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Electricity Provision with Intermittent Sources of Energy

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Abstract

We analyze the interaction between a reliable source of electricity production and intermittent sources such as wind or solar power. We first characterize the optimal energy mix, emphasizing the availability of the intermittent source as a major parameter for the optimal investment in capacity. We then analyze decentralization through competitive market mechanisms. We show that decentralizing the efficient energy mix requires electricity to be priced contingently on the availability of the intermittent source. By contrast, traditional meters impose uniform pricing, which distorts the optimal mix of energy sources. Decentralizing the efficient energy mix with uniform prices requires either cross-subsidies from the intermittent source to the reliable source of energy or structural integration of the two types of technology.

Keywords: renewable resources, wind electricity, solar energy, global warming

JEL codes: D24, D61, Q27, Q32, Q42

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1 Introduction

The substitution of renewable sources of energy such as wind and photovoltaic power for fossil fuel in electricity production is one of the key technological solutions to mitigate global warming. It is currently pushed forward by many scientists and policy makers in the debate on greenhouse gas emission reduction. Environmental policies now support less carbon-intensive renewable sources of energy by means of subsidies, feed-in tariffs and mandatory minimal installed capacity. In all developed countries, the generation of electricity from geothermal, wind, solar and other renewable sources is increasing by more than 20% a year. There is however a large difference between OECD Europe where renewable sources of energy account for 6% of electricity generation and OECD North America where the ratio is 2.5%.¹ The difference mainly comes from the policy of the European Commission, which has fixed a minimum target of a 20 % share of energy from renewable sources in the overall energy mix for 2020. If all Member States could achieve their national targets fixed in 2001, 21 % of overall electricity consumption in the EU would be produced from renewable energy sources by 2010.²

An essential feature of most renewable sources of energy is intermittency. Electricity can be produced from wind turbines only on windy days, from photovoltaic cells on sunny days and certainly not at night, and from waves and swell when the sea is rough. All these intermittent sources of energy rely on an input (wind, sun, waves, tide) whose supply depends on out-of-control conditions. Some of these conditions are perfectly predictable, for example the seasonal duration of daylight for sun power or the tide level. Others like wind and sunshine intensity can only be forecast a few days in advance and even then with some degree of uncertainty.

Yet a particular feature of the electricity industry is the commitment of retailers to supply electricity to consumers at a given price any time for any level of demand. This

¹OECD/IEA (2010), Monthly Electric Statistics, June 2010; available at www.iea.org/stats/surveys/mes.pdf

²“Renewable Energy Road Map. Renewable energies in the 21st century: building a more sustainable future”; available at europa.eu/scadplus/leg/en/lvb/l27065.htm.

business model reflects the consumers' taste for a reliable source of energy viewed as essential, for example for lighting, cooling or heating. In developed countries, as power outage and black-outs are very costly both economically and politically, electricity production and supply are designed to match the demand of consumers any time at any location on the grid. Thus, the variability and unpredictability of intermittent sources of energy clearly conflict with a reliable supply of electricity.

One way to reconcile intermittent supply with permanent demand consists in storing either the input, or the output or both. In that respect, hydropower production is an attractive source of energy. Although it relies on uncertain rainfall and snow, water can be stored in dams to supply peak load with electricity. In northern countries, water is stored during the fall and the spring to be used in winter for heating and lighting. By contrast, input storage is not possible for two growing renewable sources of energy: wind and solar power. As regards output storage, it is also very limited. The current storage technologies using batteries are still very costly and inefficient. An intermediary solution in combination with hydropower is pumped storage.³

The introduction of a large share of intermittent and non-storable sources of energy is a new challenge for the operators and regulators of the electricity industry. In addition to difficulties of the transportation and distribution, intermittent sources raise problems at the generation stage. In this paper we are mainly interested in three of them. The first is the efficient mix of intermittent sources (wind, solar) and reliable sources such as fossil fuel (coal, oil, natural gas) or nuclear power. The second is the compatibility of intermittent sources of energy with market mechanisms. Can competitive markets decentralize the efficient mix of capacity? The third problem is the design of an environmental policy aimed at promoting low carbon technologies by relying on intermittent sources of energy and simultaneously guaranteeing security of supply.

To address those issues, we construct a stylized model of energy investment and production with two sources of energy: an intermittent and non-storable source, wind, and

³Cheap electricity is used in periods of low demand to restore water resources that can be used to generate electricity in periods of peak demand. See Crampes and Moreaux (2010).

a non-intermittent source, fossil fuel. The two sources differ in cost and availability. Of course, both sources require installed capacities at a cost. Electricity generation in plants using non-intermittent energy costs the price of fossil fuel plus a possible polluting emission tax or price. By contrast, producing electricity from wind is (almost) free once capacity is installed. Nevertheless, it is possible only in the "states of nature" where the input (wind) is available. We characterize the efficient energy mix in installed capacity and production for all the possible values of their costs. We show that wind power is used either as a substitute for or a complement to fuel power on windy days. We generalize this result with several sources of intermittent energy, e.g. wind turbines at two different locations with heterogeneous wind regimes. We determine under which conditions it is efficient to invest in both sources of intermittent energy even if one is more efficient at producing MWh.

Decentralizing the efficient energy mix requires to prices to be based on the availability of the intermittent source of energy, that is, on weather conditions. Unfortunately, this is not implementable because of the lack of price responsiveness on the consumer's side. We show that imposing the same price in all states of nature (independently of whether wind turbines are spinning or not) leads to a second-best solution characterized by under-investment in wind power and over-investment in fossil fuel. The reason is that a uniform price does not reflect energy scarcity in each state of nature. The price is too high on windy days when energy is abundant and too low on windless days when energy is scarce. Therefore, wind power production is more profitable than fossil power. As a consequence, a regulated electricity monopoly that operates the two technologies under a zero profit condition experiences a deficit on fossil power compensated by the profit from its wind power division. If, by contrast, electricity is supplied by competing firms, each owning only one of the two technologies, and there is free entry, the zero profit condition of the fossil power producers implies strictly positive profits for wind power producers. We conclude by providing policy insights for efficiency improvement and climate change mitigation.

Several papers address the issue of mixing renewable with non-renewable sources of energy. Fischer and Newell (2008) assess different environmental policies to mitigate climate change when electricity is produced from polluting fossil fuel and from non-polluting

green sources. However, they abstract from wind power and photovoltaic intermittency by assuming a reliable annual output from renewable energy. Other papers focus on the difference between hydroelectricity and generation plants using fossil fuel. They examine competition among hydropower plants (Garcia *et al.*, 2001, Ambec and Doucet, 2003) or between a hydropower and a thermal producer (Crampes and Moreaux, 2001). Again, these papers consider a deterministic supply of renewable inputs whereas here we focus on input variability.

The economics of intermittent sources of electricity production are still in their infancy. Most papers on the subject are empirical and country specific. For example, Neuhoff *et al.* (2006 and 2007) develop a linear programming model to capture the effects of the regional variation of wind output on investment planning and on dispatching in the UK when transport is constrained. Kennedy (2005) estimates the social benefit of large-scale wind power production (taking into account the environmental benefits) and applies it to the development of this technology in the South of Long Island. Bocard (2008) computes the social cost of wind power as the difference between its actual cost and the cost of replacing the produced energy. He divides the social cost into technological and adequacy components and applies the break-up to Denmark, France, Germany, Ireland, Portugal and Spain. Müsgens and Neuhoff (2006) build an engineering model representing inter-temporal constraints in electricity generation with uncertain wind output. They provide numerical results for the German power system. Coulomb and Neuhoff (2005) focus on the cost of wind turbines in relation to changes in their size using data on German prices. Butler and Neuhoff (2004) and Menanteau *et al.* (2003) consider the variety of tools available for public intervention in the development of renewable energy in general, and intermittent sources in particular.⁴ Compared to these papers, our analysis is situated upstream as it provides a microeconomic framework for the study of optimal investment and dispatching of wind or solar plants. It also allows us to determine by how much market mechanisms depart from the outcome of optimal decisions. Garcia and Alzate (2010) compare the performance of

⁴ All these papers are devoted to wind power. Borenstein (2008) proposes a deep economic analysis of solar photovoltaic electricity production with a focus on California.

two public policies: feed-in tariffs and mandatory portfolio standards. They also examine the efficient energy mix but with an inelastic demand which is nil beyond a maximal price. By contrast, we consider a standard increasing and concave consumer's surplus function which leads to a demand for electricity that smoothly decreases in price. Our framework is more appropriate for analyzing long-term decisions concerning investment in generation capacity since in the long run smart equipment will improve demand response.

Our paper is also related to the literature on electricity pricing with variable demand, in particular on peak-load pricing (see for instance Crew and Kleindorfer, 1995). In the basic peak-load pricing model, a single piece of equipment can be operated at a constant marginal cost, up to the installed capacity, to serve a demand that can be either high or low. Although we consider a deterministic demand, our model can be rephrased in terms of random demand. Since the wind (or solar) power plant has a zero operating cost, it must be dispatched before the thermal plant. Consequently, the latter faces the residual demand. This residual demand is intermittent since it is the difference between the deterministic gross demand and the intermittent energy supplied by windmills. The main difference with standard peak-load pricing models is that the magnitude of this intermittent residual demand is endogenous since it depends on the installed capacity of the intermittent source. Yet, the pricing of intermittent energy shares some similarities with the standard peak-load model. In both cases, prices should reflect the cost of capacity when used. Most of the capacity cost must be recovered during peak demand. Similarly, with intermittent energy and state-contingent prices, the entire cost of fossil fuel capacity is charged to consumers when there is no wind or sun.

The model of non-stationary demand has been extended by Chao (1985) and Kleindorfer and Fernando (1993) to include supply uncertainty. This strand of literature focuses on the effects of outage costs and rationing rules on optimal pricing policies. When all production plants generate a random output, assumptions on the adjustment process when demand exceeds capacity are essential. By contrast, as we assume that the plant using fossil fuel is fully reliable, we do not have to make any assumption on outage costs or rationing. Central in our analysis is the opposition between, on one hand, technologies with different levels

of reliability, and, on the other, consumers demanding fully reliable electricity provision.⁵ Our work is complementary to that of Chao (1985) and Kleindorfer and Fernando (1993) in several respects. First, they characterize the optimal energy mix imposing uniform pricing among states of nature. We show that the optimal energy mix actually requires state dependent prices. Second, when prices are forced to be uniform across states of nature, we show that the market implementation of the optimal energy mix requires cross-subsidies among producers or divisions using the two sources of energies. Finally, contrary to Chao (1985) and Kleindorfer and Fernando (1993) where the random outputs are independently distributed, we explicitly consider the case of correlated intermittent sources. We highlight the fact that the correlation of energy availability is an important determinant of the optimal energy mix.

The paper is organized as follows. Section 2 sets up the model with the two sources of energy (reliable and intermittent) and determines the optimal dispatch and generation capacities. Section 3 analyzes the decentralization of the efficient energy mix with state-contingent prices. It also addresses the issue of the optimal energy mix and its market implementation when consumers are not reactive to state contingent prices. In Section 4 we extend the model to two different sources of intermittent energy. Section 5 discusses policy insights based on our main results, namely the development of smart technologies necessary for the implementation of the first best energy mix, the structural or financial links between technologies necessary for the implementation of the efficient solution under uniform pricing, and some aspects of environmental policy. Section 6 concludes.

⁵In developed countries, the reliability standard is that power should not be cut for more than 1 day every 10 years on average, which means a probability of supply above 99.97%. In France, the average duration of an energy cut (obtained by dividing the non-distributed energy by the average distributed power) in 2010 was 2 minutes and 53 seconds (source: Réseau de Transport d'Electricité, 2010 Report)

2 Optimal energy mix

We consider an industry where consumers derive gross utility $S(q)$ from the consumption of q kWh of electricity. This function is unchanged throughout the period considered.⁶ It is a continuous derivable function with $S' > 0$ and $S'' < 0$.

Electricity can be produced by means of two technologies. First, a fully controlled technology (e.g. coal, oil, gas, nuclear, hydropower from reservoirs) allows for the production of q_f at unit cost c as long as production does not exceed the installed capacity, K_f . The unit cost of capacity is r_f . We call this source of electricity the "fossil" source. We assume $S'(0) > c + r_f$, meaning that the production of electricity from fossil energy is efficient when it is the only energy source.

The second technology relies on an intermittent source of energy. It allows for the produce of q_i kWh at 0 cost as long as q_i is smaller than the installed capacity K_i , whose unit cost is r_i , and the primary energy is available. We assume two states of nature: "with" and "without" intermittent energy. The state of nature with (respectively without) intermittent energy occurs with probability ν (respectively $1 - \nu$). It is denoted by the superscript w (respectively \bar{w}).

To simplify, we abstract from environmental issues related to electricity production by assuming that c , r_f and r_i include the environmental marginal costs to society. More specifically, burning fossil fuel to produce q_f kWh requires the purchase of carbon emission permits or the payment of a carbon tax. We assume that they are part of the marginal cost c and reflect the marginal damage due to climate change by one kWh produced from fossil energy once capacity has been installed. Given this assumption and given the technological expectations, it is not unrealistic to consider a scenario where wind or solar power technologies are more competitive than thermal plants in the near future. Therefore, it makes sense to take all combinations of cost parameters into consideration.

⁶Actually, demand is changing along the daily cycle so that there is some positive correlation between demand for electricity and the supply of solar energy. As regards wind, one can find examples of negative correlation due to the demand for air conditioning; for a description of the California case, see NERC (2009). Depending on season and location, all cases are possible.

The first-best problem to solve is twofold. First, the central planner determines the capacities K_i , K_f to install. This is the long-term commitment of the decision process. Second, it chooses how to dispatch the capacities in each state of nature q_i^w , q_f^w and $q_i^{\bar{w}}$, $q_f^{\bar{w}}$, depending on the availability of the intermittent source. It is a short-term decision constrained by the installed capacities. When deciding on the dispatch of plants, the planner knows the state of nature.

Although the problem *a priori* has six decision variables, three can easily be determined, leaving us with only three unknowns. Indeed,

i. $q_i^{\bar{w}} \equiv 0$: windmills cannot produce if there is no wind and solar batteries cannot produce without any sun's rays;

ii. $q_i^w \equiv K_i$: since the installation of the capacity for producing with the intermittent source is costly, it would be inefficient to install idle capacity.⁷

iii. $q_f^{\bar{w}} = K_f$: without intermittent sources of energy, since demand is unchanged and the available capacity is reduced from $K_f + K_i$ to K_f , it would be inefficient to leave some production capacity idle.⁸

For the three remaining decision variables K_i , K_f and q_f^w , the planner's program can be written as follows:⁹

$$(P1) \quad \max_{K_i, K_f} \nu \left[\max_{q_f^w} S(K_i + q_f^w) - cq_f^w \right] + (1 - \nu)[S(K_f) - cK_f] - r_f K_f - r_i K_i$$

subject to

$$q_f^w \geq 0 \quad , \quad q_f^w \leq K_f \quad , \quad K_i \geq 0.$$

As proven in the Appendix, we can establish the following:

⁷We discard the necessary maintenance operations, for example assuming that they can be performed during type \bar{w} periods.

⁸Here again, we discard maintenance operations, for example by assuming that capacity is measured in terms of available plants.

⁹Note that it is not necessary to write the constraint $K_f \geq 0$ explicitly because $K_f > 0$ is implicit in the assumption $S'(0) > c + r_f$.

Proposition 1 *The optimal capacities and outputs are such that*

a) for $\frac{r_i}{\nu} > c + r_f$

$$q_f^w = q_f^{\bar{w}} = K_f = S'^{-1}(c + r_f), \quad q_i^w = K_i = 0$$

b) for $c > \frac{r_i}{\nu}$

$$q_f^w = 0 < q_f^{\bar{w}} = K_f = S'^{-1}\left(c + \frac{r_f}{1-\nu}\right) < q_i^w = K_i = S'^{-1}\left(\frac{r_i}{\nu}\right)$$

c) for $c + r_f > \frac{r_i}{\nu} > c$

$$q_f^w = q_f^{\bar{w}} = K_f = S'^{-1}\left(\frac{c+r_f-r_i}{1-\nu}\right), \quad q_i^w = K_i = S'^{-1}\left(\frac{r_i}{\nu}\right) - S'^{-1}\left(\frac{c+r_f-r_i}{1-\nu}\right)$$

In case a) the intermittent energy is so scarce (small ν) and/or the technology using this energy is so costly (high r_i) that no plant using intermittent energy should be installed. Then, whatever the state of nature, the fossil plant is used at full capacity. The capacity is determined by the equality between the marginal utility of electricity and its long-run marginal cost $c + r_f$. In case b) wind is so abundant and wind turbines so cheap that the intermittent energy totally replaces fossil energy in state of nature w . The capacity to install makes marginal utility equal to the unit cost of capacity r_i discounted by the probability of availability ν . Fossil energy capacity is only used in state of nature \bar{w} . Its long-run marginal cost is c plus the capacity cost r_f corrected by the probability of using it $1 - \nu$ since it is dispatched only when the intermittent source is not available.

In the intermediary case c), fossil energy is used at full capacity jointly with intermittent energy. This case is illustrated in Figure 1. The merit order in state of nature \bar{w} consists in dispatching fossil energy up to K_f , determined by the equality between marginal utility and long-run marginal cost. The latter is equal to the cost of the marginal technology in state \bar{w} that is $\frac{c+r_f}{1-\nu}$ reduced by the saving on the cost of developing the other technology. This is because, in periods w , f is the marginal technology to dispatch (since $c > 0$) but i is the one to develop (since $\frac{r_i}{\nu} < c + r_f$). Then $\frac{r_i}{\nu}$ is the long-run marginal cost of the whole system and it determines the total capacity to install $K_i + K_f$ by $S'(K_i + K_f) = \frac{r_i}{\nu}$.

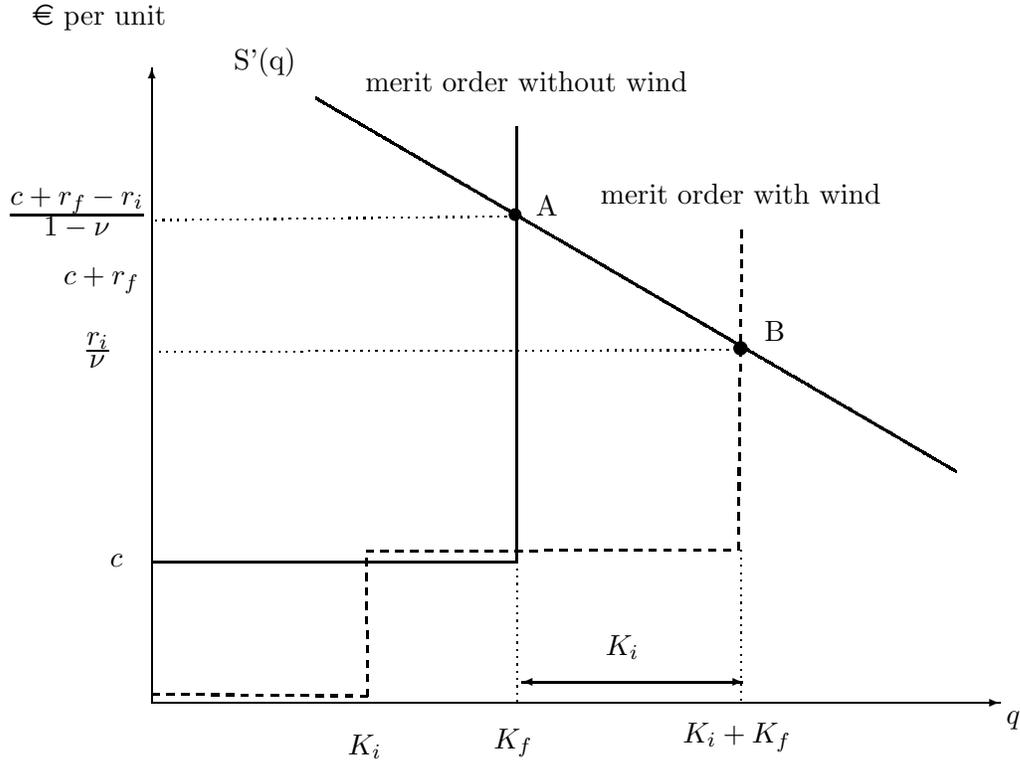


Figure 1: *Optimal energy mix when the two technologies are used*

We have depicted K_i , K_f and the sum $K_i + K_f$ as functions of r_i in Figure 2. It clearly shows that when the intermittent technology i becomes attractive in the sense that $\frac{r_i}{\nu} \leq c + r_f$, it is not simply substituted for fossil energy f . As r_i decreases, it is true that some substitution occurs since K_f decreases. Nevertheless, the total capacity $K_f + K_i$ increases. Substitution cannot be done on a one-to-one basis since nothing can be produced with technology i in the state of nature \bar{w} . It remains true that there is some substitution, with the consequence that, as compared with a world without technology i , there is less energy available in the state of nature \bar{w} than in state w .

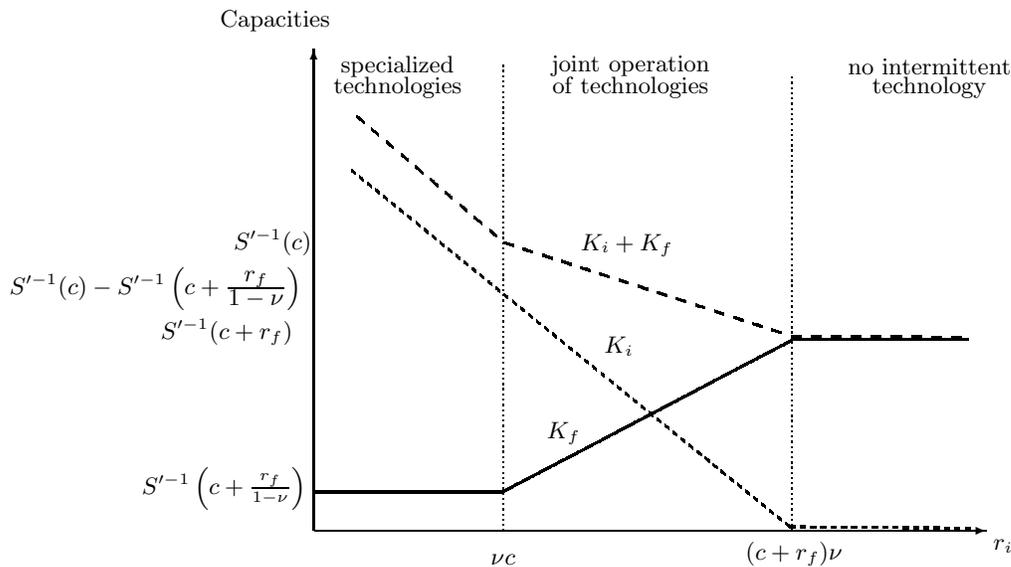


Figure 2: *Capacities with different costs of intermittent energy*

3 Decentralized Production and Consumers' Reactivity

For the past decade, in most developed countries, regulation authorities have been promoting renewable sources of energy for electricity production in addition to liberalizing of the industry. To assess the consequences of these separate policies, we now consider the decentralization of the optimal energy mix by market mechanisms, taking into account either consumers' reactivity to price variations (Section 3.1) or their lack of reactivity (Section 3.2).

3.1 Market implementation with reactive consumers

Assume that consumers and firms are price-takers. Suppose also that they are equipped to be price sensitive. The price-reactiveness property requires high-tech equipment informing the consumer about the spot price, or software that switches electrical appliances on and off according to information received on spot prices, or a load-shedding service provider. With such devices, the optimal outcome can be decentralized with prices contingent on

states of nature p^w and $p^{\bar{w}}$. In practice, this means that electricity prices should depend on the presence or absence of the intermittent source of energy.

In each state of nature $s \in \{w, \bar{w}\}$, consumers facing price p^s solve $\max_q S(q^s) - p^s q$. They demand q^s kWh in state s where $S'(q^s) = p^s$ (marginal utility equals price) for $s = w, \bar{w}$.

First consider case a) whereby $\frac{r_i}{\nu} > c + r_f$. The prices that decentralize the optimal outcome are $p^w = p^{\bar{w}} = c + r_f$. Consumers react to that price by consuming the efficient productions $q^w = q^{\bar{w}} = S'^{-1}(c + r_f)$. Producers owning the intermittent technology i invest nothing since the long term marginal cost r_i of each kWh exceeds the expected unit benefit $p^w \nu$. Producers endowed with the fossil technology f invest to supply all consumers $K_f = q^w = q^{\bar{w}}$. Since the long run marginal cost of each kWh $c + r_f$ equals the market price in both states of nature $p^w = p^{\bar{w}}$, they make no profit. Clearly, the prices that decentralize the optimal energy mix are unique. With lower prices, fossil electricity producers would not recoup their investment and would consequently invest nothing. Symmetrically, with higher prices, more fossil fuel capacities would be installed and competitive entry would reduce prices to the long-term marginal cost.

Second, in case b) where $c > \frac{r_i}{\nu}$, the prices that decentralize the optimal energy mix are $p^w = \frac{r_i}{\nu}$ and $p^{\bar{w}} = c + \frac{r_f}{1-\nu}$ per kWh. Consumers react to those prices by consuming $q^w = S'^{-1}(\frac{r_i}{\nu})$ in state of nature w and $q^{\bar{w}} = S'^{-1}(c + \frac{r_f}{1-\nu})$ in state \bar{w} . In state w , firms producing energy from fossil sources cannot compete with those producing from intermittent sources. They therefore specialize in producing only during state of nature \bar{w} . Their expected return on each unit of capacity is thus $(1 - \nu)(p^{\bar{w}} - c) = r_f$. Since it exactly balances the marginal cost of capacities, the plants using fossil sources have a zero expected profit. Similarly, firms with intermittent technology obtain an expected return $\nu p^w = r_i$ per unit of investment and thus zero profit on average. In other words, under those prices, each type of producer recoups exactly its expected long-term marginal cost.

Third, in case c) where $c + r_f > \frac{r_i}{\nu} > c$, with prices $p^w = \frac{r_i}{\nu}$ and $p^{\bar{w}} = \frac{c + r_f - r_i}{1 - \nu}$ the market quantities are also at first-best levels. Consumers' demand is $q_f^{\bar{w}} = S'^{-1}(\frac{c + r_f - r_i}{1 - \nu})$ when the wind is not blowing and $q_f^w + q_i^w = S'^{-1}(\frac{r_i}{\nu})$ when it is. In state w , competing producers

are ordered on the basis of their bids which are equal to short-run marginal costs under perfect competition, that is, 0 for i -producers and c for f -producers. The investment in capacity depends on expected returns and long-run marginal costs. Since fossil electricity firms produce in both states of nature, the return per unit of capacity is $\nu p^w + (1 - \nu)p^{\bar{w}} - c$ which matches exactly the capacity unit cost r_f . Thus f -producers make zero profit. On the other hand, i producers obtain an expected return of νp^w per unit of investment which also matches exactly the cost r_i . They therefore make zero profit as well, which is the equilibrium under free entry conditions. In conclusion, we posit the following proposition.

Proposition 2 *State contingent prices p^w and $p^{\bar{w}}$ with $p^{\bar{w}} \geq p^w$ and free entry allow market mechanisms to implement the optimal energy mix. The price inequality is strict ($p^{\bar{w}} > p^w$) whenever the intermittent source of energy is installed and operated.*

3.2 Optimal energy mix and market implementation with non-reactive consumers

The decentralization process described in the former section faces a serious hurdle. The optimal energy dispatch and investment can be driven with state contingent prices only if consumers have smart meters signaling scarcity values and if they are able to adapt to price signals. However, most consumers, particularly households, are equipped with traditional meters and are therefore billed at a price independent of the state of nature determining production.

To assess the consequences of a uniform pricing constraint, we determine the efficient production and investment levels constrained by uniform delivery. With a stationary surplus function as assumed here, consumers react to uniform prices by consuming the same amount of electricity in both states of nature. Because our model focuses on the complementarity between intermittent and non-intermittent sources, we exclude the easy solution of brownouts and blackouts, a simple way to replace the wind in state \bar{w} . By introducing a "value of lost load", we would have two complements to wind energy: electricity from fossil fuel and outages. However, as explained above, in developed countries public opinion

refuses to accept outages and the provision of electricity is (close to) 100% guaranteed. We therefore prefer to stick to the no-rationing hypothesis. Formally, non-state-contingent pricing implies the constraint $q_i^w + q_f^w = q_f^{\bar{w}}$. Yet, since the intermittent (resp. reliable) technology is to be used at full capacity in state w (resp. \bar{w}) the latter constraint leads to $K_i + q_f^w = K_f$.

As shown in the Appendix, the main consequence of this restriction is that the intermittent source of energy will never be used to complement fossil energy in state w . More precisely, case c) of Proposition 1 where both technologies are operated in state w (namely for $c < \frac{r_i}{\nu} < c + r_f$) disappears. This is because the constraint of uniform provision $K_i + q_f^w = K_f$ makes the two technologies perfect substitutes in state w . This results in a bang-bang solution. If $c < \frac{r_i}{\nu}$ only technology f is installed and $S'(K_f) = c + r_f = \tilde{p}^w = \tilde{p}^{\bar{w}}$. The uniform price only just matches the long-term marginal cost of the f technology. In the opposite case, $c > \frac{r_i}{\nu}$, both technologies are installed but only technology i is used when possible, i.e. in state w with $S'(K_f) = S'(K_i) = (1 - \nu)c + r_f + r_i = \tilde{p}^w = \tilde{p}^{\bar{w}}$. The uniform price equals the long-term marginal cost of each kWh, namely $(1 - \nu)c + r_f + r_i$, taking into account that the two technologies are developed to ensure uniform delivery and c is incurred only in state \bar{w} which arises with probability $1 - \nu$. We therefore can assert the following proposition.

Proposition 3 *With non-reactive consumers, the optimal capacities and outputs are such that:*

a) for $\frac{r_i}{\nu} > c$

$$\tilde{q}_f^w = \tilde{q}_f^{\bar{w}} = \tilde{K}_f = S'^{-1}(c + r_f), \quad \tilde{q}_i^w = \tilde{K}_i = 0$$

b) for $\frac{r_i}{\nu} < c$,

$$\tilde{q}_f^w = 0 < \tilde{q}_f^{\bar{w}} = \tilde{K}_f = \tilde{q}_i^w = \tilde{K}_i = S'^{-1}((1 - \nu)c + r_f + r_i)$$

The disappearance of the possibility of jointly operating the two technologies in state w can be illustrated using Figure 1. The fact that consumers are weakly price-sensitive can

be viewed as if their marginal surplus curve $S'(q)$ were more vertical.¹⁰ Consequently the horizontal difference between points A and B is smaller and smaller, which means that K_i converges to zero in this interval of cost when all consumers are under the non-contingent retail price regime.

Figure 3 shows how the uniform-pricing constraint modifies the capacities as functions of the cost of the intermittent energy, r_i . In the left part of the graph where it is socially profitable to invest in technology i , the two technologies become strict complements, contrary to what we have observed in Figure 2. The consequence is that, except if $\nu = 1$ where $\tilde{K}_f = 0$ the smaller r_i is, the larger $\tilde{K}_f = \tilde{K}_i$ will be. In other words, even if the renewable source has a very high probability of availability but cannot be totally guaranteed, when prices are not state-contingent the whole capacity has to be duplicated. Actually, the problem is the same as for reserve capacities that must be available to replace failing plants or to supply unexpected demand, except that in state \bar{w} the whole type- i capacity is failing and must be replaced. If brownouts and blackouts were taken into consideration, the strict equality $\tilde{K}_f = \tilde{K}_i$ would not hold any longer since energy outages and electricity from fossil sources would compete to replace windmills when they do not produce. It nevertheless remains true that \tilde{K}_f would increase as r_i decreases.

¹⁰Note that consumers are not inelastic to price since their demand (marginal utility) function has a finite negative slope. They would change their consumption if they could receive state-contingent price signals. But they cannot react since they receive a uniform price signal.

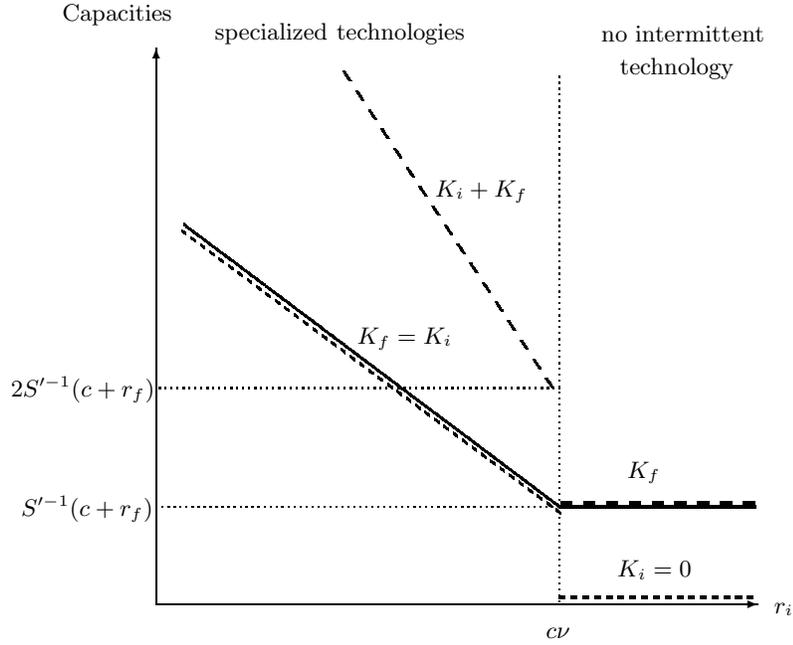


Figure 3: *Capacities with different costs of intermittent energy under uniform price*

A major drawback of the optimal solution with non-reactive consumers when the two technologies are installed (that is when $c > \frac{r_i}{\nu}$) is that it requires some form of subsidy from technology i to technology f to secure non-negative profits. Without any external financial transfer, the optimal energy mix under uniform prices can be decentralized only under certain conditions, for example a regulated electricity monopoly or competitive firms owning the two technologies.

To prove this surprising result, we first observe that the expected unit profit of a firm using the two technologies is nil since $\nu\tilde{p}^w - r_i + (1 - \nu)(\tilde{p}^w - c) - r_f = 0$. Given the uniform prices $\tilde{p}^w = \tilde{p}^w = (1 - \nu)c + r_f + r_i$ the division operating technology i obtains positive cash flows $\nu\tilde{p}^w - r_i = \nu[(1 - \nu)(c - \frac{r_i}{\nu}) + r_f] > 0$ whereas the fossil energy division incurs financial losses $(1 - \nu)(\tilde{p}^w - c) - r_f < 0$. These losses are obviously larger and larger when ν increases since: *i*) the price of electricity decreases, *ii*) the type- f technology is less often utilized and *iii*) the capacity to install increases. This can result in huge financial amounts so that transfers from division i towards division f are necessary to sustain the efficient

energy mix with non-reactive consumers.

What occurs when the two technologies are owned by separate operators and transfers are not allowed? In a competitive industry with free entry, the fossil energy-based electricity producers will exit the market under the second-best electricity price. This will reduce the supply of energy in state \bar{w} and therefore increase the price of electricity in both states of nature to above the second best level. The free entry equilibrium price in a competitive industry with a single price in the two states of nature is such that firms with fossil technology make zero profit. It therefore matches the fossil energy producer's long-term marginal cost $c + r_f$. The firms with intermittent energy technology i enjoy strictly positive profits. They free-ride on the uniform price constraint.

Finally, note that since $p^{\bar{w}} = c + \frac{r_f}{1-\nu} > \tilde{p}^w = \tilde{p}^{\bar{w}} > p^w = \frac{r_i}{\nu}$, and prices signal investment opportunities, the capacity of intermittent energy installed under uniform prices is smaller than at first-best, whereas the opposite stands for fossil energy, i.e. $\tilde{K}_i < K_i$ and $\tilde{K}_f > K_f$. This is true when $\tilde{K}_i = \tilde{K}_f$, that is, when $\frac{r_i}{\nu} < c$, but it is also obviously true when $c < \frac{r_i}{\nu} < c + r_f$ since $\tilde{K}_i = 0 < K_i$ and $\tilde{K}_f < K_f + K_i$. We summarize the above results in the following proposition.

Proposition 4 *When consumers are not reactive to price variations, the second best energy mix is implementable in a market economy only if the two sources of energy are owned by the same financial entity or if the government transfers revenues from intermittent sources to reliable sources. Otherwise, free entry with uniform prices leads to: (i) over-investment and zero profits in the fossil-fuel production plants; and (ii) under-investment and strictly positive profits in the plants using the intermittent source.*

To sum up this section, we can say that the decentralization of the efficient energy mix calls for a lower price when intermittent sources of energy are available. If not feasible for technological or institutional reasons, efficient production under uniform pricing distorts prices by increasing the price of intermittent energy p^w and reducing the price of fossil energy when intermittent energy is not available $p^{\bar{w}}$. Intermittent energy is therefore overvalued and fossil-fuel electricity undervalued compared to first-best. This results in

under-investment in intermittent energy and over-investment in fossil-fuel electricity. In a nutshell, since a uniform price does not reflect state-of-nature marginal costs, consumers tend to over-consume electricity when it is costly to produce (in state \bar{w}) and under-consume it when it is cheap (in state w). Compared to first-best, this increases demand for fossil energy and reduces it for intermittent energy. Long-run supply through investment in capacities is adapted accordingly.

4 Two sources of intermittent energy

The former results can be generalized to cases where several sources of intermittent energy are available. Assume there are two sources, 1 and 2. The two sources can be of a different kind, e.g. wind and solar. They also can be of the same kind but at different locations e.g. turbines facing different wind conditions. As a consequence, the two sources differ potentially both in their occurrence and in the energy produced when available. For instance, they might face different dominant winds (north versus south), one being stronger on average than the other.

The results of the former sections can be extended to the multiplicity of sources by increasing the number of states of nature. For example, with two turbines located at different places, we have four states of nature: in state 1 only the intermittent source of energy 1 is available, in state 2 only the intermittent source of energy 2 is available, in state 12 both are available and, as before, in state \bar{w} none is available (and therefore electricity can only be produced from fossil energy). These states of nature occur with probabilities ν_1 , ν_2 , ν_{12} and $1 - \nu$ respectively where $\nu = \nu_1 + \nu_2 + \nu_{12}$. Let us denote by K_i the investment into intermittent source of energy i for $i = 1, 2$. The unit cost of capacity of source i is denoted $r_i > 0$ for $i = 1, 2$ where $r_2 > r_1$. For instance, if wind turbines are at different locations and the mean wind is stronger¹¹ at location 1, than at 2 when it is windy, then with a smaller number of wind turbines at location 1 one can produce the same amount of electricity at location 1 and at location 2. Yet the occurrence of the two sources

¹¹The wind should however not be “too strong” because windmills could not withstand it.

of intermittent energy might make location 2 attractive.

The planner must determine the capacity of the two intermittent sources of energy K_1 and K_2 in addition to the fossil source K_f and the production levels q_f^s , q_1^s and q_2^s in states $s = \bar{w}, 1, 2$ and 12 . Using notations similar to those of the former section, we can easily determine that for $i = 1, 2$, $q_i^{\bar{w}} \equiv 0$, $q_i^s = K_i$ in states $s = 1, 2, 12$ and $q_f^{\bar{w}} = K_f$. The remaining decision variables K_1 , K_2 , K_f , q_f^1 , q_f^2 and q_f^{12} are determined by solving program (P2) below:

$$(P2) \max_{K_1, K_2, K_f} \nu_1 \max_{q_f^1} [S(K_1 + q_f^1) - cq_f^1] + \nu_2 \max_{q_f^2} [S(K_2 + q_f^2) - cq_f^2] \\ + \nu_{12} \max_{q_f^{12}} [S(K_1 + K_2 + q_f^{12}) - cq_f^{12}] + (1 - \nu)[S(K_f) - cK_f] \\ - r_f K_f - r_1 K_1 - r_2 K_2$$

subject to

$$0 \leq q_f^s \leq K_f \text{ for } s = 1, 2, 12; \quad K_i \geq 0 \text{ for } i = 1, 2.$$

Depending on the cost parameters and the probability of each state of nature, we can obtain a large spectrum of results, some with only one intermittent source of energy to operate, others combining the two sources. In each case we can derive the capacity to install and the dispatch that maximize the net social welfare. As an illustration, we establish the following proposition (see the proof in Appendix).

Proposition 5 *The optimal capacities in the intermittent sources of energy 1 and 2 are such that*

a) For $\nu_1 > 0$ and $\nu_2 > 0$, $K_1 = K_2 = 0$ if and only if $c + r_f < \frac{r_i}{\nu_i + \nu_{12}}$ for $i = 1, 2$.

b) For $c + r_f > \frac{r_i}{\nu_i + \nu_{12}}$

1) $K_1 > 0$ and $K_2 = 0$ if $\nu_1 = \nu_2 = 0$ and $\nu_{12} > 0$,
(perfect positive correlation),

- 2) $K_1 > 0$ and $K_2 > 0$ if $\nu_1 > 0$, $\nu_2 > 0$ and $\nu_{12} = 0$
(perfect negative correlation).

As in investment portfolios, the decision to invest in various intermittent technologies does not only depend on the return on investment but also on the risk associated to each return. According to *a)* in Proposition 5, a necessary and sufficient condition for investing in an intermittent source of energy is $\frac{r_i}{\nu_i + \nu_{12}} < c + r_f$ for one $i \in \{1, 2\}$ at least: the long-run marginal cost of electricity produced from source i discounted by the probability of its availability $\nu_i + \nu_{12}$ must be lower than the long-run marginal cost of electricity produced from fossil energy (which is very similar to parts *b* and *c* of Proposition 1. Depending on the probabilities of each event, in some cases the two sources of intermittent energy are installed and, in other cases, only one is installed. For instance, consider the extreme cases *b.1)* and *b.2)* of perfectly positive and negative correlations, respectively. If sources 1 and 2 are available only in the same states of nature (perfect positive correlation), we have $\nu_1 = \nu_2 = 0$. Then only the more efficient source of intermittent energy should be installed. Even if $\frac{r_i}{\nu_{12}} < c + r_f$ for $i = 1, 2$ so that the two sources of intermittent energy have a lower discounted marginal cost than fossil energy, only source 1 is installed since we have assumed $r_1 < r_2$. By contrast, if sources 1 and 2 are never available at the same time (perfect negative correlation which translates formally into $\nu_{12} = 0$), then as long as $\frac{r_i}{\nu_i} < c + r_f$ for $i = 1, 2$, both sources of intermittent energy are installed. In particular, source 2 is installed even if it is more costly ($r_2 > r_1$) and/or less frequent ($\nu_2 < \nu_1$). Concretely, if wind turbines can be developed at two different locations, one being superior in terms of wind speed and frequency, it is efficient to install turbines at both locations to exploit the complementarity of the two sources of energy as long as their discounted long-run marginal costs are lower than the fossil energy cost. As in all portfolio management problems, negative correlation allows some form of insurance. Nevertheless, as long as $\nu_1 + \nu_2 + \nu_{12} < 1$, it is necessary to install reliable capacity to replace the intermittent technologies in “bad” states of nature. This cost should be internalized by the builders and operators of the plants using intermittent sources. Note that the multiplicity of locations

increases the transmission costs of any signal as to which source of energy is currently generating electricity. Our above arguments on the difficulty of implementing the optimal energy mix are therefore even more relevant.

5 Policy insights

The model developed in the above sections analyzes cases where intermittent technologies can compete against fossil fuel technologies. This will be the case in the future after a technological breakthrough or some drastic learning effect, or due to more stringent climate change mitigation policies (higher carbon taxes or fewer emission permits) leading to a higher marginal cost for fossil combustion.¹² Meanwhile, intermittent technologies are sustained by public aid (e.g. certificates, feed-in tariffs) or purchase requirements that are a financial burden for society. These costs are well known. They are so high that at the beginning of 2010, some governments (in particular France and Germany) decided to step back from a blind policy of support for photovoltaic energy. By contrast, the back-up costs we have identified with our model have received less emphasis and still have important policy implications as regards the future of the energy industry. In Section 5.1, we examine the cost of adapting both the electrical appliances used for consumption and the network to decentralize the optimal energy mix with reactive consumers. We then consider in Section 5.2 the structural or institutional arrangement required to decentralize the optimal energy mix with non-reactive consumers. Finally, in Section 5.3 we discuss the conflict between the environmental regulation of fossil sources and the financial regulation of the intermittent sources.

¹²According to the Royal Swedish Academy of Sciences (2010), the full cost for electricity generated by wind power, excluding the costs for expanding the electricity power line network and the back-up power, is currently 6-10c€/kWh. This is slightly higher than the generation cost in coal power plants but twice that in nuclear plants.

5.1 Smart consumers

Our first-best analysis suggests that intermittent technologies should be promoted in parallel with smart meters and/or smart boxes. These intelligent devices can make electricity consumption dependent on the state of nature that prevails at the location of production plants. By controlling in real time some programmed electric equipment such as boilers and heaters, disconnecting them when the intermittent source of energy is not available, smart meters and smart boxes will make electricity demand sensitive to energy scarcity across time and space. They are likely to be more receptive and reactive than consumers exposed to messages such as “the wind turbines you are connected to are currently running; therefore the price of electricity is low” (or the opposite). The smart meters and boxes that would dispatch consumption automatically across time need to be connected with information technologies to be installed in the shadow of the energy network. More generally, the growth of intermittent energy calls for further investment in the network, to increase both connection and information processing. Indeed, compared to thermal power plants, wind and solar power plants are more likely to be scattered on a given territory. This has two consequences. First, connection requires large investment in small-scale lines, transformers and two-way meters. This obviously makes coordination necessary between producers, transmitters and system and market operators. Second, random local injections radically modify the business model of distributors since they now have to balance the flows on the grid under their responsibility and in some cases to install new lines to guarantee the reliability of the local system under the constraint of accepting all injections by authorized generators. The adaptation of networks to the development of intermittent sources has been underestimated so far. In most developed countries making the grid smart is now a priority, which means huge investments for embedding Information and Communication Technologies (ICT) into the grid.¹³

¹³See for example www.smartgrids.eu.

5.2 Structural arrangements

The huge cost of installing smart appliances at consumption nodes, coupled with ICT devices all along the grid, is still too high compared to the welfare increase from introducing state-contingent prices. Consequently, consumers continue to be offered only one price, whether wind turbines are producing or not. Compared to the optimal energy mix with state-contingent prices, they demand too little energy when the intermittent source is available and too much when it is not. If the resulting equilibrium price were an average value of the marginal costs of production in the different types of generation plants, the generators using fossil energy would lose money. They would prefer to leave the industry. We therefore have to consider several structural and legal solutions to implement the optimal energy mix constrained by non-contingent prices. Under free entry and exit, in order to keep generators using fossil fuel in the market, the price should be equal to the long run marginal cost of their MWh. It is as if consumers had to pay for a guarantee of services. The drawback of this solution is that the owners of plants using intermittent energy pocket a profit equal to the difference between the long-run marginal cost of electricity from fossil fuel and the long-run marginal cost of electricity from intermittent energy. Consumers pay for being insured against random supply and the money they pay accrues those who create randomness. Two public policies can reduce the rent assigned to intermittent energy producers. The first one consists in taxing wind turbines to subsidize thermal plants in order to balance the budget of all producers. The second policy consists of a mandatory technological mix or insurance. Each producer should either control the two technologies or buy an insurance contract that guarantees energy supply any time. These legal arrangements would force any new entrant to guarantee production in all states of nature. Both policies have their drawbacks. The first one, more “market-based”, comes at the cost of levying and redistributing public funds. The second one, more in line with “command-and-control”, restricts the firms’ flexibility in their technological choices.

5.3 Environmental policy

In our model, the cost of energy production from the different sources includes all social costs. In particular, it includes the environmental costs of air pollution, resource uses and wastes, for present and future generations. When externalities from energy production are not internalized by producers, environmental policies should be enforced in addition to the regulation devoted to implement the optimal energy mix when consumers are non-reactive. In this case, the regulator must mitigate two market failures. The first market failure is the standard externality due to pollution emitted by electricity generation in plants using fossil fuel. Its mitigation requires to favor less dirty sources of energy such as wind and solar power. Examples of such policies include subsidies and feed-in tariffs for renewables, carbon tax or emission permits for CO_2 and SO_2 . This obviously reduces the competitiveness of the reliable sources of energy (coal, oil, nuclear) compared to the intermittent ones.

The second market failure is due to the non-reactivity of consumers to energy intermittency. When consumers do not react to state-contingent pricing, the demand of electricity is independent of the energy availability. It implies that producers supply the same quantity at the same price in all states of nature, i.e. with and without wind. Under perfect competition with free entry, thermal plants meet the zero profit condition while windmills and photovoltaic plants experience strictly positive profits at the detriment of consumers. To increase consumers' surplus, the regulator should set the average profit of the electricity industry to zero in the long run by taxing windmills to subsidize thermal power. Hence the two market failures call for opposite policies. The net tax or subsidy on each source of energy would depend on the magnitude of each market failure. If pollution damages are more important than the surplus loss due to consumers' non-reactivity, fossil fuel power should be taxed and wind power should be subsidized, while money should go the other way in the opposite case. Furthermore, when the non-reactivity market failure is ignored by the regulator, environmental policies increase the rent assigned to intermittent energy producers even under perfect competition at the cost of a lower consumers' surplus.

6 Conclusion

The development of intermittent sources of energy to produce electricity creates a series of difficulties as regards the adaptation of behavior, structures and institutions to the characteristics of these sources. Satisfying the demand for non-contingent electricity at a non-contingent price without blackouts clearly requires an installed capacity of non-intermittent sources matching the capacity built for processing the intermittent source, whatever the availability duration of the intermittent source. Actually, because availability periods are not known with certainty and fossil fuel plants cannot be dispatched instantaneously when necessary to replace intermittent sources, the back-up capacity must even be larger. In Ireland for example, *“incorporating 30GW of additional renewable capacity into the grid, to meet EU’s 2020 target, will require a further 14-19GW of new fossil fuel and nuclear capacity to replace plants due to close and to meet new demand (almost doubling the total new installed electricity generating capacity required by 2020, compared to a scenario where renewable generation was not expanded)”*.¹⁴ In our model we have analyzed the basic parameters that should be considered to determine the optimal capacity of intermittent and non-intermittent production plants anticipating their efficient dispatch. To implement the optimal energy mix in a market economy, prices should vary with the availability of the intermittent source of energy and consumers should react accordingly. We also have shown that the optimal energy mix constrained by fixed delivery is not financially feasible. The conclusion is that an electricity industry with a large share of intermittent sources is not sustainable without some form of integration in production, either structural or financial. This will bring about additional problems that should be put in the balance with the gains of integration.

¹⁴Northern Ireland Assembly (2009).

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A Proof of Proposition 1

Denoting $\xi_f^w \geq 0$, $\eta_f^w \geq 0$ and $\xi_i \geq 0$ the multipliers respectively associated to $q_f^w \geq 0$, $q_f^w \leq K_f$ and $K_i \geq 0$, the Lagrange function corresponding to problem (P1) in the text is

$$\begin{aligned} \mathcal{L} = & \nu [S(K_i + q_f^w) - cq_f^w + \xi_f^w q_f^w + \eta_f^w (K_f - q_f^w) + \xi_i K_i] \\ & + (1 - \nu) [S(K_f) - cK_f] - r_f K_f - r_i K_i \end{aligned}$$

Given the linearity of technologies and the concavity of the surplus function, the following first order conditions are sufficient to determine the optimal capacities and outputs:

$$\frac{\partial \mathcal{L}}{\partial q_f^w} = \nu [S'(K_i + q_f^w) - c + \xi_f^w - \eta_f^w] = 0 \quad (\text{A1})$$

$$\frac{\partial \mathcal{L}}{\partial K_f} = \nu \eta_f^w + (1 - \nu) [S'(K_f) - c] - r_f = 0 \quad (\text{A2})$$

$$\frac{\partial \mathcal{L}}{\partial K_i} = \nu [S'(K_i + q_f^w) + \xi_i] - r_i = 0 \quad (\text{A3})$$

plus the complementary slackness conditions derived from the three constraint of (P1).

We first identify conditions for $K_i > 0$.

From (A3), if $K_i > 0$, $S'(K_i + q_f^w) = \frac{r_i}{\nu}$ and we can write from (A1) $\frac{r_i}{\nu} - c = \eta_f^w - \xi_f^w$. Then, we face two possibilities:

1. If $\frac{r_i}{\nu} > c$, $\eta_f^w > 0$ so that $q_f^w = K_f > 0$ and $\xi_f^w = 0$.

Plugging $\eta_f^w = S'(K_i + K_f) - c$ into (A2) we obtain

$$\nu S'(K_i + K_f) + (1 - \nu)S'(K_f) = c + r_f$$

From $K_i > 0$ and $S'' < 0$, we have $S'(K_i + K_f) < S'(K_f)$ so that

$$S'(K_i + K_f) = \frac{r_i}{\nu} < c + r_f$$

2. In the second possibility, $\frac{r_i}{\nu} < c$, the condition $\frac{r_i}{\nu} < c + r_f$ is obviously satisfied.

We conclude that $\frac{r_i}{\nu} > c + r_f$ is sufficient for $K_i = 0$. We can therefore partition the set of parameters as follows:

- a) for $\frac{r_i}{\nu} > c + r_f$, $K_i = 0$. As regards the output of the reliable technology in state of nature w , we have $q_f^w = K_f$. Indeed, assume $q_f^w < K_f$. Then $\eta_f^w = 0$ and from (A1) $S'(q_f^w) - c = -\xi_f^w \leq 0$. Similarly from (A2) $S'(K_f) - c = \frac{r_f}{1-\nu} > 0$. But since $S'' < 0$ these two inequalities are not compatible. It results that $q_f^w = \bar{q}_f^w = K_f$ and combining (A1) and (A2), $S'(K_f) = c + r_f$.
- b) for $c > \frac{r_i}{\nu}$, since from (A1) $\frac{r_i}{\nu} - c = \eta_f^w - \xi_f^w$, we have $\xi_f^w > 0$ so that $q_f^w = 0$. Knowing that $K_f > 0$, this implies $\eta_f^w = 0$. Then, from (A2) we have

$$S'(K_f) = c + \frac{r_f}{1 - \nu}$$

and from (A3) and $K_i > 0$

$$S'(K_i) = \frac{r_i}{\nu}.$$

c) for the intermediary case $c + r_f > \frac{r_i}{\nu} > c$, we have seen that $K_i > 0$ and $q_f^w = K_f$.
From equation (A3),

$$S'(K_i + K_f) = \frac{r_i}{\nu}$$

and combining (A1) and (A2)

$$\nu S'(K_i + K_f) + (1 - \nu)S'(K_f) = c + r_f$$

Plugging the first equation into the second,

$$S'(K_f) = \frac{c + r_f - r_i}{1 - \nu}.$$

B Proof of Proposition 3

Adding the constraint $K_i + q_f^w = K_f$ and the multiplier γ to the Lagrange function of the optimal mix, the first order conditions become

$$\frac{\partial \mathcal{L}}{\partial q_f^w} = \nu[S'(K_i + q_f^w) - c + \xi_f^w - \eta_f^w + \gamma] = 0, \quad (\text{B1})$$

$$\frac{\partial \mathcal{L}}{\partial K_f} = \nu(\eta_f^w - \gamma) + (1 - \nu)[S'(K_f) - c] - r_f = 0, \quad (\text{B2})$$

$$\frac{\partial \mathcal{L}}{\partial K_i} = \nu[S'(K_i + q_f^w) + \xi_i + \gamma] - r_i = 0. \quad (\text{B3})$$

We already know that $K_i = 0$ when $\frac{r_i}{\nu} > c + r_f$. We then focus on $\frac{r_i}{\nu} < c + r_f$. Combining (B1) and (B3) we have that

$$\frac{r_i}{\nu} - c = \eta_f^w + \xi_i - \xi_f^w.$$

There are two cases.

1. If $\frac{r_i}{\nu} < c$, $\xi_f^w > 0$ so that $q_f^w = 0$ and $K_i = K_f$. Because $K_f > q_f^w = 0$, $\eta_f^w = 0$ and $\xi_i = 0$. Consequently we can combine (B2) and (B3) to get

$$S'(K_i = K_f) = (1 - \nu)c + r_f + r_i.$$

2. If $\frac{r_i}{\nu} > c$, $\eta_f^w > 0$ so that $q_f^w = K_f$ and $\xi_i > 0$ so that $K_i = 0$. In effect, we cannot have $K_i > 0$ because, it would entail $\xi_i = 0$ and $\eta_f^w > 0$ so that $q_f^w = K_f$. The uniform delivery constraint would give $K_i + K_f = K_f$ which cannot be true for $K_i > 0$. Then, second best commands $S'(K_f) = c + r_f$ like for $\frac{r_i}{\nu} > c + r_f$.

C Proof of Proposition 5

Assume a one-to-one relationship between states of nature and the technology using the energy available in this state of nature. This allows to simplify notations by dropping the index naming states of nature s . Using the same notation as before for the multipliers, the first-order conditions of problem (P2) yield:

$$\frac{\partial \mathcal{L}}{\partial q_f^i} = \nu_i [S'(K_i + q_f^i) - c + \xi_f^i - \eta_f^i] = 0 \text{ for } i = 1, 2$$

$$\frac{\partial \mathcal{L}}{\partial q_f^{12}} = \nu_{12} [S'(K_1 + K_2 + q_f^{12}) - c + \xi_f^{12} - \eta_f^{12}] = 0$$

$$\frac{\partial \mathcal{L}}{\partial K_f} = (1 - \nu) [S'(K_f) - c] + \nu_1 \eta_f^1 + \nu_2 \eta_f^2 + \nu_{12} \eta_f^{12} - r_f = 0$$

$$\frac{\partial \mathcal{L}}{\partial K_i} = \nu_i [S'(K_i + q_i^f) + \xi_i] + \nu_{12} [S'(K_1 + K_2 + q_f^{12}) + \xi_i] - r_i = 0 \quad \text{for } i = 1, 2$$

plus the complementary slackness conditions derived from the constraints of (P2).

In states of nature with intermittent energy, one can always use the fossil fuel equipment. Therefore production in state 12 cannot be lower than in states with only one source and in state without any intermittent source: $K_1 + K_2 + q_{12}^f \geq K_i + q_i^f \geq K_f$ for $i = 1, 2$. Since S' is decreasing, these inequalities imply $S'(K_1 + K_2 + q_{12}^f) \leq S'(K_i + q_i^f) \leq S'(K_f)$ for $i = 1, 2$.

Proof of a). We show that $K_i = 0$ if and only if $c + r_f < \frac{r_i}{\nu_i + \nu_{12}}$ for $i = 1, 2$. Suppose that $c + r_f < \frac{r_i}{\nu_i + \nu_{12}}$ for $i = 1, 2$. By (C5-6),

$$\frac{r_i}{\nu_i + \nu_{12}} = \frac{\nu_i S'(K_i + q_i^f) + \nu_{12} S'(K_1 + K_2 + q_f^{12})}{\nu_i + \nu_{12}} + \xi_i$$

Moreover, by (C4),

$$c + r_f = (1 - \nu) S'(K_f) + \nu c + \nu_1 \eta_f^1 + \nu_2 \eta_f^2 + \nu_{12} \eta_f^{12}$$

Using (C1-3), we substitute for η_f^j ($j = 1, 2, 12$) to obtain:

$$c + r_f = E[S'(K_i + q_i^f)] + \nu_1 \xi_f^1 + \nu_2 \xi_f^2 + \nu_{12} \xi_f^{12}$$

where $E[S'(K_i + q_i^f)] \equiv \nu_1 S'(K_1 + q_1^f) + \nu_2 S'(K_2 + q_2^f) + \nu_{12} S'(K_1 + K_2 + q_f^{12}) + (1 - \nu) S'(K_f)$ is the expected marginal surplus. The assumption $c + r_f < \frac{r_i}{\nu_i + \nu_{12}}$ for $i = 1, 2$ combined with (C7), (C8) and the non-negativity of ξ_f^j for $j = 1, 2, 12$ leads to

$$E[S'(K_i + q_i^f)] < \frac{\nu_i S'(K_i + q_i^f) + \nu_{12} S'(K_1 + K_2 + q_f^{12})}{\nu_i + \nu_{12}} + \xi_i,$$

for $i = 1, 2$. Suppose first that $K_1 + q_1^f \leq K_2 + q_2^f$ then $S'(K_1 + K_2 + q_{12}^f) \leq S'(K_2 + q_2^f) \leq S'(K_1 + q_1^f) \leq S'(K_f)$ which implies:

$$E[S'(K_i + q_i^f)] \geq \frac{\nu_2 S'(K_2 + q_2^f) + \nu_{12} S'(K_1 + K_2 + q_f^{12})}{\nu_2 + \nu_{12}},$$

for $i = 1, 2$. For the last inequality to be consistent with (C9) for $i = 2$, it must be true that $\xi_2 > 0$ which implies $K_2 = 0$. Since by assumption $K_1 + q_1^f \leq K_2 + q_2^f = q_2^f = K_f$, we must have $K_1 = 0$ and $q_1^f = K_f$.

Suppose now that $K_i = 0$ for $i = 1, 2$ then $\xi_i > 0$ for $i = 1, 2$ in (C5-6) which leads to

$$(\nu_i + \nu_{12})S'(K_f) < r_i$$

for $i = 1, 2$. Moreover by (C1-3), $\eta_f^j = S'(K_f) - c$ for $j = 1, 2, 12$ which combined with (C4) leads to $S'(K_f) = c + r_f$. The last equality joint with (C10) leads to $c + r_f < \frac{r_i}{\nu_i + \nu_{12}}$ for $i = 1, 2$.

Proof of *b.1*). Suppose $\nu_1 = \nu_2 = 0$, $\nu_{12} > 0$ and $c + r_f > \frac{r_i}{\nu_{12}}$ for $i = 1, 2$. The first-order conditions simplify to:

$$S'(K_1 + K_2 + q_f^{12}) = c - \xi_f^{12} + \eta_f^{12} \quad (\text{C}'3)$$

$$(1 - \nu) [S'(K_f) - c] = r_f - \nu_{12}\eta_f^{12} \quad (\text{C}'4)$$

$$S'(K_1 + K_2 + q_f^{12}) = \frac{r_i}{\nu_{12}} - \xi_i \quad \text{for } i = 1, 2 \quad (\text{C}'5-6)$$

Conditions (C'5-6) lead to $\xi_2 - \xi_1 = \frac{r_2}{\nu_{12}} - \frac{r_1}{\nu_{12}} > 0$ where the last inequality is due to the assumption $r_2 > r_1$. Therefore $\xi_2 > 0$ which implies $K_2 = 0$. Since there are only two states of nature with only one source of intermittent energy in one state like in Section 2, Proposition 1 holds. In particular, with our notation we have $K_1 > 0$ for $c + r_f > \frac{r_1}{\nu_{12}}$.

Proof of *b.2*). Suppose that $\nu_1 > 0$, $\nu_2 > 0$ and $\nu_{12} = 0$ and $c + r_f > \frac{r_i}{\nu_i}$ for $i = 1, 2$. The first-order conditions simplify to:

$$S'(K_i + q_f^i) = c - \xi_f^i + \eta_f^i \quad \text{for } i = 1, 2 \quad (\text{C}''1-2)$$

$$(1 - \nu) [S'(K_f) - c] = r_f - \nu_1\eta_f^1 - \nu_2\eta_f^2 \quad (\text{C}''4)$$

$$S'(K_i + q_f^i) = \frac{r_i}{\nu_i} - \xi_i \quad \text{for } i = 1, 2 \quad (\text{C}''5-6)$$

Case 1: $\frac{r_i}{\nu_i} < c$ for one $i \in \{1, 2\}$. The conditions (C''1-2) and (C''5-6) imply $c - \frac{r_i}{\nu_i} = \xi_f^i - \eta_f^i - \xi_i > 0$ which implies $\xi_f^i > 0$ and therefore $q_f^i = 0$, i.e. no fossil power in state i . As long as $\nu_i > 0$ and $S'(0) = +\infty$, $q_f^i = 0$ is optimal only if $K_i > 0$.

Case 2: $\frac{r_i}{\nu_i} > c$ for $i = 1, 2$. Suppose first that $K_1 = K_2 = 0$. Then $K_i + q_i^f = K_f$ for $i = 1, 2, 12$ (use of fossil power under full capacity in all states of nature). Moreover, $K_i = 0$ implies $\xi_i > 0$ and therefore $S'(K_f) < \frac{r_i}{\nu_i}$ by (C''5-6). The first-order conditions (C''1-2) and (C''4) imply $S'(K_f) = c + r_f$ which combined with the last inequality contradicts the assumption $c + r_f > \frac{r_i}{\nu_i}$. Suppose now that $K_1 > 0$ and $K_2 = 0$ which implies $K_2 + q_2^f = K_f$ and $\xi_2 > 0$. The first-order conditions (C''4) and (C''5-6) imply respectively $(1 - \nu_1)[S'(K_f) - c] = r_f - \nu_1 \eta_f^1$ and $S'(K_f) < \frac{r_2}{\nu_2}$. The two last relations lead to $\nu_1 \eta_f^1 > r_f + c - \frac{r_2}{\nu_2} + \nu_1 \left[\frac{r_2}{\nu_2} - c \right]$. Since by assumption $r_f + c > \frac{r_2}{\nu_2} > c$, $\eta_f^1 > 0$ and therefore $q_1^f = K_f$. The first-order conditions imply $E[S'(K_i + q_i^f)] = r_f + c$. Since $S'(K_1 + K_f) < E[S'(K_i + q_i^f)] < S'(K_f)$, the last equality combined with $S'(K_f) < \frac{r_2}{\nu_2}$ contradicts our starting assumption $c + r_f > \frac{r_2}{\nu_2}$.