbo equalizer for these codes. Several papers have studied the turbo equalization principle for MIMO systems [8–11], where the great majority deals with spatial multiplexing schemes (BLAST) first proposed by Bell Labs [12], and few others consider the Alamouti scheme like [9]. However, the turbo equalization proposed in [9] can only be applied to the Alamouti code. The advantage of our generalized turbo equalizer is that it can be used for any OSTBCB and any number of receiver antenna. Our generalized receiver achieves near-optimal performance while reducing complexity at the same time.

This paper is organized as follows: in Section II. presents the notations and the system model, then the ML detector structure is introduced. In Section III. the frequency domain generalized turbo equalizer based on the MMSE criterion for OSTBCB is derived. Finally, in Section IV., simulations show the receiver efficiency over selective-frequency block fading channels.

## II. SYSTEM MODEL

### A. Orthogonal frequency STBC per block



Figure 1: Discrete-time equivalent transmitter model

Let us consider the transmitter given in Figure 1. At the transmitter the incoming input bit stream  $\{d(n)\}$  is encoded by a forward error correction (FEC) code. A random interleaver  $\Pi$  shuffles the resulting coded data sequence,  $c(n) = 0, 1$ . Each packet of  $m$  coded and interleaved bits is mapped onto an  $M$ -ary complex symbol with variance  $\sigma_s^2$  and belonging to a signal set  $\mathcal{S}$  and  $s$  represents a symbol of the set. The symbols  $\{s(n)\}$  are split into  $K$  packets of size  $P$  denoted  $\mathbf{s}_k = [s_k(0) \dots s_k(P-1)]$ ,  $1 \leq k \leq K$ . The transmitted symbols are distributed between the  $n_T$  transmit antennas by a matrix  $\mathbf{c}_{ST}$  of size  $T \times n_T$ , defined by

$$\mathbf{c}_{ST} = \sum_{k=1}^K [\mathbf{s}_k(n)]^T \otimes \beta_k^- + [\mathbf{s}_k^*(p_n)]^T \otimes \beta_k^+ \quad (1)$$

where  $[\mathbf{a}]^*$  denotes the conjugate of  $[\mathbf{a}]$ ,  $[\mathbf{a}]^T$  its transpose matrix,  $(p_n) = (P-n) \bmod(P)$ ,  $\otimes$  denotes the Kronecker (tensor)

product, and  $\beta_k^-$  and  $\beta_k^+$  are  $T \times n_T$  complex matrices characteristic of the considered OSTBCB which properties are given in [4].

Let us consider the frequency properties of the OSTBCB given in [5], and let  $S_k(p)$  be the  $p$ -th coefficient of the  $P$ -FFT of  $\{s_k(n)\}$ . The  $P$ -FFT applied to  $c_{ST}$  along the time dimension (vertical) leads to  $P$  complex matrices  $C_{ST}(p)$  of size  $T \times n_T$  [5]

$$C_{ST}(p) = \sum_{k=1}^K S_k(p)\beta_k^- + S_k^*(p)\beta_k^+, \quad 0 \leq p \leq P-1 \quad (2)$$

Subsequently, each transmitted block is preceded by a cyclic prefix CP which is the repetition of the last  $G$  samples of each block. These CPs guarantee circular convolution in the time-domain, *i.e.* a scalar product in the frequency domain. The length of the CP is at least equal to the channel ISI order to avoid the inter-block interference (IBI).

Assuming perfect carrier and time synchronizations, the received samples  $\{y_j(n)\}$  on the receive antenna  $j$  can be expressed as

$$y_j(n) = \sum_{i=1}^{n_T} \sum_{l=0}^{L-1} h_{j,i}(n-l)c_{ST}^i(n) + \phi_j(n) \quad (3)$$

where  $\{h_{j,i}(n)\}$  are the coefficients of the channel impulse response between transmit antenna  $i$  and receive antenna  $j$ ,  $c_{ST}^i(n)$  corresponds to the  $m$ -th OSTBCB element sent from the  $i$ -th transmit antenna,  $\phi_j(n)$  are complex circular AWGN noise coefficients on received antenna  $j$  with zero mean and variance  $\sigma_\phi^2$ .

After eliminating the CP,  $\{y_j(n)\}$  is split into  $T$  packets of length  $P$ . Then, a  $P$ -FFT is applied on each block to obtain the  $P$  matrices  $Y(p)$  of size  $T \times n_R$ .

$$Y(p) = H(p)C_{ST}(p) + \Phi(p) \quad (4)$$

where  $\{\Phi(p)\}$  denotes the  $P$ -FFT of the noise  $\{\phi(n)\}$ ,  $H(p) = \{H_{j,i}(p)\}$  with  $1 \leq i \leq n_T$ ,  $1 \leq j \leq n_R$  and  $\{H_{j,i}(p)\}$  is the  $P$ -FFT of the time channel impulse response coefficients  $\{h_{j,i}(n)\}$ .

$$H_{j,i}(p) = \sum_{n=0}^{P-1} h_{j,i}(n) \exp(-j2\pi \frac{np}{P}) \quad (5)$$

### B. The ML-detector

From now on, soft decisions variables for each  $S_k(p)$  resulting from the application of the maximum likelihood criterion to matrices  $Y(p)$  are denoted  $X_k(p)$  [5]. In [5], it is shown that the soft decisions variables can be described by

$$X_k(p) = G_k^*(p)[G_k(p)S_k(p) + \Lambda_k(p)] \quad (6)$$

where  $G_k^*(p)G_k(p) = \text{Tr}[H(p)^\dagger H(p)]$ ,  $H^\dagger$  denotes the Hermitian of  $H$  and  $\Lambda_k(p)$  is the equivalent complex AWGN of variance  $\sigma_\Lambda^2 = \sigma_\Phi^2 = P\sigma_\phi^2$ . Let us denote  $B_k(p) = G_k^*(p)\Lambda_k(p)$ . Its variance is defined by  $\sigma_B^2 = P|G_k(p)|^2\sigma_\phi^2$ . From now on, we define and consider  $\sigma_b^2 = \sigma_B^2/P$ .

## III. FREQUENCY DOMAIN MMSE TURBO EQUALIZATION IC-LE

Let us consider the finite-length MMSE interference cancellation (IC) structure given in Figure 3. After the  $P$ -FFT, the ML-detector detailed in [5] provides  $X_k(p)$ . Using equivalent model [7] and the FD-MMSE-IC-LE described in [7] and [13] for single-input single-output systems, an interference cancellation structure consisting of a feedforward filter  $F_k(p)$  and a feedback filter  $Q_k(p)$  delivers estimated symbols  $Z_k(p)$  *cf.* Figure 2.

$$Z_k(p) = F_k(p)X_k(p) - Q_k(p)\bar{X}_k(p) \quad (7)$$

where  $\{\bar{X}_k(p)\}$  is the estimated ISI. The  $P$ -IFFT of  $\{Z_k(p)\}$  denoted  $\{z_k(n)\}$  is used in the soft SBC to deliver lineary reliability values used as input of the decoder. The decoder feed the soft-input soft-output (SISO) bit to symbol converter (BSC) with log-likelihood ratios (LLR) denoted  $L_{OUT}^D(n)$  or  $L_a(n)$  after interleaving.

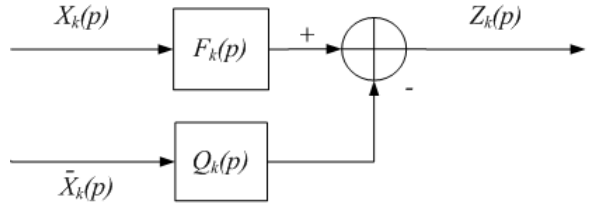


Figure 2: Structure of the equalizer

The SISO BSC module generates soft symbol estimates. They are computed as the expected value of the transmitted symbols, with respect to the *a priori* probabilities derived from the soft value delivered by the decoder at the previous iteration.

$$\bar{x}_k(n) = E[s_k(n)|L_a(n)] \quad (8)$$

Under the assumption of *i.i.d*<sup>1</sup> transmitted symbols  $\{s_k(n)\}$ ,  $\sigma_{\bar{x}}^2 = E[|s_k(n) - \bar{x}_k(n)|^2|L_a(n)]$ . Those soft symbol estimates are exploited as *a priori* information to cancel the ISI in MMSE-IC-LE.

Let us now detail the equations defining the turbo equalizer. The MMSE IC-LE aims at minimizing the mean square error  $\varepsilon_{LE} = E[|z_k(n) - s_k(n)|^2]$ .  $\{z_k(n)\}$  is the IFFT  $Z_k(p)$ . Due to the FFT properties combined and assuming *i.i.d.* of  $\{s_k(n)\}$ , minimizing  $\varepsilon_{LE}$  is equivalent to minimizing each  $E[|Z_k(p) - S_k(p)|^2]$  under the constraint  $\sum_{p=0}^{P-1} Q_k(p) = 0$  *i.e.*  $q_k(n=0) = 0$ . The Lagrangian resolution of this optimization leads to

$$Q_k(p) = F_k(p)|G_k(p)|^2 - \mu \quad (9)$$

where  $\mu = \frac{1}{P} \sum_{q=0}^{P-1} F_k(q)|G_k(q)|^2$ .

Finally, the frequency domain equalizer coefficients  $F_k(p)$  are given by

<sup>1</sup>independent and identically distributed

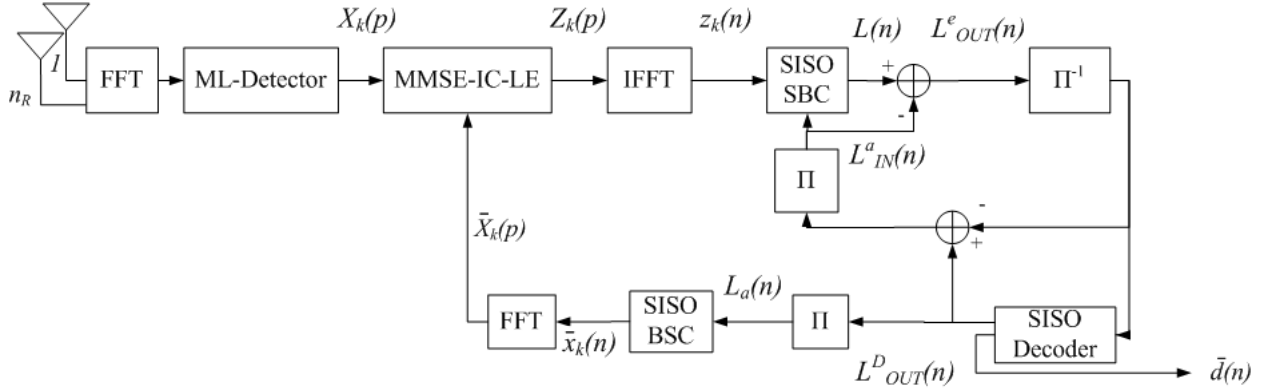


Figure 3: Frequency domain turbo equalizer structure

$$F_k(p) = \frac{(\sigma_s^2 - \mu\sigma_x^2)}{|G_k(p)|^2(\sigma_s^2 - \sigma_x^2) + \sigma_b^2} \quad (10)$$

The output of the SBC denoted  $L(n)$  is equal to

$$L(n) = \ln \frac{\sum_{(s \in \mathcal{S} | c(n)=1)} Pr(z_k(n) | s_k(n) = s) P_{a,n}(s)}{\sum_{(s \in \mathcal{S} | c(n)=0)} Pr(z_k(n) | s_k(n) = s) P_{a,n}(s)} \quad (11)$$

where

$$Pr(z_k(n) | s_k(n) = s) = \frac{1}{\pi\sigma_b^2} \exp - \frac{|z_k(n) - \mu s_k(n)|^2}{\sigma_b^2} \quad (12)$$

and

$$P_{a,n}(s) = \prod_{i=1}^m P_{a,n}(c_n^i) \quad (13)$$

with  $\{c_n^i\}_{\{1 \leq i \leq m\}}$  are the interleaved encoded bit values mapped to  $s \in \mathcal{S}$ .

$$P_{a,n}(c_n^i) = \frac{\exp \frac{(2c_n^i - 1)L_T^q N(n)}{2}}{\exp \frac{L_T^q N(n)}{2} + \exp - \frac{L_T^q N(n)}{2}} \quad (14)$$

$L(n)$  values are used to conjugate the extrinsic information  $L^e_{out}(n)$  defined by

$$L^e_{OUT}(n) = L(n) - L^a_{IN}(n) \quad (15)$$

#### IV. SIMULATION RESULTS

The MIMO system consists of 2 transmit antennas and 1 receive antenna. The OSTBCB considered is derived from the Alamouti scheme and is defined by

$$c = \begin{bmatrix} s_k(n) & -s_{k+1}^*(p_n) \\ s_{k+1}(n) & s_k^*(p_n) \end{bmatrix} \quad (16)$$

The FEC code is a 1/2 rate convolutional code with generator polynomials  $(13, 15)_8$ . The pseudo random interleaver length is 1024. Interleaved data is QPSK modulated. The total average transmitted power is normalized to unity. The frequency selective channel consists of 5 *i.i.d* paths of equal average power such that the total average received power is equal

to one (EQ-5).  $h_{1,i}(l)$  is a complex Gaussian random variable of zero mean and variance  $\frac{1}{5}$ . We further assume that the channel coefficients are spatially uncorrelated and remain constant during the transmission of one OSTBCB. At the receiver, we assume perfect channel state information and perfect synchronization. In our simulations, 5 iterations of the turbo equalizer are considered. The outer decoder is a BCJR decoder [14].

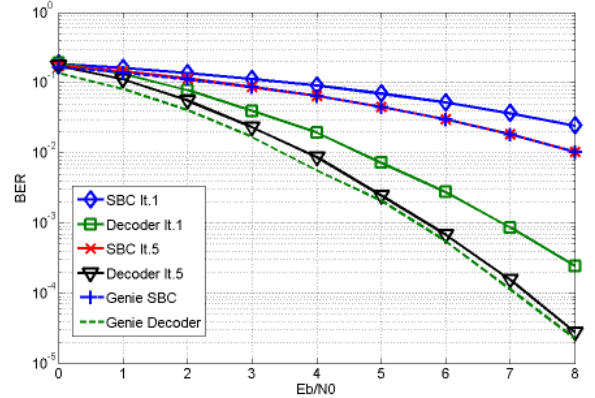


Figure 4: MMSE-IC-LE performance of OSTBCB ( $n_T = 2, n_R = 1$ ) with QPSK over the EQ-5 block fading channel

In Figure 4, the BER is given as a function of  $E_b/N_0$  where  $E_b$  is the average energy per transmitted bit and  $N_0$  is the unilateral noise power spectral density. The genie equalizer and decoder bounds (obtained with perfect ISI knowledge at the equalizer and SBC input) are plotted as references. The SBC and decoder output BER at first and fifth iterations are given. In comparison with the first iteration, the fifth iteration exhibits a gain of 1.5 dB for a BER of  $2.10^{-2}$  (resp. 1 dB for a BER of  $2.10^{-4}$ ) at the SBC output (resp. decoder output). Moreover the equalizer achieves the genie equalizer bound after 5 iterations and the decoder performs at 0.2 dB of the genie decoder bound. One can check that the genie bound coincides with the theoretical bit error probability of a QPSK modulated SISO system over the Rayleigh frequency-selective fast fading

channel with  $n_T \times L$  taps [15].

## V. CONCLUSIONS

In this paper we have generalized the turbo equalizer structure of [13] for the generalized OSTBCB proposed in [5]. This structure can be applied to any OSTBCB and any receive antenna number. Simulations carried out over a multipath block fading channel have shown the efficiency of the proposed receiver. Future work will consider the turbo equalizer structure optimization, including synchronization and channel estimation.

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