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Edith Petitfour

► **To cite this version:**

Edith Petitfour. Teaching geometry to visual-spatial dyspraxic pupils. 13th International Congress on Mathematical Education, Jul 2016, Hamburg, Germany. hal-02119848

**HAL Id: hal-02119848**

**<https://hal.science/hal-02119848>**

Submitted on 4 May 2019

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## TEACHING GEOMETRY TO VISUAL-SPATIAL DYSPRAXIC PUPILS

Edith Petitfour LDAR (EA 4434) UA UCP UPD UPEC URN

*The aim of our study is to present an overview of an approach to the teaching of elementary plane geometry to dyspraxic fifth and sixth-grade pupils. Dyspraxic pupils face some important difficulties in using geometrical instruments (ruler, compass and set square) with required precision. They present a lack of organizational and fine motor skills. The approach is based on semiotics and two disciplines cognitive sciences: cognitive ergonomics according to an instrumental approach (Rabardel, 1995) and neuropsychology (Mazeau & Pouhet, 2014). This approach allowed us to identify and categorize the actions required in elementary geometric tasks to better understand the pupils' difficulties and potential didactic actions to overcome them.*

### INTRODUCTION

In Grades 5 and 6 in France, the teaching and learning of geometry is based on the handling of objects (folding, assembling geometric shapes or drawing with ruler, compass and set square) and on a sharp perception of figures. Indeed school curriculum recommend situations which require actions with instruments such as to describe, to reproduce, to represent, to construct geometric figures. Learning geometry may hence appear to be inaccessible to visual-spatial dyspraxic pupils. As a consequence of such teaching, these pupils fail in geometry despite the fact that construction with instruments, a task that they are unable to complete, is not a teaching objective in itself: it is only a means that must lead the pupil to the conceptualization of geometric notions. In our research, we suggest the hypothesis that dyspraxic pupils are able to learn geometry by using their preserved abilities (language, memory and reasoning). To capitalize on these preserved capabilities, we had recourse to a classroom social organization based on pair work: the dyspraxic pupil worked in interactions with a peer (standard pupil), the teacher or a school special education assistant. The dyspraxic pupil gave the instructions to the other individual about what had to be done with the instruments (e.g., tracing a segment, measuring a length, etc.).

In the first part of this text, we present a few elements of a framework intended to analyze the process of accessing geometrical concepts (e.g., alignment, equality of length, right angle, etc.) through construction with instruments. In the second part we briefly discuss an application of the theoretical framework with special attention to gestures and language.

### ELEMENTS OF THE THEORETICAL FRAMEWORK

#### Theoretical background

Elementary geometric activity regarding non-ostensive entities (intuition, concepts, ideas) can only develop through a plurality of different palpable registers (oral, written, graphical, gestural, material) (Bosch & Chevillard, 1999). These ostensive objects, signs produced through different intentional actions (speaking, writing, drawing, gesticulating, handling an artifact), with their production and transformation modes, are constituents of semiotic sets, as well as the relationships between these signs and their meanings (Radford, 2002). Considering that learning processes are multimodal, we

rely on concept of semiotic bundle (Arzarello, 2006), in order to consider the dynamic development of interactions between semiotic sets. During a pair-work activity based on construction with instruments, these sets are composed of one or several ostensive objects (actions with artifact, language, gestures and graphical objects). Our analysis framework aims at studying their articulations and their links to geometrical learning.

The approach is based on semiotics, on two disciplines in cognitive sciences: cognitive ergonomics according to an instrumental approach (Rabardel, 1995) and neuropsychology (Mazeau & Le Lostec, 2010; Mazeau & Pouhet, 2014) and on observations of several fifth and sixth-grade visual-spatial dyspraxic pupils, in interaction with others, involved in different tasks based on constructions with instruments.

### **Analysis framework to study actions with instruments**

In his/her working environment, the individual must implement a complex sequence of *actions with instruments* in order to obtain the graphical representation of geometric objects (e.g., tracing the symmetric of a point with respect to an axis, using square and compass). The effective implementation of an action with instruments requires the initiation of effector, sensory and motor organs: it is the neuromotor and muscular aspect of the action. Every action with instrument is preceded by its representation, which is its cognitive aspect.

At a cognitive level, we first consider that the project to perform an action with instrument is constituted by a dual intent: *intent to act* and *motor intention*. Intention to act relates to an anticipation of the action in which individual himself represents the final purpose and the theoretical project of the action with the instrument. Motor intention relates to planning and programming of the action in concrete implementation, taking into account the context (environment and material instrument). Moreover, we consider that the action with the instrument is divided into four components characterized by relationships activated between geometric objects, graphical objects, instruments, individual's body and environment:

- the *semiotic* one is the set of relationships between graphical objects and geometric objects,
- the *technical-figural* one is the set of relationships between instruments and graphical objects,
- the *motor manipulation* one is the set of relationships between the individual's body and instruments,
- the *organizational* one consists of the interaction of the individual with his/her environment's artifacts related to the practical organization of the action.

We call *aim* how the individual's intention manifests itself in these four components.

For the individual, the *intention to act* is generated by the project to represent a geometric object by a graphical object (e.g., tracing a straight line). This project corresponds to the *semiotic aim* of the action. This intention is part of the semiotic component, focused on the action effects (geometric object or graphical object) and is independent of the working environment and instruments to be used. It involves geometric knowledge about objects, relationships and properties.

The project to represent a geometric object leads to elaborate the project of an action with a technical tool. The *technical-figural aim* of the action consists of the intention of executing the action to be

performed this time taking into consideration the instrument in order to obtain the graphical object. But it remains a step of a technical construction developed *theoretically* by the individual, regardless of practical aspects of implementation concerning body's abilities and the hardware features of the technical tools. For instance, in paper-pencil environment, the three following elementary actions can be considered: taking the instrument, positioning the instrument, drawing. (e.g., drawing a line through two given points leads to: taking a ruler and a pencil, positioning the ruler on the two points, making sure to leave a portion of the ruler beyond each point, drawing a line along the ruler edge with the pencil, starting before one of the point and finishing after the other). Such a construction involves technical knowledge about the instrument functionality, its use to get the graph and the theoretical relationship between the graphical trace and part(s) of the instrument. For example, a ruler edge enables to draw a straight line. For that purpose, the ruler edge must be placed on the site of the wanted line before drawing the line. Moreover, the aforementioned construction also involves semiotic knowledge about the links between the geometrical objects or properties and their graphical representations (a line is represented by a straight line whose length does not matter, the line may be extended on both sides, the line is infinite: just a part of it may be represented, when you draw a line through two points, the line that you draw must go beyond these points). The construction thus relies on geometrical knowledge carried by the instruments (a line is defined by two points, a line is infinite).

The intention to act generates a *motor intention*, which leads to carry out in practical terms the project of the action with instrument. The *motor manipulation aim* of the action relates to corporeal aspects of the action to be performed with the instrument: movements to grasp the instrument and body manipulation with instruments (body posture, hands position, press hold, speed, etc.). In the previous example, motor manipulation relates to movements to grasp the ruler and the pencil, movements to position the ruler (successive adjustments by sliding the ruler held with the two hands or placing the pencil's mine at one precise point, put the ruler against the mine and make it pivot to the other point). Furthermore, the individual must maintain the ruler with the non-dominant hand, fingers wide apart in the middle of the ruler, without protruding over the edge, while the dominant hand draws along. At last, a stronger pressure of the non-dominant hand must be done. Such a contextualized construction involves specific praxis skills: it is up to the individual to organize and coordinate his/her movements in time and space in order to carry out the expected action. Practical skills are required too: the individual has to be able to carry out practical knowledge to efficiently handle technical tool considering its hardware features.

A part of the motor intention is in the organizational component too. The *organizational aim* of the action consists of the conception of the elementary actions organization in the working environment. The individual should be able to plan basic actions. At the same time he/she must cope with the organization of those actions according to a defined plan in order to promote the realization of the complex action project. Moreover, this organizational component is constituted of necessary interactions between the individual and the artifacts of his/her working environment to establish good conditions to obtain a graphical object: searching and finding or borrowing the technical objects (e.g., a ruler), making them fully operational (e.g., take the ruler out of its bag, sharpen the pencil), having a flat working place (e.g., positioning the working sheet correctly – not half on the notebook, half on the table). Organizational skills are required: the individual has to be able to plan and to organize basic actions in order to promote complex actions achievement. Visuo-spatial skills are involved in

the motor intention of the individual about the implementation of the action: he/she must mentally anticipate the spatial dimensions and the relative position of the graphical objects, artifacts and body in the environment.

At last, intention to act and motor intention lead to the decision to implement the action: the individual can choose to do the action with instrument mentally or in concrete terms. The previous components of action with instrument are summarized in Figure 1.

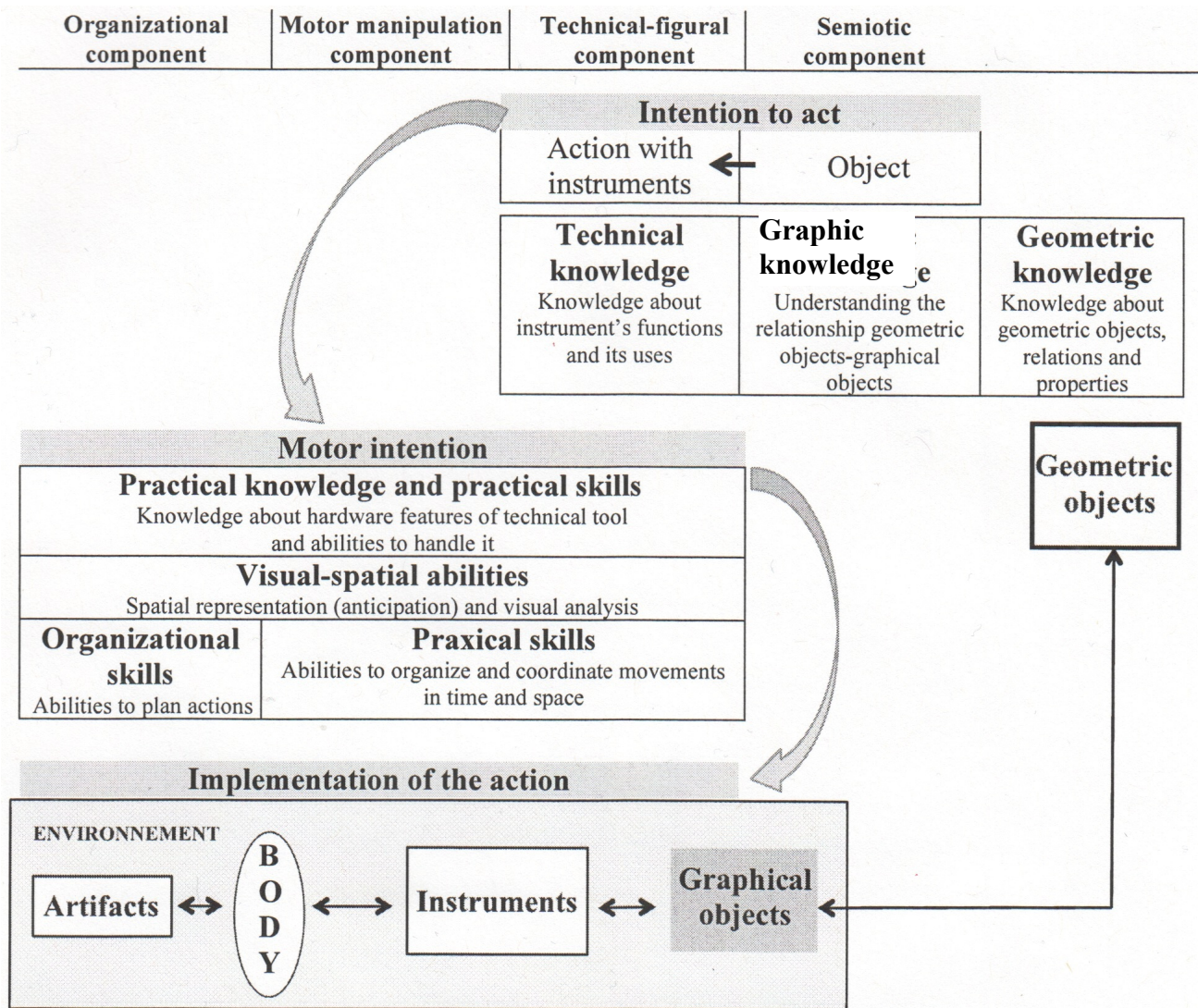


Figure 1: Actions with instruments

## APPLICATION OF THE THEORETICAL FRAMEWORK

The technical-figural aim and the semiotic aim of the action (the intention, the project, the decision) are conscious for the individual and dyspraxic pupils have no problem in this respect. The other two aims (motor manipulation and organizational) are automatic for the standard individual, whereas they are not for the individual with dyspraxia, despite the repetition and training, which leads him/her very often to failure in the effective implementation of his/her actions (see an example of constructions realized by a dyspraxic pupil in Figure 2).



Figure 2: Left, a square construction; right, a rhombus construction

We therefore suggest abandoning these practical aspects of the action (organizational and motor manipulation), which have no contribution in terms of geometric knowledge, and focus on the technical-figural component of the action representation, by using the language skills of the dyspraxic pupil, through pair work, where the actual manipulation is carried out by the other pupil. In this context of oral communication, the dyspraxic pupil is brought to give instruction pertaining to the technical-figural component to whoever is realizing the actions with instruments. She/he can do it in a verbal language, accompanied by gestures.

### Language

In our study, we consider different languages to communicate about the action with instrument based on the individual's aims: semiotic, technical-figural, motor manipulation and organizational.

In the semiotic component, we have two kinds of verbal language. First, the *geometric language* consists of the mathematical tongue with its lexicon and its specific expressions (Laborde, 1982), for example: "circle with centre A and radius r," "line constructed through a point and parallel to another line," etc. This language aims to define geometric objects, relationships and properties. Second, the *common language* consists of lexicon on spatial relationships (e.g., obliquely, next to, above) and on graphical objects (e.g., a stroke, a cross). This language aims to define graphical objects and their spatial relationships.

In the technical-figural component, we define the *geometric technical language* and the *common technical language*. The former uses a geometric lexicon; the latter uses a common lexicon. These languages are used to inform about the actions to be performed with the instruments in order to obtain the graphical representations of the geometric objects in question: these actions are related to *unexpressed* geometric properties (for instance: put the ruler over the two points, as opposed to: draw the straight line passing through the two points).

In the motor manipulation component, the language expresses corporeal action to be performed with instruments in order to obtain graphical objects in practical terms. (e.g., "Hold your compass by the top, not by the legs!" "Press more on the ruler than on the pencil!")

In the organizational component, the language relates to peripheral actions (to the principal action with instruments) and to temporal organization of the action with instruments. This language uses a common lexicon with time indicators as "first," "then," "after," "to begin," "to continue," "to finish," etc.

### Gestures

Gestures can complement or reinforce the oral language in communication. Moreover, according to McNeill (1992) gesture plays a role in the process of conceptualization, not just in communication:



“gestures, together with language, help constitute thought” (McNeill, 1992, p. 245). We call mathematical gestures any body movement, spontaneous or deliberate, performed in the air or on a support, and in relation to the geometric activity. We consider the following gesture categories, in support of communicative gestures (Kendon, 1988; McNeill, 1992): deictic, mimetic, iconic, metaphoric and beats. These gestures play a role not only in communication but also in the process of conceptualization, by allowing the pupil for example to convey information that he/she is not yet able to express in a purely verbal manner. Thus, these gestures are bearers of geometrical knowledge.

The gesture is *deictic* when the finger moves along or points a graphical object or part of an instrument. This gesture is prototypically performed with the pointing finger, although any body part or manipulated artifacts can be used. For example, on Figure 3, you can see deictic gestures: left, in the motor manipulation aim, the finger is pointing the right place where the compass has to be held; in the middle, in the technical-figural aim, the pencil is moving along set square right-angle sides, then is moving along the straight line (d) and is pointing the point A (in order to construct the perpendicular line to the line (d) through the point A); right, in the semiotic aim, the finger is moving along the symmetry axis.

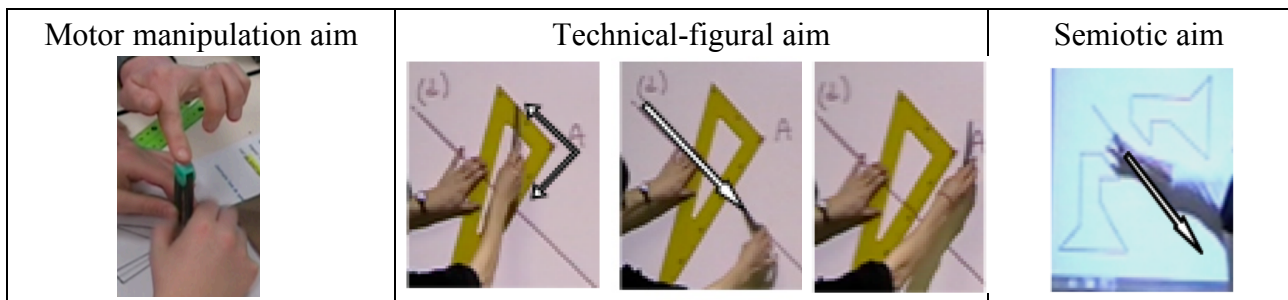


Figure 3: Examples of deictic gestures

The gesture is *mimetic* when the handling of the instrument is mimicked. For example, on Figure 4, you can see mimetic gestures: left, in the motor manipulation aim, right and wrong hand positions to draw along the ruler; right, in the technical-figural aim, mime of using the compass in order to draw a circle.

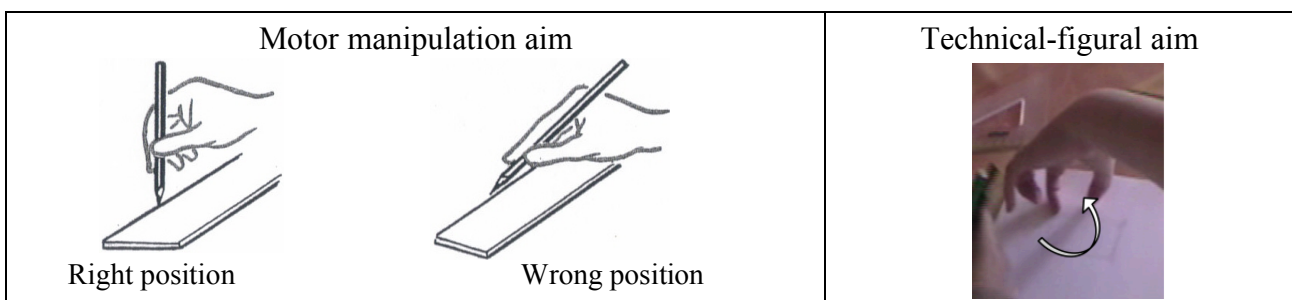


Figure 4: Examples of mimetic gestures

The gesture is *iconic* in the semiotic aim when it is the gestural representation of a geometric object or when it expresses a mathematical property in relation to graphical objects and the way to draw them with instruments. For example, on Figure 5, you can see iconic gestures: left, triangle gestural representation, static or dynamic in the air; right, gesture about the measure transport of a segment over a line.

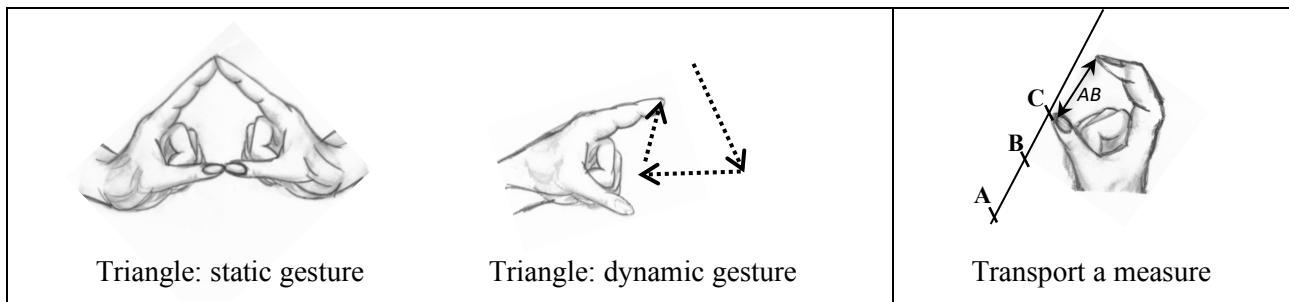


Figure 5: Examples of iconic gestures (semiotic aim)

The gesture is *metaphoric* in the semiotic aim when it expresses a mathematical concept. This type of gesture, closely related to mimetic and iconic gestures, exhibits decontextualized images of abstract concept. The same gesture may refer both to the activity with the instrument, their meanings are personal and related to the experience of the individual, but also to the geometric concept represented thanks to the construction with instruments. As a consequence these gesture can be considered as a pivot sign that mark a process of generalization (Bartolini Bussi & Mariotti, 2008). For example, on Figure 6, you can see metaphoric gestures: left, gestures with fingers represent the conservation of length property; right, gestures with the hands represent symmetrical figures. These gestures can be based on mimetic gestures, the former for using compass and the latter for folding a sheet along a line in order to see the superposition of symmetrical figures.

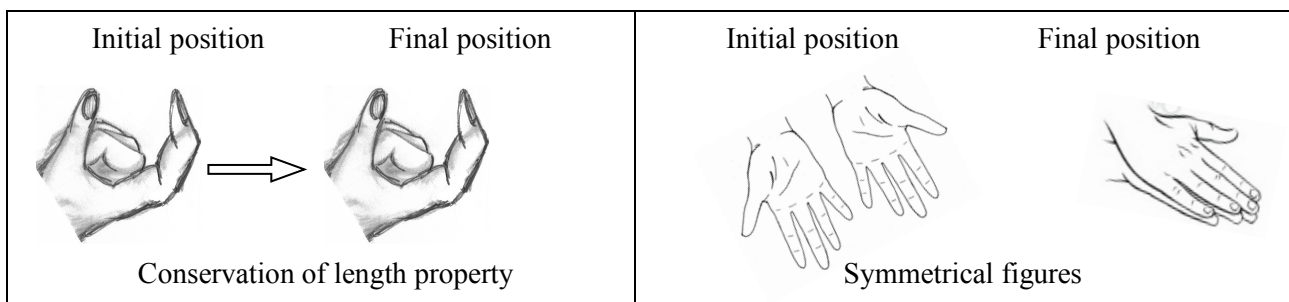


Figure 6: Examples of metaphoric gestures (semiotic aim)

*Beats* are gestures that emphasize off-propositional relations. These gestures are mere flicks of the hand(s) up and down or back and forth that give rhythm to the speech. For example, on Figure 7, you can see beats that emphasize geometric properties to be taken into account to trace the symmetric of a point with respect to an axis, using set square and compass.

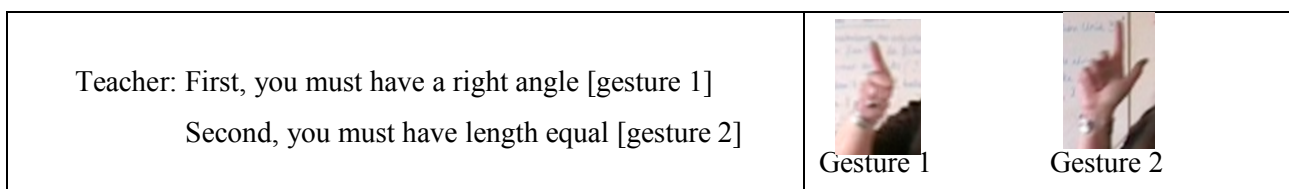


Figure 7: Examples of beats

## CONCLUSION

We have seen in this article that actions with instruments can have different aims (semiotic, technical-figural, motor manipulation and organizational) related to the individual intention (intention to act, motor intention). Besides, we have identified knowledge (geometric, graphic technical, practical) and



skills (practical, praxical, organizational, visuo-spatial) involved in component of the action. Then, we have distinguished language and gesture linked to each of them.

Our theoretical framework allows us to envision a teaching method, which is likely to lead a pupil with visual spatial dyspraxia towards geometric learning. Our hypothesis is that the dyspraxic pupil is able to go through a process of conceptualization in geometry without handling tools by him/herself, but rather by guiding someone else using a defined appropriate technical language, accompanied by mathematical gestures, in a pair-work setting. Previous experimentation out of the classroom with a dyad of fifth-graders' pupils (one dyspraxic and the other standard) allowed to see that the dyspraxic pupil is able to reason and to solve geometric problems within a context of oral communication with a peer or with a school special education assistant when he/she is freed from any organizational and motor manipulation tasks (Petitfour, 2015). We are currently leading experimentations in order to test our teaching method in the classroom.

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