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# A 3D parallel boundary element method on unstructured triangular grids for fully nonlinear wave-body interactions

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## Abstract

This paper presents the development and validation of a three-dimensional numerical wave tank devoted to studying wave-structure interaction problems. It is based on the fully nonlinear potential flow theory, here solved by a boundary element approach and using unstructured triangular meshes of the domain's boundaries. Time updating is based on a second-order explicit Taylor series expansion. The method is parallelized using the Message Passing Interface (MPI) in order to take advantage of multi-processor systems. For radiation problems, with cylindrical bodies moving in prescribed motion, the free-surface is updated with a fully Lagrangian scheme, and is able to reproduce reference results for nonlinear forces exerted on the moving body. For diffraction problems, semi-Lagrangian time-updating is used, and reproduces nonlinear effects for diffraction on monopiles. Finally, we study the nonlinear wave loads on a fixed semi-submersible structure, thereby illustrating the possibility to apply the proposed numerical model for the design of offshore structures and floaters.

*Keywords:* Nonlinear wave-structure interaction, Offshore structures, Ocean engineering, Boundary element method

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## 1 Introduction

2 The numerical modeling of fully nonlinear interactions between floating structures and  
3 waves in three dimensions (3D) is of great importance for the design of ocean engineering  
4 structures such as offshore wind turbines or wave energy converters, as realistic sea states  
5 may cause nonlinear motions of the structure. The problem is often addressed by means  
6 of the fully nonlinear potential flow (FNPF) approach, and has had broad success for  
7 both radiation and diffraction problems.

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8 Solving for FNPF involves the solution of Laplace’s equation for a velocity potential,  
9 which can be treated with the boundary element method (BEM). For one example, Fer-  
10 rant [1] was able to get very good agreement for the loads on a vertical cylinder in regular  
11 waves, using linear triangular elements. More recently High Order Boundary Element  
12 Method (HOBEM) has seen more use, for its better properties in convergence, although  
13 it can be more complicated to produce an appropriate mesh for complex geometries.

14 Boo [2] studied the effect of linear and nonlinear irregular waves on a fixed bottom  
15 mounted cylinder with an HOBEM. In the work of Liu *et al.* [3], a HOBEM with struc-  
16 tured meshes was used to compute the wave loads in forced motion and diffraction on a  
17 Wigley Hull and a truncated cylinder. A fair agreement with measurements performed  
18 at MARINTEK and the third order theory of Malenica and Molin [4], was found for  
19 the case of the truncated cylinder. Another HOBEM was recently developed by Bai and  
20 Eatock Taylor in [5] using unstructured grids and combined with a domain decomposition  
21 method in [6]. Various forced motions cases were investigated with a vertical cylinder.  
22 A remeshing step based on the Laplace smoothing technique was used. Typical nonlin-  
23 ear effects were outlined like the oscillation of the vertical force in surge at twice the  
24 frequency of the motion, occurring with symmetrical objects. An important nonlinear  
25 interaction between surge and pitch motion was also shown.

26 Other approaches have also been considered for these wave-body interaction problems  
27 with fully nonlinear potential flow. Ma *et al.* [7] developed an approach based on the  
28 finite element method (FEM) and applied it to wave loads on fixed vertical cylinders.  
29 Similar work with FEM was made by Wang *et al.* [8]. Shao and Faltinsen [9] devel-  
30 oped the harmonic polynomial cell approach, which is another solution to potential flow  
31 problems based on a volume discretization, but representing the local solution as a linear  
32 superposition of elementary solutions of the Laplace equation, resulting in improvements  
33 in accuracy and speed. Mola *et al.* [10] used a BEM, but with substantial innovations  
34 with adaptive mesh refinement and stabilized by a Streamline Upwind Petrov–Galerkin  
35 (SUPG) scheme applied to the dominant transport term of the boundary condition, in  
36 case of a non-negligible advancing velocity.

37 While theoretically much of the physics is well understood even for cases with free  
38 motion, numerical complexities of working with higher-order methods mean that they  
39 are more difficult to work with in 3D for complex body shapes, hence even some recent  
40 works focus on 2D simulations, such as the FEM work by Yan and Ma [11] and by Wang  
41 and Wu [12]. Industrial design work can be done now though using such tools; the free  
42 motion of a simplified Floating Production Storage and Offloading (FPSO) structure  
43 was studied in [13]. Free motion was also addressed in [6], where the effect of the shape  
44 of the cylinder was investigated. An extension of the QALE-FEM method developed  
45 in [11] has also been applied to free motions in [14]. Similar to the Laplace smoothing  
46 technique mentioned above used by Bai and Eatock Taylor [6], the QALE-FEM avoids  
47 remeshing of the free surface, but an adaptive mesh strategy is used based on a spring  
48 analogy for moving interior nodes of the free surface and the body surface. A validation  
49 of this scheme is performed with comparisons to experimental data for a barge-type and  
50 a spar-type floating structure with a good agreement.

51 Despite the many methods which exist, due to the numerous difficulties involved in  
52 fully nonlinear potential flow modeling, the state-of-the-art has not yet converged on a  
53 single approach, and industry models such as AEGIR [15] are normally used to only solve  
54 linear or second-order flow.

55 This paper presents a variation of the 3D model of Grilli *et al.* [16], focusing on  
 56 working with surface-piercing bodies with arbitrary geometry. Additionally, the code is  
 57 parallelized to work on modern computer clusters. Notably, we continue to use Taylor  
 58 series expansions for the time-stepping, shown by Machane and Canot [17] to be faster  
 59 than Runge-Kutta for the same accuracy. In the approach of Grilli *et al.*, however, they  
 60 used higher-order elements on a structured grid, which they refer to as mid-interval inter-  
 61 polation (MII). Unfortunately, this is best suited for simple wave propagation problems,  
 62 whereas complex surface piercing objects may not always be well-suited for structured  
 63 grids. As a result, this is reformulated for unstructured 3D meshes; although often struc-  
 64 tured high-order grids will be more accurate, we believe that it will be important to have  
 65 the capability to consider any mesh, as increasingly complex offshore structures may not  
 66 be easily adapted to a structured or block structured mesh.

67 For the present study, the general theory is presented in Section 2, and the discrete  
 68 equations are detailed in Section 3, including the derivation of the time-stepping scheme,  
 69 the assembly of the BEM system matrix, and the representation of rigid body surfaces  
 70 in our numerical model. Section 4 shows applications including the verification and the  
 71 validation of the numerical model with a bottom-mounted cylinder, a truncated cylinder  
 72 and finally a complex structure representing a simplified semi-submersible floater.  
 73 Concluding remarks follow in Section 5.

## 74 2. Mathematical model

75 We assume the fluid to be incompressible and inviscid and the flow to be irrotational.  
 76 We thus introduce a velocity potential  $\phi$  which satisfies mass conservation, expressed as  
 77 Laplace's equation within the entire fluid domain  $\Omega_f(t)$ . We assume that the boundary of  
 78 the fluid domain  $\partial\Omega_f(t)$  is divided into four parts, on which different types of boundary  
 79 conditions can be applied,  $\partial\Omega_f(t) = \Gamma_f(t) \cup \Gamma_c(t) \cup \Gamma_b \cup \Gamma_l(t)$  described later, including  
 80 the free-surface,  $\Gamma_f$ , the bottom boundary,  $\Gamma_b$ , the far-field edges (i.e., wavemaker or  
 81 sidewall boundary),  $\Gamma_l$ , and the surface of a fixed or floating body under consideration,  
 82  $\Gamma_c$ . The bottom boundary  $\Gamma_b$  is assumed to be time independent.

Denoting the Green's function,  $G(\mathbf{x}, \mathbf{y}) = \frac{1}{4\pi\|\mathbf{x}-\mathbf{y}\|}$ , the fundamental solution of  
 Laplace's equation in 3D (i.e.,  $\mathbb{R}^3$ ), the velocity potential obeys the following bound-  
 ary integral equation (BIE), for every point,  $\mathbf{x}$ , on the boundary:

$$c(\mathbf{x}, t)\phi(\mathbf{x}, t) = \int_{\partial\Omega_f(t)} \left( \frac{\partial\phi}{\partial n}(\mathbf{y}, t)G(\mathbf{x}, \mathbf{y}) - \phi(\mathbf{y}, t)\frac{\partial G}{\partial n}(\mathbf{x}, \mathbf{y}) \right) dS_y \quad (1)$$

83 where the function  $c(\mathbf{x}, t)$  denotes the inner solid angle seen from the boundary (field)  
 84 point  $\mathbf{x}$ , and  $\mathbf{y}$  is taken to be the source point on the boundary,  $\partial\Omega_f(t)$ .

On the free surface  $\Gamma_f(t)$ , the kinematic and dynamic boundary conditions state that:

$$\begin{cases} \phi_t(\mathbf{x}, t) = -gz - \frac{1}{2}\nabla\phi(\mathbf{x}, t) \cdot \nabla\phi(\mathbf{x}, t) & \text{for } \mathbf{x} \in \Gamma_f(t) \\ \frac{d\mathbf{x}}{dt} = \nabla\phi(\mathbf{x}, t) & \text{for } \mathbf{x} \in \Gamma_f(t) \end{cases} \quad (2)$$

85 The time-integration of these equations is described in the next section.

On the solid boundary of the floating body  $\Gamma_c(t)$ , we specify a free-slip condition, which expresses the normal derivative of the potential equal to the normal component of the body velocity on that boundary:

$$\phi_n(\mathbf{x}, t) \equiv \frac{\partial \phi}{\partial n}(\mathbf{x}, t) = \mathbf{v}_b(\mathbf{x}, t) \cdot \mathbf{n}_b(\mathbf{x}, t), \quad \forall \mathbf{x} \in \Gamma_f(t) \quad (3)$$

where  $\mathbf{n}_b(\mathbf{x}, t)$  denotes the unit normal vector pointing inward to the solid surface  $\Gamma_c(t)$ , at point  $\mathbf{x}$ , and  $\mathbf{v}_b(\mathbf{x}, t)$  is the body velocity. This condition remains valid on the fixed bottom and lateral boundaries,  $\Gamma_b \cup \Gamma_l$ , using a zero velocity, i.e.,  $\frac{\partial \phi}{\partial n} = 0$ .

To avoid the evaluation of the time derivative of the potential by use of a finite difference scheme, we apply the same BIE technique for computing  $\phi_t$ . Indeed  $\phi_t$  satisfies the same field equation, and requires the associated boundary conditions. Following Dombre *et al.* [18], the Neumann boundary condition satisfied by  $\phi_t$  on  $\Gamma_c(t)$  is expressed as:

$$\phi_{tn}(\mathbf{x}, t) \equiv \frac{\partial \phi_t}{\partial n}(\mathbf{x}, t) = \frac{d\mathbf{n}}{dt} \cdot (\mathbf{v}_b(\mathbf{x}, t) - \nabla \phi) + \mathbf{a}_b(\mathbf{x}, t) \cdot \mathbf{n} - \mathbf{v}_b(\mathbf{x}, t) \cdot (\nabla \nabla \phi \cdot \mathbf{n}) \quad (4)$$

with  $\mathbf{a}_b(\mathbf{x}, t)$  the solid acceleration vector at the position  $\mathbf{x}$  and time  $t$ .

### 3. Numerical scheme

#### 3.1. Boundary Element Discretization

At each time-step, we solve the BIE problems associated to  $\phi$  and to  $\phi_t$  by using an isoparametric BEM with flat triangles. The whole set of boundaries of the domain is meshed with non-overlapping triangles. On each triangular element  $\Gamma^k$ , we assume the field variables and the geometry to have linear variations, described as:

$$\begin{cases} \phi^k(\xi_1, \xi_2) = \sum_{\{j: \mathbf{x}_j \in \mathcal{V}^k\}} \phi_j N_j(\xi_1, \xi_2) \\ \mathbf{x}^k(\xi_1, \xi_2) = \sum_{\{j: \mathbf{x}_j \in \mathcal{V}^k\}} \mathbf{x}_j N_j(\xi_1, \xi_2) \end{cases} \quad (5)$$

where  $\mathcal{V}^k$  is the set of the vertices of  $\Gamma^k$  and  $(\xi_1, \xi_2)$  denotes the co-ordinates in the reference element  $\Gamma_\xi$ . The functions  $N_j$  are the so-called shape functions, i.e.,  $N_1(\xi_1, \xi_2) = \xi_1$ ,  $N_2(\xi_1, \xi_2) = \xi_2$ , and  $N_3(\xi_1, \xi_2) = 1 - \xi_1 - \xi_2$ .

Using a collocation method, we write that for any  $\mathbf{x}_i$  belonging to the discrete boundary of the fluid domain  $\Gamma_j = \partial\Omega_j$  at time  $t_j$ , we have:

$$c(\mathbf{x}_i, t_j) \phi(\mathbf{x}_i, t_j) = \int_{\Gamma_j} \left( \phi_n(\mathbf{y}_k, t_j) G(\mathbf{y}_k, \mathbf{x}_i) dS_k - \phi(\mathbf{y}_k, t_j) \frac{\partial G}{\partial n}(\mathbf{y}_k, \mathbf{x}_i) \right) dS_k \quad (6)$$

which can be, upon replacing the integral by a discrete sum over the vertices of the mesh, rewritten as:

$$\left( \delta_{ij} c_i + \sum_{j=1}^{N_{dof}} K_{ij}^n \right) \phi_j = \sum_{j=1}^{N_{dof}} K_{ij}^d \phi_j^d \quad (7)$$

with  $N_{dof}$  the number of vertices of the mesh. Adopting notations used in [19], we can show that the sub-matrices of this system are defined as:

$$\begin{aligned} K_{ij}^n &= \sum_{k \in \mathcal{S}_j} \int_{\Gamma_\xi} \left( N_{l_k(j)}(\xi) \frac{\partial G}{\partial n}(\mathbf{x}^k(\xi), \mathbf{x}_i) J^k(\xi) \right) d\xi \\ K_{ij}^d &= \sum_{k \in \mathcal{S}_j} \int_{\Gamma_\xi} \left( N_{l_k(j)}(\xi) G(\mathbf{x}^k(\xi), \mathbf{x}_i) J^k(\xi) \right) d\xi \end{aligned} \quad (8)$$

where  $l_k(j)$  is a local index varying from 1 to 3,  $\mathcal{S}_j$  denotes the set of elements containing the node of global index  $j$  and  $J^k(\xi)$  is the Jacobian of the transformation which maps  $\Gamma_\xi$  to  $\Gamma_k$ .  $l_k(j)$  thus associates to each  $\Gamma_k$  in  $\mathcal{S}_j$  the local index of the node  $\mathbf{x}_j$  in  $\Gamma_k$ . We also define the notation  $S_k$  as the set of vertices belonging to the element  $\Gamma_k$ . When assembling the matrix of the system (7), two situations may arise. If the source point  $\mathbf{x}_p$  is not belonging to the element  $\Gamma^k$ , both  $K_{ij}^d$  and  $K_{ij}^n$  are regular and we use a quadrature formula defined by a Dunavant rule [20] in order to perform numerical integration. Otherwise, the source point is belonging to  $\Gamma^k$ , *i.e.* there exists  $\mathbf{x}_j \in S_k$  such that  $\mathbf{x}_i = \mathbf{x}_j$ , in which case we treat the singularity by applying a Duffy transformation, similar to the polar change of coordinates presented in Grilli *et al.* [16].

In case of body motions, the velocity  $\mathbf{v}_b$  is non zero and the present model requires to evaluate the high-order term  $\mathbf{v}_b \cdot (\nabla \nabla \phi \cdot \mathbf{n})$  in Eq.(4), which is not trivial. This calculation may be simplified by transforming the second-derivative when computing the integral  $\int_{\Gamma_b(t)} G \phi_{tn} dS_y$ . This integral is decomposed into  $I_1 = \int_{\Gamma_b(t)} G \phi_{tn}^{(1)} dS_y$  and  $I_2 = \int_{\Gamma_b(t)} G \phi_{tn}^{(2)} dS_y$  with  $\phi_{tn}^{(1)} = \frac{d\mathbf{n}}{dt} \cdot (\mathbf{v}_b - \nabla \phi) + \mathbf{a}_b \cdot \mathbf{n}$  and  $\phi_{tn}^{(2)} = \mathbf{v}_b \cdot (\nabla \nabla \phi \cdot \mathbf{n})$ . Following Bai and Teng [21], the use of the Stokes formula and basic vector analysis manipulations lead to the relationship:

$$\begin{aligned} I_2 &= \int_{\Gamma_b(t)} G (\mathbf{v}_b \cdot \nabla (\nabla \phi)) \cdot \mathbf{n} dS_y = \int_{\partial \Gamma_b(t)} G (\nabla \phi \times \mathbf{v}_b) \cdot d\mathbf{y} + \\ &\int_{\Gamma_b(t)} G [(\boldsymbol{\Omega} \times \nabla \phi) + (\nabla G \cdot \nabla \phi) \mathbf{v}_b - (\nabla G \cdot \mathbf{v}_b) \nabla \phi] \cdot \mathbf{n} dS_y \end{aligned} \quad (9)$$

assuming the rigid body velocity is given by  $\mathbf{v}_b(\mathbf{x}) = \mathbf{v}_G + \boldsymbol{\Omega} \times (\mathbf{x} - \mathbf{x}_G)$  with  $\mathbf{v}_G$  the translational and  $\boldsymbol{\Omega}$  the rotational velocity vectors of the rigid body. We recall that, by convention, the unit normal vector  $\mathbf{n}$  is oriented towards the inside of the rigid body. In this respect, the line integral  $\int_{\partial \Gamma_b(t)} G (\nabla \phi \times \mathbf{v}_b) \cdot d\mathbf{y}$  must be evaluated considering that the tangent vector to the waterline  $d\mathbf{y}$  is such that the vector  $d\mathbf{y} \times \mathbf{n}(\mathbf{y})$  points outside of the rigid body.

### 3.2. Time-stepping

As in the original 2D-FNPF code of [22], we update both the position and the potential on the free surface  $\Gamma_f(t)$  by an explicit scheme based on a second-order explicit Taylor series expansion. In this scheme, the values of the potential  $\phi$  and the position  $\mathbf{x}$  at time

$t^{i+1}$  may be expressed as follows:

$$\begin{cases} \phi^{i+1} = \phi(\mathbf{x}^{i+1}, t^{i+1}) \\ = \phi(\mathbf{x}^i, t^i) + \frac{d\phi}{dt}(\mathbf{x}^i, t^i)\Delta t + \frac{d^2\phi}{dt^2}(\mathbf{x}^i, t^i)\frac{\Delta t^2}{2} \\ \mathbf{x}^{i+1} = \mathbf{x}^i + \frac{d\mathbf{x}}{dt}(\mathbf{x}^i, t^i, \phi^i, \phi_n^i)\Delta t + \frac{d^2\mathbf{x}}{dt^2}(\mathbf{x}^i, t^i, \phi^i, \phi_n^i)\frac{\Delta t^2}{2} \end{cases} \quad (10)$$

From system (10), several numerical schemes can be devised according to the choice of the advection velocity used in the material derivative  $\frac{d}{dt}$ . Let  $\mathbf{v}_p$  be a velocity field which is chosen to advect the free surface particles. When using the velocity  $\mathbf{v}_p$  for moving the free surface nodes, we obtain the following first-order derivatives:

$$\begin{cases} \frac{d\phi}{dt} = \phi_t + \mathbf{v}_p \cdot \nabla\phi \\ \frac{d\mathbf{x}}{dt} = \mathbf{v}_p \end{cases} \quad (11)$$

Then, if we take again the material derivative of the first-order coefficients along the velocity vector  $\mathbf{v}_p$ , we obtain:

$$\begin{cases} \frac{d^2\mathbf{x}}{dt^2} = \frac{\partial\mathbf{v}_p}{\partial t} + \nabla\mathbf{v}_p \cdot \mathbf{v}_p \\ \frac{d^2\phi}{dt^2} = \frac{d\phi_t}{dt} + \frac{d\mathbf{v}_p}{dt} \cdot \nabla\phi + \mathbf{v}_p \cdot \frac{d\nabla\phi}{dt} \end{cases} \quad (12)$$

Given the free surface dynamic boundary condition  $\phi_t = -gz - \frac{1}{2}\nabla\phi \cdot \nabla\phi$ , we can derive:

$$\frac{d\phi_t}{dt} = \mathbf{g} \cdot \mathbf{v}_p - \nabla\phi \cdot (\nabla\phi_t + \nabla\nabla\phi \cdot \mathbf{v}_p) \quad (13)$$

We also have:

$$\begin{aligned} \frac{d\mathbf{v}_p}{dt} \cdot \nabla\phi + \mathbf{v}_p \cdot \frac{d\nabla\phi}{dt} &= \frac{\partial\mathbf{v}_p}{\partial t} \cdot \nabla\phi + (\nabla\mathbf{v}_p \cdot \mathbf{v}_p) \cdot \nabla\phi \\ &\quad + \mathbf{v}_p \cdot \nabla\phi_t + (\nabla\nabla\phi \cdot \mathbf{v}_p) \cdot \mathbf{v}_p \end{aligned} \quad (14)$$

Summing all of these contributions, we obtain:

$$\begin{aligned} \frac{d^2\phi}{dt^2} &= \mathbf{g} \cdot \mathbf{v}_p + (\mathbf{v}_p - \nabla\phi) \cdot \nabla\phi_t \\ &\quad + \frac{\partial\mathbf{v}_p}{\partial t} \cdot \nabla\phi + (\nabla\nabla\phi \cdot \mathbf{v}_p) \cdot (\mathbf{v}_p - \nabla\phi) + (\nabla\mathbf{v}_p \cdot \mathbf{v}_p) \cdot \nabla\phi \end{aligned} \quad (15)$$

One can easily check that setting  $\mathbf{v}_p = \nabla\phi$  in Eq. (15) gives the Lagrangian second-order development:

$$\frac{d^2\phi}{dt^2} = \mathbf{g} \cdot \nabla\phi + \nabla\phi_t \cdot \nabla\phi + (\nabla\nabla\phi \cdot \nabla\phi) \cdot \nabla\phi \quad (16)$$

112 as can be found *e.g.* in Appendix A of [23].

We recall that the nonlinear conditions on the free surface read:

$$\begin{cases} \frac{\partial \eta}{\partial t} = \phi_z - \eta_x \phi_x - \eta_y \phi_y \\ \frac{\partial \phi}{\partial t} = -g\eta - \frac{1}{2} \nabla \phi \cdot \nabla \phi \end{cases} \quad (17)$$

113 When dealing with fixed vertical cylinders or fully submerged bodies (or cases without  
114 any body), it can be useful to consider a semi-Lagrangian scheme, allowing only vertical  
115 motion of the fluid particles. For this, we consider the definition  $\mathbf{v}_p = \frac{\partial \eta}{\partial t} \mathbf{e}_z$ . (For the  
116 equivalent material derivative with curved structures, see Zhang and Kashiwagi [24].)

While expanding this scheme, we first make the assumption that the function  $(x, y) \in \mathbb{R}^2 \mapsto \eta \in \mathbb{R}$  is single-valued (i.e., the waves are not overturning), which allows to get the relationships:

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} - \frac{\partial \eta}{\partial x} \frac{\partial \phi}{\partial x} - \frac{\partial \eta}{\partial y} \frac{\partial \phi}{\partial y} = \frac{\partial \phi}{\partial z} + \frac{n_x}{n_z} \frac{\partial \phi}{\partial x} + \frac{n_y}{n_z} \frac{\partial \phi}{\partial y} \quad (18)$$

where  $\mathbf{n} = (n_x, n_y, n_z)$  is the outward unit normal vector to the free surface. We can thus work out the following formula:

$$\frac{\partial \eta}{\partial t} = \frac{\phi_n}{n_z} \quad (19)$$

117 which is simpler to evaluate numerically as the present model relies on the distribution  
118 of the variables  $\phi$  and  $\phi_n$  on the boundary.

### 119 3.2.1. Discrete velocity on unstructured grids

For first-order elements, the velocity in a triangle is computed using a relationship found in Meyer *et al.* [25]. In the triangle number  $j$ , called  $T_j$ , the gradient of any linear function is calculated as follows:

$$\begin{aligned} \nabla \phi_{T_j} = \frac{1}{2A_j} & ((\phi_{j,i+1} - \phi_{j,i}) \mathbf{n}_j \times (\mathbf{x}_{j,i} - \mathbf{x}_{j,i-1}) \\ & + (\phi_{j,i-1} - \phi_{j,i}) \mathbf{n}_j \times (\mathbf{x}_{j,i+1} - \mathbf{x}_{j,i})) \end{aligned} \quad (20)$$

where  $A_j$  denotes the area of  $T_j$ ,  $\mathbf{n}_j$  the unit outward normal vector to  $T_j$  and the index  $(j, i)$  corresponds to the local index of the node number  $i$  located on the triangle  $T_j$ . The indices  $(j, i-1)$  and  $(j, i+1)$  refer to local indices of the nodes in the triangle  $T_j$  such that the arc  $\overline{\mathbf{x}_{j,i-1} \mathbf{x}_{j,i} \mathbf{x}_{j,i+1}}$  is positively oriented with respect to the local normal vector  $\mathbf{n}_j$ . According to these definitions, we can further derive the following relationships:

$$\begin{aligned} \mathbf{n}_j &= \frac{(\mathbf{x}_{j,i+1} - \mathbf{x}_{j,i}) \times (\mathbf{x}_{j,i-1} - \mathbf{x}_{j,i})}{\|(\mathbf{x}_{j,i+1} - \mathbf{x}_{j,i}) \times (\mathbf{x}_{j,i-1} - \mathbf{x}_{j,i})\|} \\ A_j &= \frac{1}{2} \|(\mathbf{x}_{j,i+1} - \mathbf{x}_{j,i}) \times (\mathbf{x}_{j,i-1} - \mathbf{x}_{j,i})\| \end{aligned} \quad (21)$$

We then take an average of Eq. (20) over the 1-ring neighborhood of the vertex  $i$ , which is the set of triangles containing  $\mathbf{x}_i$  and denoted by  $\mathcal{S}_i$  (see Fig. (1)). Taking into account

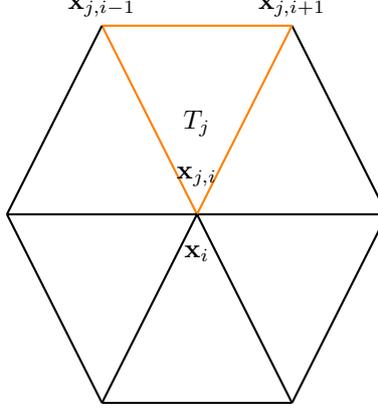


Figure 1: Sketch of the set of elements  $S_i$ , also called the 1-ring neighborhood of the vertex  $\mathbf{x}_i$ .

the contribution of  $\phi_n$  to the gradient, we obtain an approximation of the velocity vector:

$$\nabla\phi_i = \frac{1}{\sum_{j \in S_i} A_j} \sum_{j \in S_i} A_j \nabla\phi_{T_j} + \phi_n \frac{1}{\sum_{j \in S_i} A_j} \sum_{j \in S_i} A_j \mathbf{n}_j \quad (22)$$

Eq. (20) has been, for example, used in the field of computer graphics [25] in order to evaluate geometric quantities such as the principal curvature on irregular meshes. It is shown in [25] to achieve a comparable accuracy to finite differences schemes and has the advantage to be less sensitive to the mesh configuration.

### 3.2.2. Discrete acceleration on unstructured grids

For evaluating the terms related to second-order derivatives of  $\phi$ , we try to take advantage of the previous methodology. This is made possible by expressing the second-order tensor  $\nabla\nabla\phi$  in a local basis attached to the vertex under consideration. In matrix form, this term reads:

$$V_{kl}^i = (\nabla\nabla_{\mathcal{B}(\mathbf{x}_i)}\phi)_{kl} = \nabla(\nabla\phi \cdot \mathbf{s}_k) \cdot \mathbf{s}_l \quad (23)$$

with  $\mathcal{B}(\mathbf{x}_i) = (\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3)$  an orthonormal basis such that  $\mathbf{s}_3$  is the normal vector to the discrete surface at the node  $\mathbf{x}_i$ . We define the vector:

$$\begin{aligned} \beta_{ijk} = \frac{1}{2A_j} & ((\nabla\phi_{j,i+1} \cdot \mathbf{s}_k - \nabla\phi_{j,i} \cdot \mathbf{s}_k) \mathbf{n}_j \times (\mathbf{x}_{j,i} - \mathbf{x}_{j,i-1}) \\ & + (\nabla\phi_{j,i-1} \cdot \mathbf{s}_k - \nabla\phi_{j,i} \cdot \mathbf{s}_k) \mathbf{n}_j \times (\mathbf{x}_{j,i+1} - \mathbf{x}_{j,i})) \end{aligned} \quad (24)$$

The components of the matrix  $V^i$  can be calculated as:

$$\begin{cases} V_{kl}^i = \frac{1}{\sum_{j \in S_i} A_j} \left( \sum_{j \in S_i} A_j \beta_{ijk} \right) \cdot \mathbf{s}_l, & k \in \{1, 2, 3\}, l \in \{1, 2\} \\ V_{13}^i = V_{31}^i \\ V_{23}^i = V_{32}^i \\ V_{33}^i = -V_{11}^i - V_{22}^i \end{cases} \quad (25)$$

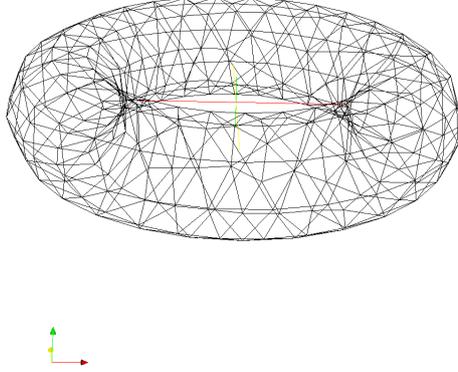


Figure 2: Irregular triangular grid of a torus with a mean radius of  $R_1 = 2\text{m}$  and a cross-section of radius  $R_2 = 1\text{m}$  generated by the algorithm NETGEN2D in the SALOME platform [26].

125 The relationships  $V_{13}^i = V_{31}^i$  and  $V_{23}^i = V_{32}^i$  are obtained by symmetry of the matrix  $V^i$   
 126 which comes from the fact that its components are second-order partial derivatives. The  
 127 last relationship  $V_{33}^i = -V_{11}^i - V_{22}^i$  stems from the Laplace equation.

This scheme consists therefore in interpolating linearly the projections of  $\nabla\phi$  on the local basis  $\mathcal{B}(\mathbf{x}_i)$  over the 1-ring neighbourhood of  $\mathbf{x}_i$ . The outward normal vector at  $\mathbf{x}_i$  is evaluated by means of the following weighted sum:

$$\mathbf{s}_3(\mathbf{x}_i) = \frac{\sum_{j \in \mathcal{S}_i} A_j \mathbf{n}_j}{\sum_{j \in \mathcal{S}_i} A_j} \quad (26)$$

with the vector  $\mathbf{n}_j$  previously defined on each triangle of  $\mathcal{S}_i$ . This vector is then scaled to define a unit normal vector, and from this we define two tangential vectors to the discrete surface at  $\mathbf{x}_i$ . For this purpose, we need to define a first tangential vector by projecting any point  $\mathbf{x}_j$  belonging to a triangle of the set  $\mathcal{S}_i$  and different from the point  $\mathbf{x}_i$  onto the tangential plane oriented by  $\mathbf{s}_3$  and passing through  $\mathbf{x}_i$  by means of the following formula:

$$\mathbf{s}_1(\mathbf{x}_i) = \frac{\mathbf{x}_j - \mathbf{x}_i - \mathbf{s}_3(\mathbf{x}_i) \cdot (\mathbf{x}_j - \mathbf{x}_i)}{\|\mathbf{x}_j - \mathbf{x}_i - \mathbf{s}_3(\mathbf{x}_i) \cdot (\mathbf{x}_j - \mathbf{x}_i)\|} \quad (27)$$

128 A last vector of the basis  $\mathcal{B}(\mathbf{x}_i)$  is then given by  $\mathbf{s}_2(\mathbf{x}_i) = \mathbf{s}_3(\mathbf{x}_i) \times \mathbf{s}_1(\mathbf{x}_i)$ . The performance  
 129 of these discrete differential operators is finally assessed on the case of the irregular mesh  
 130 of a torus with a mean radius of  $R_1 = 2\text{m}$  and a cross-section of radius  $R_2 = 1\text{m}$ . An  
 131 example of discretization for the torus is represented on Fig. 2. We use the function  
 132  $\phi = \exp k_z z \sin(k_x x + k_y y)$  as a test case. We consider the following error indicators:

- 133 • The normalized maximum error of the vector  $\nabla\phi$  (denoted by  $L^\infty$  in the figures)  
 134 and defined as  $\epsilon = \frac{\max_{i=1..N} \|\nabla\phi - \nabla\phi_{ref}\|_i}{\max_{i=1..N} \|\nabla\phi_{ref}\|_i}$
- 135 • The normalized average error of the vector  $\nabla\phi$  (denoted by  $L^1$  in the figures) and  
 136 defined as  $\epsilon = \frac{\sum_{i=1}^N \|\nabla\phi - \nabla\phi_{ref}\|_i}{\sum_{i=1}^N \|\nabla\phi_{ref}\|_i}$

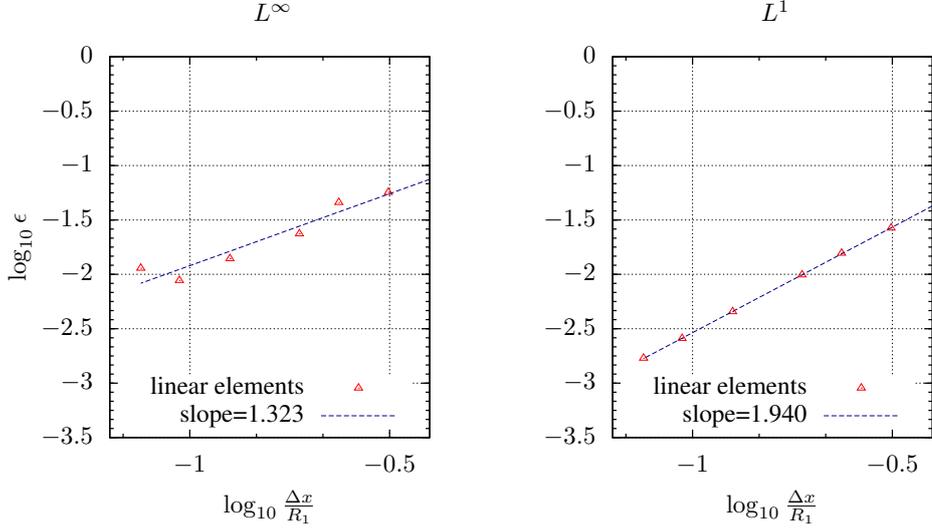


Figure 3: Maximum normalized error (left) and average normalized error (right) for the velocity vector  $\mathbf{v}$  associated to the potential  $\phi = \exp(k_z z) \sin(k_x x + k_y y)$  with  $k_x = 1\text{m}^{-1}$ ,  $k_y = 0.5\text{m}^{-1}$  and  $k_z = \sqrt{k_x^2 + k_y^2}$  on a torus.

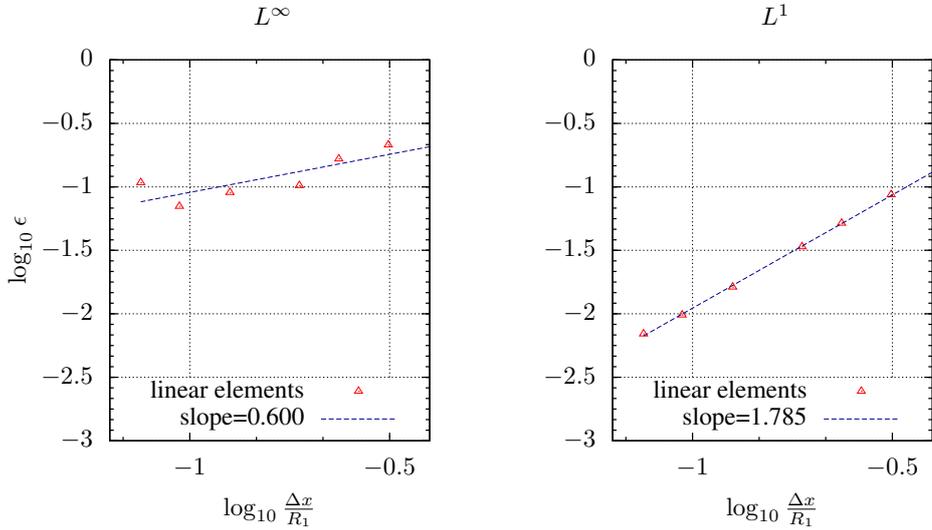


Figure 4: Maximum normalized error (left) and average normalized error (right) for the acceleration vector  $\mathbf{a}$  associated to the potential  $\phi = \exp(k_z z) \sin(k_x x + k_y y)$  with  $k_x = 1\text{m}^{-1}$ ,  $k_y = 0.5\text{m}^{-1}$  and  $k_z = \sqrt{k_x^2 + k_y^2}$  on a torus.

137 The errors for the tested velocity field are represented on Fig. (3). The ones for the  
138 acceleration is represented on Fig. (4). The parameters of the tested potential are set to  
139  $k_x = 1\text{m}^{-1}$ ,  $k_y = 0.5\text{m}^{-1}$  and  $k_z = \sqrt{k_x^2 + k_y^2}$ . It can be concluded that the proposed  
140 method converges in  $L^1$  norm on a smooth surface. In this norm, the error is scaling, in  
141 the root mean square sense, as  $\left(\frac{\Delta x}{R_1}\right)^{-1.940}$  for the velocity vector and as  $\left(\frac{\Delta x}{R_1}\right)^{-1.785}$   
142 for the acceleration vector.

### 143 3.3. Parallel assembly of the system matrices

144 In order to speed-up the simulations for large-size problems, the nodes which make  
145 up the BEM mesh are subdivided into nearly equal subsets based on a domain decom-  
146 position. That is to say, the main simulation (e.g., time-stepping) is handled on a single  
147 processor, but when many processors are available, the workload for solving the Laplace  
148 equation is divided over the available processors. This is done by taking the linear system  
149 of equations shown in Eq. 7, and only computing (and storing) part of this coefficient  
150 matrix on any given processor. The resulting system is then solved with an iterative  
151 linear solver, BiCGSTAB. The Message Passing Interface (MPI) is used to exchange  
152 data across the distributed computer cluster.

153 We test the efficiency of our parallelization by computing the solution of a mixed  
154 Boundary Value Problem for the velocity potential  $\phi(x, y, z) = x$ . The geometry is a  
155 box of size  $L_x \times L_y \times L_z = 1\text{m} \times 1\text{m} \times 1\text{m}$ . On the top surface, we impose a Dirichlet  
156 boundary condition  $\phi = x$ . On the remaining surfaces, we impose a Neumann boundary  
157 condition  $\phi_n = n_x$ ,  $n_x$  being the  $x$ -component of the outward normal vector to the box.  
158 We test the solution process for a moderate grid of 16,743 collocation nodes with different  
159 numbers of processors  $n_p \in \{1, 4, 16, 64\}$ . The results are represented on Fig. 5.

160 This approach is in contrast to Bai and Eatock Taylor [27], for example, where they  
161 implement domain decomposition by adding extra boundary elements within the domain  
162 to create individual subdomains. They therefore only scale up to around a 10x speed-up  
163 compared to their solution on a single processor. This means that the discrete equations  
164 that are being solved can change depending on the number of processors used. Here,  
165 on the other hand, only the coefficient matrix is shared, so the same equation is being  
166 considered. This is possible because the amount of information that needs to be shared  
167 between processors is small compared to the size of the overall system matrix.

### 168 3.4. Mesh deformation

#### 169 3.4.1. Time-updating of deformable surfaces

The velocity potential  $\phi$  and the position  $\mathbf{x}$  on the free-surface are updated using  
Eq.(10). In order to avoid an incompatibility of velocity on the far-field edges, the free-  
surface boundary conditions are corrected in a certain area near the exterior boundary  
of the free surface. For a cylindrical fluid domain, as those which will be considered in  
the validation presented hereafter, we consider an inner cylindrical domain of radius  $R_\lambda$   
and an outer cylindrical domain of radius  $R_{ext}$  (Fig. (6)). Then, the absorbing area is  
defined by the set  $D_\lambda = \{\mathbf{x} = (r, \theta, z) \in \Gamma_f \text{ such that } (r, \theta) \in [R_\lambda, R_{ext}] \times [0, 2\pi]\}$ . In

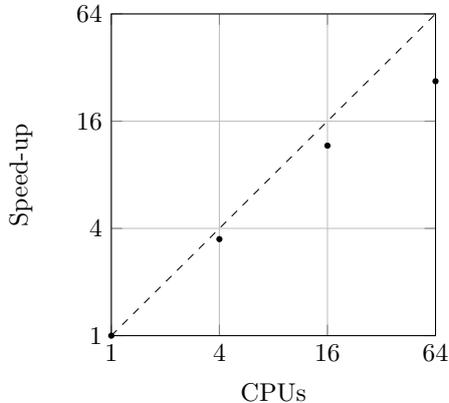


Figure 5: Speed-up  $\frac{T_{n_p=1}}{T_{n_p}}$  of the assembling step plus the solution of the system matrix with respect to the number of processors  $n_p$ , invoked with MPI, with a mesh of 16,743 collocation nodes.  $T_{n_p}$  is the CPU time of the process when using  $n_p$  processors.

the set  $D_\lambda$ , the free surface and the potential are corrected as:

$$\begin{aligned}
 z^* &\leftarrow z - \Delta t \omega_i \left( \frac{r - R_\lambda}{R_{ext} - R_\lambda} \right)^2 (z - D(t) \eta_i(r, \theta, t)) \\
 \phi^* &\leftarrow \phi - \Delta t \omega_i \left( \frac{r - R_\lambda}{R_{ext} - R_\lambda} \right)^2 (\phi - D(t) \phi_i(r, \theta, z^*, t))
 \end{aligned} \tag{28}$$

170 where  $\eta_i$  and  $\phi_i$  correspond to a theoretical wave profile. For the absorption of the  
 171 incident waves, other techniques like that developed by Clamond *et al.* [28] could be  
 172 more efficient for irregular waves. The implementation of this method would require to  
 173 solve (in parallel with MPI) an additional linear system corresponding to the vertices of  
 174 the free surface mesh. This is left for future works.

175 For the bottom mounted cylinder, we will use a rectangular domain. In this case, the  
 176 absorbing beach defined above is divided into two parts. The first part lies in front of the  
 177 entrance of the tank and is defined as:  $D_{\lambda,1} = \{\mathbf{x} = (x_1, x_2, x_3) \in \Gamma_f \text{ such that } (x_1, x_2) \in$   
 178  $[0, L_y] \times [0, l_\lambda]\}$  where  $l_\lambda$  is the length of the absorbing beach. In this domain,  $\eta_i$  and  
 179  $\phi_i$  will be chosen to correspond to a desired incident wave profile. At the end of the  
 180 domain, a second beach is defined:  $D_{\lambda,2} = \{\mathbf{x} = (x_1, x_2, x_3) \in \Gamma_f \text{ such that } (x_1, x_2) \in$   
 181  $[0, L_y] \times [L_x - l_\lambda, L_x]\}$ . In this second domain, we set  $\eta_i = 0$  and  $\phi_i = 0$ .

### 182 3.4.2. Time-updating of rigid surfaces

183 The nodes associated to the fluid particles on the body surface are updated following  
 184 the rigid body motion. We restrict ourselves to the forced motion of a rigid body. We  
 185 recall that, with a rigid body, for any material points  $\mathbf{x}_A$  and  $\mathbf{x}_B$  belonging to the body,  
 186 we have the relationship  $\left\{ \frac{d(\mathbf{x}_B - \mathbf{x}_A)}{dt} \right\}_B = 0$  in the reference frame of the body  $\mathcal{B}$ .

Rotation matrices around the axes of the fixed global reference frame are defined in the following fashion:

$$R_{\theta_x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_x & -\sin \theta_x \\ 0 & \sin \theta_x & \cos \theta_x \end{pmatrix} \tag{29}$$

$$R_{\theta_y} = \begin{pmatrix} \cos \theta_y & 0 & \sin \theta_y \\ 0 & 1 & 0 \\ -\sin \theta_y & 0 & \cos \theta_y \end{pmatrix} \quad (30)$$

$$R_{\theta_z} = \begin{pmatrix} \cos \theta_z & -\sin \theta_z & 0 \\ \sin \theta_z & \cos \theta_z & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (31)$$

At any time, the link between the position expressed in  $\mathcal{R}$  and the one expressed in  $\mathcal{B}$  is made with the equation:

$$(\mathbf{x}(t) - \mathbf{x}_G(t))_{\mathcal{R}} = R_{\theta_x} R_{\theta_y} R_{\theta_z} (\mathbf{x}(t) - \mathbf{x}_G(t))_{\mathcal{B}} \quad (32)$$

187 with  $(\mathbf{x}(t) - \mathbf{x}_G(t))_{\mathcal{B}}$  a time-invariant vector. We define the fixed basis  $\mathcal{R} = (\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$   
 188 and the moving basis  $\mathcal{B} = (\mathbf{e}_{xb}, \mathbf{e}_{yb}, \mathbf{e}_{zb})$ . The latter can be obtained by the transforma-  
 189 tion  $R_{\theta_x} R_{\theta_y} R_{\theta_z}$  applied to the basis  $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$ .

### 190 3.4.3. Merging of the meshes

191 As the free surface vertices are tracked in a Lagrangian manner during their motion,  
 192 the meshes of the free-surface and the body need to be reconnected. Since the distance  
 193 between the displaced waterline and the displaced body appears to be small, similarly to  
 194 Bai and Eatock Taylor [5], we make a projection of the nodes located on the waterline  
 195 onto the body geometry. Here we consider an application of this technique in the case of  
 196 a cylindrical body surface. The following procedure is used for each node of the waterline:

1. Apply the MEL time-stepping to the position  $\mathbf{x}_f^*$ :

$$\mathbf{x}_f^*(t + \Delta t) = \mathbf{x}_f(t) + \frac{d\mathbf{x}_f}{dt} \Delta t + \frac{d^2\mathbf{x}_f}{dt^2} \frac{\Delta t^2}{2} \quad (33)$$

2. Express the vector  $\mathbf{x}_f^*$  in the body reference frame  $\mathcal{B}$ :

$$\begin{aligned} \mathbf{x}_f^*(t + \Delta t)_{\mathcal{B}} &= \mathbf{x}_G(t + \Delta t)_{\mathcal{B}} \\ &+ R_{\theta_z}^{-1} R_{\theta_y}^{-1} R_{\theta_x}^{-1} (\mathbf{x}_f^*(t + \Delta t) - \mathbf{x}_G(t + \Delta t))_{\mathcal{R}} \end{aligned} \quad (34)$$

3. Express  $\mathbf{x}_f^*(t + \Delta t)_{\mathcal{B}}$  in a coordinate system suited to the shape under consideration (for a cylinder, we choose the polar coordinate system):

$$\begin{aligned} \mathbf{x}_f^*(t + \Delta t)_{\mathcal{B}} &= \mathbf{x}_G(t + \Delta t)_{\mathcal{B}} \\ &+ r_f (\cos \theta_f \mathbf{e}_{xb} + \sin \theta_f \mathbf{e}_{yb}) + z_f \mathbf{e}_{zb} \end{aligned} \quad (35)$$

4. Replace the radial coordinate  $r_f$  by the radius of the cylinder  $R_b$  in the reference frame  $\mathcal{B}$ :

$$\begin{aligned} \mathbf{x}_f(t + \Delta t)_{\mathcal{B}} &= \mathbf{x}_G(t + \Delta t)_{\mathcal{B}} \\ &+ R_b (\cos \theta_f \mathbf{e}_{xb} + \sin \theta_f \mathbf{e}_{yb}) + z_f \mathbf{e}_{zb} \end{aligned} \quad (36)$$

5. Transform the new coordinate  $\mathbf{x}_f(t + \Delta t)_{\mathcal{B}}$  into the global reference frame  $\mathcal{R}$ :

$$\begin{aligned} \mathbf{x}_f(t + \Delta t)_{\mathcal{R}} &= \mathbf{x}_G(t + \Delta t)_{\mathcal{R}} \\ &+ R_{\theta_x} R_{\theta_y} R_{\theta_z} (\mathbf{x}_f(t + \Delta t) - \mathbf{x}_G(t + \Delta t))_{\mathcal{B}} \end{aligned} \quad (37)$$

197 For each geometric node of the waterline, we denote by  $\mathbf{x}_f$  the position vector of  
 198 the node belonging to the mesh of the free surface and by  $\mathbf{x}_b$  the position vector of  
 199 the node belonging to the mesh of the body surface. Prior to updating the position  
 200 of the nodes for the next time-step,  $\mathbf{x}_f$  and  $\mathbf{x}_b$  are equal. An interpolation of the free  
 201 surface elevation angular distribution found with the procedure described above, is made  
 202 on a fixed uniform angular distribution. In other words, for each double-node of the  
 203 intersection, we first express  $\mathbf{x}_f$  and  $\mathbf{x}_b$  in the basis  $(\mathbf{e}_{xb}, \mathbf{e}_{yb}, \mathbf{e}_{zb})$ :

$$\begin{aligned} (\mathbf{x}_f - \mathbf{x}_G)_B &= R_b(\cos \theta_f \mathbf{e}_{xb} + \sin \theta_f \mathbf{e}_{yb}) + z_f \mathbf{e}_{zb} \\ (\mathbf{x}_b - \mathbf{x}_G)_B &= R_b(\cos \theta_b \mathbf{e}_{xb} + \sin \theta_b \mathbf{e}_{yb}) + z_b \mathbf{e}_{zb} \end{aligned} \quad (38)$$

Then, for each angle  $\theta_{bj}$ , we compute the indices:

$$\begin{aligned} i_1 &= \operatorname{argmin}_{i \in \llbracket 1, N_d \rrbracket} [|\theta_{fi} - \theta_{bj}|; \theta_{bj} \geq \theta_{fi}] \\ i_2 &= \operatorname{argmin}_{i \in \llbracket 1, N_d \rrbracket} [|\theta_{fi} - \theta_{bj}|; \theta_{bj} < \theta_{fi}] \end{aligned} \quad (39)$$

204 with  $N_d$  the number of double-nodes at the intersection of the mesh of the body surface  
 205 and the mesh of the free surface. Afterwards, we interpolate linearly the values of the  
 206 potential  $\phi_j$  and the values of the local free-surface elevation  $(\mathbf{x}_f - \mathbf{x}_b)_B \cdot \mathbf{e}_{zb}$  in the  
 207 interval  $[\theta_{i_1}, \theta_{i_2}]$ .

#### 208 3.4.4. Filtering of the waterline

When considering steep sea states, one often observes instabilities near the waterline,  
 which eventually propagate throughout the domain. As in many publications, we thus  
 filter, at each time step, the variables of the waterline defined previously. We apply,  
 similarly to [29, 30], a filtering formula with a moving stencil of 7 points for the variables  
 $\phi$  and  $(\mathbf{x}_f - \mathbf{x}_b)_B \cdot \mathbf{e}_{zb}$  along the waterline only. For the potential  $\phi$ , this formula reads:

$$\phi_i = \frac{1}{32}(-\phi_{i-3} + 9\phi_{i-1} + 16\phi_i + 9\phi_{i+1} - \phi_{i+3}). \quad (40)$$

#### 209 3.4.5. Remeshing step

210 In time-domain simulations with floating bodies, the geometry is always changing.  
 211 As a consequence it is necessary to change the mesh of the geometry. On the one hand,  
 212 in order to prevent the triangles to be distorted in the vicinity of the piercing surface  
 213 cylinder, a Laplace smoothing technique is applied, for the mesh of the free surface, on  
 214 the projection of the nodes onto an horizontal plane. This technique has also been used  
 215 by [7, 5]. On the other hand, the mesh of the body is generated with regular quadrangles  
 216 divided into triangles by their diagonal. For this part of the mesh, we modify only the  
 217 vertical position of the nodes to get a uniform vertical distribution.

## 218 4. Numerical results

### 219 4.1. Sway motion of a truncated vertical cylinder

#### 220 4.1.1. Problem setup

We are concerned in this section with the imposed periodic motion in sway (*i.e.* along  
 $x_1$  axis) of a truncated cylinder. The position vector of the center of gravity is given the  
 form:

$$\mathbf{x}_G(t) = (x_1(t), x_2(t), x_3(t)) = (A \sin \omega t, 0, 0) \quad (41)$$

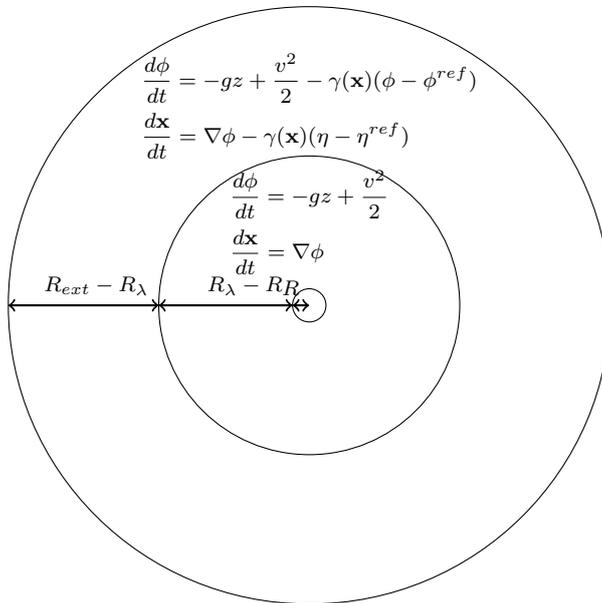


Figure 6: Sketch of the decomposition of the domain with respect to the free surface boundary conditions, noting boundary conditions for fully Lagrangian time-updating. In this case, Laplace smoothing is used to maintain a well formed mesh on the free-surface.

221 with  $\omega$  the angular frequency of motion. The water depth of the numerical wave tank  
 222 is set to  $d = 1.5R$ , with  $R$  the radius of the cylinder, and the draft of the cylinder  
 223 is set to  $B = 0.5R$ . The shape of the fluid domain is also cylindrical, with a radius  
 224  $R_{ext}$ . Moreover, the fluid domain is divided into two cylindrical parts, as represented on  
 225 Fig. (6).

226 An outer sub-domain is defined, where the boundary conditions include corrective  
 227 terms, e.g.,  $\gamma(\mathbf{x})(\eta - \eta^{ref})$  for the free surface elevation, with  $\eta^{ref}$  an analytical free  
 228 surface elevation and  $\gamma$ , a space-dependent factor. For the radiation,  $\eta^{ref} = 0$  and  
 229  $\phi^{ref} = 0$  in order to damp the reflected waves radiated by the body motion. In the inner  
 230 domain, the fully nonlinear boundary conditions are used for determining the evolution  
 231 of the free surface elevation and potential. The radius of the inner cylinder is denoted  
 232 by  $R_\lambda$ . Such a configuration has been employed for instance by Ferrant [31].

#### 233 4.1.2. Mesh convergence

234 In this section, we study the mesh convergence of the model for one of the frequencies  
 235 tested hereafter. We note that by having a damping region such as Eq. 28, as used  
 236 before by Cointe [32], volume conservation is not inherently guaranteed, as the free-  
 237 surface boundary conditions are modified. We consider the non-dimensional wavenumber  
 238  $kR = 1.4$  (with a cylinder radius of 1 m), which gives a wavelength  $\lambda \approx 4.48\text{m}$  and a  
 239 period  $T \approx 1.72\text{s}$ . The radius of the external domain is chosen as  $R_{ext} = 8\text{m}$ . An  
 240 absorbing beach of radial length 4.5m is used. We check in this section the convergence  
 241 by looking at the conservation of the discrete water volume in the wave tank. The  
 242 characteristics of the meshes used for the convergence study are recalled in Table 1.

	Mesh <i>a</i>	Mesh <i>b</i>	Mesh <i>c</i>	Mesh <i>d</i>	Mesh <i>e</i>
$\frac{2\pi R_{ext}}{\Delta r_e}$	100	150	200	250	300
$\frac{2\pi R}{\Delta r_e}$	20	30	40	50	60
$\frac{h}{\Delta z_e}$	4	6	6	8	10
$\frac{B}{\Delta z_i}$	8	12	14	18	20
min $l_{1D}$ [m]	0.313	0.209	0.157	0.126	0.105
max $l_{1D}$ [m]	0.505	0.335	0.251	0.201	0.168
$N_{FS}$	856	3141	3286	12437	12576
$N_{body}$	210	535	760	1116	1940

Table 1: Truncated cylinder in sway motion along the axis  $Ox$ : characteristics of the meshes used in the volume conservation study.  $\Delta r_e$  (respectively  $\Delta r_i$ ) is the space-step of the mesh on the circumference of the outer (respectively inner) cylinder.  $\Delta z_e$  (respectively  $\Delta z_i$ ) is the space-step of the mesh on the vertical direction of the outer (respectively inner) cylinder.  $N_{FS}$  is the number of nodes on the free surface.  $N_{body}$  is the number of nodes on the truncated cylinder surface.  $l_{1D}$  is the length of the edges on the mesh of the free surface.

In Fig. 7, we represent the volume error defined as:

$$\epsilon(t^+) = \frac{V(t^+) - V(t^+ = 0)}{V(t^+ = 0)} \quad (42)$$

as a function of the non-dimensional time  $t^+ = \frac{t}{T}$  and the spatial discretization. For each mesh, we enforce a Courant–Friedrichs–Lewy (CFL) condition by automatically adapting the time-step as follows:

$$\Delta t = C_0 \frac{\min_{(i,j); i < j; (\mathbf{x}_i, \mathbf{x}_j) \in \Gamma_f(t)^2} \|\mathbf{x}_i - \mathbf{x}_j\|}{\sqrt{gd}} \quad (43)$$

243 where we choose  $C_0 = 0.4$ , following Grilli *et al.* [33].

244 It is seen in Fig. 7 that the error  $\epsilon_V$  stabilizes after 6 periods and decreases with the  
245 time-step. More specifically, after 7 periods, the mean relative error is around  $8 \cdot 10^{-5}$   
246 for the time-step  $\Delta t_a = 0.0122T$ , while it is around  $4 \cdot 10^{-5}$  for the time-step  $\Delta t_b =$   
247  $0.0069T \approx \frac{1}{2}\Delta t_a$ . This shows that the error is decreasing linearly with the time-step.  
248 Moreover we can observe that the amplitude of the high frequency oscillations decreases  
249 also with the time-step.

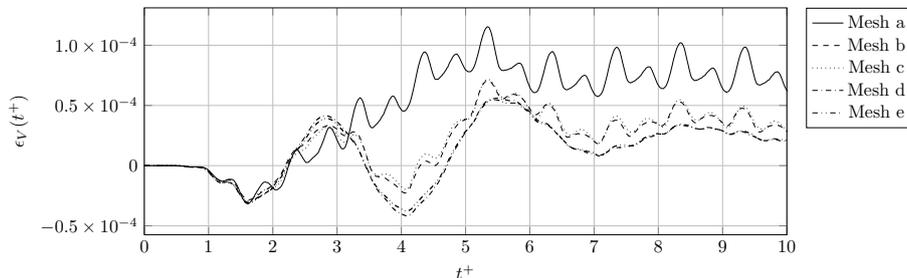


Figure 7: Truncated cylinder in sway motion along the axis  $Ox$ : computation of the relative volume error  $\epsilon_V(t^+)$  for various spatial and time discretizations.

250 *4.1.3. Force coefficients*

251 The radius of the outer cylinder is set to  $R_{ext} = 2\lambda$  where  $\lambda$  is the wavelength expected  
 252 from the linear theory at the water depth  $d$  and the angular frequency  $\omega$  (Fig. 8). The  
 253 entrance of the absorbing beach is located at a distance  $R_\lambda = \lambda$  from the origin  $(0, 0, 0)$ .  
 254 This problem has been recently addressed by Zhou *et al.* [29], using a HOBEM devised  
 255 for structured quadrangular meshes. Our results are compared to the linear BEM of  
 256 Yeung [34], the second-order frequency BEM model of Teng *et al.* [35], and the nonlinear  
 257 method of Zhou *et al.*

258 For the amplitude of motion under consideration,  $\frac{A}{R} = 0.15$ , we observe on Fig. (9)  
 259 a fair agreement between the added-mass computed with our model and the results of  
 260 Zhou *et al.* [29]. The discrepancy with the linear results of Yeung is reduced as the non-  
 261 dimensional wavenumber  $kR$  decreases. This behavior is not surprising as, for a given  
 262 choice of non-dimensional amplitude  $\frac{A}{R}$ , the associated wave steepness will increase with  
 263 increasing value of  $kR$ , resulting in larger nonlinear effects.

264 For the linear radiation damping coefficient  $B_{11}$ , represented in Fig. 10, a still closer  
 265 agreement with the linear theory is found. A similar agreement is observed for the non-  
 266 dimensional first-order coefficient of the horizontal force  $\frac{F_x^{(1)}}{\rho A R^2}$ , represented on Fig. 11,  
 267 except a slight discrepancy which appears at  $kR = 1$ .

268 Despite the apparently linear behavior of the horizontal force, this case clearly shows  
 269 the interest of using a nonlinear model. As mentioned by Zhou *et al.* [29], according to  
 270 a theoretical result demonstrated by Wu [36], the vertical force oscillates at twice the  
 271 frequency of the motion  $2\omega$ . As before, the discrepancy between the weakly nonlinear  
 272 model of Teng *et al.* [35] and the fully nonlinear model of Zhou *et al.* increases with  
 273 increasing values of the parameter  $kR$ . The existence of the second-order coefficient  $F_z^{(2)}$   
 274 is well captured with our model as can be seen in Fig. 12. We see that our method allows  
 275 us to find a very good agreement with the frequency analysis, for wavenumbers such that  
 276  $kR \leq 0.8$ . For decreasing values of  $kR$ , the present results exhibit a closer convergence  
 277 towards the results of the second-order model in comparison with the model of Zhou *et al.*  
 278 *al.* For larger  $kR$ , our nonlinear model deviates from the weakly nonlinear model and  
 279 predicts higher values of  $F_z^{(2)}$ , which is not the case of the model of Zhou *et al.*

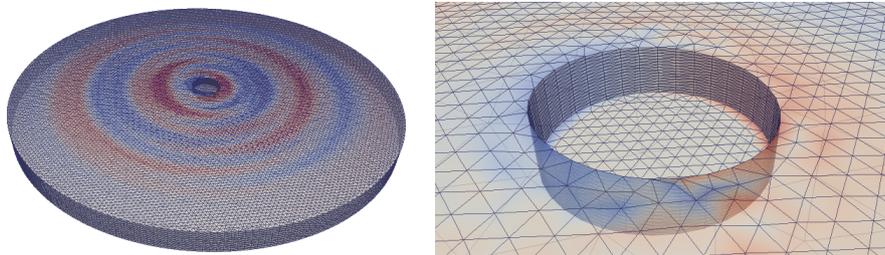


Figure 8: Computational domain (left panel) and close-up of grid near cylinder (right panel) for test cases of waves moving cylinder.

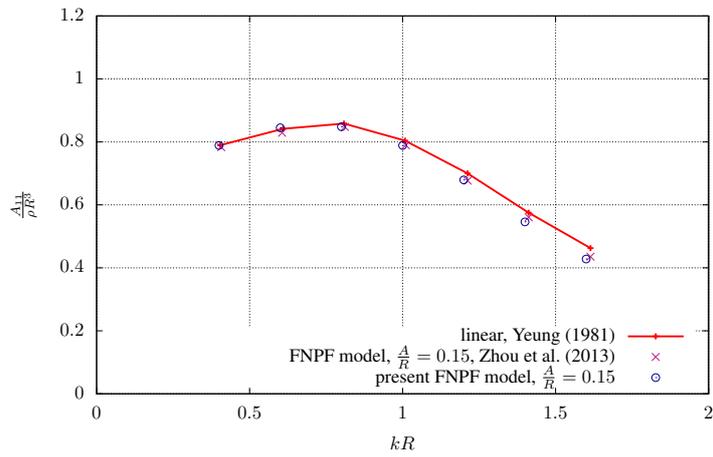


Figure 9: Truncated cylinder in sway motion along the axis  $Ox$ : computation of the added-mass with respect to the non-dimensional wavenumber  $kR$  for various numerical models.

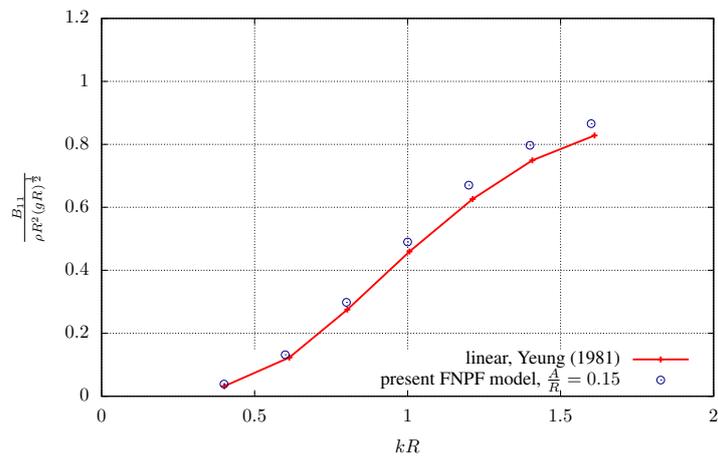


Figure 10: Truncated cylinder in sway motion along the axis  $Ox$ : computation of the radiation damping coefficient with respect to the non-dimensional wavenumber  $kR$  for various numerical models.

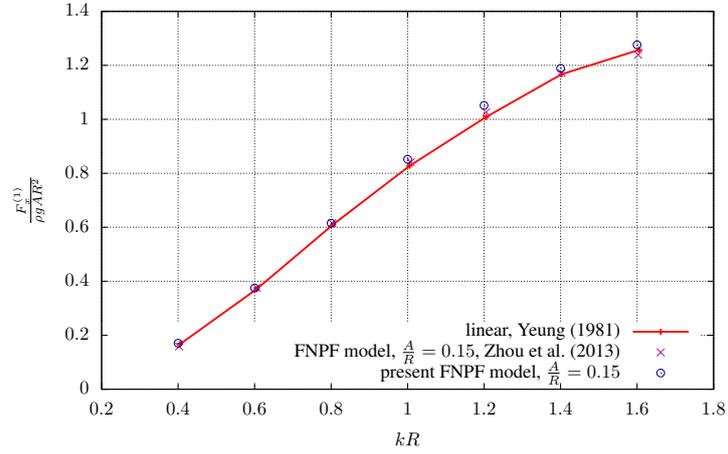


Figure 11: Truncated cylinder in sway motion along the axis  $Ox$ : first-order Fourier coefficient associated to the horizontal force  $F_x$  with respect to the non-dimensional wavenumber  $kR$  for various numerical models.

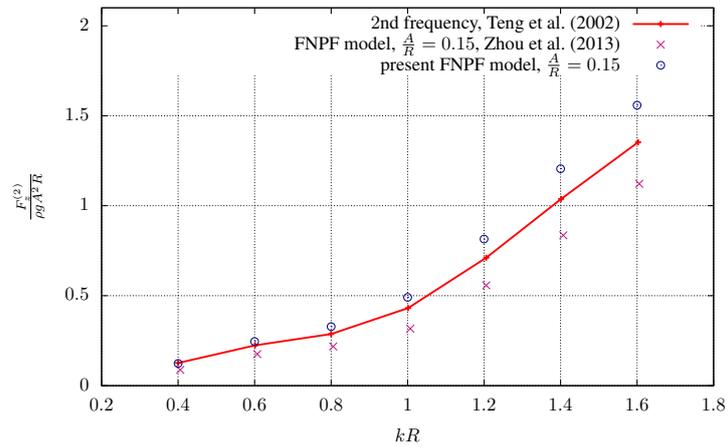


Figure 12: Truncated cylinder in sway motion along the axis  $Ox$ : second-order Fourier coefficient associated to the vertical force  $F_z$  with respect to the non-dimensional wavenumber  $kR$  for various numerical models.

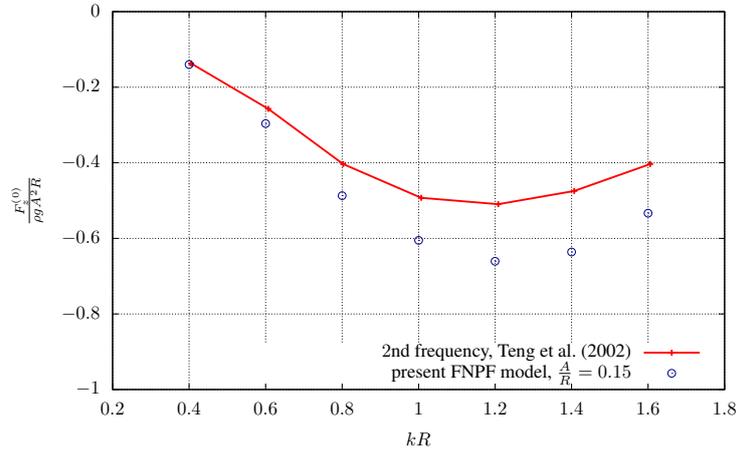


Figure 13: Truncated cylinder in sway motion along the axis  $Ox$ : zero-order Fourier coefficient associated to the vertical force  $F_z$  with respect to the non-dimensional wavenumber  $kR$  for various numerical models.

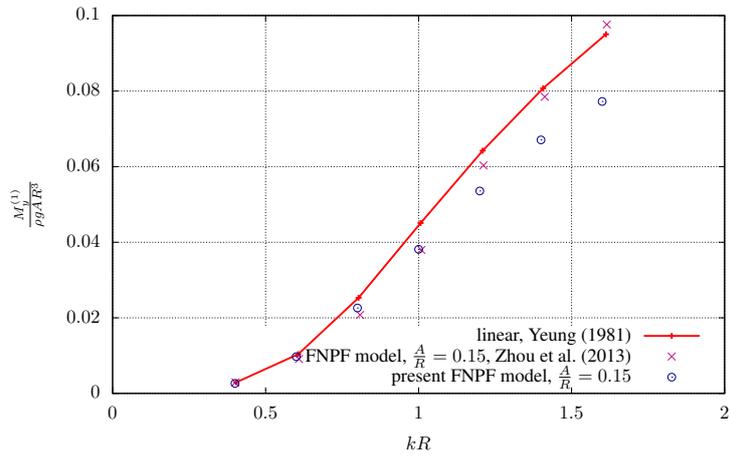


Figure 14: Truncated cylinder in sway motion along the axis  $Ox$ : first-order Fourier coefficient associated to the overturning moment  $M_y$  with respect to the non-dimensional wavenumber  $kR$  for various numerical models.

280 On Fig. 13, the non-dimensional difference between the zero-order coefficient  $F_z^{(0)}$   
 281 and the hydrostatic force  $\rho g V_b$ ,  $V_b$  being the discretized wet volume of the body, is  
 282 represented. Our method predicts very well the load for  $kR = 0.4$  and the trend of this  
 283 quantity is correctly reproduced.

284 Finally, we represent on Fig. 14 the variation of the first-order coefficient associated  
 285 to the overturning moment  $\frac{M_y^{(1)}}{\rho A R^3}$ . Our results are very close to those of Zhou *et al.* [29]  
 286 for wavenumbers  $kR$  smaller than  $kR = 1$ . For larger values of  $kR$ , we observe with  
 287 our model a clear deviation from the linear theory, suggesting a significant influence of  
 288 nonlinear effects at high frequencies, although this is contrary to the results of Zhou *et*  
 289 *al.*.

#### 290 4.2. Diffraction of long waves on a bottom-mounted cylinder

291 In this section, we analyze the diffraction of a long wave on a bottom mounted vertical  
 292 cylinder of radius  $R = 0.25\text{m}$  (Fig. 15). The wavenumber  $k$  is chosen such that  $kR =$   
 293  $0.245$ , which gives for linear theory a wave period  $T = 2.03\text{s}$ . Simulations are repeated  
 294 for 6 different wave amplitudes such that  $kA = \{0.025, 0.05, 0.075, 0.10, 0.125, 0.150\}$ .  
 295 The signal is analyzed with a discrete Fast Fourier Transform on the time-interval  $t^+ =$   
 296  $\frac{t}{T} \in [5, 13]$ . For this case, the wavelength in deep-water may be estimated as  $\lambda \approx 6.41\text{m}$ .  
 297 The domain is a box with the dimensions  $L_x \times L_y \times L_z = 26\text{m} \times 6\text{m} \times 3.2\text{m}$ . The  
 298 mesh used for that study contains  $N_{dof} = 16,743$  nodes. When generating the mesh  
 299 of the free surface with the algorithm NETGEN2D in the SALOME platform [26], we  
 300 used  $N_x = 150$  segments in the longitudinal direction,  $N_y = 45$  segments in the lateral  
 301 direction and  $N_r = 20$  segments around the circumference of the cylinder. This makes,  
 302 for the direction of wave propagation, a discretization of approximately 30 nodes per  
 303 wavelength. In addition,  $N_z = 10$  nodes are used in the vertical direction on the tank  
 304 sides and  $N_z^* = 30$  nodes are used for the vertical discretization of the cylinder. Incident  
 305 wave conditions are imposed using the fifth-order Stokes theory. Fourier coefficients of  
 306 the potential up to fourth-order are represented on Fig. 16, where they are compared to  
 307 the experiments carried out by Huseby and Grue [37]. Results match the experiments  
 308 well, with a slight difference seen in the second harmonic, but this is quite consistent with  
 309 other fully nonlinear computations, such as those by Ferrant [1] and Shao and Faltinsen  
 310 [9]. Obtaining such an agreement for harmonics of the wave force signal up to the fourth-  
 311 order clearly demonstrates the nonlinear capabilities of the proposed modeling approach.  
 312 Note that contrary to Ferrant [1], we do not decompose the velocity potential into an  
 313 incident component plus a diffraction component.

314 Note that the simulations performed here lead to simulated values for the forces  
 315 which vary smoothly in time. The free surface pattern, can be noisy near the waterline  
 316 without the filtering described by Eq. 40. As this test case is for a fixed body, we do  
 317 not need to invoke the Laplace smoothing or regridding described earlier, simply this  
 318 filtering along the waterline. Recent experimental tests of interactions between regular  
 319 waves and vertical cylinders [38], however, suggest that waves near the waterline may  
 320 physically be causing local breaking, or be damped by viscous effects. Calibrating the  
 321 amount of dissipation required will be further investigated in future work; here we simply  
 322 remove high-frequency waves, similar to Longuet-Higgins and Cokelet [39], which does  
 323 not significantly affect waves which are well-resolved.

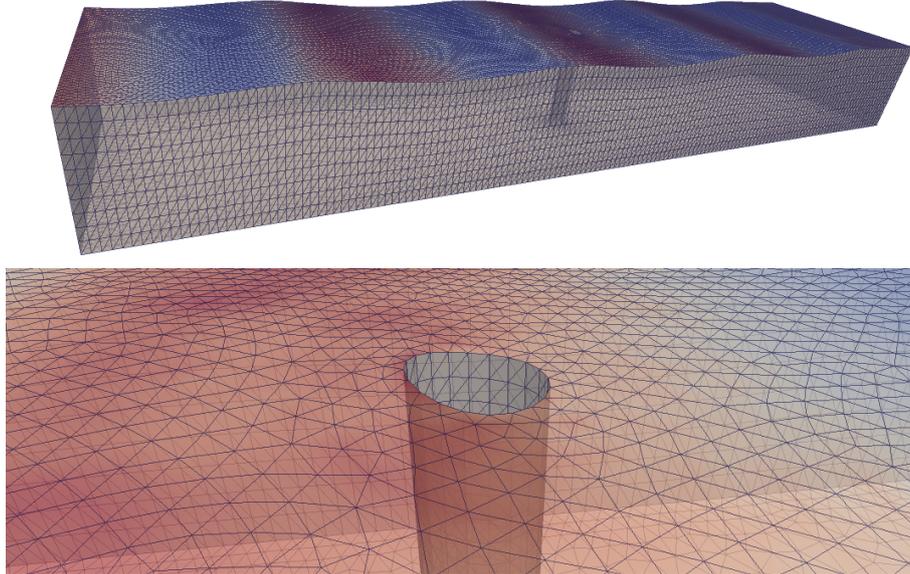


Figure 15: Computational domain (upper panel) and close-up of grid near cylinder (lower panel) for test cases of waves interacting with a fixed cylinder, in the case of  $kR = 0.245$  and  $kA = 0.10$ .

324 *4.3. Diffraction on a modified fixed Dutch Tri-floater platform*

325 *4.3.1. Setup of the model*

326 We now consider a more complicated test case with a semi-submersible structure,  
 327 hereafter referred to as the Dutch Tri-floater [40], which is a proposed structure for  
 328 offshore wind turbines. The shape of the floater involves three surface piercing cylinders,  
 329 rigidly connected. Moreover, each cylinder is equipped at its base with a heave plate.  
 330 This geometry is a slight simplification of original the Dutch Tri-floater geometry [40],  
 331 as we do not consider here the smaller supporting struts, but they should not constitute  
 332 a significant source of hydrodynamic forces with an inviscid model. A sketch of the  
 333 geometry adopted in this work, also studied by Antonutti *et al.* in [41], is represented  
 334 on Fig. (17).

335 For the numerical wave tank, we again use a cylindrical domain such as the one  
 336 represented in Fig. (6). This time, the reference free surface elevation  $\eta^{ref}$  and the  
 337 reference potential  $\phi^{ref}$  correspond to the solution of the fifth-order Stokes theory. The

Design draft (m)	12.0
Column centre-to-center spacing (m)	68.0
Column diameter (m)	8.0
Column depth including plate (m)	24.0
Plate diameter $2r$ (m)	18.0
Plate thickness (m)	1.0

Table 2: Geometric parameters of the *Dutch Tri-floater* chosen as in the study [41].

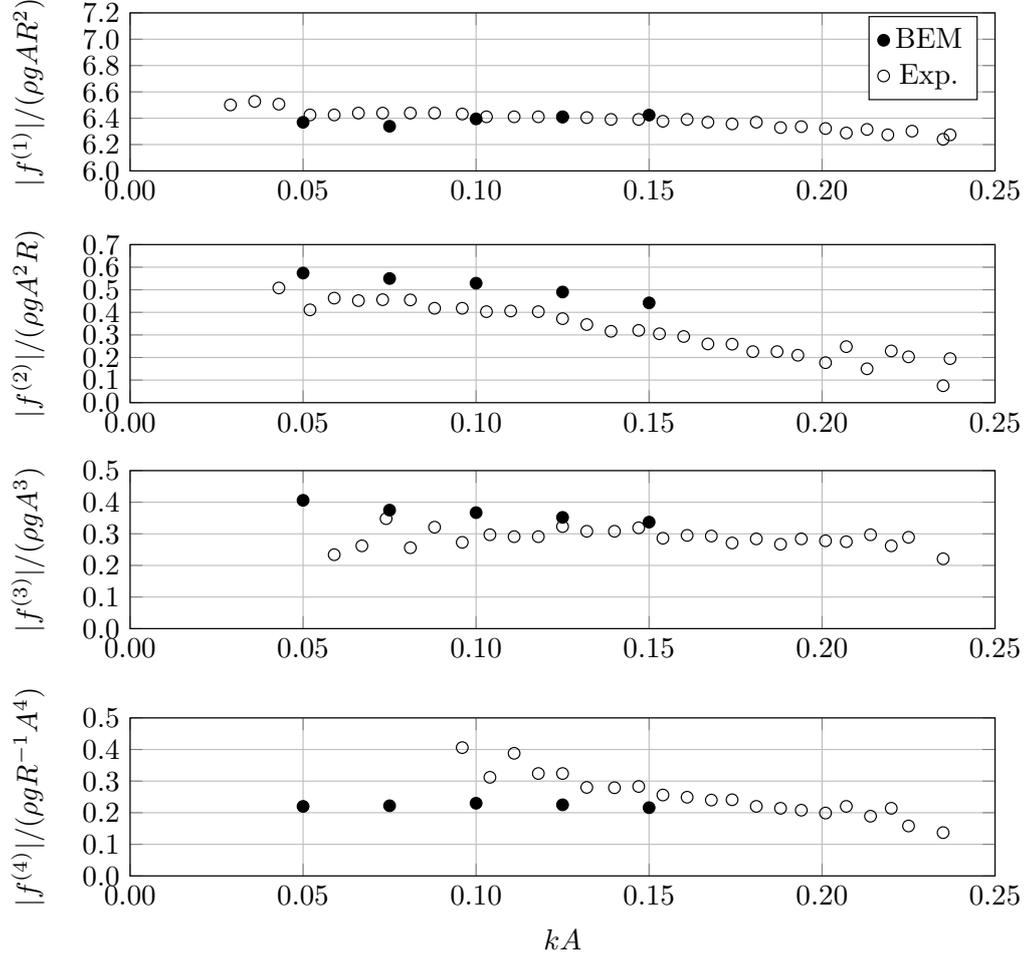


Figure 16: First through fourth-order Fourier coefficients associated to the time-series of  $F_x$  for a bottom-mounted cylinder with a period  $T = 2.03\text{s}$ ,  $kR = 0.245$  and the water depth  $kd = \pi$ .

338 coefficients requested to evaluate this solution are computed at the beginning of the  
 339 simulation.

340 For the size of the domain, trial and error showed that a choice of parameters  $R_{ext} =$   
 341  $3\lambda + \Lambda$ ,  $R_\lambda = \lambda + \Lambda$  with  $\Lambda = 70\text{m}$  and  $\gamma_0 = 1$ , leads to a stable value of wave loads  
 342 after a duration of 40 wave periods, except for nondimensional wavenumbers  $kr \geq 2.5$ ,  
 343 for which longer time simulations are needed. On the contrary, by choosing  $(R_\lambda, R_{ext}) =$   
 344  $(2\lambda + \Lambda, 3\lambda + \Lambda)$  or  $(R_\lambda, R_{ext}) = (\lambda + \Lambda, 2\lambda + \Lambda)$ , that is, with an absorbing beach of  
 345 radial length  $\lambda$ , we cannot obtain a convergence of the wave loads towards a periodic  
 346 state.

347 We study below the diffraction of monochromatic waves on the floater. The waves  
 348 are generated by imposing the velocity and acceleration of fifth-order Stokes waves on  
 349 the far-field lateral sides. In addition, as already mentioned earlier, the wave profile

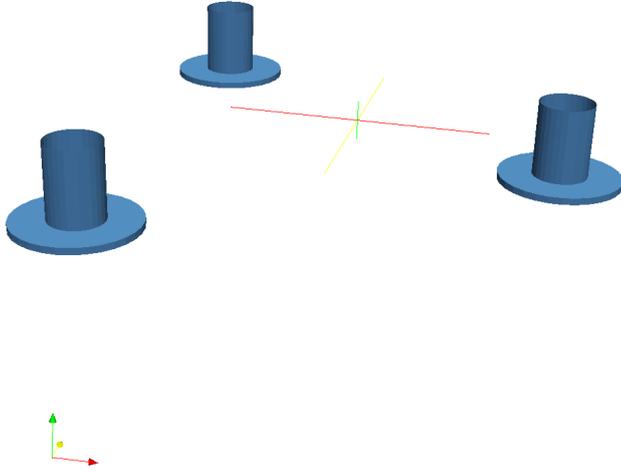


Figure 17: Geometry of the modified *Dutch Tri-floater* taken into account in the potential flow model. Above the mean free surface level, each cylinder is assumed to be infinite.

350 is damped out near the external boundary. With the methodology exposed above, we  
 351 were able to simulate successfully the waves whose characteristics are given in Table (3)  
 352 of the Appendix A. We note that for the steepest waves, the convergence of the wave  
 353 loads toward a periodic state is slower, which suggests that in the future, it could be  
 354 worth computing the Response Amplitude Operator (RAO) curve with a fixed steepness  
 355 rather than with a fixed wave height (as done here in the traditional manner). The  
 356 meshes used in this study are always adapted to the wavelength under consideration.  
 357 The unstructured mesh of the free surface is done with the algorithm NETGEN2D in  
 358 the SALOME platform [26]. Each simulation used a mesh with a number of degrees of  
 359 freedom in the range:  $N_{dof} \in [18000, 23000]$ .

#### 360 4.3.2. Zero and first order loads

361 We analyze the time-series of the forces  $F_x$  and  $F_y$ , and the overturning moment  
 362  $M_y$ , over a window of 6 periods. Details on the time intervals considered for the Fourier  
 363 analysis are given in Table 3 of Appendix A. For each frequency, we define the RAO of  
 364 the temporal signal  $F$  as the ratio between the first-order Fourier coefficient  $F^{(1)}$  and  
 365 the wave height  $H$ , and we assume that the density of the water is  $\rho_f = 1025 \text{ kg/m}^3$ .

366 The RAO of the longitudinal force,  $F_x$ , agrees very well with the open-source linear  
 367 wave model NEMOH [42] (Fig. 18). In particular, we find that the oscillations of the  
 368 curve are well reproduced for the small periods. Similar agreements are found for the  
 369 vertical force (Fig. 19), using reference values published in [41]. There are some deviations  
 370 between the two results, even for these moderate wave amplitudes, at the peaks, as  
 371 one might expect between linear and nonlinear models. Similarly, the nonlinear model  
 372 described here predicts a lower maximum overturning moment, as compared to the linear  
 373 model NEMOH (Fig. 20).

374 In addition, we show in Fig. (21) the horizontal drift force, defined as the zero-order  
375 coefficient  $F_x^{(0)}$  of the Fourier series  $F_x$ , with respect to the wave period  $T$ . Our results are  
376 compared to a frequency model computing the Quadratic Transfer Functions described  
377 in [43]. Except for the smallest periods, a very good agreement with the weakly nonlinear  
378 model is observed. For the highest frequencies tested in this study, our semi-Lagrangian  
379 scheme exhibits important deviations from the theory. This calculation provides further  
380 confidence in the ability of our numerical model to represent nonlinear features of the  
381 wave-structure interaction problem.

382 As an illustration of the importance of nonlinear effects, we choose to represent on  
383 Fig. (22), the ratio of second-order to first-order coefficients for the quantities of interest:  
384  $F_x$ ,  $F_z$  and  $M_y$ . Whereas this ratio does not exceed 20% for  $F_x$ , it may reach 60% for  
385  $F_z$  and  $M_y$ , for the smallest periods of this study. This shows that nonlinearities become  
386 important for these shorter wavelengths with the Dutch Tri-floater.

## 387 5. Conclusions

388 We presented in this paper an implementation of a fully nonlinear potential wave  
389 model to simulate wave-structure interactions using unstructured triangular meshes, im-  
390 portant for being able to handle future industrial applications with arbitrary problem  
391 geometry. The assembling of the system matrix is made with an efficient use of paral-  
392 lelization on distributed computer systems. Two time-stepping schemes, based on dis-  
393 crete derivatives with first-order shape functions, are derived. The accuracy of the whole  
394 algorithm could be easily enhanced by using high-order elements.

395 The model is applied to various problems involving surface piercing cylinders. The  
396 forces on a truncated cylinder in finite water depth, subjected to a prescribed sway mo-  
397 tion, using a fully Lagrangian motion of the free surface mesh, is found to agree reasonably  
398 well with reference results. Similarly, forces on a bottom-mounted vertical cylinder re-  
399 sulting from diffraction of regular waves, using a semi-Lagrangian time-stepping scheme,  
400 capture higher-order nonlinear effects (i.e., up to the fourth harmonic of the horizontal  
401 force).

402 Finally, in order to show the potential of the method in dealing with complex struc-  
403 tures, we also successfully compute both linear diffraction loads and nonlinear drift forces  
404 for a geometry inspired by the Dutch Tri-floater [40]. Other important features, such as  
405 improving computational speed (i.e., with the fast multipole method), such as started by  
406 Harris *et al.* [44], will be considered in upcoming works, as well as using improved accu-  
407 racy (i.e., cubic B-spline elements), or including more physics (i.e., coupling to Navier-  
408 Stokes solvers). Moreover, the case of arbitrary geometries for the rigid body needs to  
409 be addressed.

## 410 Acknowledgements

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415 ANR11-MONU-018-01 MONACOREV.

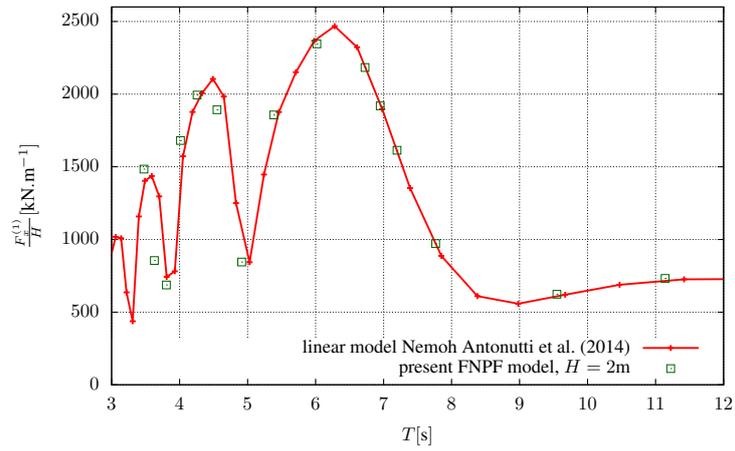


Figure 18: Nonlinear diffraction of a wave of height  $H = 2$  m around the modified Dutch Tri-floater for various periods  $T$  with a water depth of  $d = 50$  m: RAO of the horizontal force  $F_x$ .

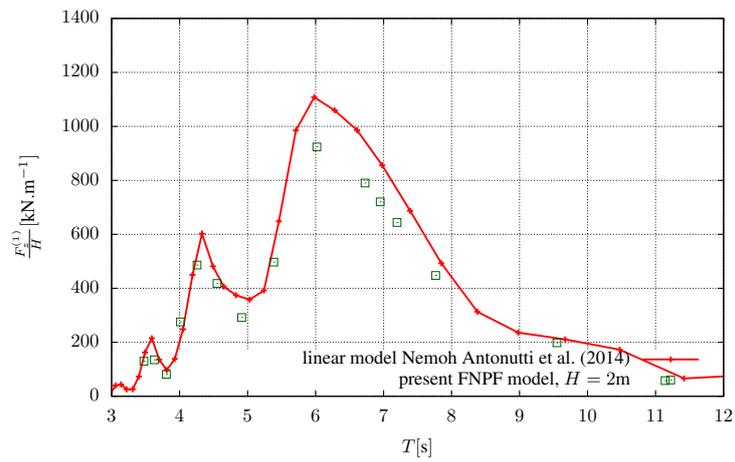


Figure 19: Nonlinear diffraction of a wave of height  $H = 2$  m around the modified Dutch Tri-floater for various periods  $T$  with a water depth of  $d = 50$  m: RAO of the vertical force  $F_z$ .

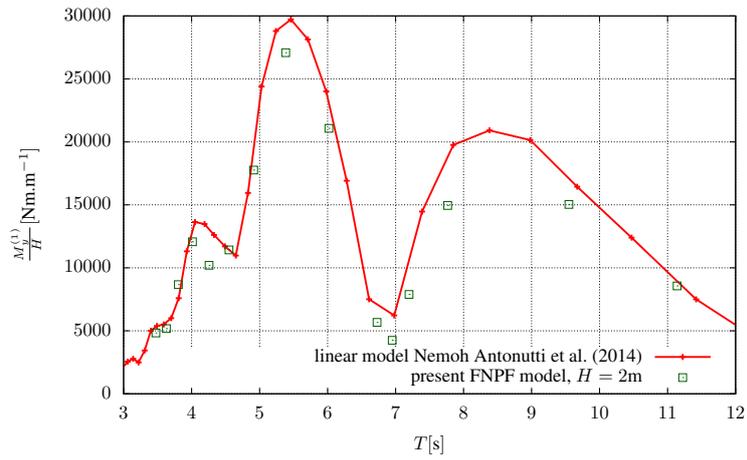


Figure 20: Nonlinear diffraction of a wave of height  $H = 2$  m around the modified Dutch Tri-floater for various periods  $T$  with a water depth of  $d = 50$  m: RAO of the overturning moment  $M_y$ .

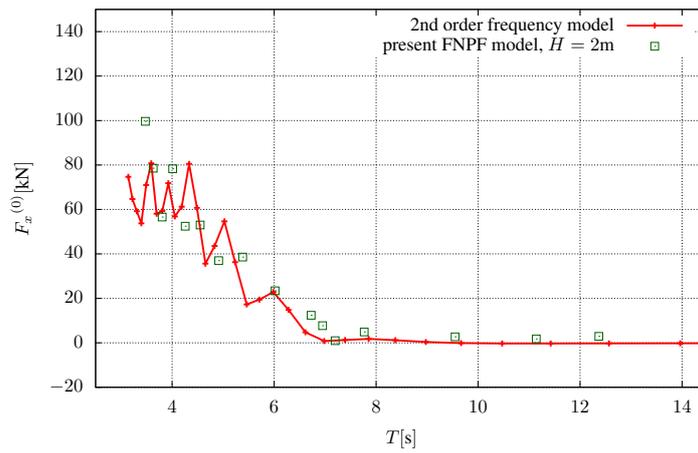


Figure 21: Nonlinear diffraction of a wave of height  $H = 2$  m around the modified Dutch Tri-floater for various periods  $T$  with a water depth of  $d = 50$  m: horizontal drift force  $F_x^{(0)}$ .

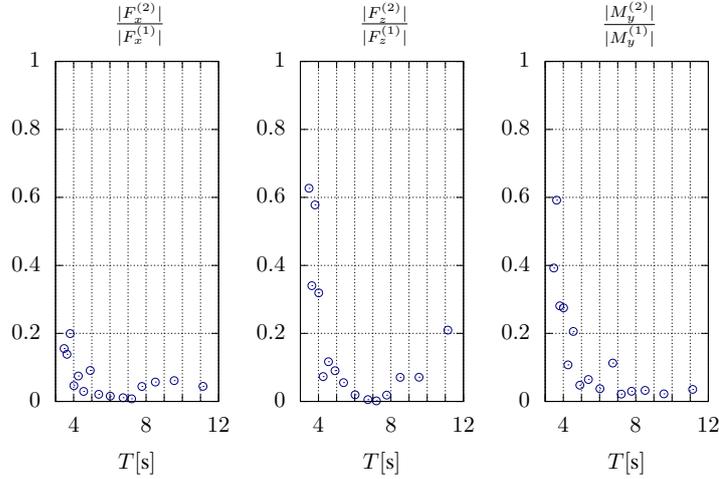


Figure 22: Nonlinear diffraction of a wave of height  $H = 2$  m around the modified Dutch Tri-floater: ratio between second-order to first-order coefficients for the quantities  $F_x$ ,  $F_z$  and  $M_y$ .

416 **Appendix A. Characteristics of the waves and the wave load analysis for the**  
417 **nonlinear diffraction around the modified Dutch Tri-floater.**

418 In Table (3), the parameters of the waves used in the model and the time-interval of  
419 the Fourier analysis are detailed for the test-case of the Dutch Tri-floater.

$kr$	$\omega$ [rad/s]	$\frac{2\pi H}{gT^2}$	$[t_1^+, t_2^+]$
0.3	0.564	0.010	[13.47, 19.47]
0.4	0.658	0.014	[15.71, 21.71]
0.6	0.809	0.021	[12.88, 18.88]
0.7	0.873	0.025	[13.90, 19.90]
0.75	0.904	0.027	[14.39, 20.39]
0.8	0.934	0.028	[16.35, 22.35]
1.0	1.044	0.036	[24.93, 30.93]
1.25	1.167	0.044	[24.15, 30.15]
1.50	1.279	0.053	[24.43, 30.43]
1.75	1.381	0.063	[26.38, 32.38]
2.0	1.476	0.071	[29.36, 35.36]
2.25	1.566	0.080	[31.16, 37.16]
2.50	1.651	0.089	[63.07, 69.07]
2.75	1.731	0.098	[63.37, 69.37]
3.00	1.808	0.108	[63.31, 69.31]

Table 3: Nonlinear diffraction for a wave height  $H = 2$  m around the modified Dutch Tri-floater: characteristics of the waves simulated with respect to the non-dimensional wavenumber. The parameter  $r$  denotes the radius of the heave plates and is set to  $r = 9$  m.  $d$  is the water-depth, such that  $\frac{H}{d} = 0.04$ .  $\omega$  is the angular wave frequency.  $k$  is the wave number.  $T$  is the wave period.  $t_1^+ = \frac{t_1}{T}$  and  $t_2^+ = \frac{t_2}{T}$  with  $[t_1, t_2]$  the time interval on which the Fourier analysis is performed.

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