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# A NEW PATH-INDEPENDENT INTEGRAL GENERALIZED FOR THREE-DIMENSIONAL CRACK PROBLEM: ANALYTICAL AND NUMERICAL RESULTS

S. EL KABIR<sup>a</sup>, F. DUBOIS<sup>b</sup>, R. MOUTOU PITTI<sup>c</sup>, N. RECHO<sup>d</sup>, Y.  
LAPUSTA<sup>e</sup>,

- a. Université de Poitiers, Institut PPRIME, Bd Marie et Pierre Curie, 86962 Chasseneuil  
Cedex, France + soliman.elkabir@yahoo.fr  
b. Université de Limoges, GC2D, 19300 Egletons, France + frederic.dubois@unilim.fr  
c. Université Clermont Auvergne, CNRS, Institut Pascal, BP 10448, 63000 Clermont-Ferrand,  
France + rostand.moutou\_pitti@uca.fr  
d. Université Clermont Auvergne, CNRS, Institut Pascal, BP 10448, 63000 Clermont-  
Ferrand, France + naman.recho@moniut.univ-bpclermont.fr  
e. French Institute of Advanced Mechanics, Institut Pascal, 63171 Aubière, France +  
yuri.lapusta@sigma-clermont.fr

## Résumé :

*Prédire le comportement des structures face à la fissuration dans le cas des chargements mécanique sous environnements variables, implique de considérer une répartition tridimensionnelle des champs hydrique et nécessite donc de généraliser une approche tridimensionnelle de la mécanique de la rupture pour évaluer les risques de fissuration dans les sections transversales.*

*L'objectif principal de ce travail est de présenter une nouvelle intégrale invariante adaptée aux problèmes de fissurations dans le cas tridimensionnel. Ce travail est basé sur le développement numérique de l'intégrale  $J$  pour des problèmes tridimensionnel. L'intégrale  $G_{\theta}^{3D}$  est développée grâce à la méthode du champ  $\theta$  et implémenté dans un logiciel d'éléments finis.*

*Pour cela l'éprouvette appelée DCB (Double Cantilever Beam) est utilisée. Cette éprouvette permet d'observer une plage importante de stabilité le long de la fissure en mode d'ouverture (mode I). Le travail réalisé est une étude numérique appliquée à l'éprouvette DCB afin de valider cette nouvelle intégrale et de vérifier son indépendance vis-à-vis du domaine d'intégration. Le taux de restitution d'énergie est calculé le long du front de la fissure. L'intégrale  $J$  tridimensionnelle est comparé à l'intégrale de Bui.*

## Abstract :

*The complex mechanical loading and high climatic variations on structures implies having a better understanding of their fracture mechanical behavior by considering humidity heterogeneity. In this case, it is necessary to consider three-dimensional case in the study of reel crack growth problems.*

*For it the most common approach is the determination of the energy release rate by energetic method. Most of the studies carried out deal with two-dimensional case. The studies of complex structures request the development of specific tools for three-dimensional configurations.*

*This paper present a new analytical formulation and its numerical application for three-dimensional crack growth problem. This work is based on a generalization of the Rice's integral for three-dimensional crack problem. A new integral parameter in real three-dimensional case, which computes the energy release rate combining an arbitrary crack front, is developed. A physical interpretation allows to evaluate the efficiency of the proposed integral. The new integral is compared with the three-dimensional Bui's integral.*

*Applying the theta method a  $G_{\theta}^{3D}$  integral, generalized to a volume domain, is implemented into a finite element software. The non-path dependence is proved with the use of numerical application. The energy release rate distribution along the crack front line is obtained, and compared to Bui's integral. A numerical validation, in terms of energy release rate, is carried out on a DCB (Double Cantilever Beam) specimen under opening mode loading for wood material. Various visions are also proposed to evaluate integral parameter and the energy release rate distribution along the crack front line.*

**Key words : Crack problem ; Energy release rate ; Path-independent integral ; J integral ; Three dimensional**

## **1 Introduction**

The complex mechanical loading and high climatic variations on structures implies having a better understanding of their fracture mechanical behavior. According a no heterogeneity of mechanical fields in the cross section, it is necessary to consider three-dimensional case in the study of reel crack growth problem. For it the most common approach is the determination of the energy release rate by energetic approach.

Most of the studies carried out deal with two-dimensional case. The studies of massive elements request the development of specific tools for three-dimensional configurations.

In this context, two approaches can be developed [1]. The first one is based on the subdivision of a three-dimensional problem by some two-dimensional ones [1, 2, 3]. This approach doesn't need much more effort because all tools developed for two-dimensional problems can be easily reused. However, even if this technique can include a non-linear crack front, it cannot allow considering an external out-of-plane loading inducing, for instance, torsion effects.

The second approach is to rewrite the J-integral formalism, starting from the very beginning with a three-dimensional analysis. The topic of this paper deals with the generalisation of the J-integral formalism to a three-dimensional problem and its adaptability of the G-theta method for a finite element implementation.

## **2 $J^{3D}$ -integral description**

For plan problems and for static cracks, Rice (1968) [4] has defined a path independent integral which allows to computes energy release rate around the crack tip. For cracked linear elastic material, Rice

[4] have used J-integral to compute energy release rate for curvilinear contour.  $J$ -integral takes the following definition :

$$J^{2D} = \int_{\Gamma} (W \cdot n_1 - (\sigma_{ij} \cdot n_j \cdot u_{i,1})) \cdot d\Gamma \quad (1)$$

Where  $W$  denotes the strain energy density,  $\Gamma$  is arbitrary curvilinear contour oriented by its normal vector,  $u_i$  is the displacement component and  $\sigma_{ij}$  is the stress component.

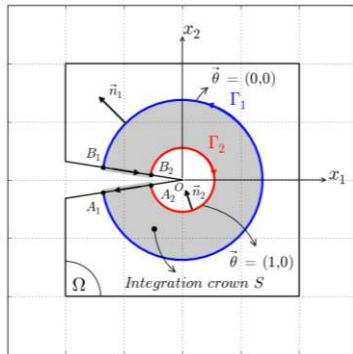


Fig. 1: Description integral domain of  $J$  and  $G_{\theta}$

For three-dimensional problem, the crack front line is defined as the intersection of two surface (figure 2).

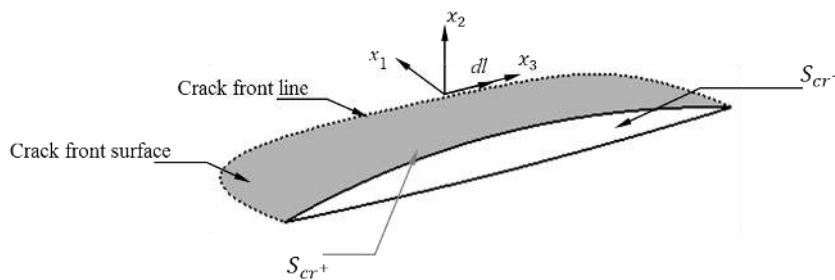


Fig. 2: Crack surface with crack system coordinate

To take into account in global formalism the most general case in which the crack front line is arbitrary, we consider some volumes and surfaces integration domains as described in Fig. 3.

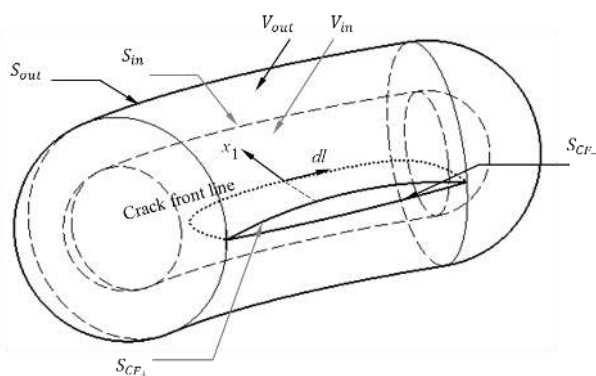


Fig. 3: Closed volume and surfaces integration domains

The new  $J^{3D}$ -integral formulation is based on an energetic approach using consideration described in figure 3. After some mathematical developments, we obtain:

$$J^{3D} = \int_{S_{out}} (W \cdot n_k - (\sigma_{ij} \cdot n_j \cdot u_{i,k})) \cdot dS - \int_{S_{CF}} \sigma_{ij} \cdot n_j \cdot u_{i,k} \cdot dS + \int_{V_{out}} (\sigma_{ij} \cdot (\epsilon_{ij})_{,k} - W_{,k}) \cdot dV \quad (2)$$

the  $J^{3D}$ -integral is composed by three separated terms. The first one designates the classical part used for the determination of the crack growth initiation. It can be completed by the effects of a crack lips pressure introduced by the second term. The last part allows the generalization for the crack

propagation ensuring the non-path dependence when the crack tip moves inside the integral domain. The  $J^{3D}$  is interpreted as the integration of the  $J_{Bui}^{3D}$ -integral along the crack front line:

$$J^{3D} = \lim_{A(T) \rightarrow 0} \left( \int_{cfl} J_{Bui}^{3D} \cdot dl \right) \quad (3)$$

$J_{Bui}^{3D}$  denotes the Bui's integral for three-dimensional crack problem. The  $J^{3D}$ -integral can be used for the evaluation of the average value of the energy release rate  $\tilde{G}$  along the crack front line:

$$\tilde{G} = \frac{J^{3D}}{\int_{cfl} dl} \quad (4)$$

To implement this integral in a finite element software, it is easier to consider a volume domain integral [5, 6].

$$G_{\theta}^{3D} = - \int_V (W \cdot \theta_{k,k} - (\sigma_{ij} \cdot u_{i,k}) \cdot \theta_{k,j}) \cdot dS - \int_{S_{CF+} + S_{CF-}} \sigma_{ij} \cdot u_{i,k} \cdot n_j \cdot \theta_k \cdot dS - \int_{V_{in}} (W_{,k} - \sigma_{ij} \cdot (\varepsilon_{ij})_{,k}) \cdot \theta_k \cdot dV \quad (5)$$

The  $G_{\theta}^{3D}$  allows to compute the distribution of energy release rate along the crack front line per slim thickness. We can establish a relation between  $G_{\theta}^{3D}$  and  $J_{Bui}^{3D}$  as follows:

$$J_{Bui}^{3D} = G_{\theta}^{3D}(\omega) \quad (6)$$

The average energy release rate per every slim thickness  $G_{\theta}^{3D}(\omega)$  can be equal to energy release rate per each plan  $J_{Bui}^{3D}$  in studied solid.

### 3 Finite element analysis

The numerical implementation is based on a Double Cantilever Beam (DCB) loaded in an open mode. The DCB specimen was adapted by Dubois et al to wood material. In this part, we recall the wood specimen dimensions of DCB device. Fig. 4 presents the dimensions in millimetres of the initial wood specimen. In this wood specimen, two holes are machined in order to fixe the Arcan device. This allows a loading fixations in tensile mode. The geometry of the DCB specimen has been optimized by using a finite element computation. This specimen is adapted to obtain a stable crack growth rate during propagation for tensile mode.

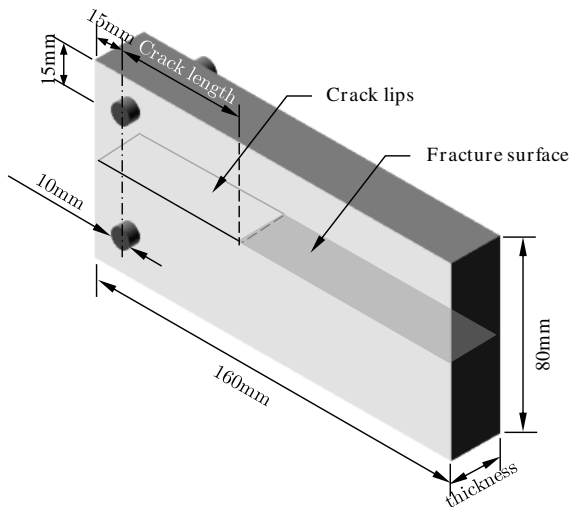


Fig. 4: DCB specimen

The finite element computation is realized for an elastic isotropic behavior. Wood material used is Douglas fire and has the following elastic characteristics  $E=14100$  MPa, Poisson ratio  $\nu=0.3$ . The initial crack length is fixed to 60mm.

In the follows, the results of numerical study are exposed. In order to observe the effect of thicknesses on the DCB specimen we plot the evolution of the energy release rate as function of the crack front line.

The description of  $\theta$  field around the crack front line is shown in Fig. 4. The  $\theta$  field is equal to zero on outside surface, and 1 on inside surface.

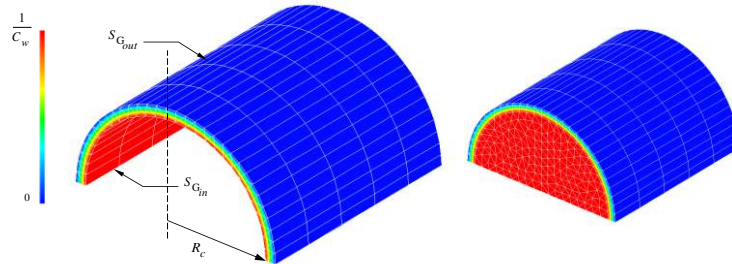


Fig. 5: Description of  $\theta$  field around the crack front line

Let us analyse now the influence of the thickness on the energy release rate. As shown in Fig. 6, the support of the theta field is supported by a cylindrical plate.

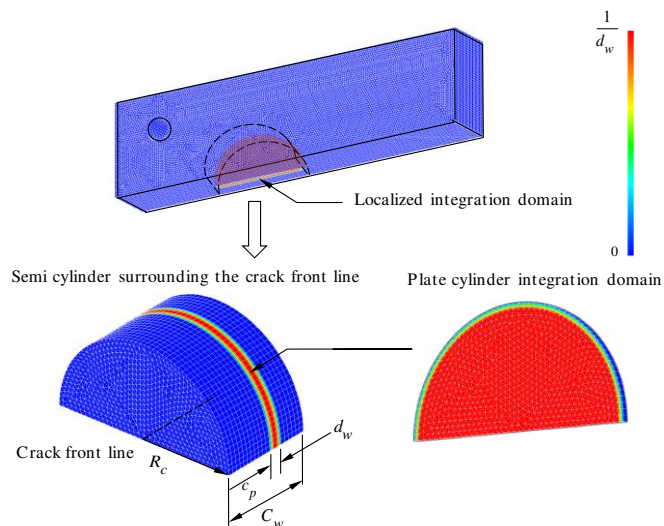


Fig. 6: Integral cylindrical plate domain around the crack tip

The average energy release rate is calculated along  $d_w$  (Fig. 6). Fig. 7 shows us the energy release rate distribution along the crack front line versus the thickness.

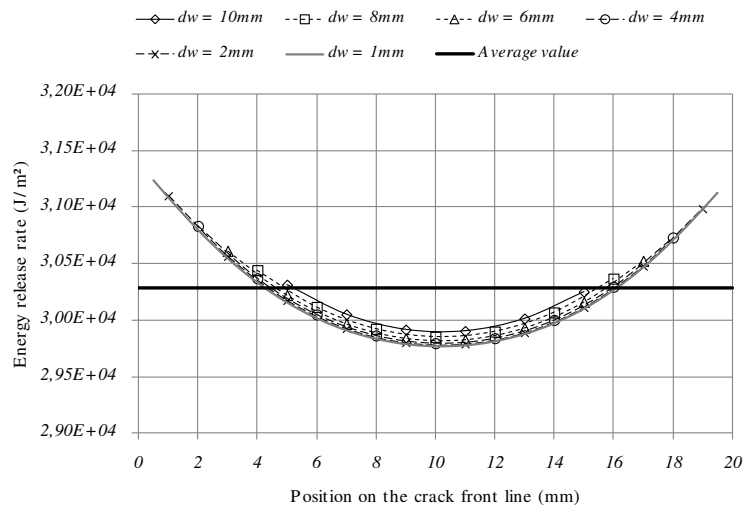


Fig. 7: Energy release rate distribution along the crack front line

Numerical approach allowed us to evaluate the distribution of energy release rate thanks to  $G_{\theta}^{3D}$ -integral in tensile mode.

## 4 Conclusions and outlooks

This paper deals with a new formulation of the J-integral for the study of fracture process in element by taking into account three dimensional effects. A theoretical and numerical approach are established. At this stages, more numerical investigations are necessary. In this case, the  $J$  integral is generalized to three-dimensional approach using the Lagrangian variation.

Also, it will be necessary to extend the  $J^{3D}$  integral to a mixed mode loading case [7] in order to introduce hydrological and thermal effects for three dimensional problems [8].

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