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MMSE turbo equalization using a low complexity series expansion to approximate the matrix inversion in frequency selective MIMO channels

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***Abstract*— The paper describes a novel approach to overcome the need for matrix inversion required by Minimum Mean Square Error (MMSE) turbo equalization in MIMO systems. In particular, turbo MMSE equalization based on an efficient series expansion from a truncated Taylor series expansion to approximate the matrix inversion is addressed. By adjusting a scaling factor, the proposed series expansion is directly optimized according to a fixed order with respect to a system performance criterion for each source to be estimated. The proposed approach enables low complexity receiver without the need for a complicated eigenvalue calculation procedure. From the first order, the proposed series expansion ensures the best bit error rate performance with a simple non iterative receiver in comparison with previous approaches. Taking advantage of the iterative process, the resulting MMSE turbo equalizer reaches the matched filter bound.**

***Index Terms*— Low complexity receiver, matrix inversion, MIMO systems, MMSE turbo equalization, series expansion.**

I. Introduction

The transmission of information for most communication channels including multiple-input multiple-output (MIMO) systems and multipath fading is subject to intersymbol interference. MMSE turbo equalization [1]–[5] has been proven effective for removing intersymbol interference. It consists of a minimum mean square error (MMSE) equalizer and a decoder. When combined with space time bit interleaved coded modulation (ST-BICM) systems [6], [7], MMSE turbo equalization achieves optimal performance by taking fully advantage of the available diversity [8]–[13]. Unfortunately, its computational cost is too high and makes its implementation difficult since it requires large matrix inversions depending on the size of the system [14], [15].

To overcome the need for matrix inversion the authors proposed in [16]–[19] an approximate MMSE equalizer employing a finite sum of weighted matrix polynomials that results from the Cayley-Hamilton theorem [20]. The coefficients are chosen to optimize some performance measure at the equalizer output. It has been extended and applied to MMSE turbo equalization in the context of multiuser detection in [21]. In particular, by taking advantage of the iterative process, the authors show that performance loss due to the polynomial approximation is negligible after few iterations. However, the complexity involved in the weight optimization problem does not seem to be easier than performing an exact matrix inversion [17], [22].

Alternatively, an approximate MMSE equalization based on an optimal series expansion from a truncated Taylor series expansion is derived in [23]. By adjusting a scaling factor, the series expansion is directly optimized according to a fixed order with respect to a system performance criterion. In comparison with previous approaches [24], [25], the proposed series expansion ensures the best performance at the equalizer output in addition to low complexity which mainly depends on the computation of only one scaling factor as a function of the equalizer coefficients. This provides a much greater potential complexity reduction in comparison with a complicated eigenvalue calculation procedure.

In this paper, the approach proposed in [23] is expanded to derive an efficient MMSE turbo equalizer based on the first order series expansion to approximate the matrix inversion in a MIMO context. Compared to previous approaches, the proposed method improves the bit error rate (BER) performance for the first iterations. By taking advantage of the iterative process, the resulting MMSE turbo equalizer exhibits sufficient performance to reach the matched filter bound with approximately the same convergence speed to that of the corresponding MMSE turbo equalizer with exact matrix inversion.

In section II, the MIMO transmission scheme is described. In section III, the proposed approximate MMSE turbo equalizer equation based on series expansion is derived. In sections V, simulation results show the receiver efficiency in comparison with that of the corresponding MMSE turbo equalizer with exact matrix inversion.

II. MIMO system description

We consider a single carrier transmission in MIMO ST-BICM system made of n_T transmit and n_R receive antennas over a frequency selective channel. As illustrated in Fig. 1, the information bit stream is encoded and then bit-interleaved

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(Π_b). The interleaved streams are then modulated and serial to parallel converted. The transmitted symbols are organized as blocks of N transmitted symbols on each antenna. Consecutive blocks of N transmitted symbols are separated by a guard interval (GI), which prevents from inter-block interference. Each guard interval is assumed to contain $L-1$ zeros (Zero-Padding). We assume a coherent symbol-spaced receiver with perfect carrier and time synchronization such that $y_n(d)$, the dth sample on the receive antenna n, can be represented by a discrete-time baseband model having $s_m(d)$, the dth transmitted symbol from the mth transmit antenna, as input. The transmitted symbols are assumed to be independent, identically distributed (i.i.d) and of variance σ_s^2. The complex additive Gaussian noise samples on the receive antenna n of variance σ_b^2 are denoted $b_n(d)$. For the equalizer derivation, we consider a sliding window of length N_F from each antenna at the receiver. By grouping the received samples from all n_R received antennas over a block of N_F symbol periods, we thus obtain N vectors $\underline{\mathbf{y}}(d)$ of size $N_F n_R \times 1$ whose expression is

$$\underline{\mathbf{y}}(d) = \underline{\mathbf{H}}\underline{\mathbf{s}}(d) + \underline{\mathbf{b}}(d) \qquad d = 0,..,N-1 \tag{1}$$

where $\underline{\mathbf{y}}(d) = \left[\{y_n(d+k)\}_{1\leq n\leq n_R}\right]_{0\leq k\leq N_F-1}$ of dimension $N_F n_R \times 1$, $\underline{\mathbf{s}}(d) = \left[\{s_m(d+k)\}_{1\leq m\leq n_T}\right]_{N_{F_1}\leq k\leq N_{F_2}+L-1}$ of dimension $(N_F+L-1)n_T \times 1$ and $\underline{\mathbf{b}}(d) = \left[\{b_n(d+k)\}_{1\leq n\leq n_T}\right]_{1\leq k\leq N_F-1}$ of dimension $N_F n_R \times 1$. $\underline{\mathbf{H}}$ represents the $N_F n_R \times (N_F+L-1)n_T$ block-Toeplitz channel matrix

$$\underline{\mathbf{H}} = \begin{bmatrix} \mathbf{H}(0) & \dots & \mathbf{H}(L-1) & 0 & \dots \\ 0 & \mathbf{H}(0) & \dots & \mathbf{H}(L-1) & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \mathbf{H}(0) & \dots & \mathbf{H}(L-1) \end{bmatrix} \tag{2}$$

consisting of blocks $\mathbf{H}(l) = \left[h_{n,m}(l)_{\substack{1\leq m\leq n_T \\ 1\leq n\leq n_R}}\right]_{0\leq l\leq L-1}$ of dimension $n_R \times n_T$ where the discrete linear filter $h_{n,m}(l)_{l=0,..,L-1}$ of length L denotes the discrete-time equivalent channel model between transmit antenna m and receive antenna n.

III. Efficient approximate MMSE turbo equalizer based on series expansion

The structure of the investigated iterative receiver is shown in Fig. 1. The receiver consists of the concatenation of a Soft Input / Soft Output equalizer and a Soft Input / Soft Output decoder exchanging information on the coded bits in an iterative manner according to the MMSE turbo equalization principle [3]–[5]. This section focuses on the derivation of the proposed MMSE turbo equalizer based on series expansion.

A. MMSE turbo equalizer derivation

When considering turbo equalization, the decoder should provide the equalizer with extrinsic information for the encoded bits. Interestingly, we have noticed performance improvements for MMSE turbo equalization when feeding back the entire *a posteriori* information. The same obervation has been reported in [26], [27]. Let us denote $\tilde{s}_m(d)$ the soft estimate of transmitted symbol $s_m(d)$ computed from the *a posteriori* probabilities provided by the decoder. As in [4],

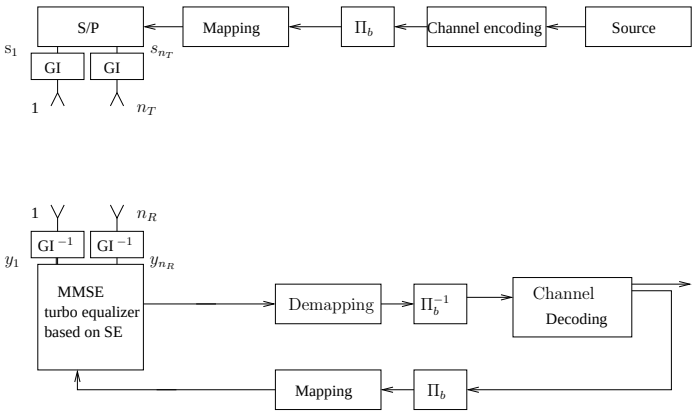


Fig. 1. Single carrier transmission in ST-BICM systems with MMSE turbo equalization based on efficient series expansion (SE) at the receiver.

[26], the soft symbols $\{\tilde{s}_m(d)\}_{m=1,..,n_T}^{d=0,..,N-1}$ are of variance $\sigma_{\tilde{s}}^2$ approximated by $\sigma_{\tilde{s}}^2 = \frac{1}{Nn_T}\sum_{m=1}^{n_T}\sum_{d=0}^{N-1}|\tilde{s}_m(d)|^2$ where a time average over a block of N symbols from all n_R received antenna is used. The resulting MMSE turbo equalizer coefficients remain invariant over a block of N transmitted symbols. Using these data estimates, and the channel coefficients, a soft replica of the multi-antenna interference (MAI) and intersymbol interference (ISI) is computed for each of the n_T antennas and then substracted from the total received signal

$$\tilde{\underline{\mathbf{y}}}(d) = \underline{\mathbf{y}}(d) - \underline{\mathbf{H}}\tilde{\underline{\mathbf{s}}}(d) \tag{3}$$

where $\tilde{\underline{\mathbf{s}}}(d) = \left[\{\tilde{s}_m(d+k)\}_{1\leq m\leq n_T}\right]_{N_{F_1}\leq k\leq N_{F_2}+L-1}$ of dimension $n_T(N_F+L-1)\times 1$. MMSE turbo equalization consists in cancelling soft ISI and MAI, and filtering based on minimum mean square error (MMSE) criteria

$$\mathrm{E}\Big[|\underline{\mathbf{w}}_m^{\mathrm{H}}\Big(\tilde{\underline{\mathbf{y}}}(d) + \underline{\mathbf{h}}_{\Delta_m}\tilde{s}_m(d)\Big) - s_m(d)|^2\Big|\tilde{\underline{\mathbf{s}}}(d)\Big],\ m = 1,..,n_T \tag{4}$$

where $\underline{\mathbf{h}}_{\Delta_m}$ is the Δ_m^{th} column vector of channel matrix $\underline{\mathbf{H}}$ corresponding to the different fadings experienced by the desired component. The MMSE filter input signal is independent of the soft estimate $\tilde{s}_m(d)$ according to the turbo principle. More explicitly, the equalizer consists of a $N_F n_R \times 1$ dimensional filter $\underline{\mathbf{w}}_m$ which minimizes the cost function (4) such that

$$\underline{\mathbf{w}}_m^{\mathrm{H}} = \alpha_m \underline{\mathbf{h}}_{\Delta_m}^{\mathrm{H}}\underline{\mathbf{\Sigma}}^{-1} \tag{5}$$

in which $\underline{\mathbf{\Sigma}}$ is the $n_R N_F \times n_R N_F$ covariance matrix of the residual interference given by $\underline{\mathbf{\Sigma}} = \sigma_s^2\underline{\mathbf{H}}\underline{\mathbf{H}}^{\mathrm{H}}(1-r) + \sigma_b^2\mathrm{I}_{N_F n_R \times N_F n_R}$ where we define $r = \frac{\sigma_{\tilde{s}}^2}{\sigma_s^2}$ as the degree of *a priori* information reliability. It can be seen that $r = 1$ (resp. $r = 0$), corresponds to perfect (resp. unavailable) *a priori* information from the decoder output. The index Δ_m is the decision delay parameter required for the derivation of the equalizer transfer function such that $\Delta_m = N_{F_1} + 1$. The intermediate variable α_m is computed as $\alpha_m = \frac{\sigma_s^2}{1+r\sigma_s^2\underline{\mathbf{h}}_{\Delta_m}^{\mathrm{H}}\underline{\mathbf{\Sigma}}^{-1}\underline{\mathbf{h}}_{\Delta_m}}$. Turbo equalization requires a minimum filter length N_F to achieve good performance [28] at the expense of high computational complexity mainly due to the calculation of $\underline{\mathbf{\Sigma}}^{-1}$ of dimension $(N_F n_R \times N_F n_R)$.

B. Previous approaches

Let λ_i, $i = 1, .., N_F n_R$ denote the positive real eigenvalues of $\underline{\mathbf{\Sigma}}$. Now consider a scaling factor $\psi \in \mathbb{R}$, which is determined for the matrix $\underline{\mathbf{\Sigma}}$ such that

$$|1 - \psi\lambda_i| < 1, \quad i = 1, .., N_F n_R \tag{6}$$

If the previous condition is satisfied, we can introduce the series expansion

$$\underline{\mathbf{\Sigma}}^{-1} = \psi \sum_{i=0}^{\infty} (\mathbf{I}_{N_F n_R} - \psi\underline{\mathbf{\Sigma}})^i \tag{7}$$

By considering the Kth order series expansion in $\underline{\mathbf{\Sigma}}$ to approximate the matrix inversion in (5), the corresponding equalizer is given by

$$\begin{aligned}\underline{\mathbf{w}}^{\mathrm{H}}_{m(K)} = \alpha_{m(K)}\psi\underline{\mathbf{h}}^{\mathrm{H}}_{\Delta_m}\Big(\mathbf{I}_{N_F n_R} + (\mathbf{I}_{N_F n_R} - \psi\underline{\mathbf{\Sigma}}) + \\ (\mathbf{I}_{N_F n_R} - \psi\underline{\mathbf{\Sigma}})^2 + ... + (\mathbf{I}_{N_F n_R} - \psi\underline{\mathbf{\Sigma}})^K\Big)\end{aligned} \tag{8}$$

which converges from the matched-filter ($K = 0$) detector to the MMSE equalizer ($K \to \infty$), as the order K grows from zero to infinity, i.e. $\lim_{K\to\infty} \underline{\mathbf{w}}^{\mathrm{H}}_{m(K)} = \underline{\mathbf{w}}^{\mathrm{H}}_m$. From this point of view, it is advantageous for computational simplicity to keep K as low as possible while still maintaining sufficient performance. Let $\lambda_{\min}$ and $\lambda_{\max}$ denote the smallest and the largest eigenvalues of $\underline{\mathbf{\Sigma}}$. A simple analysis shows that the convergence of $\underline{\mathbf{w}}^{\mathrm{H}}_{m(K)}$ to $\underline{\mathbf{w}}^{\mathrm{H}}_m$ as $K \to \infty$ is ensured by choosing ψ in the range of $0 < \psi < \frac{2}{\lambda_{\max}}$. In [24] [29], the authors propose to set

$$\psi = \frac{2}{\mathrm{trace}(\underline{\mathbf{\Sigma}})} \tag{9}$$

since $\mathrm{trace}(\underline{\mathbf{\Sigma}}) \geq \lambda_{\max}$. There is a possibility of estimating a more suitable scaling factor if the eigenvalues of $\underline{\mathbf{\Sigma}}$ are known. The fastest convergence rate takes place when the two extreme modes $|1 - \psi\lambda_{\min}|$ and $|1 - \psi\lambda_{\max}|$ are identical such that

$$\psi = \frac{2}{\lambda_{\min} + \lambda_{\max}} \tag{10}$$

These results are well-known and have previously been presented [25]. It should be noted that when the simulation context is associated with a large spread of eigenvalues, high order K may be required to keep high performance, at the expense of computational complexity.

C. Proposed approach

Instead of ensuring that $\underline{\mathbf{w}}^{\mathrm{H}}_{m(K)}$ performs the fastest convergence rate to $\underline{\mathbf{w}}^{\mathrm{H}}_m$ as K grows from zero to infinity, the proposed approach [23] consists in setting ψ such that $\underline{\mathbf{w}}^{\mathrm{H}}_{m(K)}$ achieves an optimum for some performance measure according to a fixed order K, for each source $m = 1, .., n_T$ to be estimated. In that sense, we denote $\psi_m(K)$ the new scaling factor. We propose to derive an efficient approximate MMSE turbo equalizer based on the first order series expansion of $\underline{\mathbf{\Sigma}}^{-1}$ that achieves the best signal interference to noise ratio (SINR) at the equalizer output. More explicitly, the first order approximate MMSE equalizer equation is given by

$$\underline{\mathbf{w}}^{\mathrm{H}}_{m(1)} = \alpha_{m(1)}\psi_m(1)\underline{\mathbf{h}}^{\mathrm{H}}_{\Delta_m}\left(\mathbf{I}_{N_F n_R} + (\mathbf{I}_{N_F n_R} - \psi_m(1)\underline{\mathbf{\Sigma}})\right) \tag{11}$$

By considering the expression of the SINR at the equalizer output, the optimum scaling factor $\psi_m(1)$ can then be found by solving $(\psi^{\mathrm{opt}}_m(1)) = \underset{\psi_m(1)}{\mathrm{argmax}}(\frac{||\underline{\mathbf{w}}^{\mathrm{H}}_{m(1)}\underline{\mathbf{h}}_{\Delta_m}||^2}{\underline{\mathbf{w}}^{\mathrm{H}}_{m(1)}\underline{\mathbf{\Sigma}}\underline{\mathbf{w}}_{m(1)}})$ under the constraint that $\psi_m(1)$ ensures the convergence of $\underline{\mathbf{w}}^{\mathrm{H}}_{m(K)}$ to $\underline{\mathbf{w}}^{\mathrm{H}}_m$ as $K \to \infty$. The optimum scaling factor $\psi_m(1)$ that achieves the best SINR at the equalizer output for each source $m = 1, .., n_T$ to be estimated, can be expressed as a function of the equalizer coefficients

$$\psi_m(1) = \frac{2}{\frac{\underline{\mathbf{v}}^{\mathrm{H}}_m\underline{\mathbf{\Theta}}_m\underline{\mathbf{\Sigma}}\underline{\mathbf{v}}_m}{\underline{\mathbf{v}}^{\mathrm{H}}_m\underline{\mathbf{\Theta}}_m\underline{\mathbf{v}}_m}} \tag{12}$$

where we define $\underline{\mathbf{v}}^{\mathrm{H}}_m = \mathbf{h}^{\mathrm{H}}_{\Delta_m}\underline{\mathbf{\Sigma}}$ and $\underline{\mathbf{\Theta}}_m = ||\underline{\mathbf{h}}_{\Delta_m}||^2\mathbf{I}_{N_F n_R} - \underline{\mathbf{h}}_{\Delta_m}\underline{\mathbf{h}}^{\mathrm{H}}_{\Delta_m}$ [23]. It can be shown that the resulting $\psi_m(1)$ satisfies the condition (6) if and only if $r < 1$. For the case in which $r \to 1$, *a priori* information from the decoder output is good enough so that that a matched filter approximation of the proposed MMSE turbo equalizer can efficiently reach the matched filter bound [14]. It is thus proposed that a matched filter is performed at the receiver when r is superior to a given treshold value 0.95 to ensure the convergence of $\underline{\mathbf{w}}^{\mathrm{H}}_{m(K)}$ to $\underline{\mathbf{w}}^{\mathrm{H}}_m$ as $K \to \infty$ and $r \to 1$. In comparison with previous approaches [24], [25], the proposed series expansion achieves the best SINR at the equalizer output according to a fixed order for each source to be estimated without the need for a complicated eigenvalue calculation procedure.

IV. Simulated system parameters

To evaluate the performance of the proposed approximate MMSE turbo equalizer based on series expansion, we consider a block transmission MIMO system with $n_T = n_R = 2$ and perfect channel state information at the receiver. The frame size is 512 information bits. Information bits are first encoded with a convolutional code of rate $1/2$ and octal code generator polynomials $(5, 7)$. The coded bits are then interleaved by a pseudo-random permutation, serial to parallel converted and 4-PSK modulated. The transmitted symbols from each antenna have an equal variance of $\sigma_s^2 = P_s/n_T$ where P_s denotes the total transmitted average power. We assume that channel impulse responses between n_T transmit and n_R receive antennas are uncorrelated and consist of $L = 10$ paths separated by a symbol duration and of equal average power $1/L$. The coefficients $h_{n,m}(l)_{l=0,..,L-1}$ are assumed to be complex Rayleigh-distributed, i.i.d, of zero mean and satisfying the power normalization constraint $\mathrm{E}[\sum_{l=0}^{L-1}|h_{n,m}(l)|^2] = 1$. In the MMSE equalizer, the filter coefficient number is set to $(N_F = 21,\ N_{F_1} = N_{F_2} = 10)$. Simulations have shown that setting the filter coefficient number $N_F = 21$ and the decision delay parameter $\Delta = 10$ is sufficient to achieve good BER performance [28].

V. Simulation results and discussion

In the following, TE MMSE-SEopt(K) (resp. TE MMSE-SEev(K)) denotes the Kth order approximate MMSE turbo equalizer computed from (12) (resp. (10)). To compare the interest of the proposed method with that in [24], the Kth order approximate MMSE equalizer performance obtained from (9)

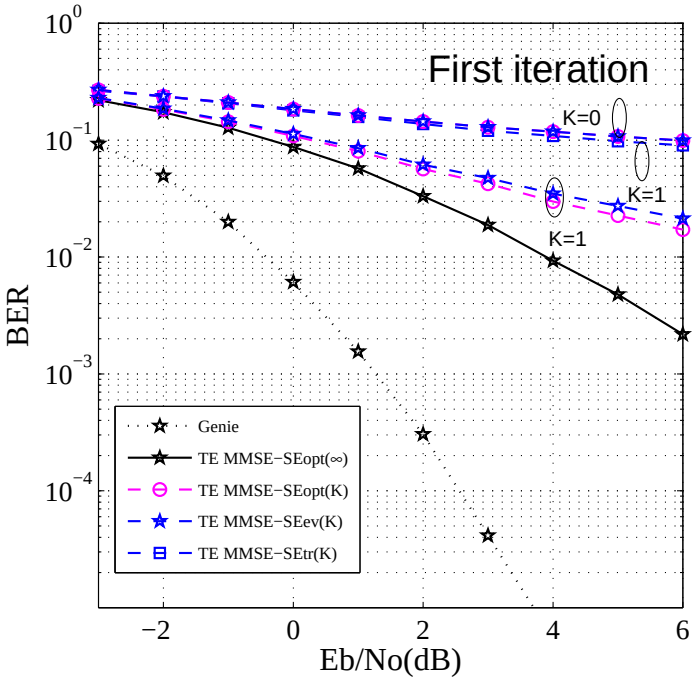


Fig. 2. BER performance after the first decoding iteration of the proposed Kth order approximate MMSE equalizer (MMSE-SEopt(K)) for $K = 0, 1$ and the corresponding MMSE equalizer with exact matrix inversion (MMSE-SEopt(∞)) as a function of $\frac{E_b}{N_0}$ - 2 Tx 2 Rx - CC(5,7)- 4-PSK - L=10.

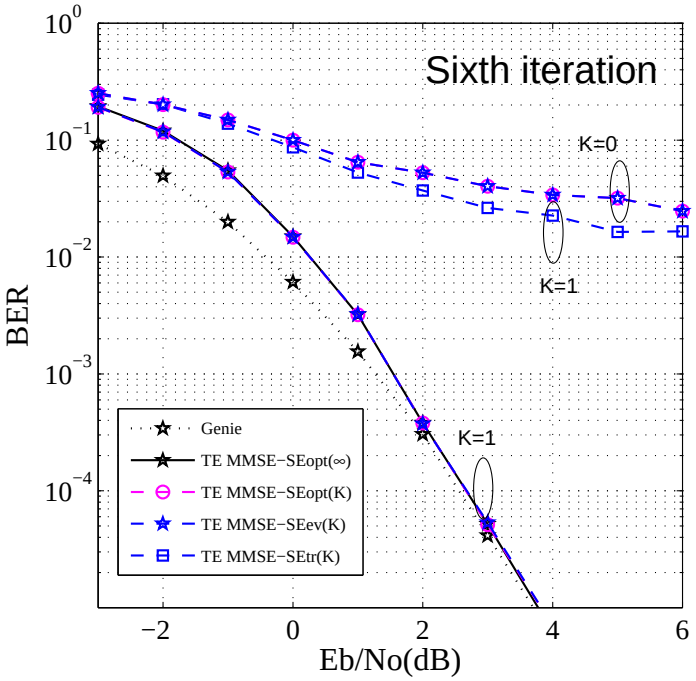


Fig. 3. BER performance after the sixth decoding iteration of the proposed Kth order approximate MMSE equalizer (MMSE-SEopt(K)) for $K = 0, 1$ and the corresponding MMSE equalizer with exact matrix inversion (MMSE-SEopt(∞)) as a function of $\frac{E_b}{N_0}$ - 2 Tx 2 Rx - CC(5,7)- 4-PSK - L=10.

is also reported (TE MMSE-SEtr(K)). In Fig. 2 and Fig. 3, the BER after the first and the sixth decoding iterations is plotted as a function of E_b/N_0 for $K = 0, 1$ and $K \to \infty$. The performance of the TE MMSE-SEopt(∞) with perfect *a priori* information from the decoder is also reported as a reference. The so-called genie MMSE turbo equalizer suppresses MAI and ISI from each transmit antenna, from the received signal.

A. Non iterative receiver performance (first iteration)

In Fig. 2, simulation results after one iteration i.e without *a priori* information stand for performance with a simple non iterative receiver. Interestingly, the TE MMSE-SEopt(1) performance enables a good compromise between the matched filter ($K = 0$) and the MMSE equalizer ($K \to \infty$) with exact matrix inversion, i.e. complexity and ISI+MAI reduction. For a BER of 1.10^{-1}, the MMSE-SEopt(1) performs better (resp. worse) by about 5.5dB (resp. 0.5dB) than the MMSE-SEopt(0) (resp. the MMSE-SEopt(∞)). Compared to the TE MMSE-SEev(1) BER, that of the TE MMSE-SEopt(1) is improved by 0.2dB. The TE MMSE-SEopt(1) equalizer is able to retrieve a large part of the ISI and MAI from the received signal and high performance can already be obtained.

At $\frac{E_b}{N_0} > 2$dB, we observe that the performance gap between the MMSE-SEopt(K) for $K = 0, 1$ and the MMSE-SEopt(∞) increases. For a BER of 2.10^{-2}, the TE MMSE-SEopt(1) performs worse by 2.5dB from the TE MMSE-SEopt(∞). The ISI phenomenon dominates over the noise such that the equalizer has to invert the channel impulse response. As a consequence, a high order series expansion is necessary to keep high performance. Compared to the TE MMSE-SEopt(1) BER, the TE MMSE-SEtr(1) BER, leads to a significant performance loss.

B. Iterative receiver performance (sixth iteration)

The Fig. 3 shows that the sixth decoding iteration corresponding to the TE MMSE-SEopt(K) (resp. TE MMSE-SEev(K)) for $K = 1$ is very close to that of the genie decoder. On the other hand, taking $K = 0$ brings a noticeable loss degradation. For $K = 0$, the BER at the TE MMSE-SEopt(0) equalizer output is exactly the same as the one available at the TE MMSE-SEev(0) (resp. TE MMSE-SEtr(0)) output (matched filter). For a BER of 2.10^{-2} at the decoder output, the TE MMSE-SEopt(0) (resp. TE MMSE-SEev(0), TE MMSE-SEtr(0)) performs worse by 5.5dB compared to the genie decoder BER. Thus, the TE MMSE-SEopt(1) enables a good compromise between the matched filter ($K = 0$) and the MMSE equalizer ($K \to \infty$) at the first iteration to reach the matched filter bound after a few iterations. Comparing the TE MMSE-SEopt(1) to the TE MMSE-SEev(1), the TE MMSE-SEopt(1) exhibits similar performance with that of the TE MMSE-SEev(1) without the need for a complicated eigenvalue calculation procedure as in (10).

C. Iterative receiver convergence

In Fig. 4, a further evaluation is performed to compare the iterative receiver convergence of the Kth order approximate MMSE turbo equalizer TE MMSE-SEopt(K) (resp. TE MMSE-SEev(K)) with that of the MMSE equalizer ($K \to \infty$). For $\frac{E_b}{N_0} = 3$dB, the BER performance is plotted as a function of the iterations. It can be seen that approximately 4 iterations are required for the MMSE-SEopt(1) (resp. TE MMSE-SEev(1)) and the TE MMSE-SEopt(∞) to reduce the performance gap by 0.2dB compared to the genie decoder output. The improvement brought by the fifth and sixth iterations is negligible. On the other hand, no performance

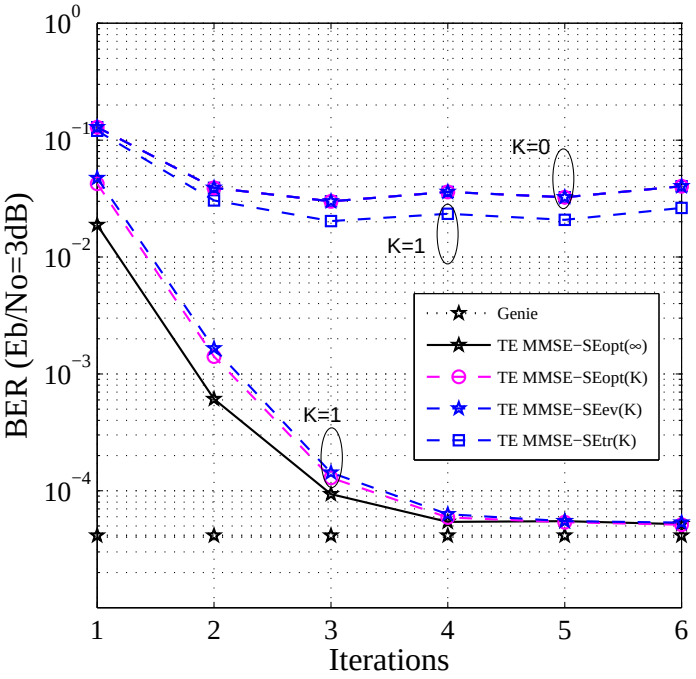


Fig. 4. BER performance of the proposed Kth order approximate MMSE equalizer (MMSE-SEopt(K)) for $K = 0, 1$ and the corresponding MMSE equalizer with exact matrix inversion (MMSE-SEopt(∞)) as a function of the iterations for $\frac{E_b}{N_0}$ = 3dB - 2 Tx 2 Rx - CC(5,7)- 4-PSK - L=10.

gain with additional iterations is observed at the TE MMSE-SEtr(1) output. This demonstrates the efficiency of the MMSE-SEopt(1) to exploit interference.

VI. Conclusion

This paper describes an innovative and low complexity approach based on series expansion to overcome the need for the complicated matrix inversion required in MMSE turbo equalization derivation. According to the first order, the proposed series expansion TE MMSE-SEopt(K) ensures the best performance with a simple non iterative receiver in comparison with previous approaches (TE MMSE-SEev(K), TE MMSE-SEtr(K)). Taking advantage of the iterative process, the resulting MMSE turbo equalizer reaches the matched filter bound with approximately the same convergence speed to that of the corresponding MMSE turbo equalizer with exact matrix inversion. Further to these promising results, important issues should be studied including the computational cost reduction of $\psi_m(K)$ derivation and making the proposed approach more widely applicable.

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