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On the mixed mode integral in three-dimensional crack problem

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Abstract

This work is based on a generalization of the mixed mode integral for three-dimensional crack problem. A new integral parameter in real three-dimensional case, which computes the energy release rate for each fracture mode, is developed. This integral provides the computation of stress intensity factors and energy release rate for each fracture mode. The virtual mechanical displacement fields is used in order to separate numerically mode I, mode II and mode III in the mixed mode ratio. A numerical validation, in terms of energy release rate, is carried out on a MMCG specimen under mixed-mode loading for different crack growth thickness. The non-dependent integration domain property is justified and the evolution of energy release rate along the thickness is evaluated.

Key words: *Crack problem, Energy release rate, Finite element, Mixed mode, M3D-integral, The non-dependent integration domain, Three-dimensional, Wood material,*

1. Introduction

In a previous work, we proposed a new three-dimensional independent integral entitled J3D and G θ (3D) which make possible studying the fracture process by considering three-dimensional effects [1]. In this paper, this is a question of extended this integral to mixed-mode loading [2] case in three-dimensional medium and takes into account different mixed-mode ratios. This integral will be entitled M3D integral.

This generalization will allow us mixed-mode separation for complex geometries and loadings, including tensile, shear and torsion fracture modes. The M3D integral is presented, and then implemented using the virtual local displacement field, to be able to separate the fracture mixed modes. The numerical computation is performed for an isotropic elastic material for static crack. This new integral is validated numerically through the verification of the invariance of the integration domain property for several thicknesses. Then, an additional verification

consists to compare the results with the two-dimensional case through the two assumptions of plane stresses and plane strain is carried out for two different thicknesses (1 and 25mm). Different results of the non-dependence of the integration domain property is presented. Afterwards, the evaluation of the energy release rate G θ GI and GII along crack front are obtained. Thanks to the mixed mode separation it become possible to obtain GIII for out of plan complex loads. In the third part, the fracture mode separation is extended to evaluate out of plan complex loads; mode III, mixed mode I + II + III and mixed mode I + III. Finally, the limits of three-dimensional modeling using the current virtual displacement field are evoked to put into perspective the need to develop a local approach adapted to the case of a three-dimensional medium.

2. Analytical formulation

For plan problem and for static crack Rice (1968) has defined A path independent integral which allows to computes energy release rate around the crack tip [1]. J-integral takes the following notation:

$$J^{2D} = \int_{\Gamma} (W \cdot n_1 - (\sigma_{ij} \cdot n_j \cdot u_{i,1})) \cdot d\Gamma$$

Where W denotes the strain energy density, Γ is arbitrary curvilinear contour oriented by its normal vector, u_i is the displacement component and σ_{ij} is the stress component.

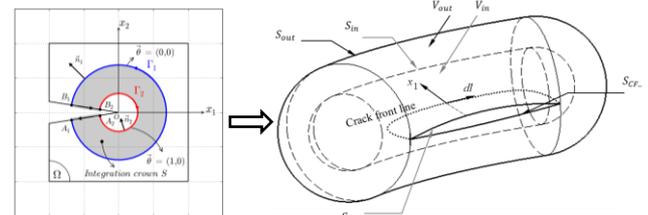


Fig. 1: Description integral domain for 2D and 3D crack

To take into account in global formalism the most general case which the crack front line is arbitrary, we consider some volume and surfaces integration domains (figure 1).

The development of the three-dimensional M-integral concept is based on a Noether's theorem [3]. The M3D-integral formulation is based on energetic approach using consideration described in figure 1 [4]. It expressed as follow:

$$M^{3D} = \frac{1}{2} \int_{S_{\Gamma_1}} \left((\sigma_{ij}^v \cdot u_{i,j} + \sigma_{ij}^u \cdot v_{i,j}) \cdot \delta_{kj} \right. \\ \left. - (\sigma_{ij}^v \cdot u_{i,k} + \sigma_{ij}^u \cdot v_{i,k}) \cdot n_j \cdot dS \right. \\ \left. + \frac{1}{2} \int_{V_{\Gamma_1}} \left((\sigma_{ij}^v \cdot (\varepsilon_{ij}^u)_{,k} + \sigma_{ij}^u \cdot (\varepsilon_{ij}^v)_{,k}) \right. \right. \\ \left. \left. - ((\sigma_{ij}^v \cdot \varepsilon_{ij}^u)_{,k} + (\sigma_{ij}^u \cdot \varepsilon_{ij}^v)_{,k}) \right) \cdot dV \right. \\ \left. - \frac{1}{2} \int_{S_{CF}} (\sigma_{ij}^v \cdot u_{i,k} + \sigma_{ij}^u \cdot v_{i,k}) \cdot n_j \cdot dS \right)$$

To implement this integral in a finite element software, it is easier to consider a volume domain integral.

$$M3D = M_{\theta}^{3D} = \frac{1}{2} \int_V P_{kj} \cdot \theta_{k,j} \cdot dV \\ + \frac{1}{2} \int_{V_{\Gamma_2}} \left((\sigma_{ij}^v \cdot (\varepsilon_{ij}^u)_{,k} + \sigma_{ij}^u \cdot (\varepsilon_{ij}^v)_{,k}) \right. \\ \left. - ((\sigma_{ij}^v \cdot \varepsilon_{ij}^u)_{,k} + (\sigma_{ij}^u \cdot \varepsilon_{ij}^v)_{,k}) \right) \cdot \theta_k \cdot dV \\ - \frac{1}{2} \int_{S_{CF}} (\sigma_{ij}^v \cdot u_{i,k} + \sigma_{ij}^u \cdot v_{i,k}) \cdot n_j \cdot \theta_k \cdot dS$$

For static crack M3D can be expressed as follow:

$$M3D = \frac{1}{2} \int_V (\sigma_{ij}^u \cdot v_{i,k} - \sigma_{ij,k}^v \cdot u_i) \cdot \theta_{k,j} \cdot dV$$

3. Numerical validation

The numerical implementation is based on a Mixed Mode Crack Growth (MMCG) specimen loaded in mixed mode. The MMCG specimen was developed by Moutou Pitti et al [5,6]. The geometry of the MMCG specimen has been optimized by using a finite element computation. This specimen is adapted to obtain a stable crack growth rate during propagation for mixed mode.

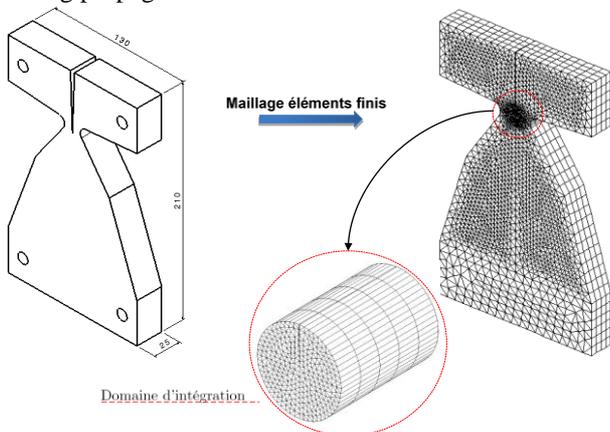


Fig. 2: MMCG specimen

The finite element computation is realized for an elastic isotropic behavior. In the follows, the results of numerical study are exposed. In order to observe the effect of thicknesses on the MMCG specimen we plot the evolution of the energy release rate as function of the crack front line. The description of θ field around the crack front line is shown in Fig. 3. The θ field is equal to zero on outside surface, and 1 on inside surface.

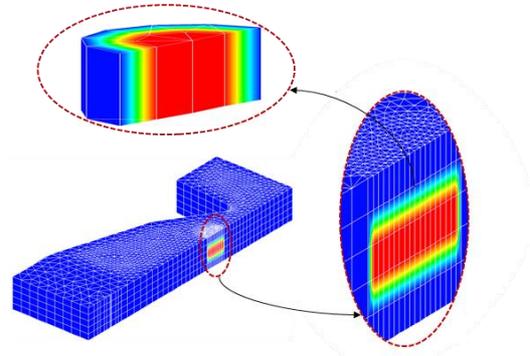


Fig. 3: Description of θ field around the crack front line

Fig. 4 shows the variations in energy release rate vs. integration domain size. Numerical results validate the non-dependence of the integration domain. Note that for $R_{ext}=1\text{mm}$, results present some artefacts because the mechanical fields present some singularity around the crack front.

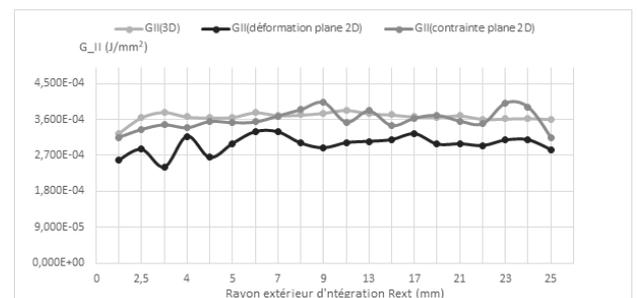
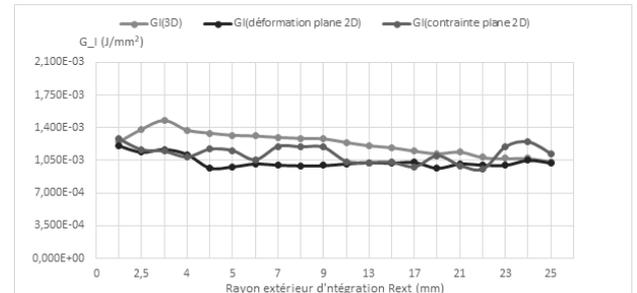


Fig. 4: Energy release rate distribution Vs integration domain around the crack front line

We observe the ability of the M3D integral to separate the fracture modes.

The second validation deals with a comparison of different thickness. In this case, two thicknesses are compared 25mm and 1mm, and energy release rate are observed for each one, Fig. 5.

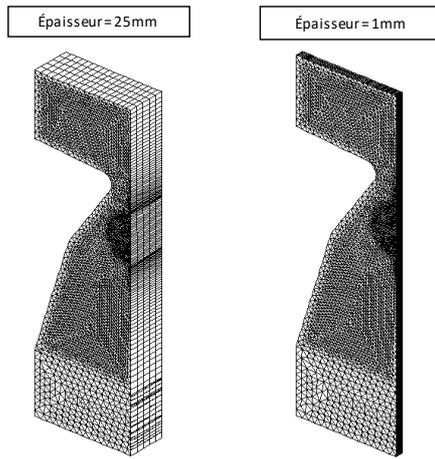


Fig. 5: Impact of the sample thickness

Fig. 6 indicates the energy release rate distribution along the crack front line vs. thickness

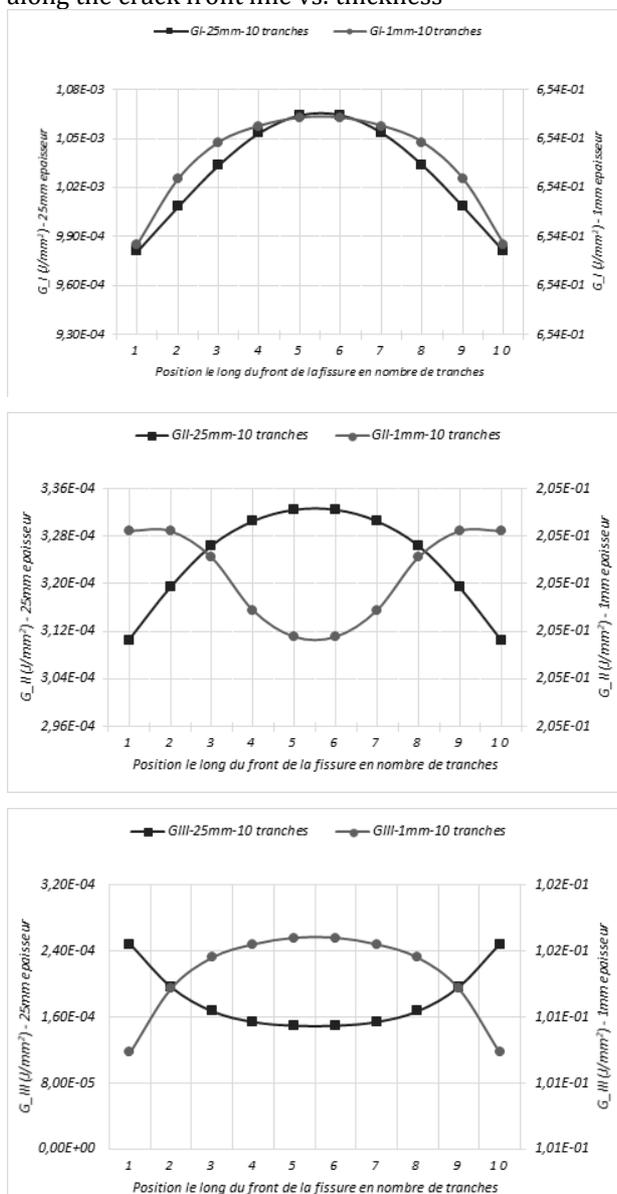


Fig. 6: Energy release rate distribution along the crack front line

This result show us in that even in the case of pure mode III loading, the Poisson effect induce systematically opening and shear mode.

4. Conclusion and outlooks

This paper deals with a new formulation of the mixed mode integral for the study of fracture process in element by taking into account three-dimensional effects. Non-path dependence is proven thanks to the use of an analytical formulation.

One promising perspective consists of completing this study by a theoretical approach proposing a three-dimensional vision of the local mechanical fields in the crack front vicinity.

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