



**HAL**  
open science

## A study of the robustness of association rules

Jérôme Azé, Philippe Lenca, Stéphane Lallich, Benoît Vaillant

► **To cite this version:**

Jérôme Azé, Philippe Lenca, Stéphane Lallich, Benoît Vaillant. A study of the robustness of association rules. DMIN' 07: International conference on data mining, Jun 2007, Las Vegas, United States. pp.132 - 137. hal-02121631

**HAL Id: hal-02121631**

**<https://hal.science/hal-02121631>**

Submitted on 29 Jun 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# A study of the robustness of association rules

Jérôme Azé, Philippe Lenca, Stéphane Lallich, Benoît Vaillant

**Abstract**— Association rules discovery aims at discovering all frequent patterns among sets of data attributes under minimum support and minimum confidence constraints. It is well known that these algorithms may produce huge amounts of rules among which a large part of is not of interest. Thus many works have focused on the selection of the best rules with objective measures. We here present an experimental study on the robustness of rules, which is strongly related to interestingness measures. We hence introduce a noise consideration in rule sets evaluations and examine how rules are impacted. We particularly present results on four measures that illustrate two different behaviors.

## I. INTRODUCTION

Association rules discovery aims at discovering all frequent patterns among sets of data attributes. As defined in [1], given a typical market-basket (transactional) database  $E$ , an association rule  $A \rightarrow B$  means that if someone buys the set of items  $A$ , then he/she will probably buy the item  $B$ , where  $B$  is a single item that is not present in  $A$ . Such sets of items are usually called itemsets. The problem of mining for association rules consists in discovering all the rules that correlate the presence of one itemset with another under minimum support and minimum confidence constraints. The support is the percentage of transactions containing  $A$  and  $B$ , the confidence of  $A \rightarrow B$  is the ratio of the number of transactions containing  $A$  and  $B$  against the number of transactions containing  $A$ . The minimum support  $\sigma_s$  and the minimum confidence  $\sigma_c$  values are two thresholds that must be defined by the user.

Since the initial works presented in [1] and the APRIORI algorithm [2], many efforts have been done in order to develop efficient algorithms, as in [3], [4], [5] and [6] for example. APRIORI-like algorithms extract rules such that their support and confidence values are above  $\sigma_s$  and  $\sigma_c$ . They are deterministic [7] and it is well known that these algorithms may produce huge amounts of rules. Moreover a large part of these rules are not interesting because the support and confidence framework is not satisfying. For example, high confidence should not be confused with high correlation, nor with causality between the antecedent and the consequent of a rule [8]. As a consequence many works

have focused on the selection of the best rules. Among these works, measuring the interestingness with objective association rule measures play a major role [9].

Objective measures are said to be data-driven and only take into account the rules cardinalities ( $n_{ab}$ , the number of examples,  $n_{a\bar{b}}$ , the number of counter-examples, etc., see table I and figure 1) and  $n = |E|$ , the number of transactions. The support is then defined as  $n_{ab}/n$  and the confidence as  $n_{ab}/n_a$ . Objective measures provide a numerical information on the quality of a rule, and the rule  $A \rightarrow B$  is said “of quality” if its evaluation by a measure is greater than a user defined threshold: let us consider an association rule  $A \rightarrow B$  whose characteristics, as presented in table I, are known. Such a rule, when evaluated by an interestingness measure  $\mu$  is said of interest according to  $\mu$  if  $\mu(A \rightarrow B) \geq \sigma_\mu$ , where  $\sigma_\mu$  has to be fixed by the user.

Such a rule may be subject to interferences. In this paper, we investigate the outcome of some kind of predefined noise introduction.

TABLE I  
CONTINGENCY TABLE OF THE RULE  $r: A \rightarrow B$

		B		
		0	1	
A	0	$n_{a\bar{b}}$	$n_{ab}$	$n_a$
	1	$n_{\bar{a}\bar{b}}$	$n_{\bar{a}b}$	$n$

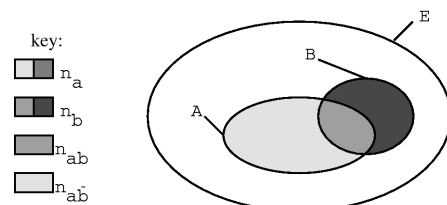


Fig. 1. Absolute characteristics of the rule  $r: A \rightarrow B$

Many objective interestingness measures have been defined and used in order to find out the best rules in a post-processing step. Interestingness measures have many different qualities or flaws. In order to select *good* rules one should use a good measure [10]. Interestingness measures have been thoroughly studied in many works. For recent overviews the reader may refer for example to [11], [12], [13], [14] and [15].

In this paper we present a study of rules robustness. This notion is strongly related to interestingness measures. Indeed we consider only association rules interestingness measures that are decreasing functions of the number of counter-

Jérôme Azé is with the Bioinformatic team, Laboratoire de Recherche en Informatique, Université Paris-Sud 11, 91405 Orsay, France (email: jerome.aze@lri.fr)

Philippe Lenca is with the GET/ENST Bretagne, CNRS TAMCIC, Technopôle de Brest Iroise, CS 83818, 29238 Brest Cedex, France (email: philippe.lenca@enst-bretagne.fr)

Stéphane Lallich is with the Université Lyon 2, Laboratoire ERIC, 5 avenue Pierre Mendès-France, 69676 Bron Cedex, France (email: stephane.lallich@univ-lyon2.fr)

Benoît Vaillant is with the IUT de Vannes, Université de Bretagne Sud, VALORIA, 8, rue Montaigne, BP 561, 56017 Vannes, France (email: benoit.vaillant@univ-ubs.fr)

examples of the rules, which is equivalent to increasing functions of the number of examples of the rules, if  $n_a$  is fixed. This is an eligibility property [16], based on the assumption that adding counter-examples to a rule should result in a worse evaluation by interestingness measures. Thus robustness is related to the number of examples an acceptable rule can loose while remaining acceptable [17]. The more this number, or the fraction of this number, is important, the more the rule is robust.

As presented in [17], when comparing two rules, the most robust one is not always the obvious one. As an illustrative example let us consider rule  $r_1$  (respectively  $r_2$ ) whose characteristics in absolute frequencies are given in table II (respectively table III). Their support, confidence and lift<sup>1</sup> [18] values are given in table IV. Remind that support and confidence are used as a first filter in APRIORI algorithms. With such high values the two rules will certainly be generated. Then in a postprocessing step, in order to select the rules, an end-user may use the lift. It is a classical approach. Both rules have the same lift value, greater than 1.0. However do they have the same interestingness? Although having a lower confidence,  $r_1$  may lose 25% of its examples while  $r_2$  may lose only 20% of them. From this point of view  $r_1$  is more robust when being evaluated in a postprocessing step with the lift and a lift threshold of 1.0.

TABLE II  
CONTINGENCY TABLE OF THE RULE  $r_1$

		B		
		0	1	
A	0	2000	2000	4000
	1	2000	4000	6000
		4000	6000	10000

TABLE III  
CONTINGENCY TABLE OF THE RULE  $r_2$

		B		
		0	1	
A	0	1800	3200	5000
	1	1000	4000	5000
		2800	7200	10000

TABLE IV  
EVALUATION OF RULES  $r_1$  AND  $r_2$

	SUP	CONF	LIFT
$r_1$	0.40	0.66	1.11
$r_2$	0.40	0.80	1.11

The paper is organised as follows. In a first section, we present some related works on association rule robustness. We then extend previous modelings to the concrete case of noise introduction. We thus propose an overall approach, taking into account three selected methods. This is illustrated on a concrete case-based study, for four pertinent measures.

<sup>1</sup>LIFT( $A \rightarrow B$ ) =  $\frac{\text{confidence}}{\text{expected confidence}} = \frac{n \times n_{ab}}{n_a \times n_b}$

## II. RELATED WORKS

As far as we know, only few and moreover recent works have assessed the association rules robustness topic.

In [19], [20] three noise modellings have been studied on artificial data. Each modelling worked directly on the data by changing some values into noisy couples (example, attribute). As no measure was found to be robust, an alternative way of analysing rules has been proposed by the authors. They suggest, when analysing a set of rules, to use noise to find those which are robust. Such subsets of rules may be of particular interest for a user.

[21] presented a simulation technique to determine if the rules are accidental or not, within a risk error lesser than a fixed value. The technique consists in generating numerous artificial data samples with the same statistical properties, as in real datasets, except the links between attributes. The frequency thresholds are then computed in order to make the difference between accidental itemsets and significant itemsets; that is to say, produce robust rules.

In [17] we define the robustness of association rules as the fraction of the number of examples an acceptable rule can lose while remaining acceptable. This notion is thus related to the threshold of acceptance and thus to the measure used. We studied nine linear and four monotonic transformations of the confidence. We provided a direct and exact way of evaluating the robustness of a rule, i.e. all our calculations are based on the rule contingency table without extra computational costs. This computation is done by moving a fraction of examples towards counter-examples. This is a classical way of studying the counter-examples, which we called modelling  $M_2$  in this paper.

In [22] we consider two other ways of adding counter-examples. For each modelling of the variation of counter-examples, we specified the provenance of counter-examples, thus identifying which quantities remain constant and which one are not in the contingency table of a rule (table I, which is a three degrees of freedom contingency table). We then studied the shape of the curve (concave, linear or convex) of ten classical measures with the apparition of news counter-examples. The decreasing rate of the measure may have an important effect on rule acceptance, as the user may tolerate or not the apparition of few counter-examples without significant loss of rule interest [23]. The behavior of the measures with respect to counter-examples is studied by expressing their second order derivatives. We particularly show that measures do not react in the same manner, depending of the modelling of the counter-examples, and should then classified between concave, linear or convex ones. [24] followed the same way to study the intensity of implication index, using modelling  $M_2$ .

## III. DEFINITION OF NOISE

Let us consider a rule  $r: A \rightarrow B$  described by its contingency matrix presented in Table I.

Three different modellings of noise are presented in this paper. These modellings have been introduced by [22].

In all three modellings, the introduction of noise into a rule  $r: A \rightarrow B$  is characterised by the diminution of the number of examples of the rule. Let  $x$  be the quantity of examples lost by the rule when noise is introduced. For all the modellings, the quantity  $n_{ab}$  becomes  $n_{ab} - x$ .

#### A. Modelling $M_1$

In the first modelling, quantities  $n_a$  and  $n_b$  are not modified when some noise is introduced into the rule  $r: A \rightarrow B$ . Table V presents the modifications of the contingency table, when such noise is introduced into a rule.

TABLE V  
CONTINGENCY TABLE OF A RULE  $r$  WHEN  $M_1$  IS APPLIED

		B		
		0	1	
A	0	$n_{\bar{a}\bar{b}} - x$	$n_{\bar{a}b} + x$	$n_{\bar{a}}$
	1	$n_{a\bar{b}} + x$	$n_{ab} - x$	$n_a$
		$n_{\bar{b}}$	$n_b$	$n$

#### B. Modelling $M_2$

In this second modelling, the quantity  $n_a$  is not affected by the noise, but the quantities  $n_b$  and  $n_{\bar{b}}$  are modified as described in table VI.

TABLE VI  
CONTINGENCY TABLE OF A RULE  $r$  WHEN  $M_2$  IS APPLIED

		B		
		0	1	
A	0	$n_{\bar{a}\bar{b}}$	$n_{\bar{a}b}$	$n_{\bar{a}}$
	1	$n_{a\bar{b}} + x$	$n_{ab} - x$	$n_a$
		$n_{\bar{b}} + x$	$n_b - x$	$n$

#### C. Modelling $M_3$

In this last modelling, the quantity  $n_b$  is not modified but other quantities are affected, as described in table VII.

TABLE VII  
CONTINGENCY TABLE OF A RULE  $r$  WHEN  $M_3$  IS APPLIED

		B		
		0	1	
A	0	$n_{\bar{a}\bar{b}}$	$n_{\bar{a}b} + x$	$n_{\bar{a}} + x$
	1	$n_{a\bar{b}}$	$n_{ab} - x$	$n_a - x$
		$n_{\bar{b}}$	$n_b$	$n$

These modellings are used to introduce noise into the set of rules extracted by the APRIORI algorithm (we used the C. Borgelt implementation [25]). In this work, we did the choice to randomly select the rules which will be modified, and we also randomly selected the modelling to apply. Once a rule is selected, at least one, or up to 5% of its examples, is balanced towards the counter-examples according to the previously selected model.

In the next sections, we will present the algorithm used to evaluate the impact of noise on a database and the experimental design we developed.

## IV. EVALUATION OF THE ROBUSTNESS OF RULES

We evaluated the robustness of different rulesets when some noise (defined by modelling  $M_1$ ,  $M_2$  or  $M_3$ ) is introduced onto them.

Let  $\mathcal{R}$  be a set of association rules extracted from a database. All the rules contained in  $\mathcal{R}$  are evaluated by different quality measures which are used to make out which are the most interesting rules. Since we wanted to evaluate the robustness of the rules for the measures used in order to select the *best* rules, we propose the following algorithm which allows one to build a new set of rules  $\mathcal{R}'$ .  $\mathcal{R}'$  is equal to  $\mathcal{R}$ , save  $\alpha\%$  of the rules on which we apply some noise, following modelling  $M_1$ ,  $M_2$  or  $M_3$ .

The procedure thus followed is:

- Create  $\mathcal{R}'$ , a copy of  $\mathcal{R}$ ,
- Compute the number of rules to be noised:  
 $n_\alpha = \alpha \times |\mathcal{R}|$ , where  $\alpha \in [0..1]$  is the noise percentage,
- Randomly select a rule amongst those having at least one example in the contingency table (no trace of previously selected rule is kept) and randomly select a noise modelling, apply this noise,
- repeat until the required  $n_\alpha$  number of rules have not been noised (see algorithm 1).

#### Input:

$\mathcal{R}$ : initial set of rules

$\alpha$ : percentage of rules affected by noise

#### Output:

$\mathcal{R}'$ : set of noisy rules

#### begin

$\mathcal{R}' = \mathcal{R}$ ;

$n_\alpha \leftarrow \alpha \times |\mathcal{R}|$ ;

**for** ( $i \in [1 : n_\alpha]$ ) **do**

    select ( $r \in \mathcal{R}'$ );

$modelling \leftarrow random(M_1, M_2, M_3)$ ;

$r \leftarrow apply(modelling, r)$ ;

**end**

**return**  $\mathcal{R}'$ ;

#### end

**Algorithm 1:** Introduction of noise in a set of association rules

Each rule of  $\mathcal{R}$  and  $\mathcal{R}'$  is evaluated on the set of measures. In order to evaluate the robustness of each rule, we determined on the initial set of rules  $\mathcal{R}$  a list of thresholds for each measure. The algorithm 2 presents how these lists and associated distribution of values are built, for a given set of measures and rules.

The randomizing process has been run 10 times, the results presented onwards correspond to the average repartition of rules for the studied measure. This repartition is highlighted using a simple split into equally spreaded measure value intervals. The lower bound (minimum measure value) is not fixed for the lowest interval, since the introduction of noise

in rules implies a lesser measure assessing of the quality of the rule.

Since the cardinality of  $\mathcal{R}'$  and  $\mathcal{R}$  are the same, the rules are only redistributed in these different intervals.

```

Input:
 $\mathcal{R}$ : set of rules
 $M$ : list of measures
Output:
thresholds: thresholds list for each measure
distribution: distribution of the rules for each
measure and thresholds
begin
  foreach ( $\mu \in M$ ) do
    evaluate  $\mu$  for each rule of  $\mathcal{R}$ ;
    get min, max values of  $\mu$  into  $\mathcal{R}$ ;
     $step = \max(0.01, \frac{max-min}{100})$ ;
    for ( $t = min; t \leq max; t += step$ ) do
       $distribution(\mu, t) = 0$ ;
      add  $t$  to thresholds( $\mu$ );
      foreach ( $r \in \mathcal{R}$ ) do
        if ( $\mu(r) \geq t$ ) then
           $distribution(\mu, t) ++$ ;
        end
      end
    end
  end
  return thresholds, distribution;
end

```

**Algorithm 2:** Computation of the thresholds list and of the associated distribution values

In order to randomly select the rule, the applied model and introduced noise, we used the perl rand() function. It is an LCG<sup>2</sup> based on the drand48() C function. The congruential process is applied as follows:

$$rand(i + 1) = (A \times rand(i) + B) \bmod M$$

mod being the modulo operator and constants  $A$ ,  $B$  and  $M$  valuing  $A = 25\,214\,903\,917$ ,  $B = 11$ ,  $M = 2^{48}$ . The periodicity of this pseudo-random number generator is thus of  $M = 2^{48} = 281\,10^{12}$  different values.

Since 10 experiments are averaged and that for each rule selected, three random draws are required, this maximal periodicity is reached at  $\frac{281\,10^{12}}{3 \times 10} \geq 9\,10^{11}$  rule selection iterations. As the noise level introduced is of 10%, the number of rules we can assess in this approach is of  $9\,10^{12}$ , which is far above those produced by APRIORI on usual datasets.

The study of the robustness of the extracted rulesets when noise is introduced is described in the following section.

## V. RESULTS

We chose to work with the census database from the UCI repository [26] since it contains a large number of instances.

<sup>2</sup>Linear Congruential Generator

This database being made of symbolic and numeric attributes and has been formatted by C. Borgelt<sup>3</sup>.

The initial database contains 48 842 instances and 14 attributes (6 continuous, 8 nominal). There also are missing attribute values. The formatted database contains only symbolic value.

The results presented in this paper were obtained by setting the support threshold to 5% and the confidence threshold to 70%. For each measure, three curves have been plotted: the frequency distributions of the values of the considered measures for the extracted rules, the distribution of the lost or won rules in each partition (number and percentage), and the distribution of the percentage of lost rules according to the threshold one can use to select rules. All of these curves correspond to 10% of noise. Other noise values were also studied, leading to similar results.

Definitely, the first partition of noised curves will always contain more rules once the noise is introduced than in the initial situation. Indeed, since we work on the extracted rules, this partition only retains rules from the upper partitions. We have carried out experiments on many measures and here propose the results for four of them in order to point out two differing behaviors.

These measures are Jaccard (JAC) and Least Contradiction (LC) *against* the Examples and Counter-examples Rate (ECR) and Loevinger (LOE). They are presented in table VIII.

TABLE VIII  
MEASURES STUDIED

	Absolute definition	Reference
JAC	$\frac{n_{ab}}{n_a + n_b - n_{ab}}$	[27]
LC	$\frac{n_{ab} - n_{a\bar{b}}}{nn_b}$	[28]
ECR	$\frac{n_{ab} - n_{a\bar{b}}}{n_{ab}}$	
LOE	$\frac{nn_{a\bar{b}} - n_a n_{\bar{b}}}{n_a n_{\bar{b}}}$	[29]

### A. JAC and LC

The JAC index and LC share the property of presenting small variations on the number of rules, for each partition. In both cases, the percentage of lost rules according to the threshold is a  $j$ -shaped distribution, the percentage being an exponential increasing function of the threshold.

1) JAC : The distribution of JAC index is a very asymmetric distribution, which is  $i$ -shaped. The variation curve of the quantity of rules per partition when 10% of noise is introduced shows that each partition loses some rules overall, but not so many. 8 rules at the most are lost, which happens for the highest partition (3% of this partition, figure 3). The curve which indicates the percentage of lost rules according to the threshold is rather regular. It indicates that less than 2% of rules are lost when the threshold is lesser or equal to 0.85. Above 0.85, the curve goes up rather stiffly up to 7%.

<sup>3</sup><http://fuzzy.cs.uni-magdeburg.de/~borgelt/data/census.zip>

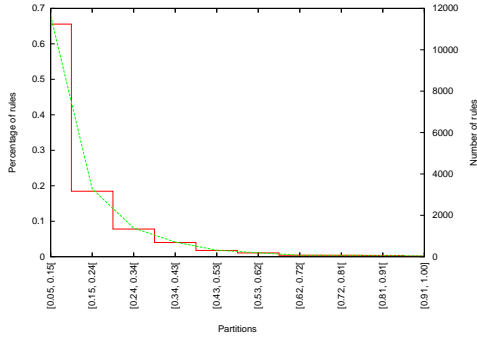


Fig. 2. Frequencies distribution of JAC

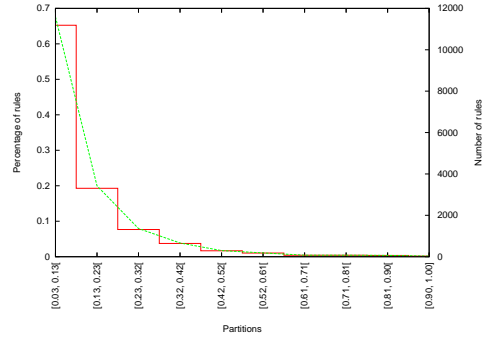


Fig. 5. Frequencies distribution of LC

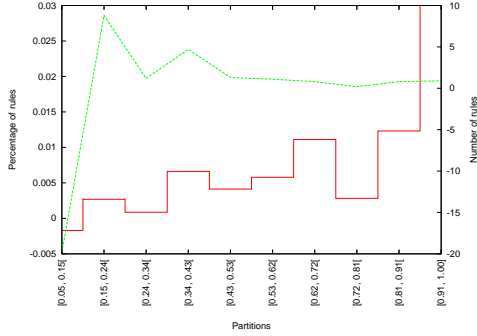


Fig. 3. Impact of 10% of noise for JAC: loss or gain of rules in each partition (number and percentage of the partition)

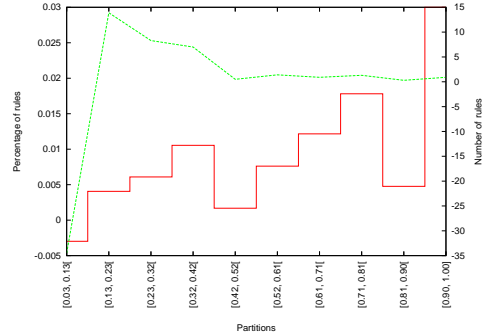


Fig. 6. Impact of 10% of noise for LC: loss or gain of rules in each partition (number and percentage of the partition)

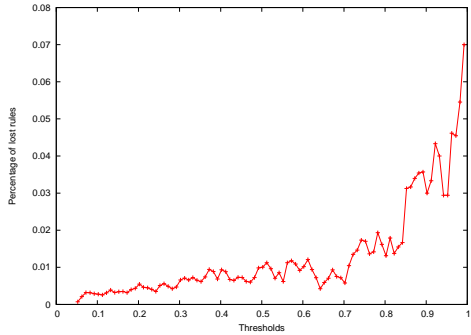


Fig. 4. Percentage of lost rules for JAC according to the thresholds

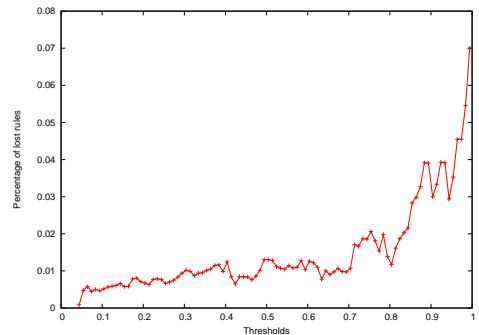


Fig. 7. Percentage of lost rules for LC according to the thresholds

2) LC : LC has the same characteristics as JAC index. Its distribution is also *i*-shaped. When 10% of noise is introduced, each partition will loose some rules overall, but little enough: 15 rules at the most (3% of a partition, see figure 6). The curve which indicates the percentage of lost rules according to the threshold is also *j*-shaped. This indicates that less than 2% of rules are lost when the threshold is lesser or equal to 0.80. Above 0.80, the curve goes up to 7% (see figure 7).

### B. ECR and LOE

The variation of the number of rules per partition is significant for both ECR and LOE measures (figures 9 and 12). The curve of the percentage of lost rules for ECR according to the threshold used to select rules is an increasing

curve in *j*-form while it is a bell-shaped curve for LOE (figures 10 and 13).

1) ECR : ECR takes its values between 0.57 (due to the confidence threshold we used, i.e. 0.7) and 1. It is a very asymmetrical distribution, in *j*-shape. Thus many rules switch to the high part of the distribution.

For 10% of noise, the lost rules are those of higher partitions but also do come from partitions containing many rules (up to 180 rules are lost for the higher partition, see figure 9). These rules moved to lower partitions, especially to central partitions (for example partition [0.74, 0.79] wins 50 rules).

Due to the distribution form, the percentage of lost rules does not exceed 4% in the highest partitions while the medium partitions may win 6% of rules. Curve 10 confirms this analysis. The percentage of lost rules quickly increases

after  $ECR = 0.75$  (i.e.  $CONF = 0.8$ ).

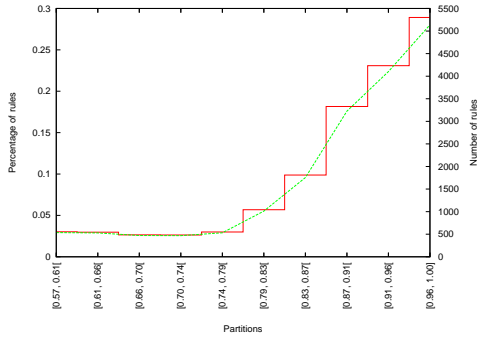


Fig. 8. Frequencies distribution of ECR

to 1 (which corresponds to logical rules or almost logical ones).

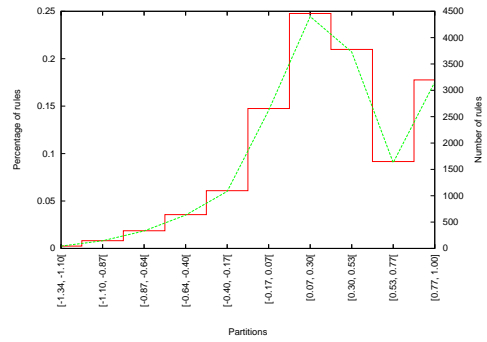


Fig. 11. Frequencies distribution of LOE

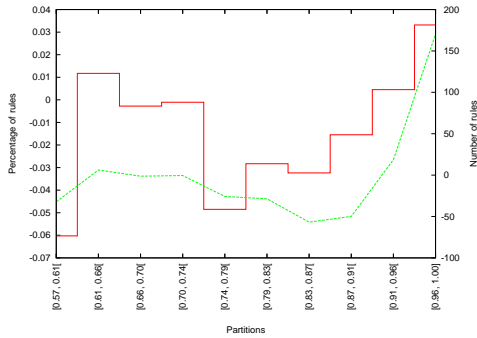


Fig. 9. Impact of 10% of noise for ECR: loss or gain of rules in each partition (number and percentage of the partition)

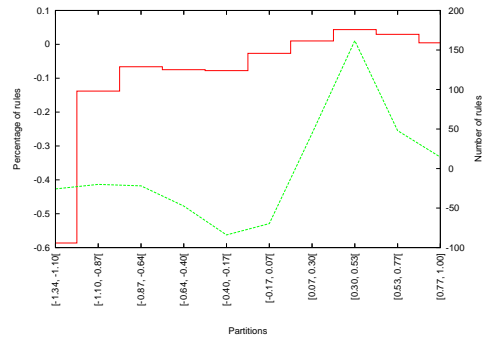


Fig. 12. Impact of 10% of noise for LOE: loss or gain of rules in each partition (number and percentage of the partition)

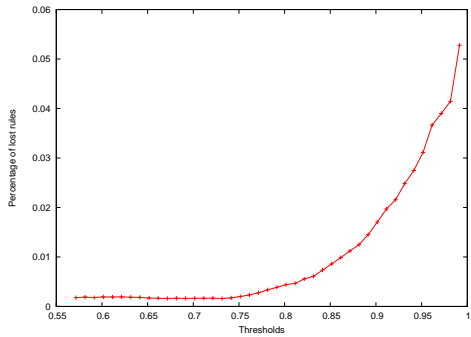


Fig. 10. Percentage of lost rules for ECR according to the thresholds

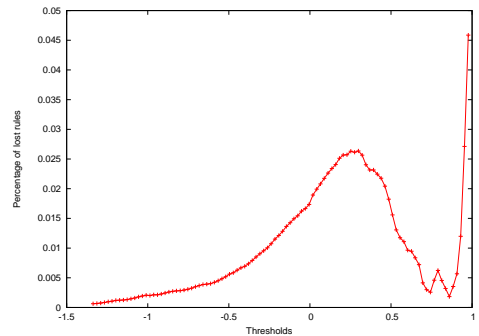


Fig. 13. Percentage of lost rules for LOE according to the thresholds

2) LOE : The LOE distribution is in  $v$ -shape between  $-0.17$  and  $1$ . This distribution presents a hole at the end, but one partition before that end goes up to  $3.000$  rules for the last partition (figure 11).

The introduction of noise strongly impacts on some rules (their evaluation shut down to  $-1$ ). The partition of rules near independency situation increases of about  $250$  rules which corresponds to  $5.5\%$  of the partition (figure 12). All other partitions do then loose rules, especially the intermediary partition that loses about  $180$  rules ( $4.4\%$  of the partition).

Figure 13 is bimodal. There is a first peak at  $0.25$  (about  $2.5\%$  of the partition) and a second one when LOE is close

## VI. CONCLUSIONS

From the best of our knowledge only few, and moreover recent, works have focussed on robustness of association rules.

In previous works, we proposed to define the robustness of a rule with respect to the number (or fraction) of examples (which is equivalent to the fraction of counter-examples;  $n_a$  being fixed) an acceptable rule may lose while remaining acceptable. The more this number is important, the more the rule is robust. Acceptability is then measured with objective interestingness measures. This way of doing is also

equivalent to examining how many counter-examples a rule can be received while remaining acceptable. We also show that counter-examples could come from different sources and that their origins have an impact on the behavior of interestingness measures, thus leading to what we consider as robust. These works allowed us to propose a theoretical framework of robustness analysis according to a random chosen modelling of counter-examples.

In concrete situations one does not know from where the counter-examples come from. We here presented an experimental study of robustness of rules by adding noise i.e. by adding counter-examples to the rules following the different randomly chosen modellings. We illustrated our purpose with four measures which show two different behavior. Of course more experimental campaigns have to be done.

A next study should propose theoretical results for a mixture model based on the different modellings. This will allow us to define an aggregate robustness measure. The interval values of measures are equally ranged, which is not always satisfactory since some measures will rank many rules as the best and others not. The stepping gap in this clustering could be adapted to the distribution values of the measures on each ruleset. What is more, a tracking of which rule has already been noised could be interesting, since we only restrict ourselves to rules having at least an example for the moment: an already noised rule could behave in a non-equiprobable manner if reselected, or rejected in the noise introduction process, depending in the previously applied modelling. Last, the noise level introduced for each selected rule could be also be randomized.

## REFERENCES

- [1] R. Agrawal, T. Imielinski, and A. Swami, "Mining association rules between sets of items in large databases," in *ACM SIGMOD International Conference on Management of Data*, P. Buneman and S. Jajodia, Eds., Washington, D.C., USA, 1993, pp. 207–216.
- [2] R. Agrawal and R. Srikant, "Fast algorithms for mining association rules," in *The 20th International Conference on Very Large Data Bases*, J. B. Bocca, M. Jarke, and C. Zaniolo, Eds. Morgan Kaufmann, 1994, pp. 487–499.
- [3] H. Toivonen, "Sampling large databases for association rules," in *The 22th International Conference on Very Large Data Bases*, T. M. Vijayaraman, A. P. Buchmann, C. Mohan, and N. L. Sarda, Eds. Morgan Kaufmann, 1996, pp. 134–145.
- [4] S. Brin, R. Motwani, J. D. Ullman, and S. Tsur, "Dynamic itemset counting and implication rules for market basket data," in *ACM SIGMOD International Conference on Management of Data*, J. Peckham, Ed. Tucson, Arizona, USA: ACM Press, 1997, pp. 255–264.
- [5] D.-I. Lin and Z. M. Kedem, "Pincer Search: A new algorithm for discovering the maximum frequent set," in *The 6th International Conference on Extending Database Technology*, ser. Lecture Notes in Computer Science, H. Schek, F. Saltor, I. Ramos, and G. Alonso, Eds., vol. 1377. Springer-Verlag, 1998, pp. 105–119.
- [6] N. Pasquier, Y. Bastide, R. Taouil, and L. Lakhal, "Discovering frequent closed itemsets for association rules," in *The 7th International Conference Database Theory*, ser. Lecture Notes in Computer Science, C. Beeri and P. Buneman, Eds., vol. 1540. Jerusalem, Israel: Springer, 1999, pp. 398–416.
- [7] A. Freitas, "Understanding the crucial differences between classification and discovery of association rules - a position paper," in *ACM SIGKDD Explorations*, vol. 2, no. 1, 2000, pp. 65–69.
- [8] T. Brijs, K. Vanhoof, and G. Wets, "Defining interestingness for association rules," *International journal of information theories and applications*, vol. 10, no. 4, pp. 370–376, 2003.
- [9] R. Hilderman and H. Hamilton, "Applying objective interestingness measures in data mining systems," in *Fourth European Symposium on Principles of Data Mining and Knowledge Discovery*, D. A. Zighed, H. J. Komorowski, and J. M. Zytkow, Eds. Lyon, France: Springer Verlag, 2000, pp. 432–439.
- [10] P. Lenca, P. Meyer, B. Vaillant, and P. Picouet, "Aide multicritère à la décision pour évaluer les indices de qualité des connaissances – modélisation des préférences de l'utilisateur," *RSTI-RIA (EGC 2003)*, vol. 1, no. 17, pp. 271–282, 2003.
- [11] R. Hilderman and H. Hamilton, "Measuring the interestingness of discovered knowledge: A principled approach," *Intelligent Data Analysis*, vol. 7, no. 4, pp. 347–382, 2003.
- [12] P.-N. Tan, V. Kumar, and J. Srivastava, "Selecting the right objective measure for association analysis," *Information Systems*, vol. 4, no. 29, pp. 293–313, 2004.
- [13] K. McGarry, "A survey of interestingness measures for knowledge discovery," *Knowledge Engineering Review Journal*, vol. 20, no. 1, pp. 39–61, 2005.
- [14] P. Lenca, B. Vaillant, P. Meyer, and S. Lallich, *Quality Measures in Data Mining*, ser. Studies in Computational Intelligence. Springer, 2007, ch. Association rule interestingness measures: experimental and theoretical studies.
- [15] F. Guillet and H. Hamilton, Eds., *Quality Measures in Data Mining*, ser. Studies in Computational Intelligence. Springer, 2007.
- [16] P. Lenca, P. Meyer, P. Picouet, B. Vaillant, and S. Lallich, "Critères d'évaluation des mesures de qualité en ECD," *RNTI (Entreposage et Fouille de Données)*, no. 1, pp. 123–134, 2003.
- [17] P. Lenca, B. Vaillant, and S. Lallich, "On the robustness of association rules," in *IEEE International Conference on Cybernetics and Intelligent Systems*, Bangkok, Thailand, 2006, pp. 596–601.
- [18] S. Brin, R. Motwani, and C. Silverstein, "Beyond market baskets: generalizing association rules to correlations," in *ACM SIGMOD/PODS Joint Conference*, 1997, pp. 265–276.
- [19] J. Azé and Y. Kodratoff, "A study of the effect of noisy data in rule extraction systems," in *The Sixteenth European Meeting on Cybernetics and Systems Research*, vol. 2, 2002, pp. 781–786.
- [20] J. Azé, "Extraction de connaissances dans des données numériques et textuelles," Ph.D. dissertation, Université de Paris 11, 2003.
- [21] M. Cadot, "A simulation technique for extracting robust association rules," in *CSDA*, Chypre, Greece, 2005.
- [22] B. Vaillant, S. Lallich, and P. Lenca, "Modeling of the counter-examples and association rules interestingness measures behavior," in *The 2006 International Conference on Data Mining*, S. Crone, S. Lessmann, and R. Stahlbock, Eds., Las Vegas, Nevada, USA, 2006, pp. 132–137.
- [23] R. Gras, R. Couturier, J. Blanchard, H. Briand, P. Kuntz, and P. Peter, "Quelques critères pour une mesure de qualité de règles d'association - un exemple : l'intensité d'implication," *RNTI (Mesures de Qualité pour la Fouille de Données)*, no. RNTI-E-1, pp. 3–31, 2004.
- [24] R. Gras, J. David, F. Guillet, and H. Briand, "Stabilité en A.S.I. de l'intensité d'implication et comparaisons avec d'autres indices de qualité de règles d'association," in *3<sup>e</sup> Atelier Qualité des Données et des Connaissances (EGC 2007)*, S. Lallich, P. Lenca, and F. Guillet, Eds., Namur, Belgium, 2007, pp. 35–43.
- [25] C. Borgelt and R. Kruse, "Induction of association rules: APRIORI implementation," in *The 15th Conference on Computational Statistics*. Heidelberg, Germany: Physika Verlag, 2002.
- [26] D. Newman, S. Hettich, C. Blake, and C. Merz, "UCI repository of machine learning databases," 1998. [Online]. Available: <http://www.ics.uci.edu/~mllearn/MLRepository.html>
- [27] P. Jaccard, "Nouvelles recherches sur la distribution florale," *Bulletin Society Vaud Science National*, no. 44, pp. 223–270, 1908.
- [28] J. Azé and Y. Kodratoff, "Evaluation de la résistance au bruit de quelques mesures d'extraction de règles d'association," in *Extraction des connaissances et apprentissage*, vol. 1, no. 4, 2002, pp. 143–154.
- [29] J. Loevinger, "A systemic approach to the construction and evaluation of tests of ability," *Psychological monographs*, vol. 61, no. 4, 1947.