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## Dynamics under location uncertainty and other energy-related stochastic subgrid schemes

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Bertrand Chapron

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# Dynamics under location uncertainty and other energy-related stochastic subgrid schemes

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L. Li, G. Jouan, P. Derian,  
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# Motivations

- More rigorously identified subgrid dynamics effects
- Quantification of modeling errors (UQ)



Ensemble forecasts and data assimilation

# Contents

- Models under location uncertainty (LU)
- Some parameterization of the models under location uncertainty
- A new energy-budget-based stochastic scheme: WaveHyperv
- Numerical comparisons



# Part I

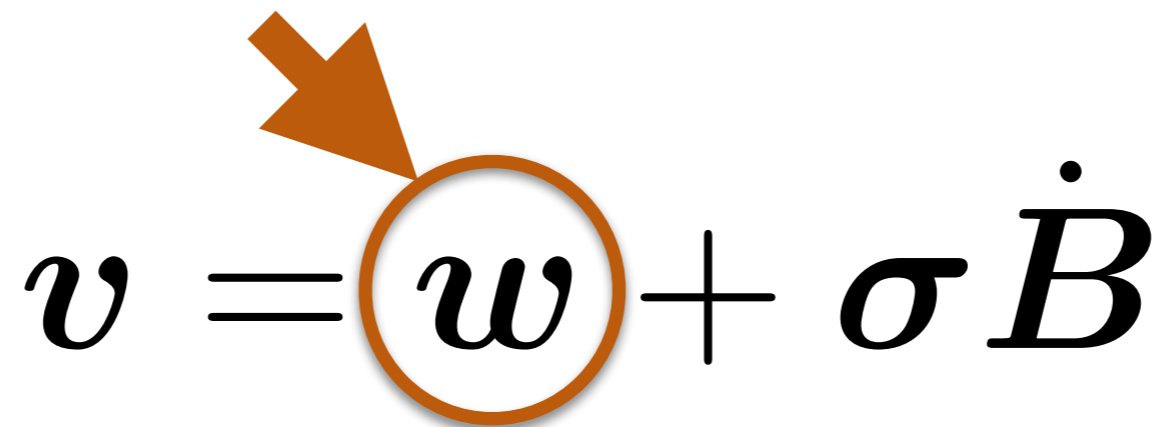
## Models under location uncertainty (LU)

LU : Adding  
random velocity

$$\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$$

# LU : Adding random velocity

Resolved  
large scales

$$\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$$
The diagram shows the equation  $\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$ . The variable  $\boldsymbol{w}$  is circled in orange, and an orange arrow points from the text "Resolved large scales" to the circle.

# LU : Adding random velocity

Resolved  
large scales

White-in-time  
small scales

$$\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$$

Large scales:

$w$

Small scales:

$\sigma \dot{B}$

Variance  
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

# LU : Adding random velocity

Resolved  
large scales

White-in-time  
small scales

$$v = w + \sigma \dot{B}$$

# LU : Adding random velocity

Large scales:

$w$

Small scales:

$\sigma \dot{B}$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Resolved large scales

White-in-time small scales

$$v = w + \sigma \dot{B}$$

References : Mikulevicius & Rozovskii, 2004  
Flandoli, 2011

**Memin**, 2014  
Resseguier et al. 2017 a, b, c  
Cai et al. 2017  
Chapron et al. 2018  
Yang & Memin 2019  
Resseguier et al. 2019 a,b

**Holm**, 2015  
Holm and Tyranowski, 2016  
Arnaudon et al. 2017  
Cotter and al. 2017

Crisan et al., 2017  
Gay-Balmaz & Holm 2017  
Cotter and al. 2018 a, b  
Cotter and al. 2019

# LU : Adding random velocity

Large scales:

$w$

Small scales:

$\sigma \dot{B}$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Resolved large scales

White-in-time small scales

$$v = w + \sigma \dot{B}$$

## LU

**Memin**, 2014  
 Resseguier et al. 2017 a, b, c  
 Cai et al. 2017  
 Chapron et al. 2018  
 Yang & Memin 2019  
 Resseguier et al. 2019 a,b

## SALT

**Holm**, 2015  
 Holm and Tyranowski, 2016  
 Arnaudon et al. 2017  
 Cotter and al. 2017  
 Crisan et al., 2017  
 Gay-Balmaz & Holm 2017  
 Cotter and al. 2018 a, b  
 Cotter and al. 2019

References : Mikulevicius & Rozovskii, 2004  
 Flandoli, 2011

Large scales:

$w$

Small scales:

$\sigma \dot{B}$

Variance  
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

# Advection of tracer $\Theta$

$$\frac{D\Theta}{Dt} = 0$$



Large scales:

$w$

Small scales:

$\sigma \dot{B}$

Variance  
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

# Advection of tracer $\Theta$

Large scales:

$w$

Small scales:

$\sigma \dot{B}$

Variance  
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

# Advection of tracer $\Theta$

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$

# Advection of tracer $\Theta$

Large scales:

$w$

Small scales:

$\sigma \dot{B}$

Variance  
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$

# Advection of tracer $\Theta$

Large scales:

$w$

Small scales:

$\sigma \dot{B}$

Variance  
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

# Advection of tracer $\Theta$

Large scales:

$w$

Small scales:

$\sigma \dot{B}$

Variance  
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

Drift correction

# Advection of tracer $\Theta$

Large scales:

$w$

Small scales:

$\sigma \dot{B}$

Variance  
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

Multiplicative  
random  
forcing

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

Drift correction

# Advection of tracer $\Theta$

Large scales:

$$w$$

Small scales:

$$\sigma \dot{B}$$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

Multiplicative random forcing

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$

The equation is presented within two colored boxes: a cyan box labeled "Advection" containing the left-hand side terms, and a green box labeled "Diffusion" containing the right-hand side term. An orange arrow points to  $w^*$  with the label "Drift correction". A purple arrow points to  $\sigma \dot{B} \cdot \nabla \Theta$  with the label "Multiplicative random forcing".

Drift correction

# Advection of tracer $\Theta$

Large scales:

$$w$$

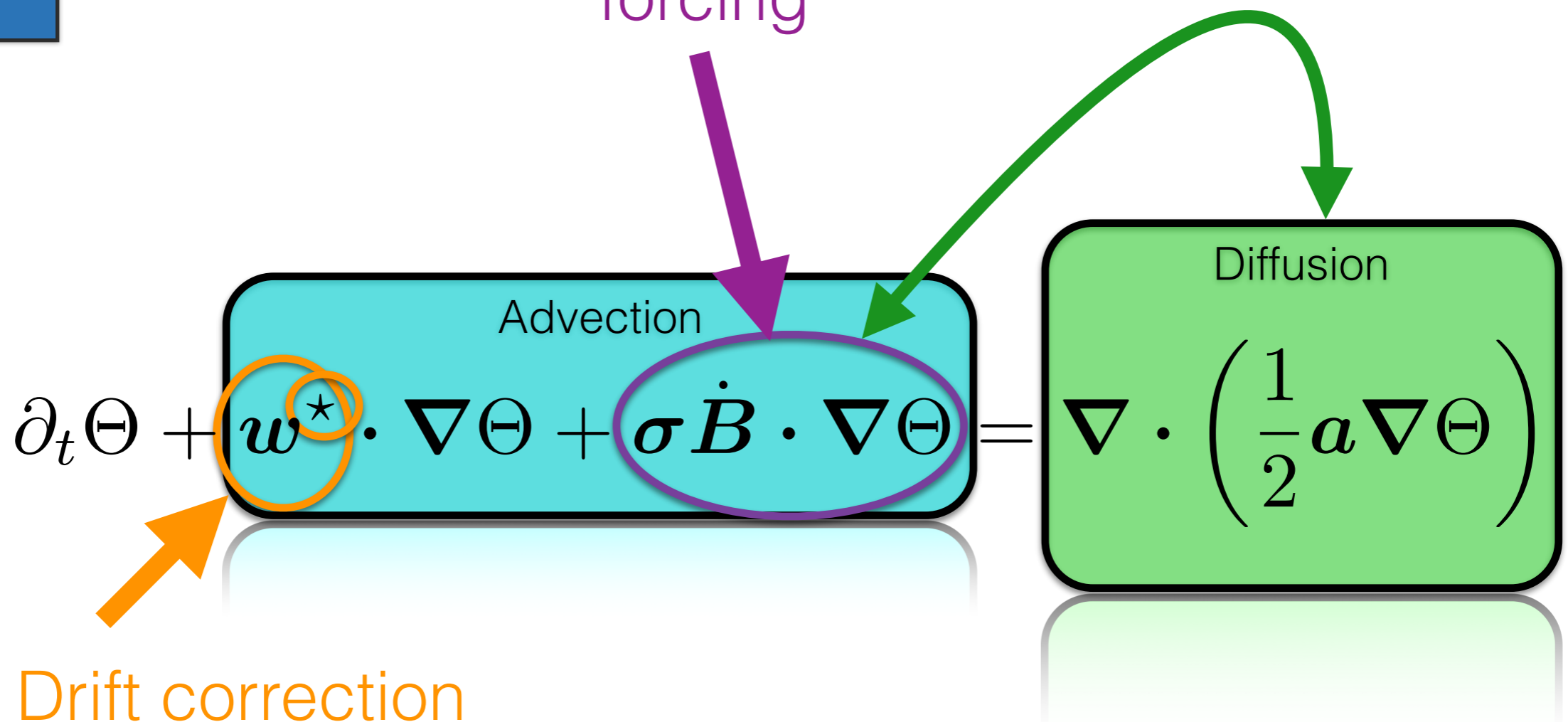
Small scales:

$$\sigma \dot{B}$$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

Multiplicative random forcing





# Advection of tracer $\Theta$

Large scales:

$w$

Small scales:

$\sigma \dot{B}$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

Multiplicative random forcing

Balanced energy exchanges

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left( \frac{1}{2} a \nabla \Theta \right)$$

Drift correction

# Part II

## Some parameterization of LU models

# Part II

Some parameterization  
of LU models

$\sigma \equiv ?$

# Spectral model

(homogeneous and stationary  $\sigma \dot{B}$ )

Large scales:

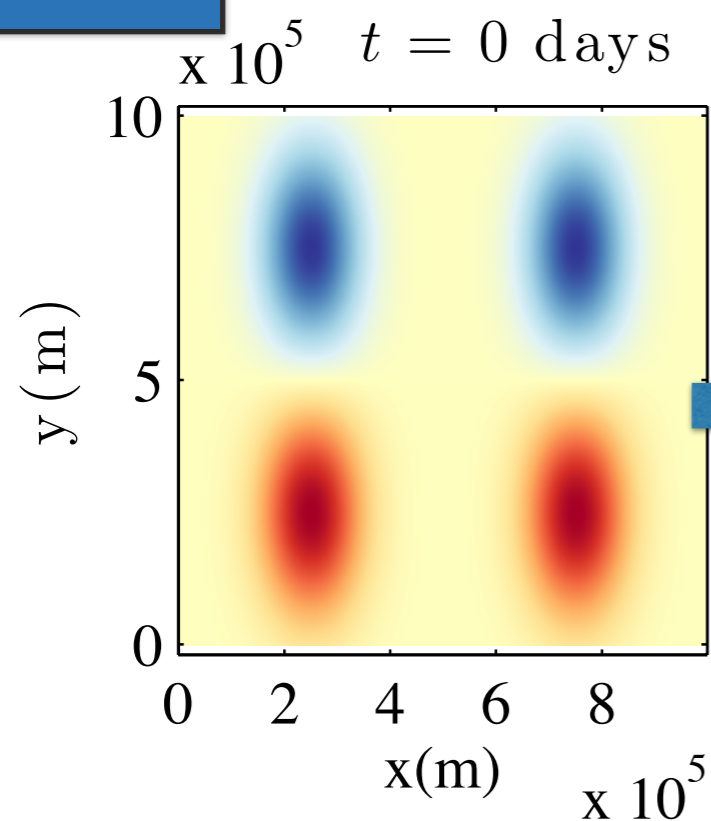
$$w$$

Small scales:

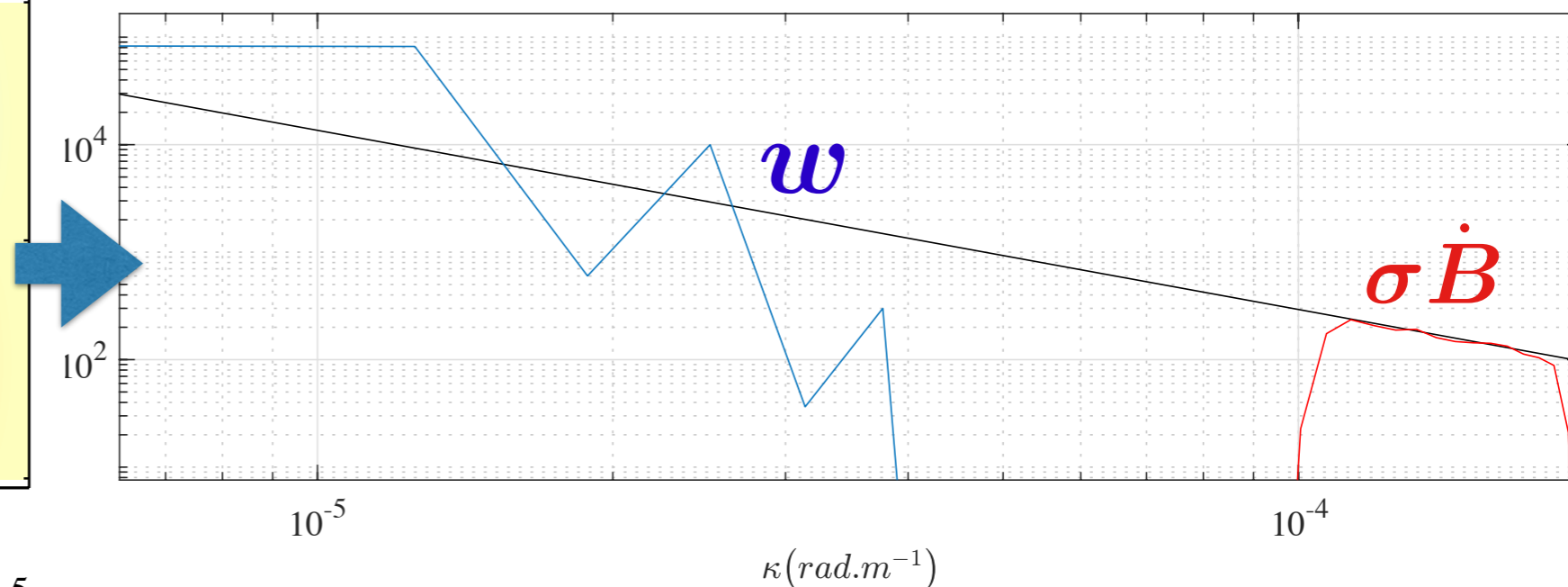
$$\sigma \dot{B}$$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$



## KE Spectrum



Code online

# Spectral model

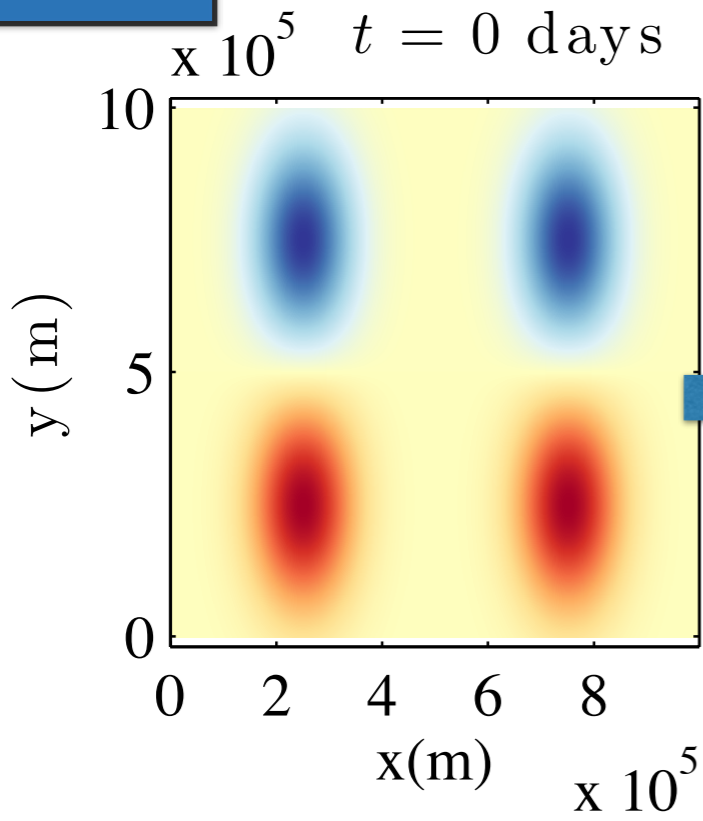
(homogeneous and stationary  $\sigma \dot{B}$ )

Large scales:  
 $w$

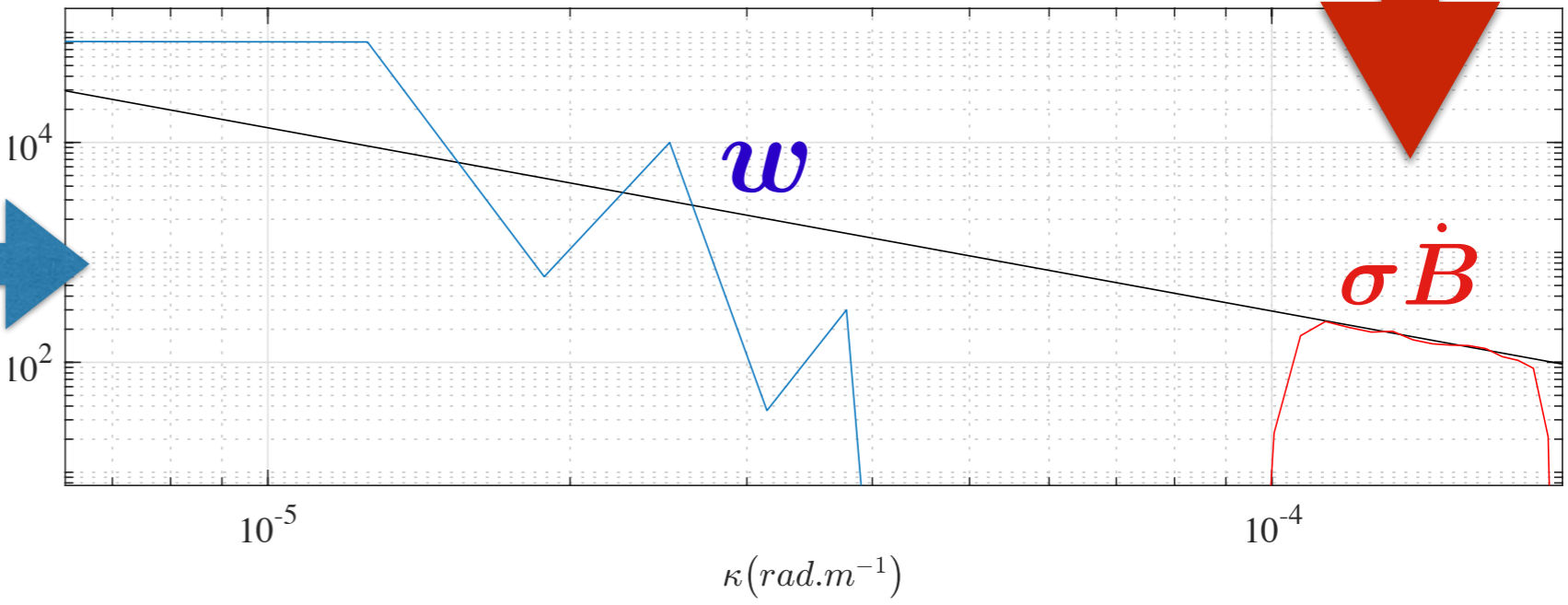
Small scales:  
 $\sigma \dot{B}$

Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

Fixed spectrum at small scales



KE Spectrum



Code online

Reference:  
Resseguier, Memin & Chapron 2017b

# Spectral model

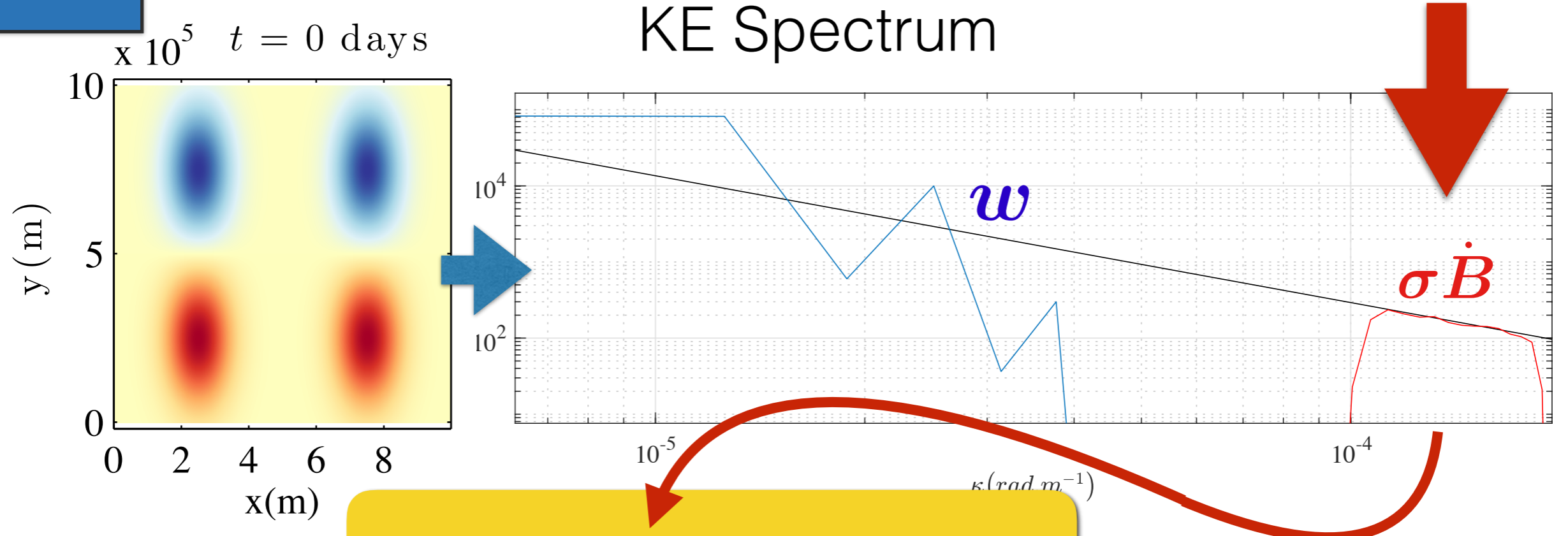
(homogeneous and stationary  $\sigma \dot{B}$ )

Large scales:  
 $w$

Small scales:  
 $\sigma \dot{B}$

Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$

Fixed spectrum at small scales



$\sigma \dot{B} = (\text{filter}) * (\text{white noise})$

Code online

Reference:  
 Resseguier, Memin & Chapron 2017b

# Absolute Diffusivity Spectral Density

MU ADSD

Large scales:

$w$

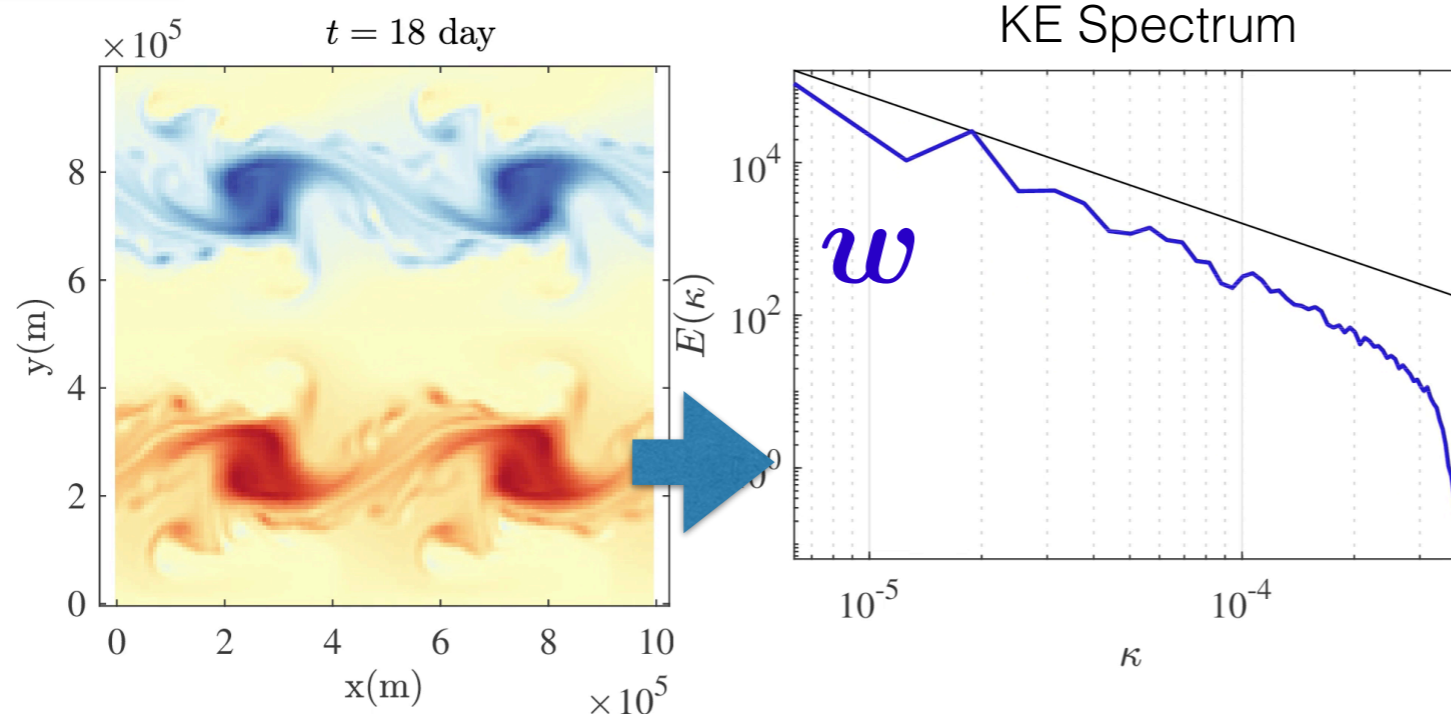
Small scales:

$\sigma \dot{B}$

Variance  
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

(homogeneous but non-stationary and tuning-free  $\sigma \dot{B}$ )



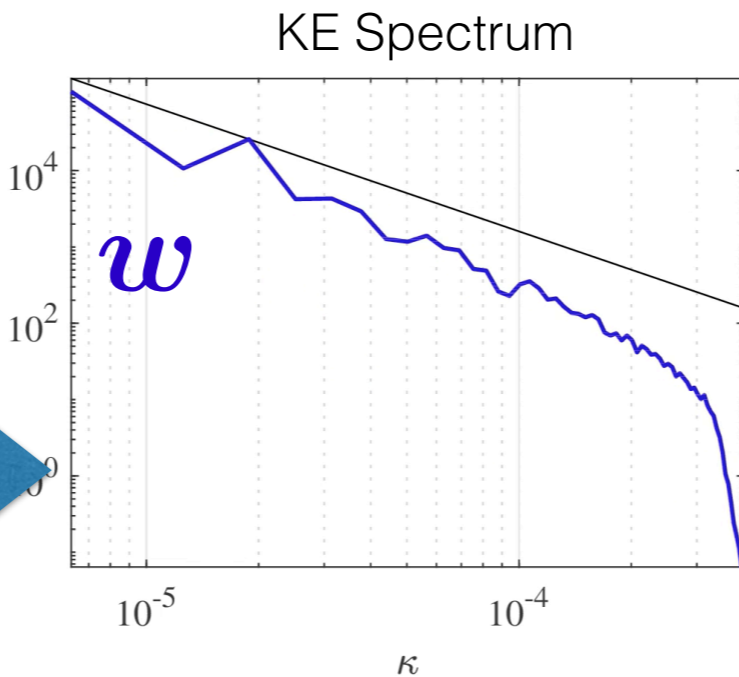
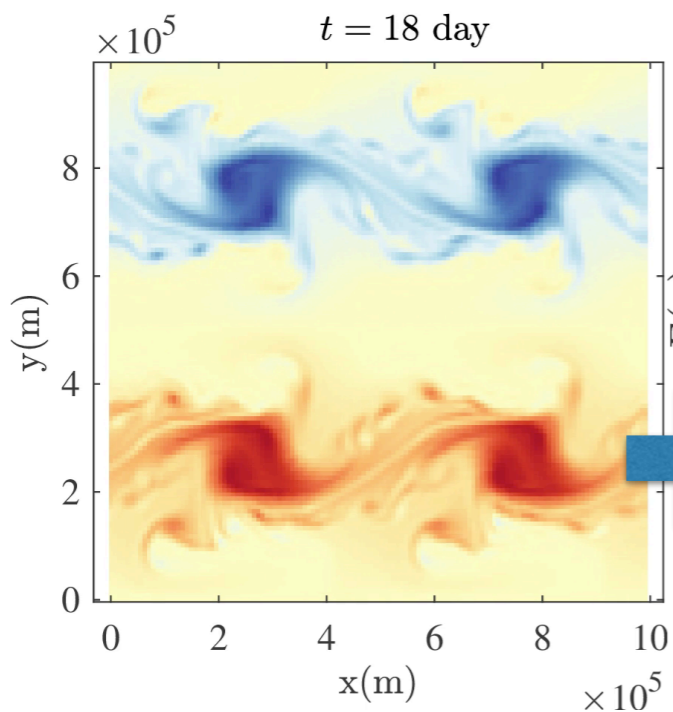
Reference:  
*Resseguier, Pan & Fox-Kemper 2019a*

# Absolute Diffusivity Spectral Density

(homogeneous but non-stationary and tuning-free  $\sigma \dot{B}$ )

Large scales:  
 $w$   
 Small scales:  
 $\sigma \dot{B}$   
 Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$

Absolute Diffusivity Spectral Density  $A(\kappa) = E(\kappa)\tau(\kappa)$





# Absolute Diffusivity Spectral Density

MU ADSD

Large scales:

$w$

Small scales:

$\sigma \dot{B}$

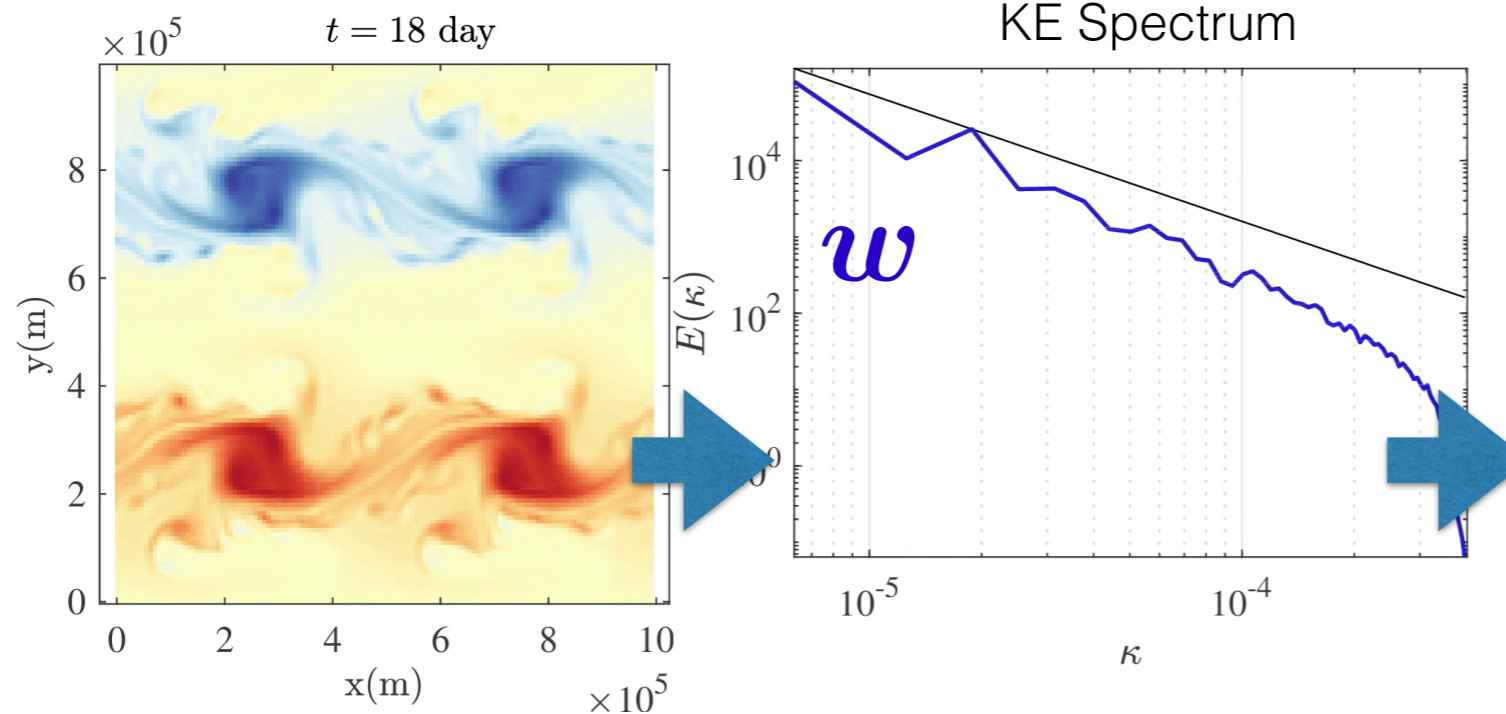
Variance  
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

(homogeneous but non-stationary and tuning-free  $\sigma \dot{B}$ )

Absolute Diffusivity  
Spectral Density

$$A(\kappa) = E(\kappa)\tau(\kappa)$$



Reference:  
*Resseguier, Pan & Fox-Kemper 2019a*

# Absolute Diffusivity Spectral Density

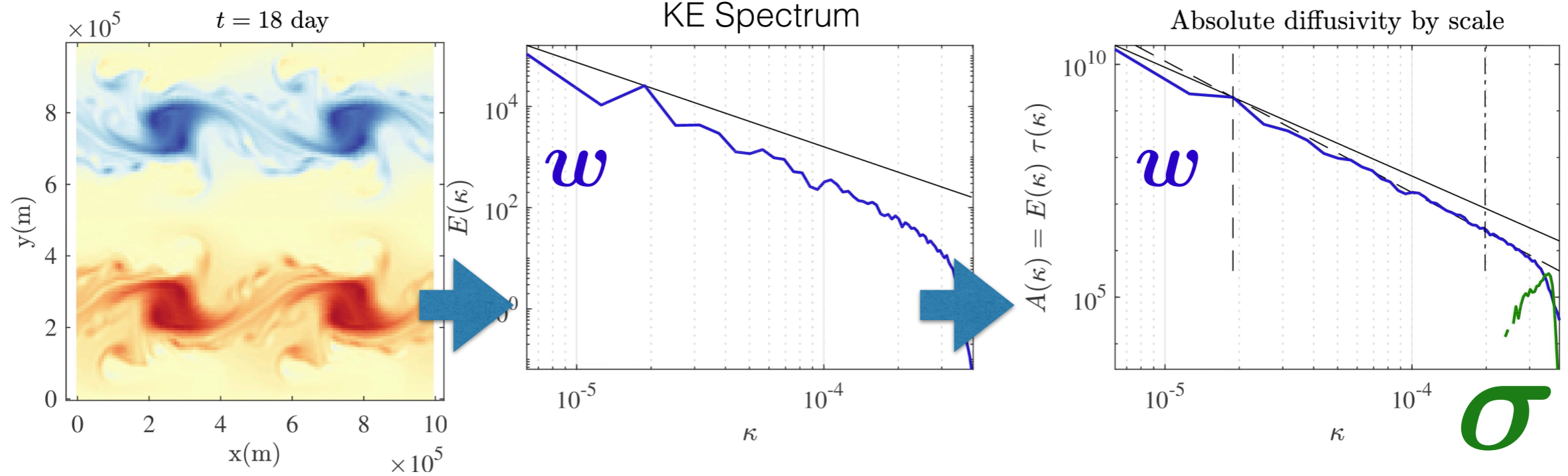
Large scales:  
 $w$

Small scales:  
 $\sigma \dot{B}$

Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

(homogeneous but non-stationary and tuning-free  $\sigma \dot{B}$ )

Absolute Diffusivity Spectral Density  $A(\kappa) = E(\kappa)\tau(\kappa)$



Reference:  
*Resseguier, Pan & Fox-Kemper 2019a*

# Absolute Diffusivity Spectral Density

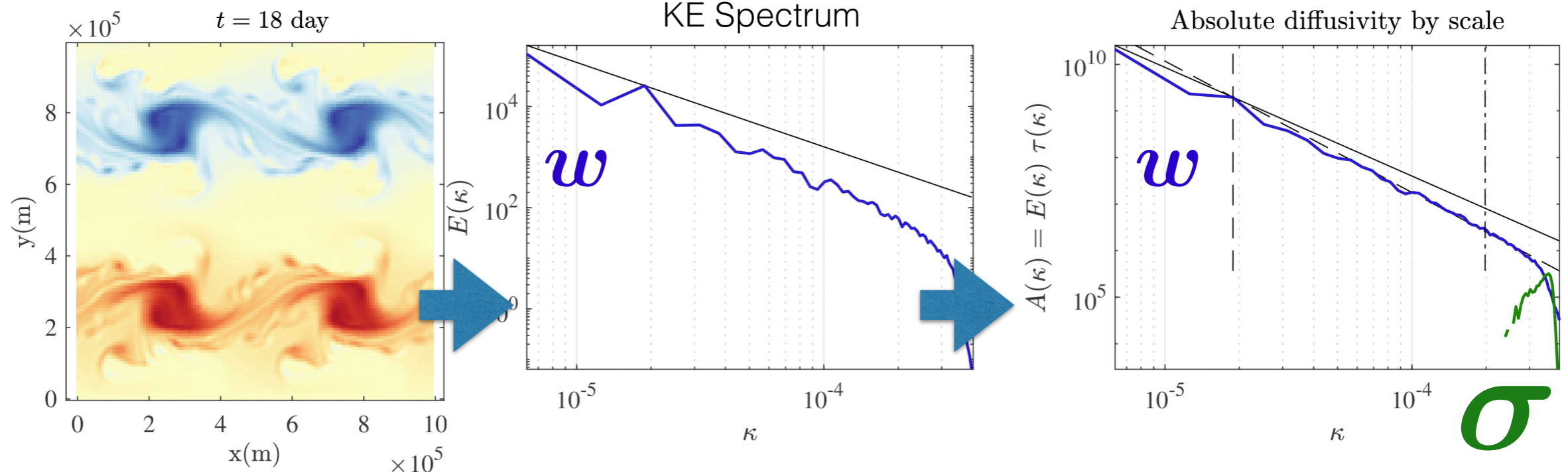
Large scales:  
 $w$

Small scales:  
 $\sigma \dot{B}$

Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

(homogeneous but non-stationary and tuning-free  $\sigma \dot{B}$ )

Absolute Diffusivity Spectral Density  $A(\kappa) = E(\kappa)\tau(\kappa)$



Reference:  
*Resseguier, Pan & Fox-Kemper 2019a*

# Absolute Diffusivity Spectral Density

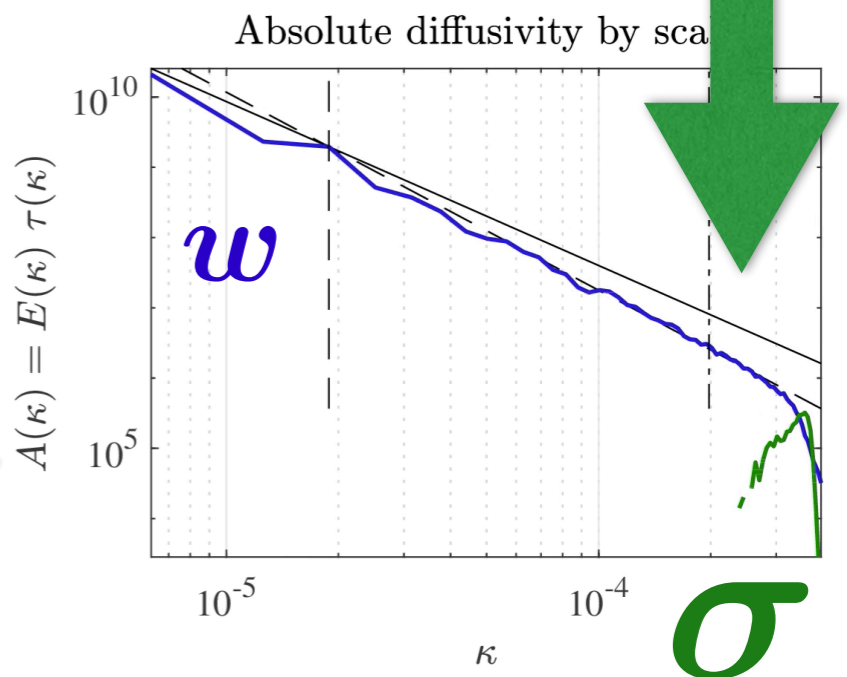
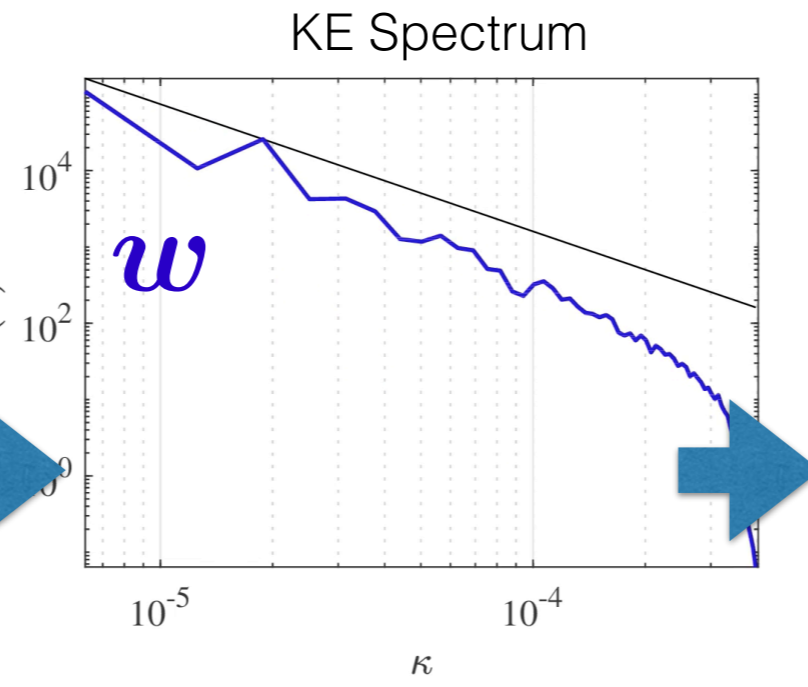
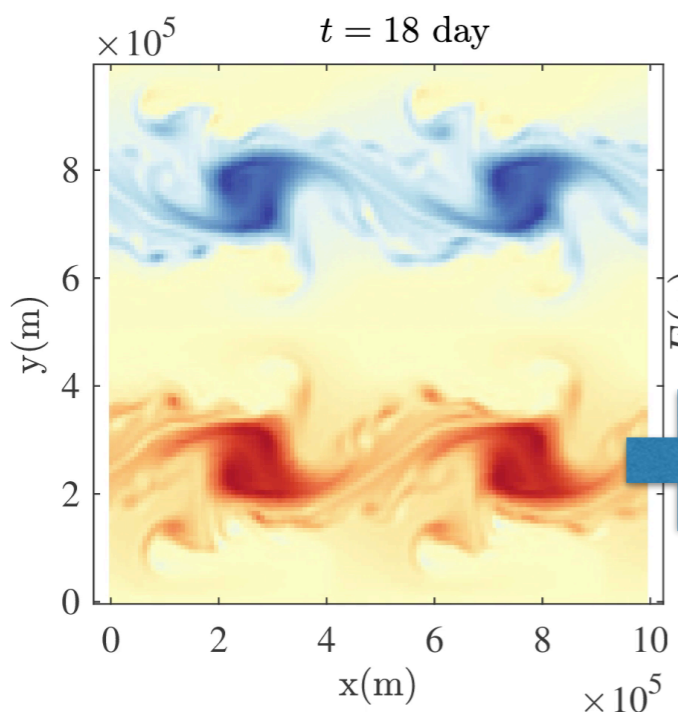
MU ADSD

Large scales:  $w$   
 Small scales:  $\sigma \dot{B}$   
 Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

(homogeneous but non-stationary and tuning-free  $\sigma \dot{B}$ )

Absolute Diffusivity Spectral Density  $A(\kappa) = E(\kappa)\tau(\kappa)$

Residual non-stationary ADSD



Reference: Resseguier, Pan & Fox-Kemper 2019a



# Absolute Diffusivity Spectral Density

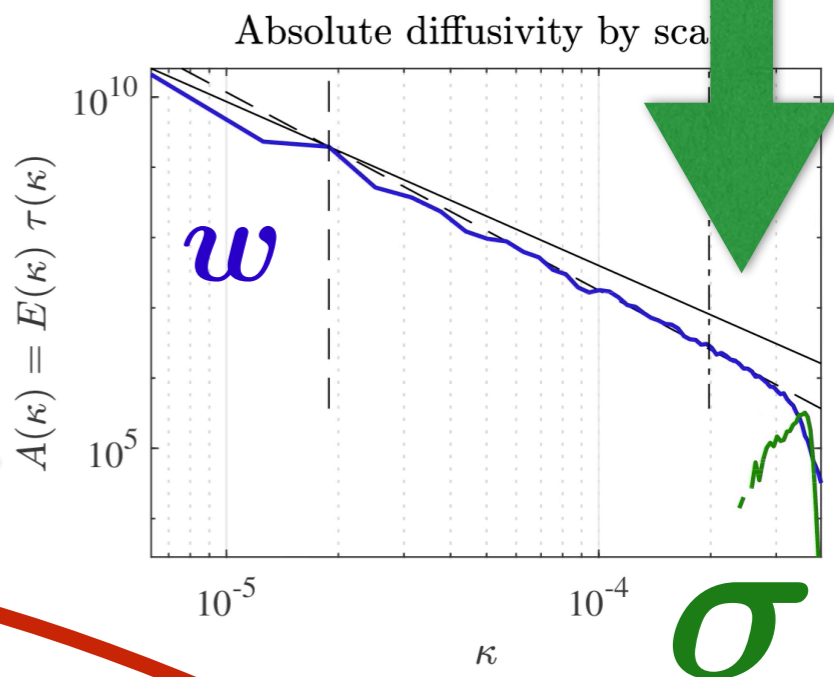
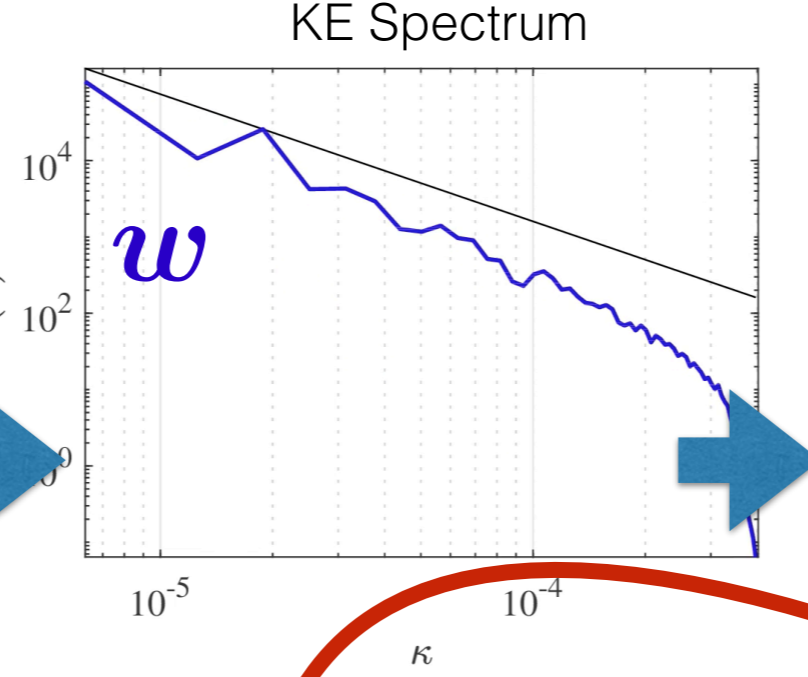
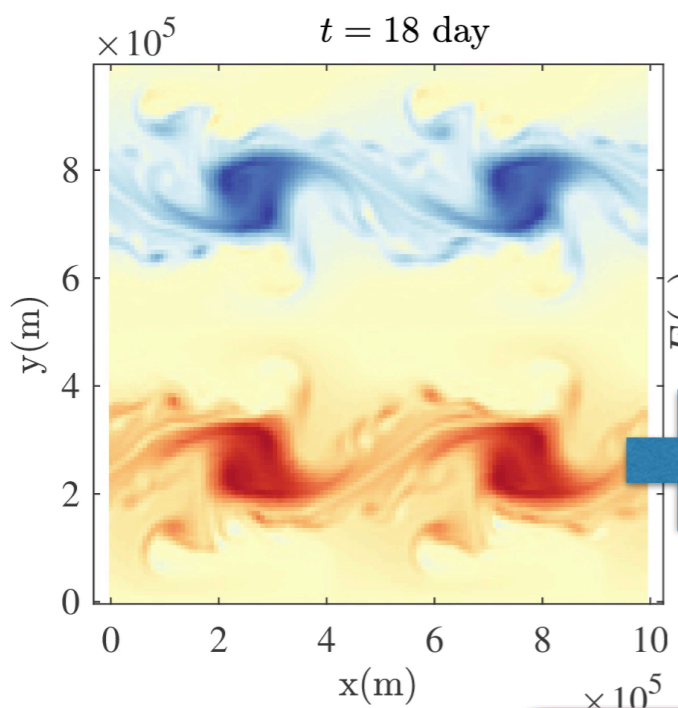
MU ADSD

Large scales:  
 $w$   
 Small scales:  
 $\sigma \dot{B}$   
 Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

(homogeneous but non-stationary and tuning-free  $\sigma \dot{B}$ )

Absolute Diffusivity Spectral Density  $A(\kappa) = E(\kappa)\tau(\kappa)$

Residual non-stationary ADSD



$\sigma \dot{B} = (\text{filter}) * (\text{white noise})$

Reference: Resseguier, Pan & Fox-Kemper 2019a

# Random switching of points

(heterogeneous and non-stationary  $\sigma \dot{B}$ )

Large scales:

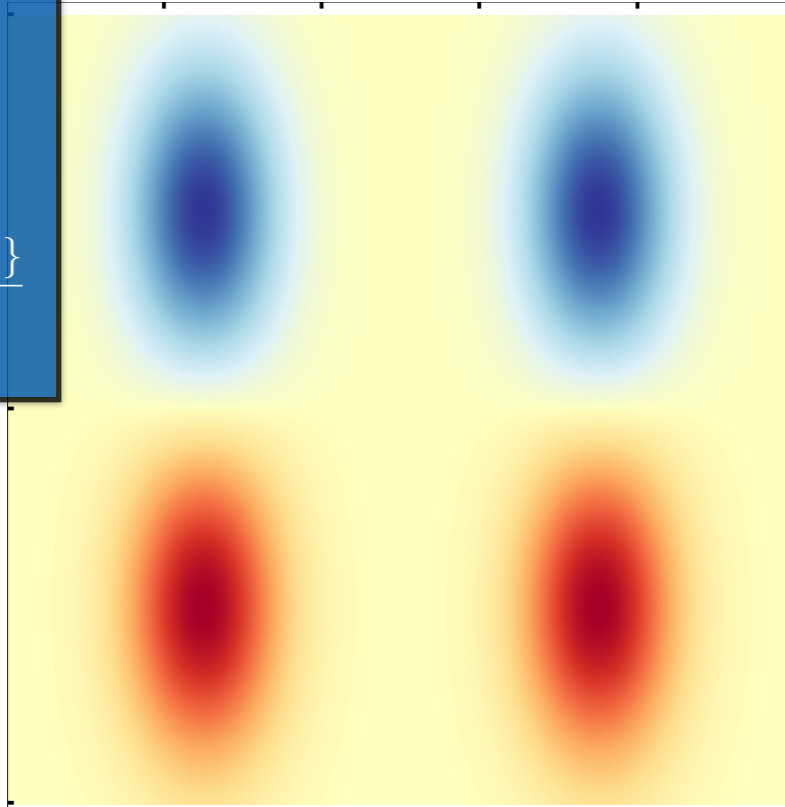
$w$

Small scales:

$\sigma \dot{B}$

Variance  
tensor:

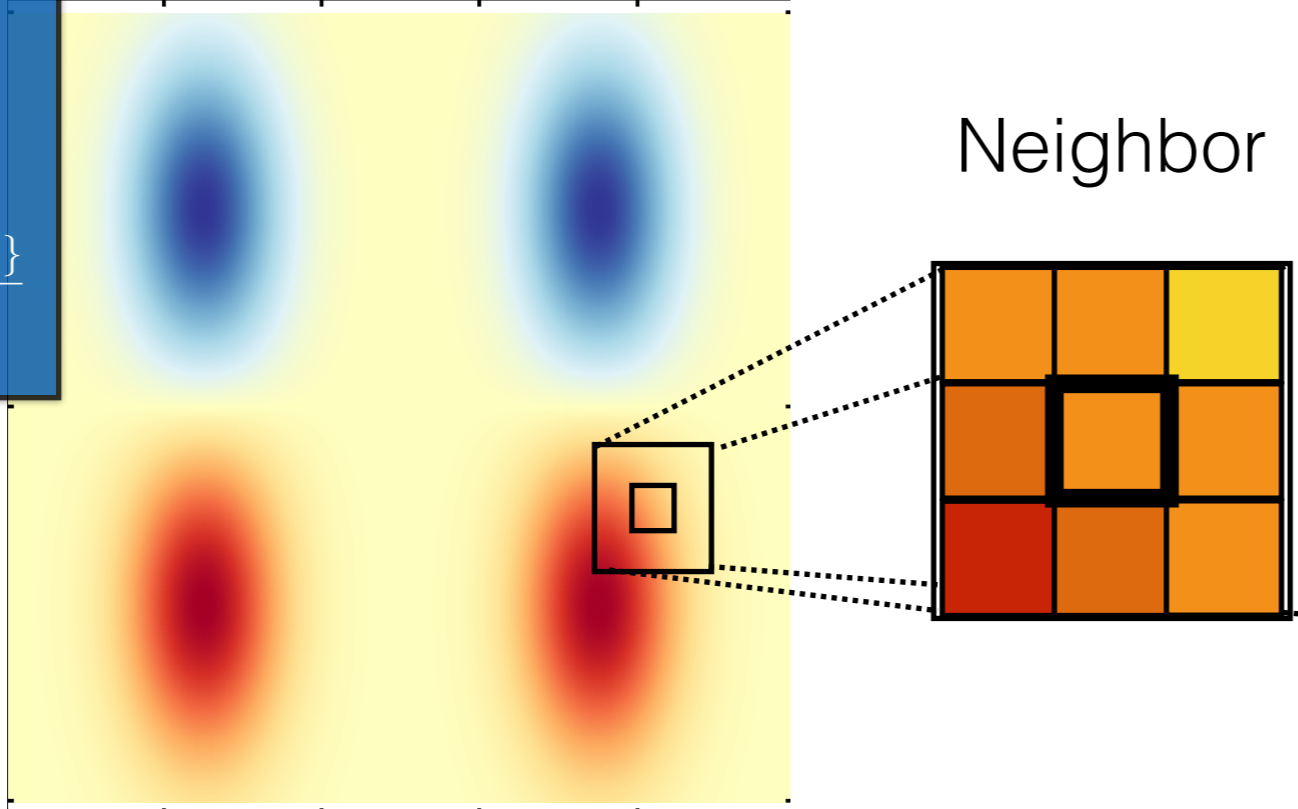
$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$



# Random switching of points

(heterogeneous and non-stationary  $\sigma \dot{B}$ )

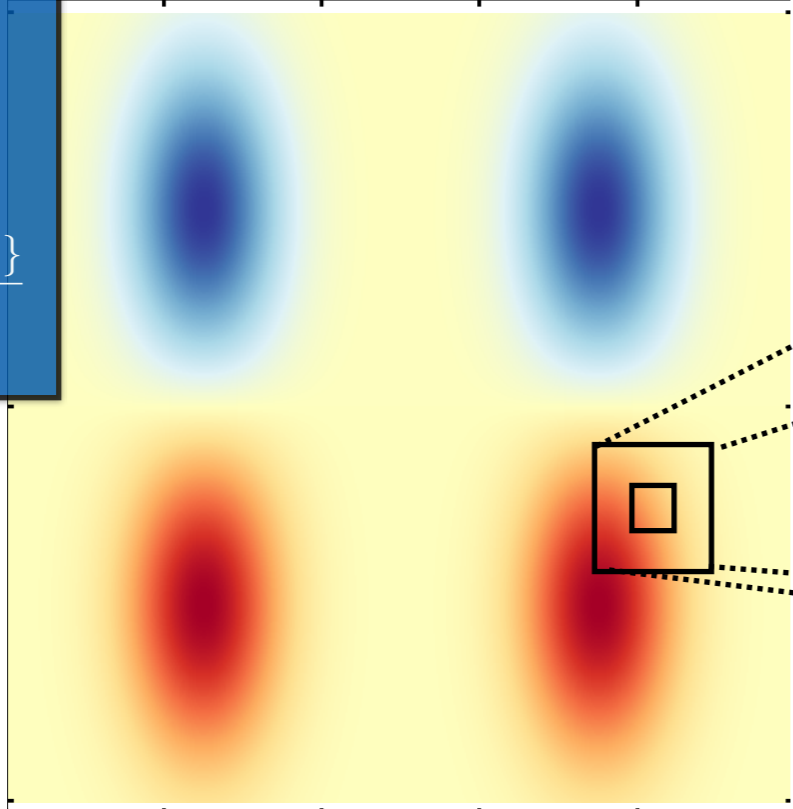
Large scales:  
 $w$   
Small scales:  
 $\sigma \dot{B}$   
Variance  
tensor:  
 $a = a(x, x) =$   
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



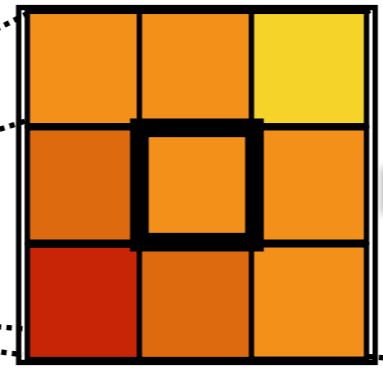
# Random switching of points

(heterogeneous and non-stationary  $\sigma \dot{B}$ )

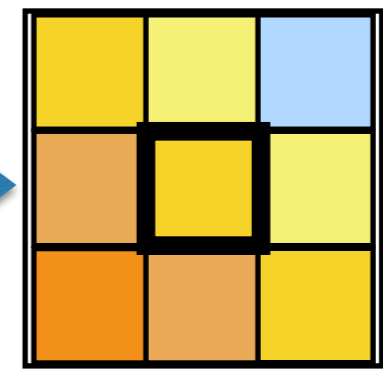
Large scales:  
 $w$   
Small scales:  
 $\sigma \dot{B}$   
Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



Neighbor



Centered neighbor



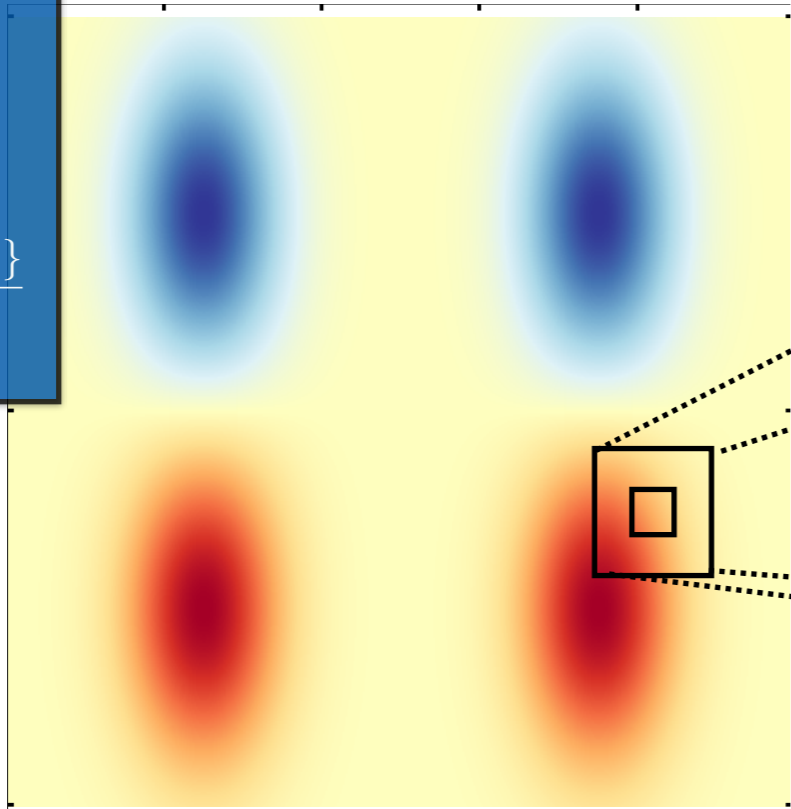


# Random switching of points

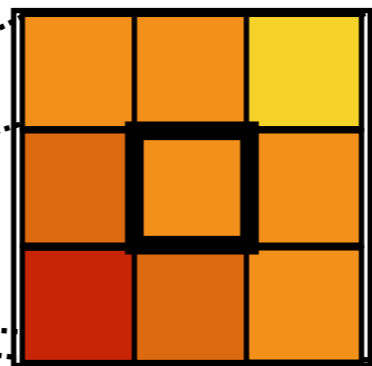
MU SVD

(heterogeneous and non-stationary  $\sigma \dot{B}$ )

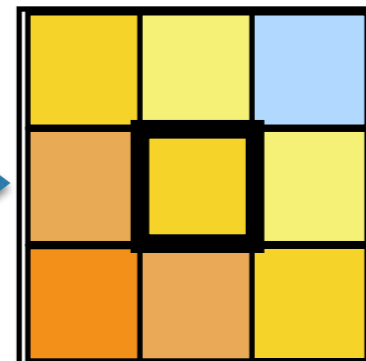
Large scales:  
 $w$   
Small scales:  
 $\sigma \dot{B}$   
Variance  
tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



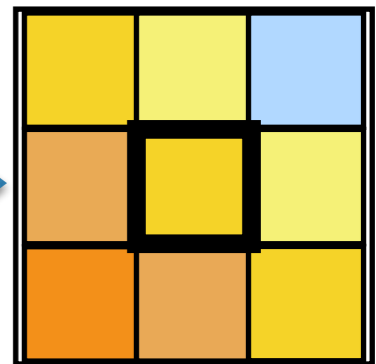
Neighbor



Centered  
neighbor



Random  
selection

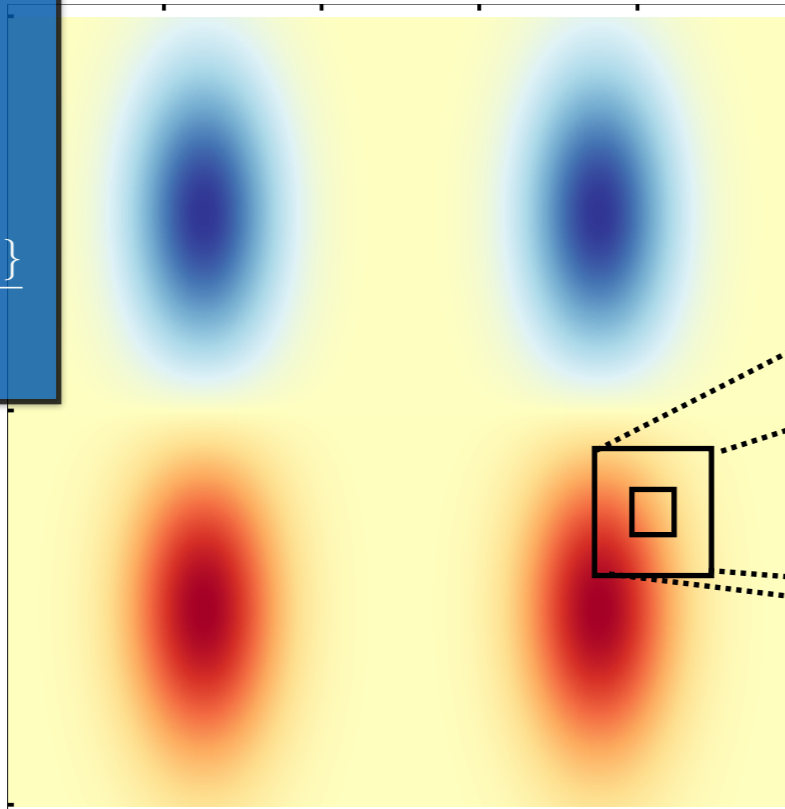


# Random switching of points

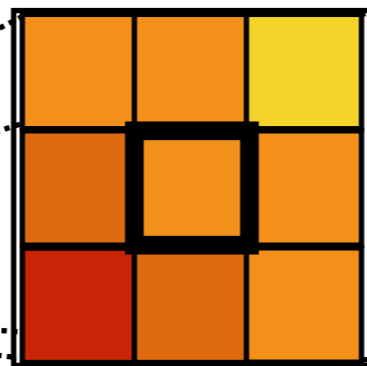
MU SVD

(heterogeneous and non-stationary  $\sigma \dot{B}$ )

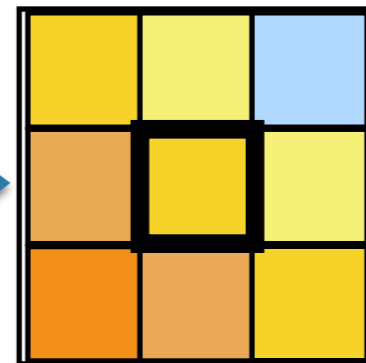
Large scales:  
 $w$   
Small scales:  
 $\sigma \dot{B}$   
Variance  
tensor:  
 $a = a(x, x) =$   
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



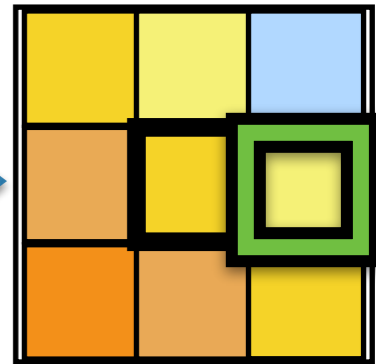
Neighbor



Centered  
neighbor



Random  
selection

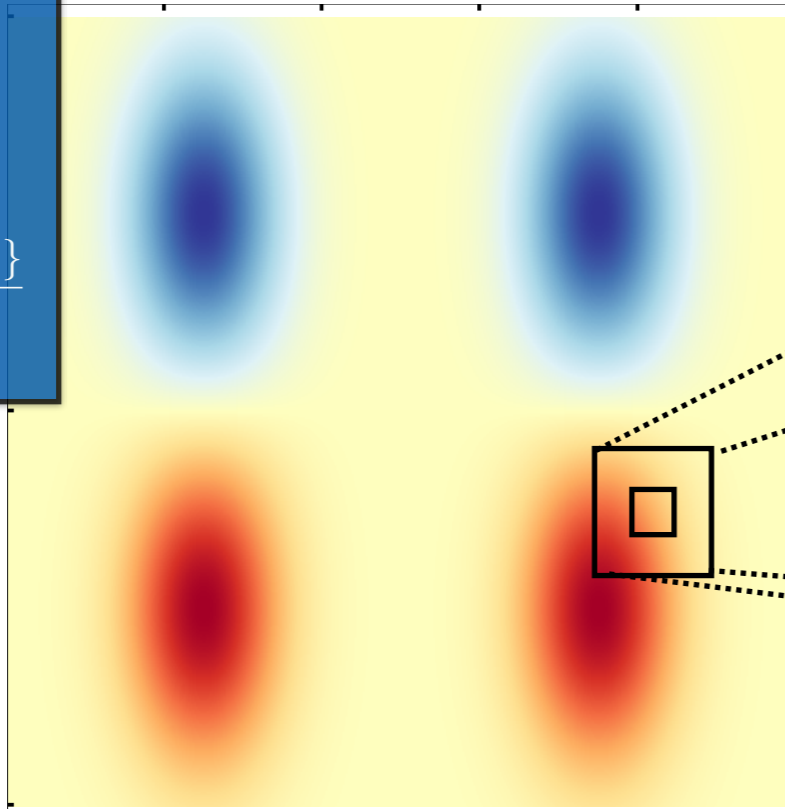


# Random switching of points

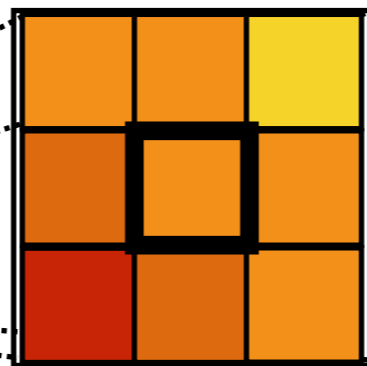
MU SVD

(heterogeneous and non-stationary  $\sigma \dot{B}$ )

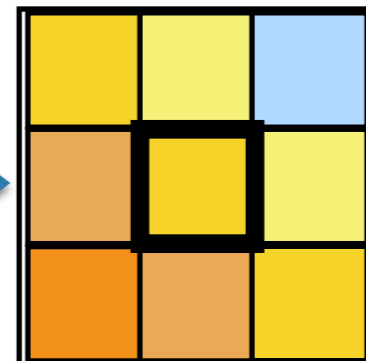
Large scales:  
 $w$   
Small scales:  
 $\sigma \dot{B}$   
Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



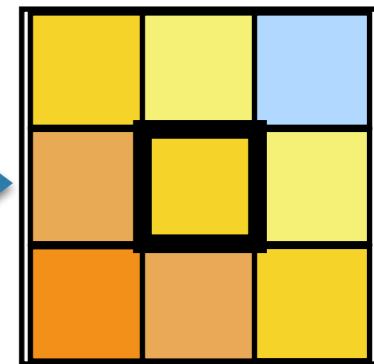
Neighbor



Centered neighbor



Random selection

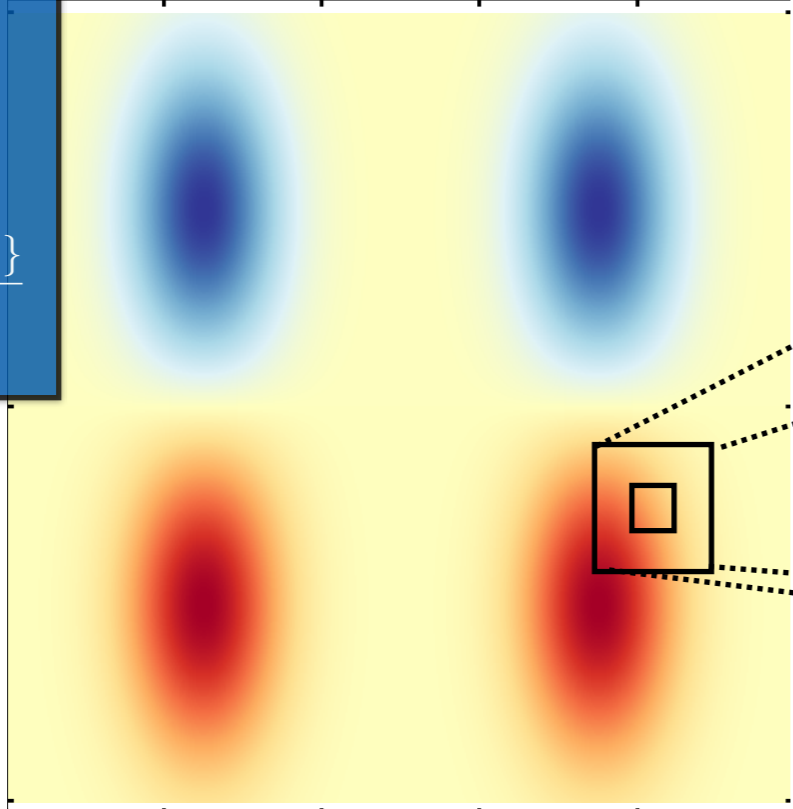


# Random switching of points

(heterogeneous and non-stationary  $\sigma \dot{B}$ )

MU SVD

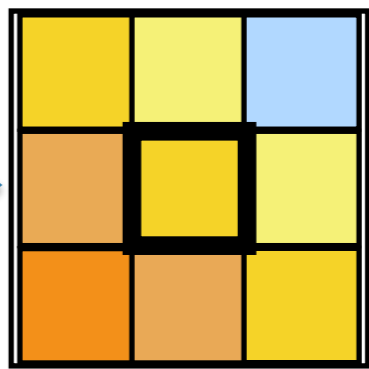
Large scales:  
 $w$   
Small scales:  
 $\sigma \dot{B}$   
Variance  
tensor:  
 $a = a(x, x) =$   
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



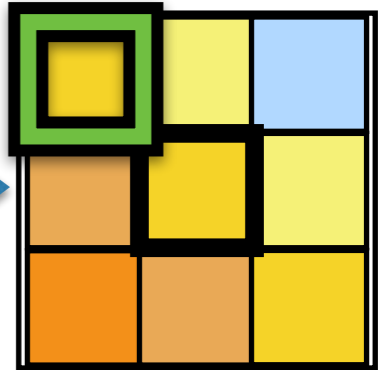
Neighbor



Centered  
neighbor



Random  
selection

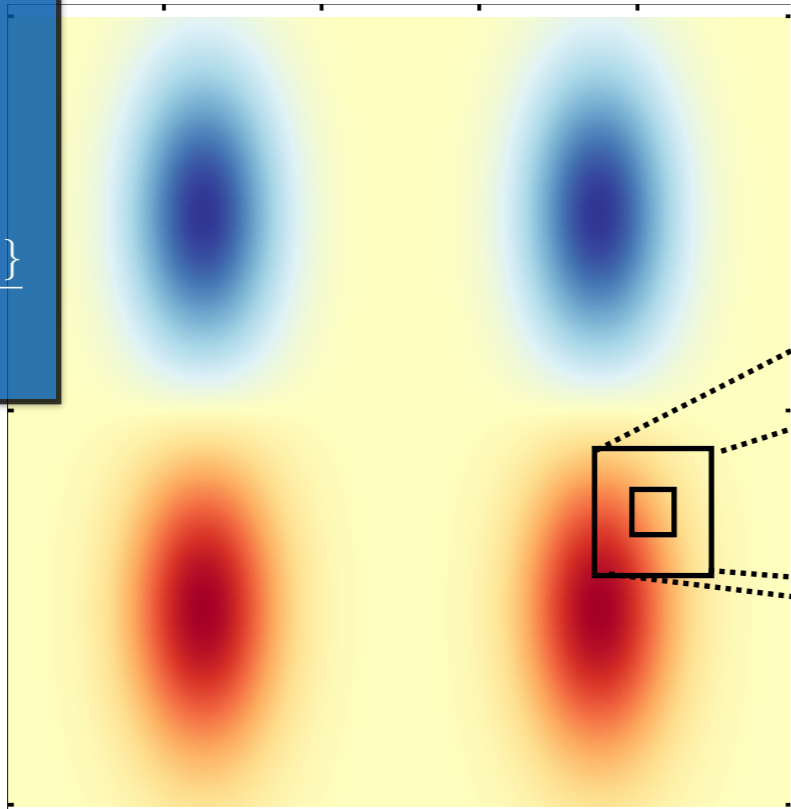


# Random switching of points

MU SVD

(heterogeneous and non-stationary  $\sigma \dot{B}$ )

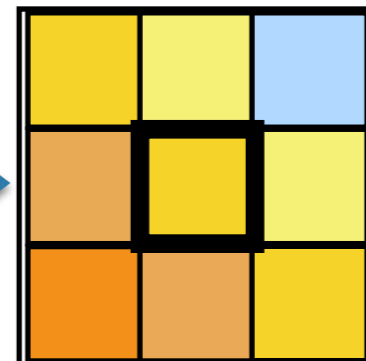
Large scales:  
 $w$   
Small scales:  
 $\sigma \dot{B}$   
Variance  
tensor:  
 $a = a(x, x) =$   
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



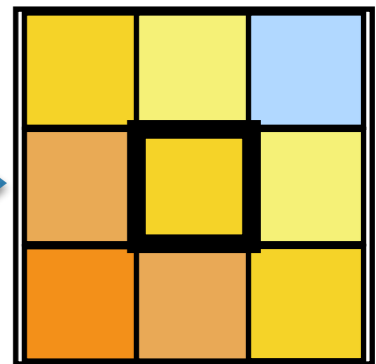
Neighbor



Centered  
neighbor



Random  
selection

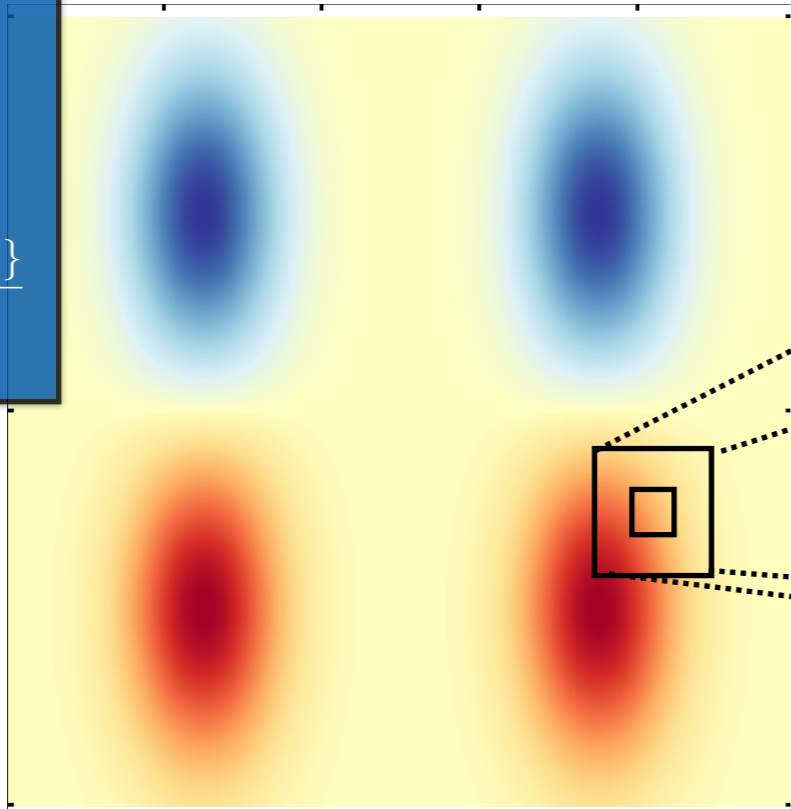


# Random switching of points

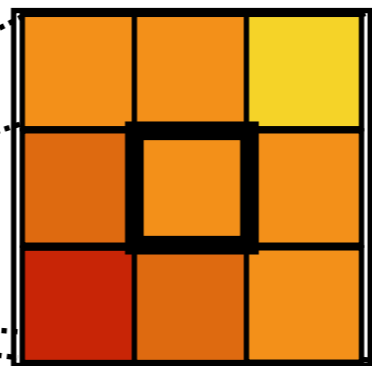
MU SVD

Large scales:  
 $w$   
Small scales:  
 $\sigma \dot{B}$   
Variance  
tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

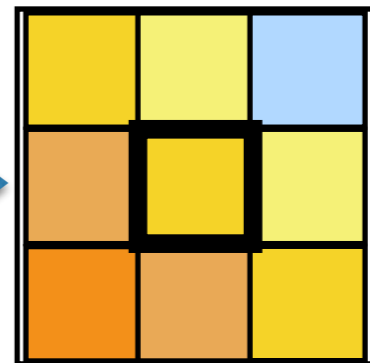
(heterogeneous and non-stationary  $\sigma \dot{B}$ )



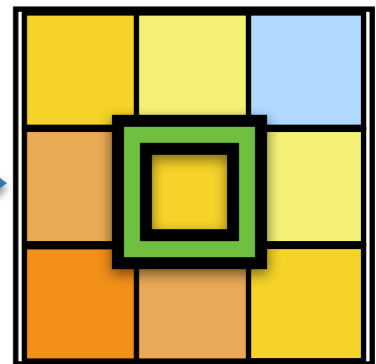
Neighbor



Centered  
neighbor



Random  
selection

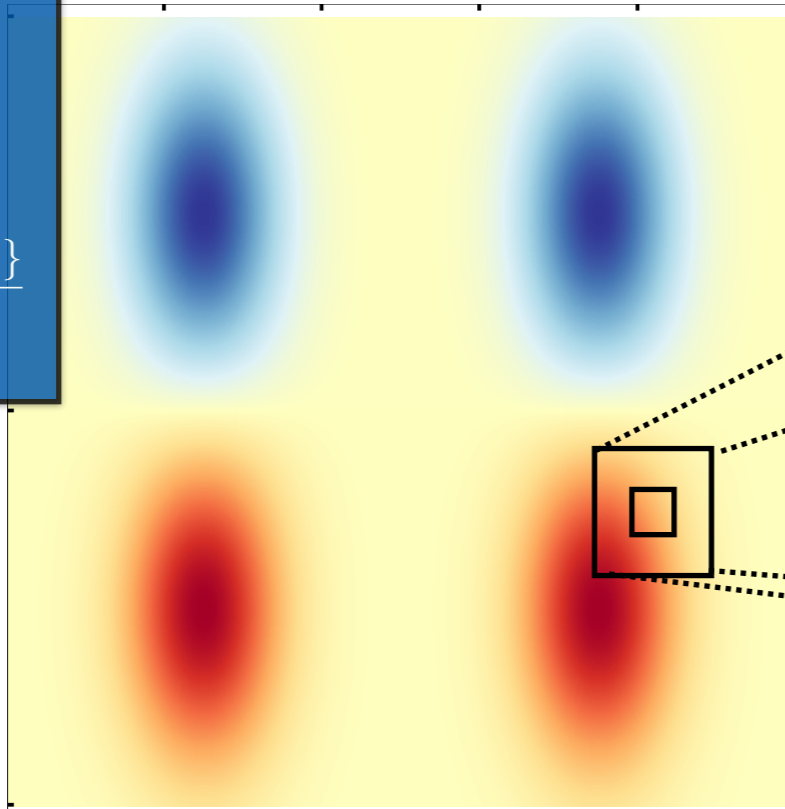


# Random switching of points

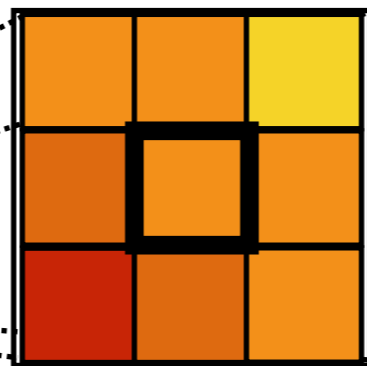
MU SVD

(heterogeneous and non-stationary  $\sigma \dot{B}$ )

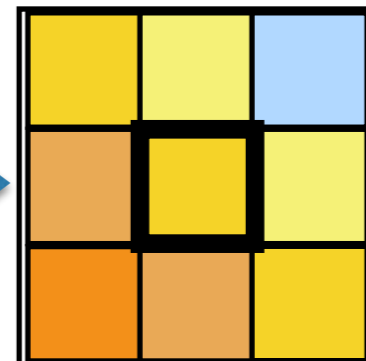
Large scales:  
 $w$   
Small scales:  
 $\sigma \dot{B}$   
Variance  
tensor:  
 $a = a(x, x) =$   
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



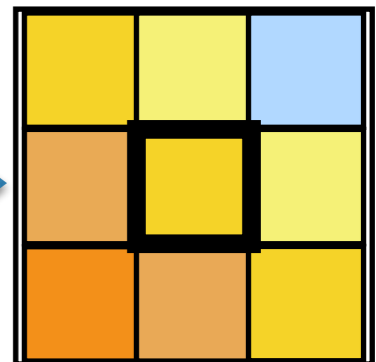
Neighbor



Centered  
neighbor



Random  
selection

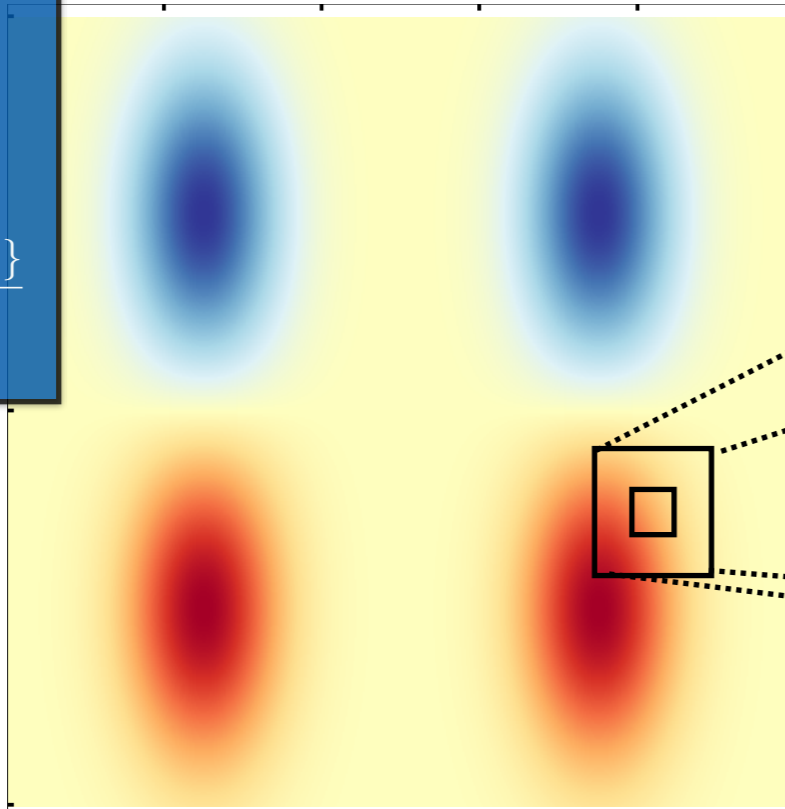


# Random switching of points

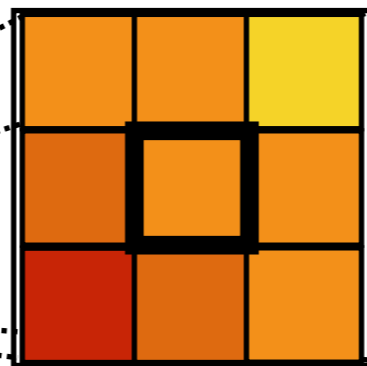
MU SVD

(heterogeneous and non-stationary  $\sigma \dot{B}$ )

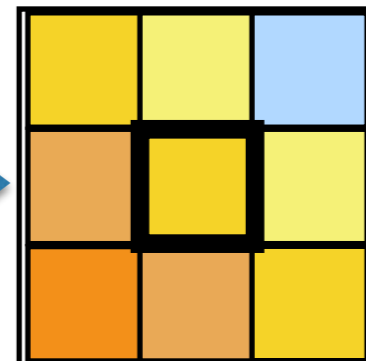
Large scales:  
 $w$   
Small scales:  
 $\sigma \dot{B}$   
Variance  
tensor:  
 $a = a(x, x) =$   
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



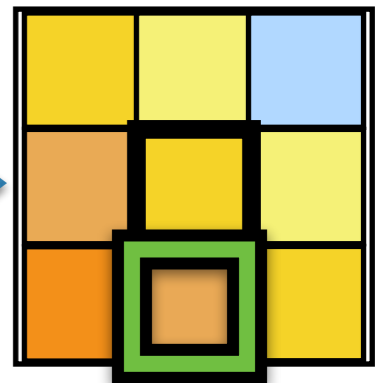
Neighbor



Centered  
neighbor



Random  
selection



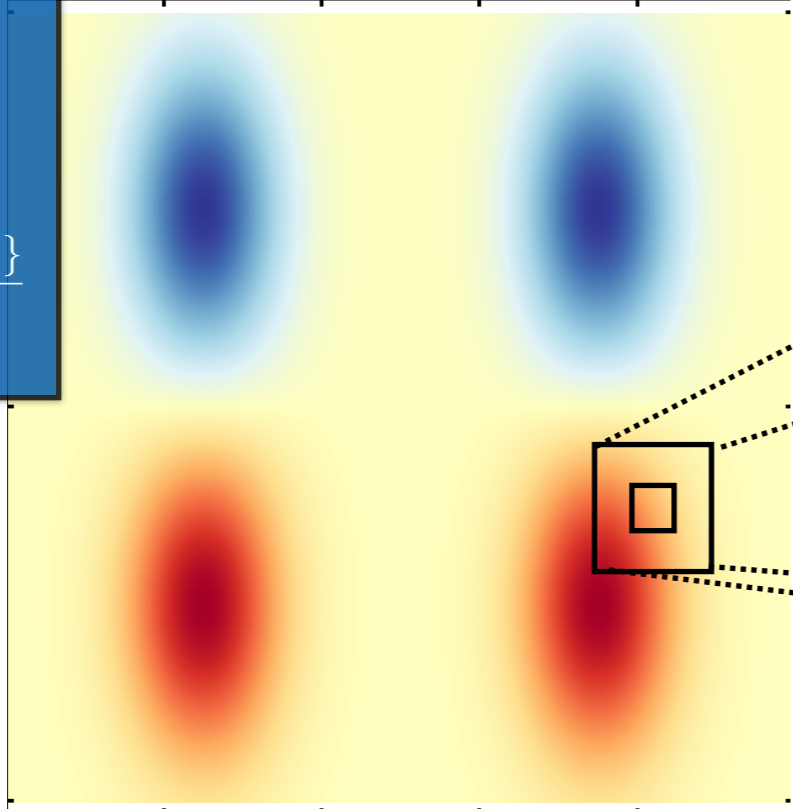


# Random switching of points

(heterogeneous and non-stationary  $\sigma \dot{B}$ )

MU SVD

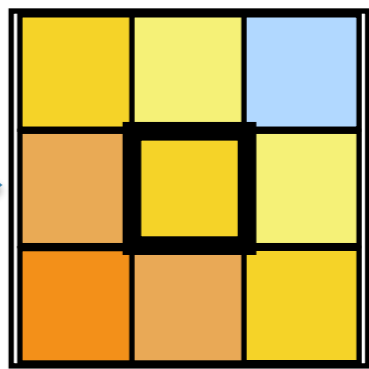
Large scales:  
 $w$   
Small scales:  
 $\sigma \dot{B}$   
Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



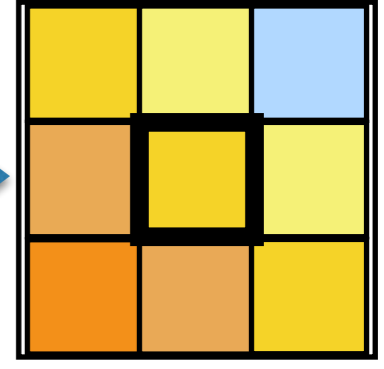
Neighbor



Centered neighbor



Random selection



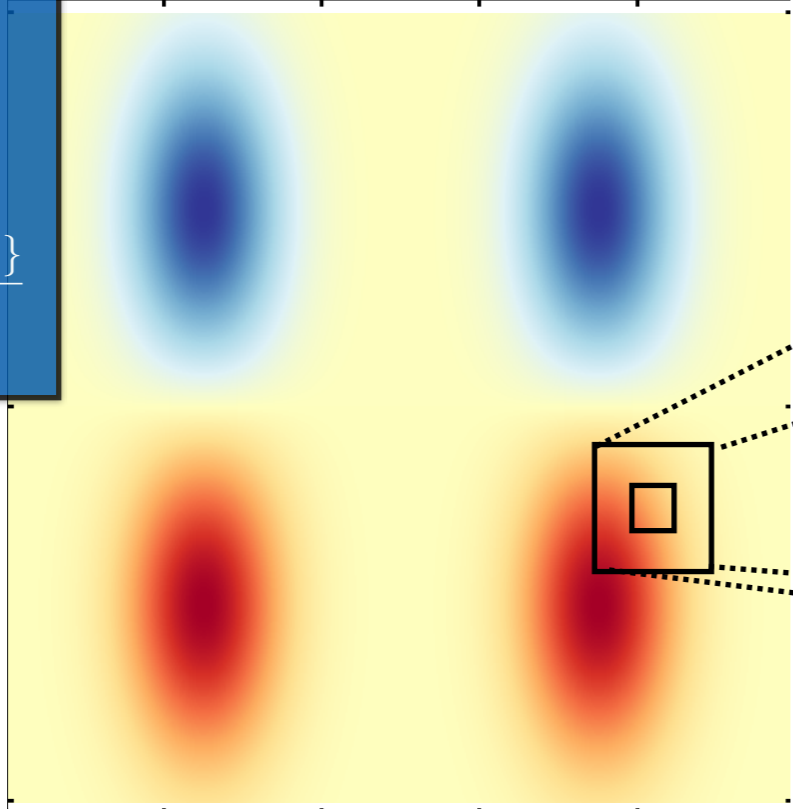
# Random switching of points

MU SVD

Large scales:  
 $w$

Small scales:  
 $\sigma \dot{B}$

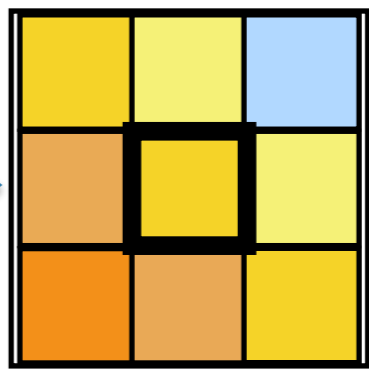
Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$



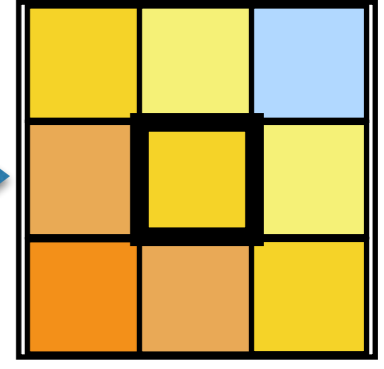
Neighbor



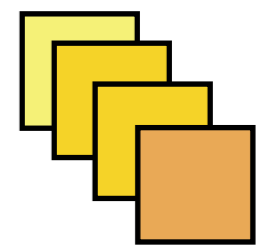
Centered neighbor



Random selection



Local ensemble :

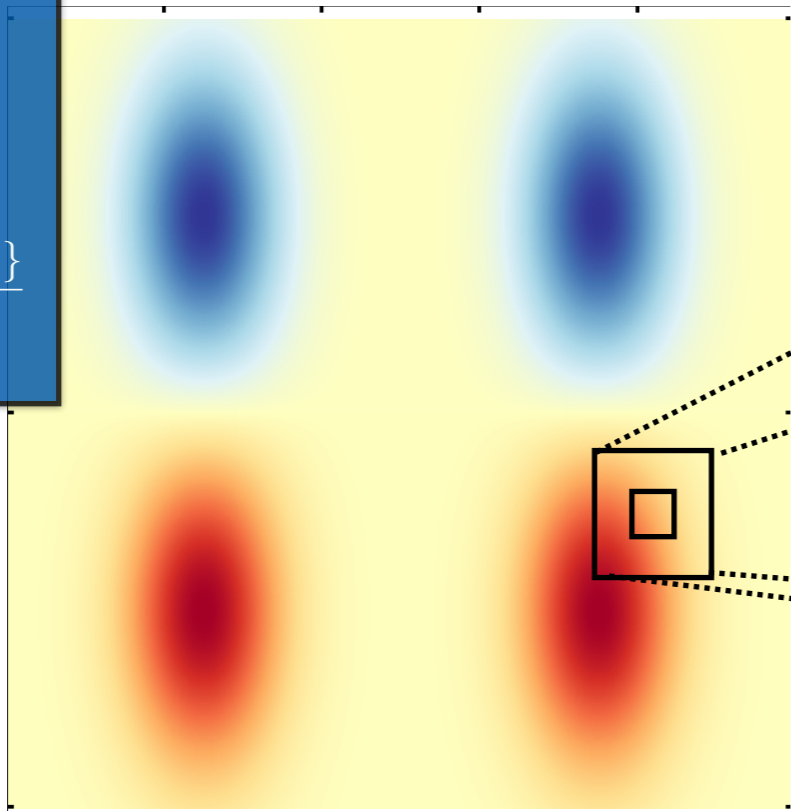


# Random switching of points

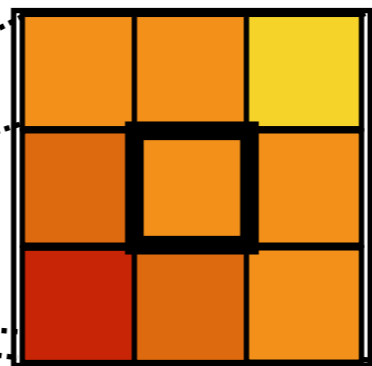
MU SVD

Large scales:  
 $w$   
Small scales:  
 $\sigma \dot{B}$   
Variance tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

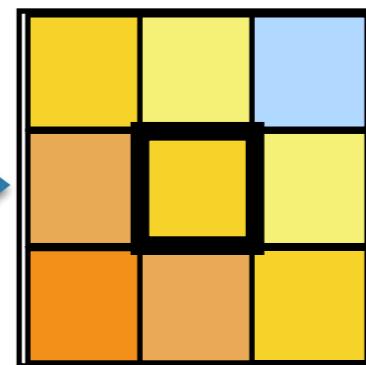
(heterogeneous and non-stationary  $\sigma \dot{B}$ )



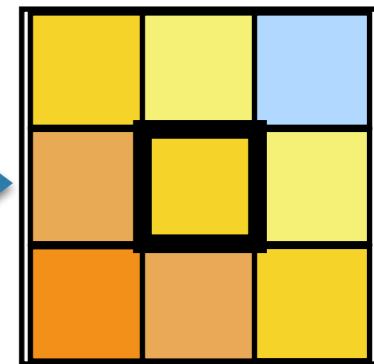
Neighbor



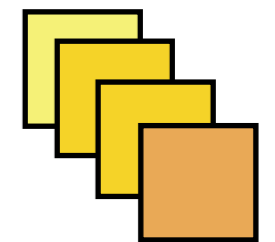
Centered neighbor



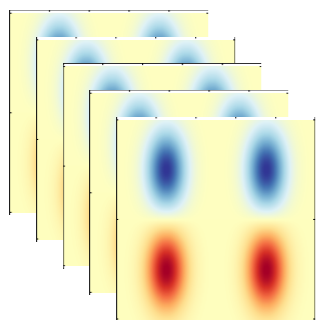
Random selection



Local ensemble :



Global ensemble



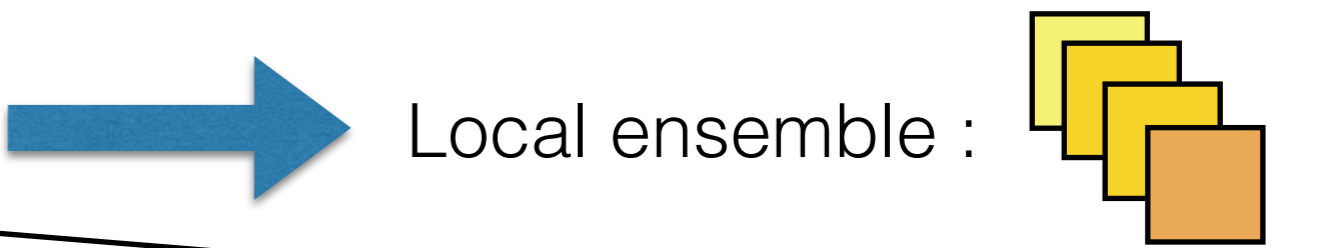
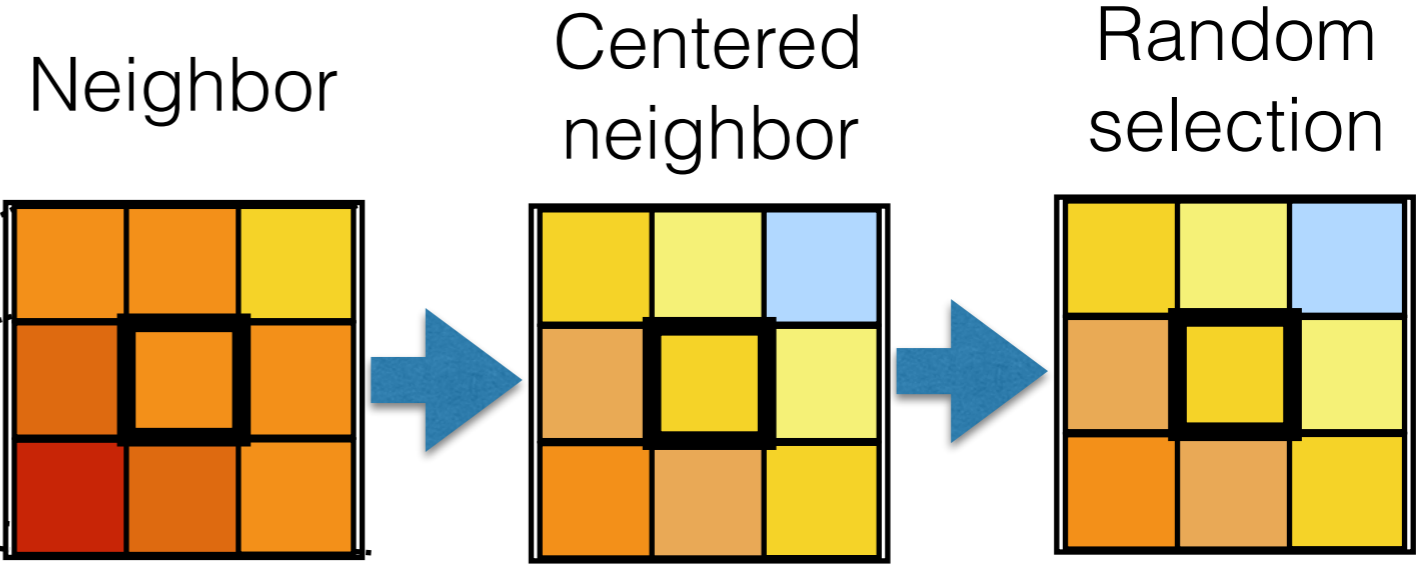
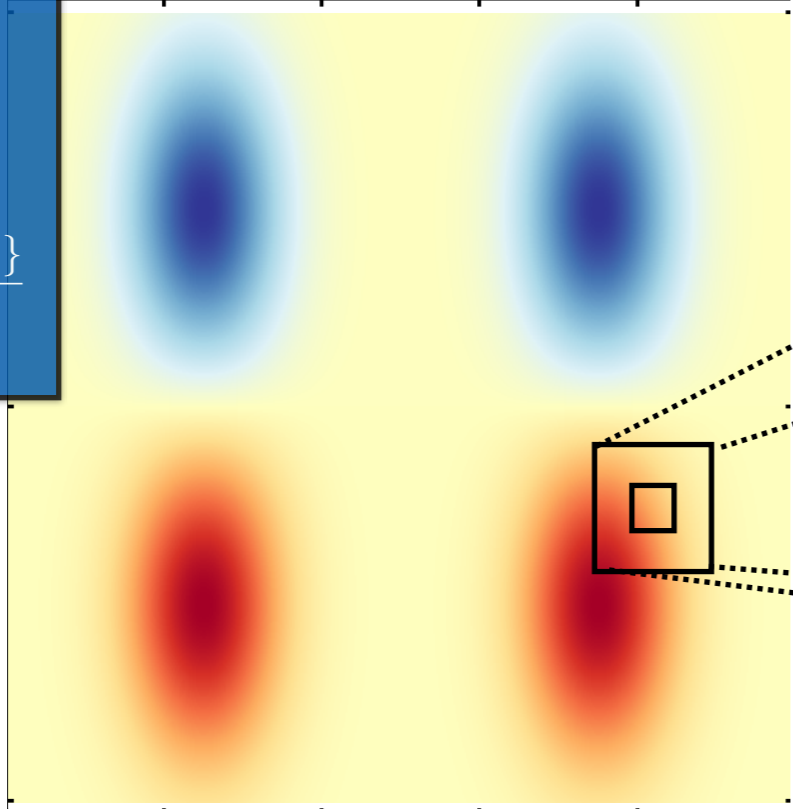
# Random switching of points

MU SVD

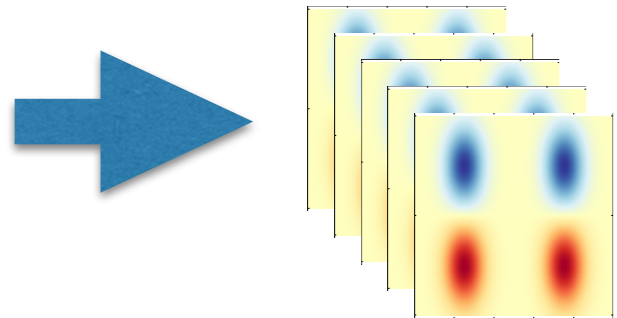
Large scales:  
 $w$

Small scales:  
 $\sigma \dot{B}$

Variance  
tensor:  
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$



Global ensemble



SVD

$\sigma \dot{B}$

# Part III

A new energy-budget-based  
stochastic scheme:

# WaveHyperv


# WaveHyperv

Transport  
equation

$$\frac{Dq}{Dt} = \mathcal{L}[q] + \eta$$

# WaveHyperv

Transport equation

$$\frac{Dq}{Dt} = \mathcal{L}[q] + \eta$$


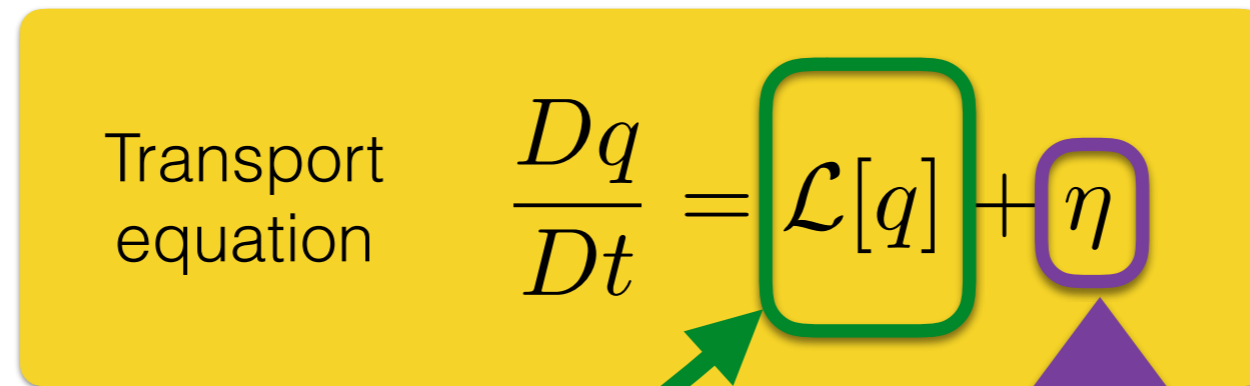
Usual  
deterministic  
subgrid tensor

e.g. Hyper-viscosity

$$\mathcal{L}[q] = -\nu \Delta^4 q$$

# WaveHyperv

Transport equation

$$\frac{Dq}{Dt} = \mathcal{L}[q] + \eta$$


Usual  
deterministic  
subgrid tensor  
e.g. Hyper-viscosity

$$\mathcal{L}[q] = -\nu \Delta^4 q$$

Random forcing

built to meet the energy budget:

$$(\text{Random energy intake}) = \zeta \times \text{Dissipation}$$



# Summary of UQ methods

Name	Method
<b>MU Spec</b>	LU with homogeneous and stationary small-scale velocity
<b>MU ADSD</b>	LU with homogeneous, non-stationary and tuning-free small-scale velocity
<b>MU SVD</b>	LU with inhomogeneous and non-stationary small-scale velocity
<b>WaveHyperv</b>	Energy-budget-based stochastic scheme
<b>PIC Spec</b>	Perturbed initial conditions with homogeneous noise
<b>PIC SVD</b>	Perturbed initial conditions with inhomogeneous noise

# Part IV

## Numerical comparisons

# Test case 1:

$t = 17$  days

**SQG**

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

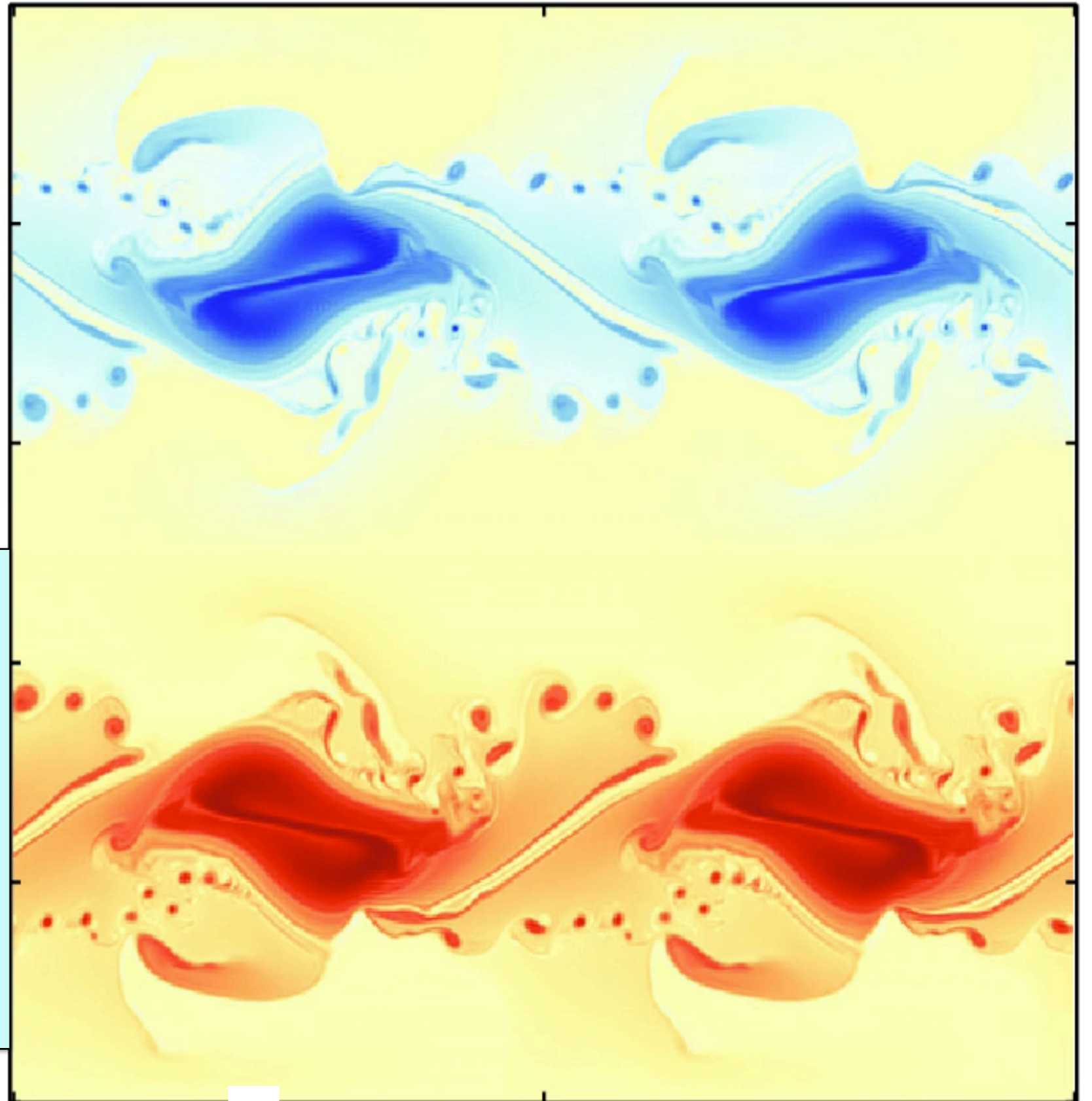
$$\mathbf{u} = \left( \text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

SQG

512 x 512



# Test case 1:

$t = 17$  days

**SQG**

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

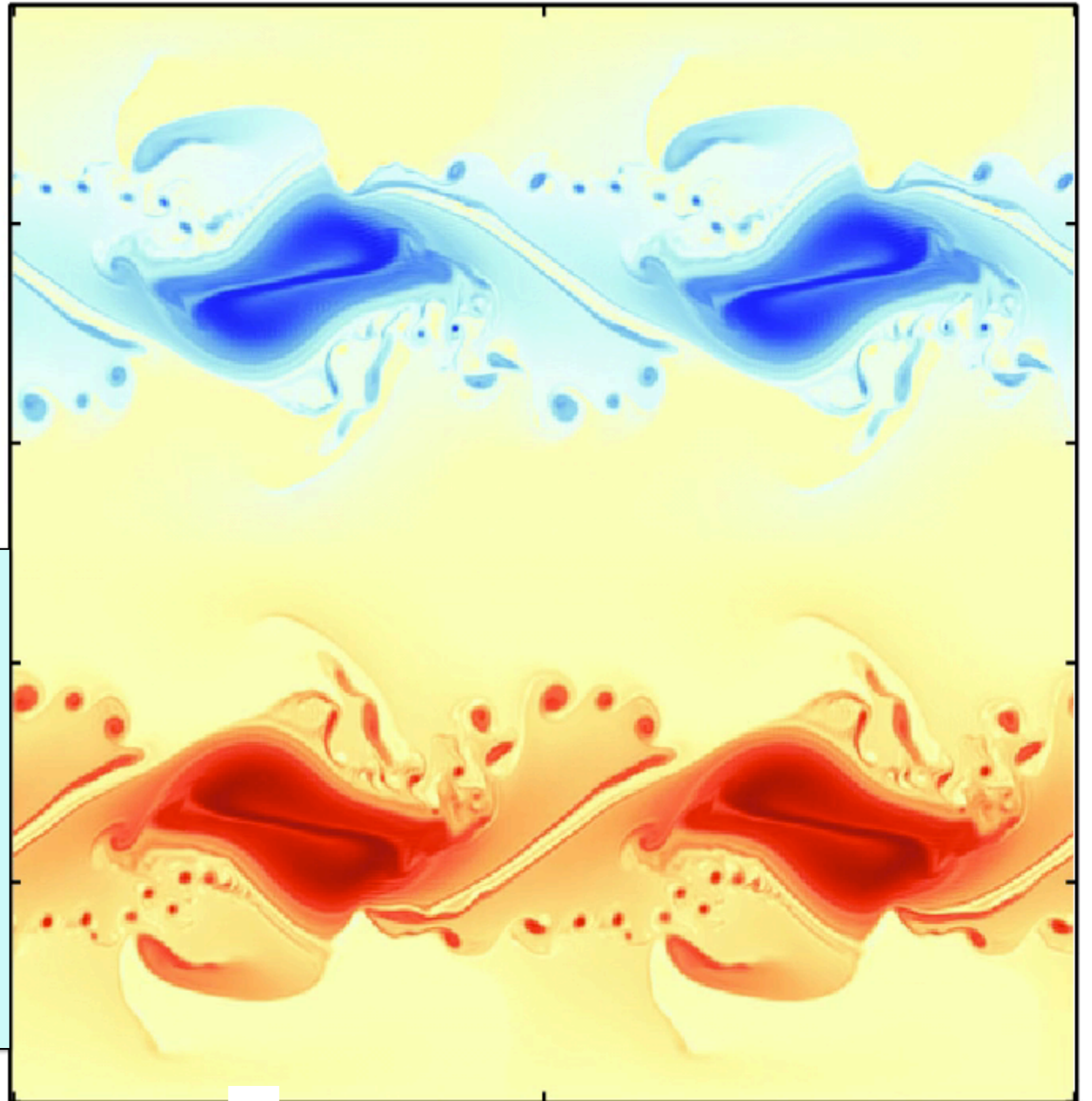
$$\mathbf{u} = \left( \text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

SQG

512 x 512





# Test case 2:

$t = 10$  days

**SQG**

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

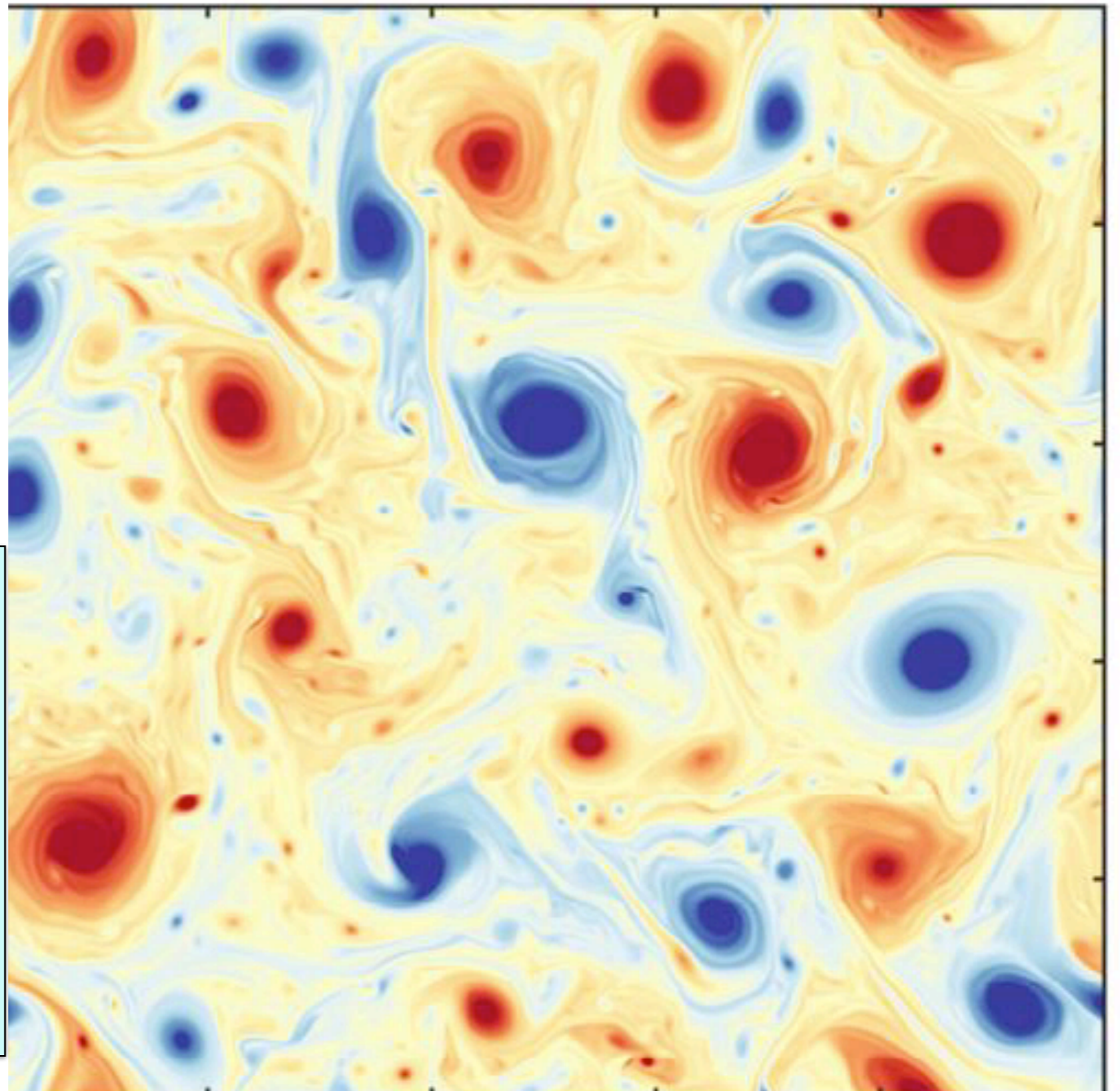
$$\mathbf{u} = \left( \text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

SQG

512 x 512





# Test case 2:

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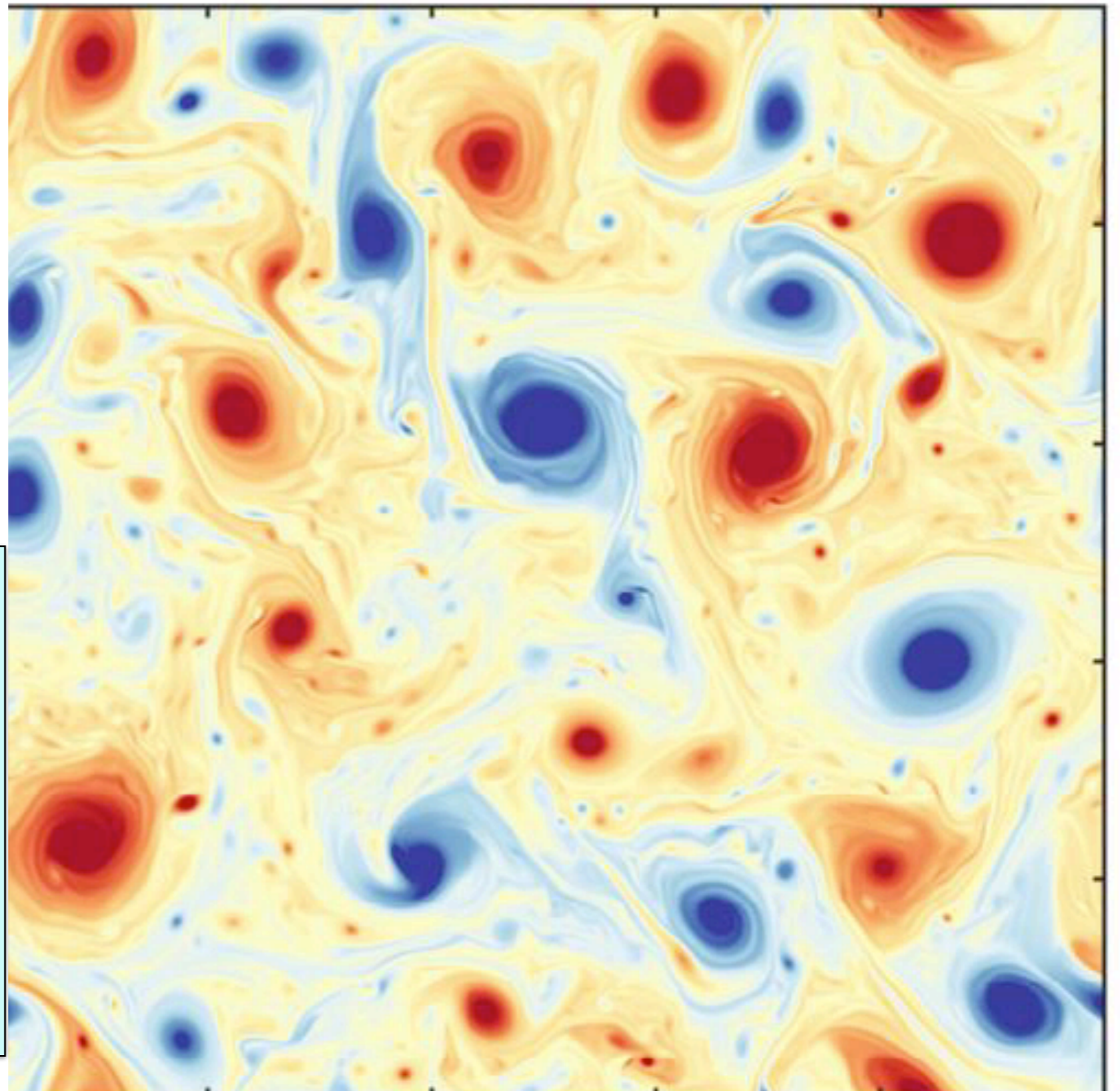
$$\mathbf{u} = \left( \text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

SQG

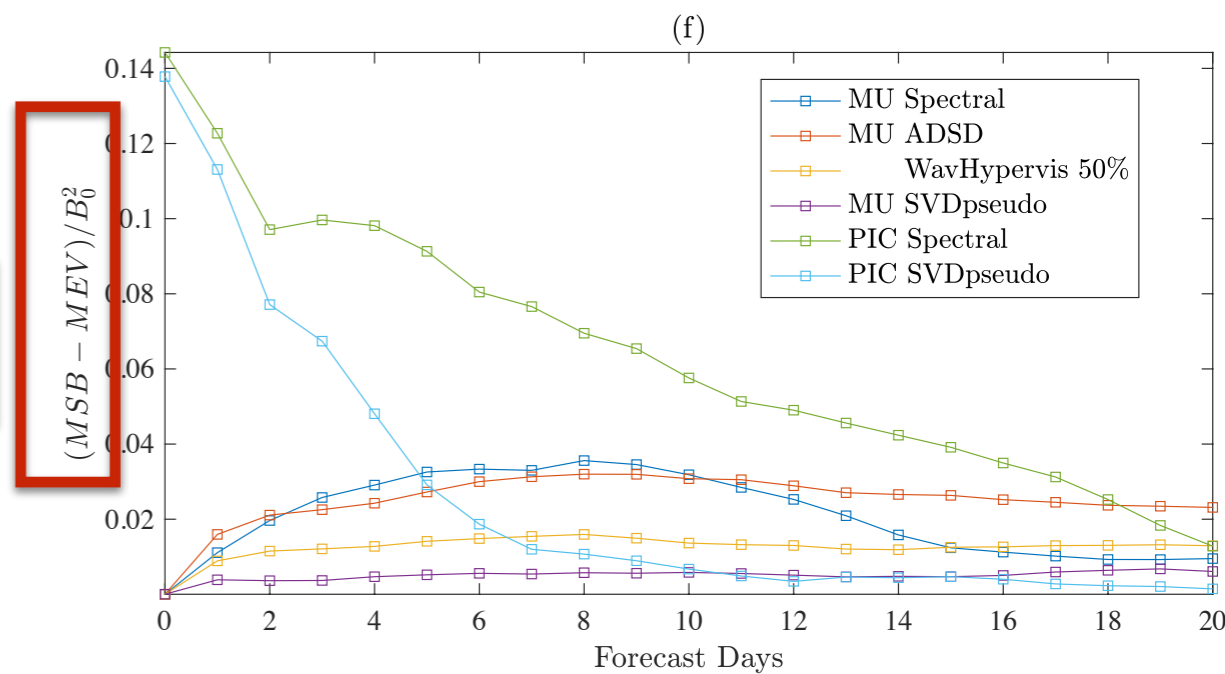
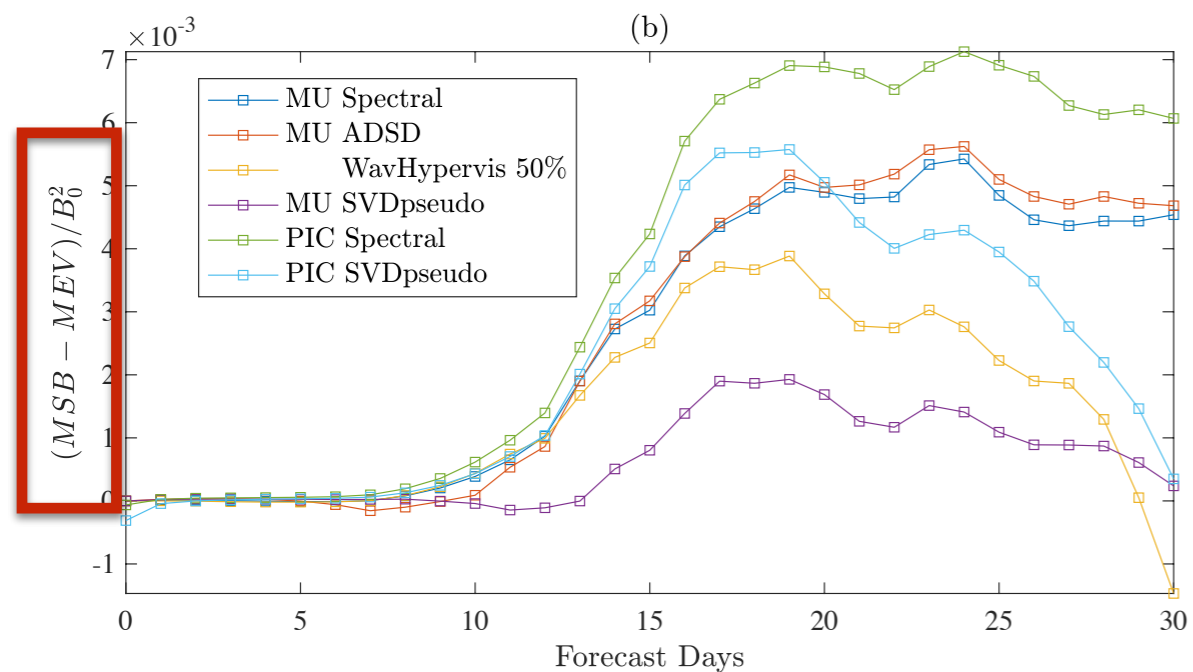
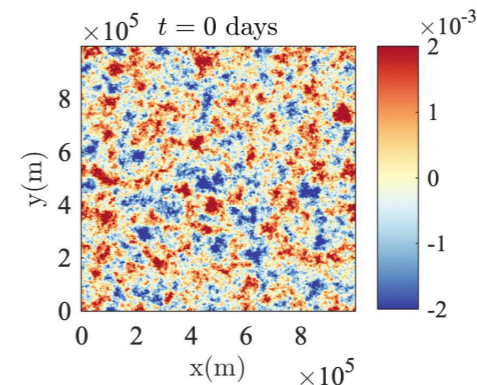
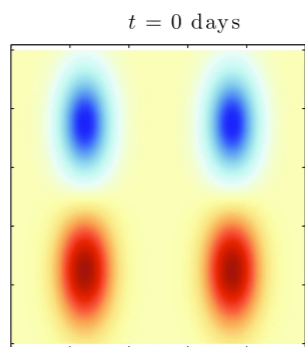
512 x 512



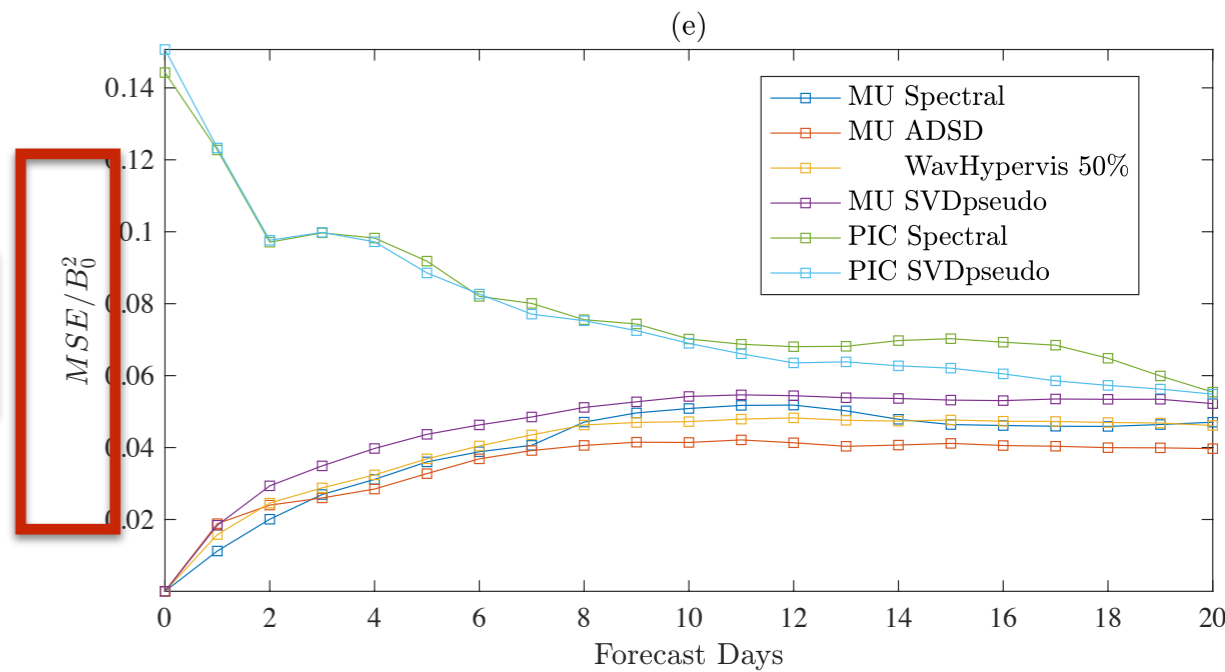
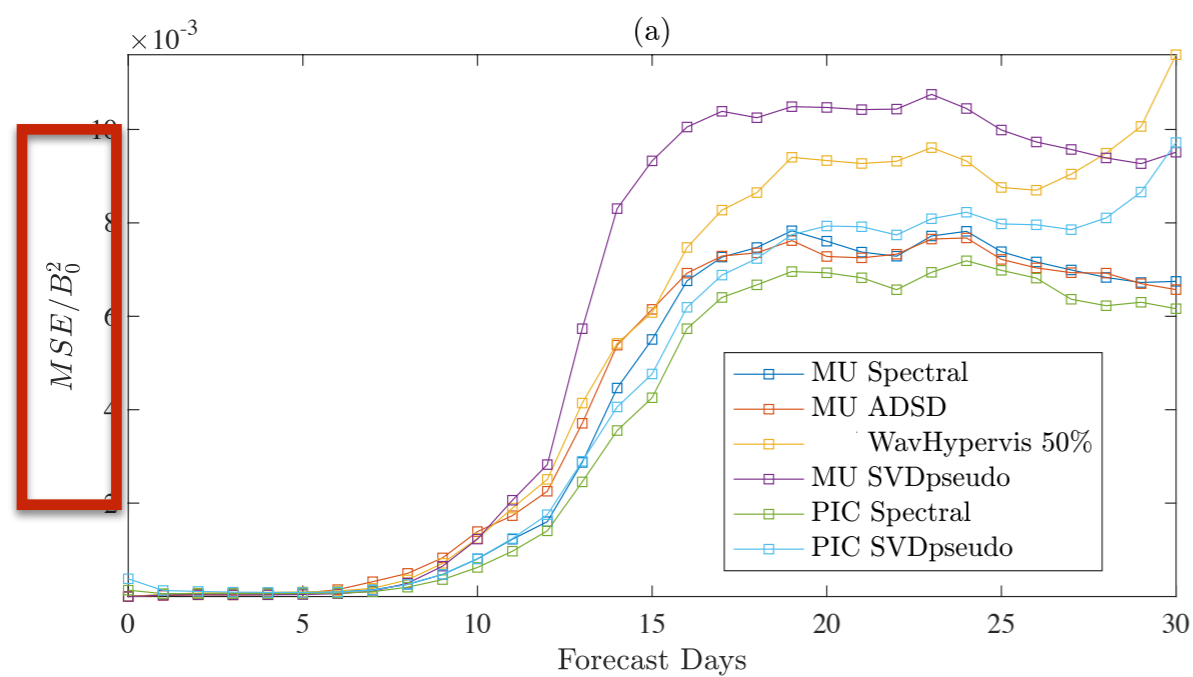
# UQ metrics

Metric	Meaning
RMSE	<u>Error</u> of ensemble members
Talagrand histogram (TH)	Capacity of the ensemble to <u>explore</u> all reference possible values
Bias <sup>2</sup> -spread	Capacity of the ensemble to <u>explore</u> all reference possible values
CRPS	Point-wise distance between the ensemble CFD and the indicator function of the event
Energy Score (ES)	Generalized CRPS for multivariate ensemble

# Spreading VS Errors



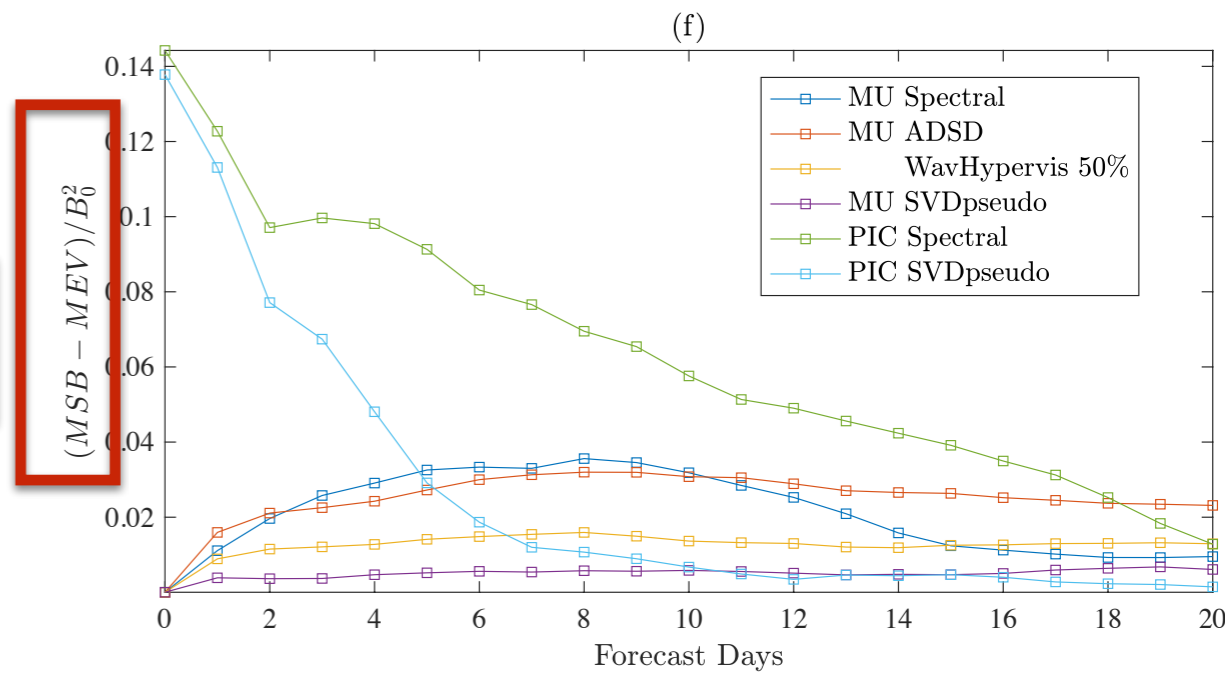
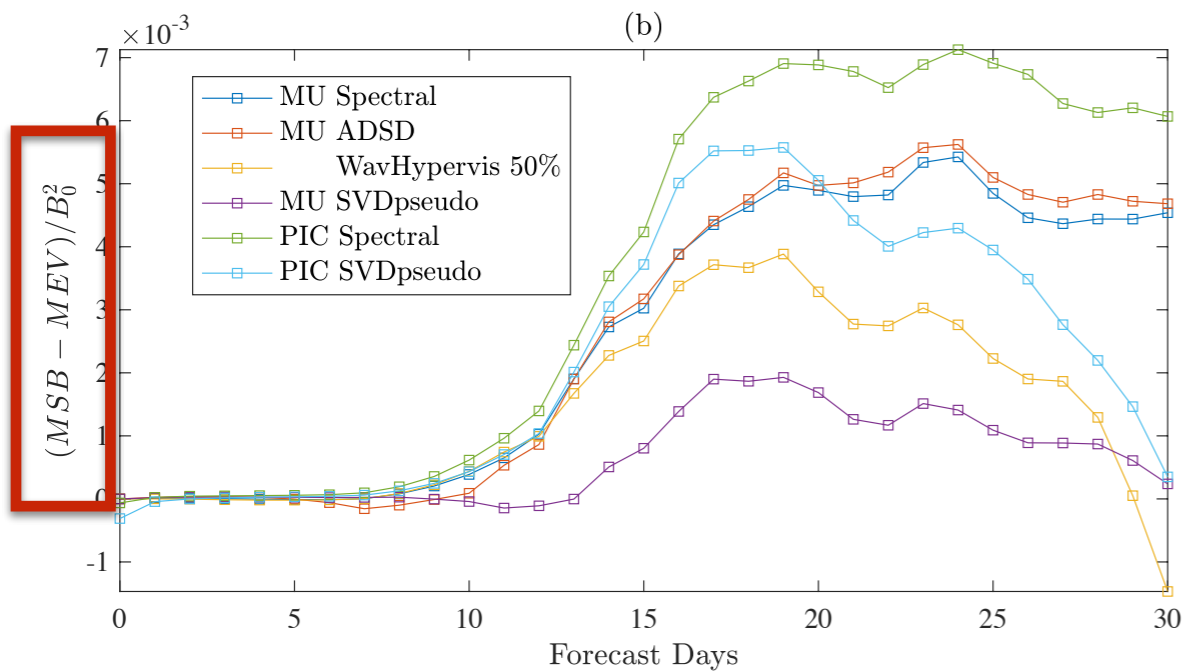
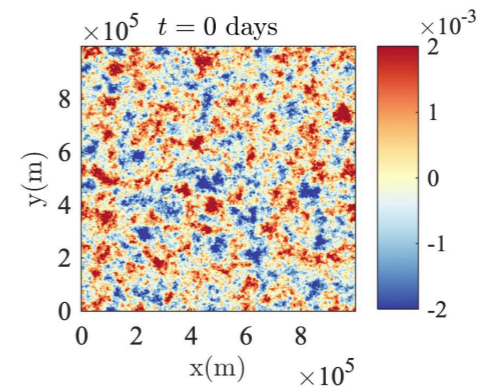
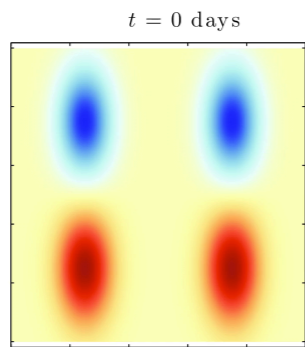
Spread



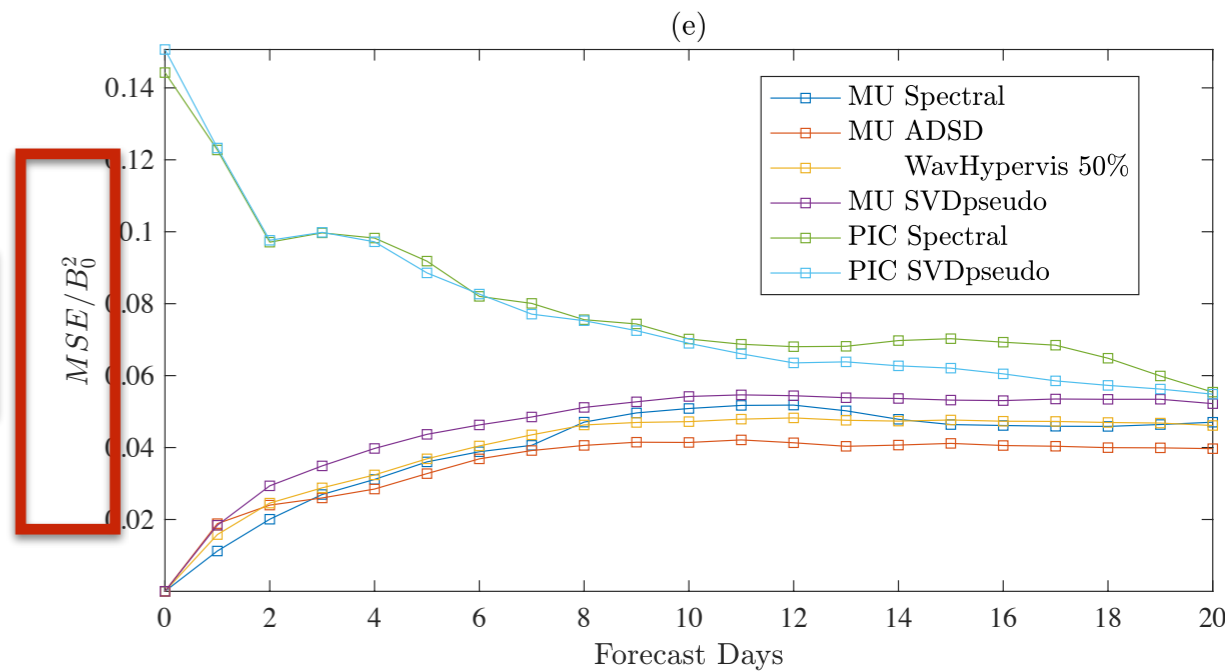
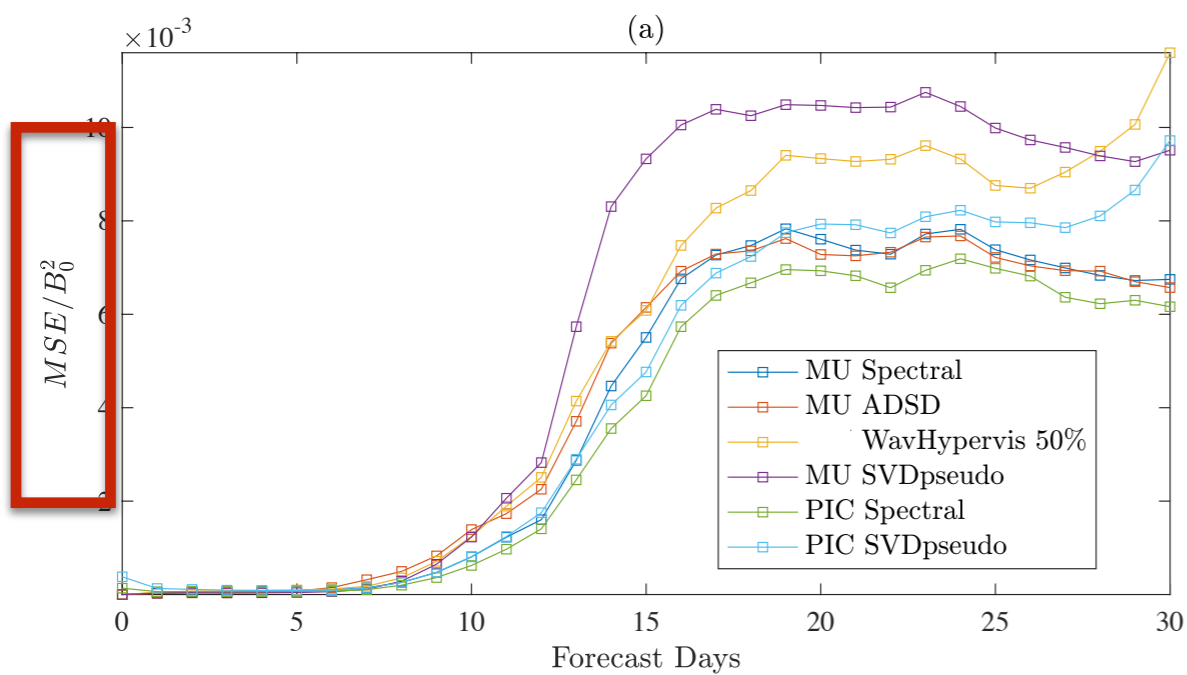
Error



# Spreading VS Errors

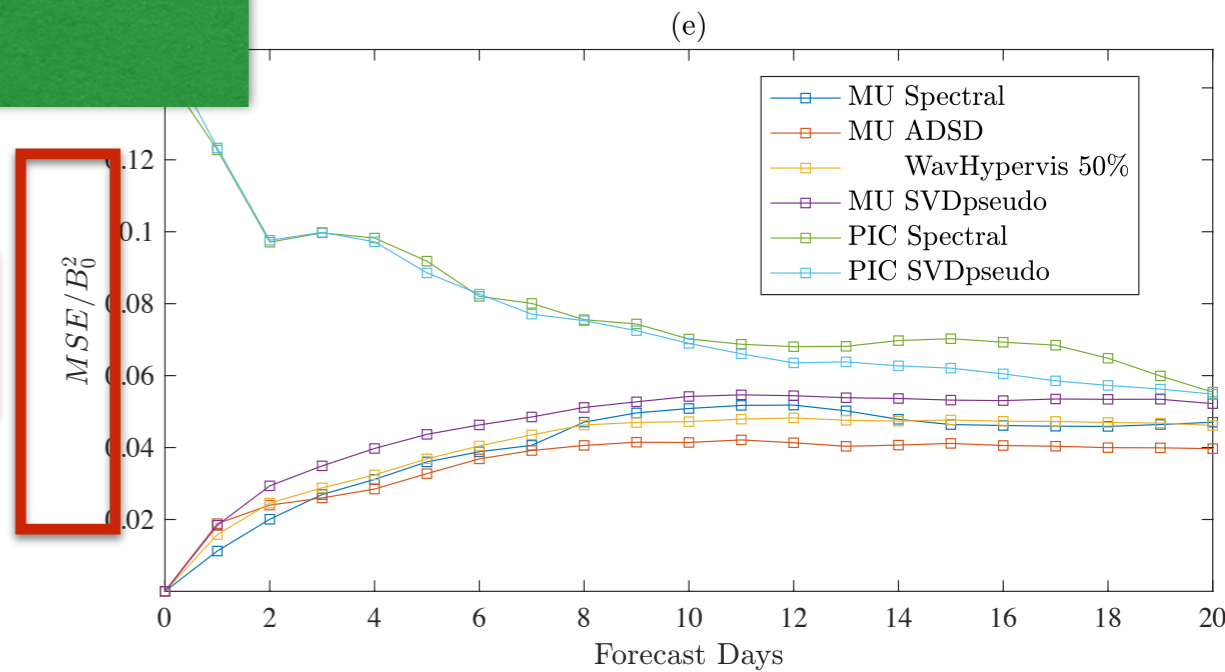
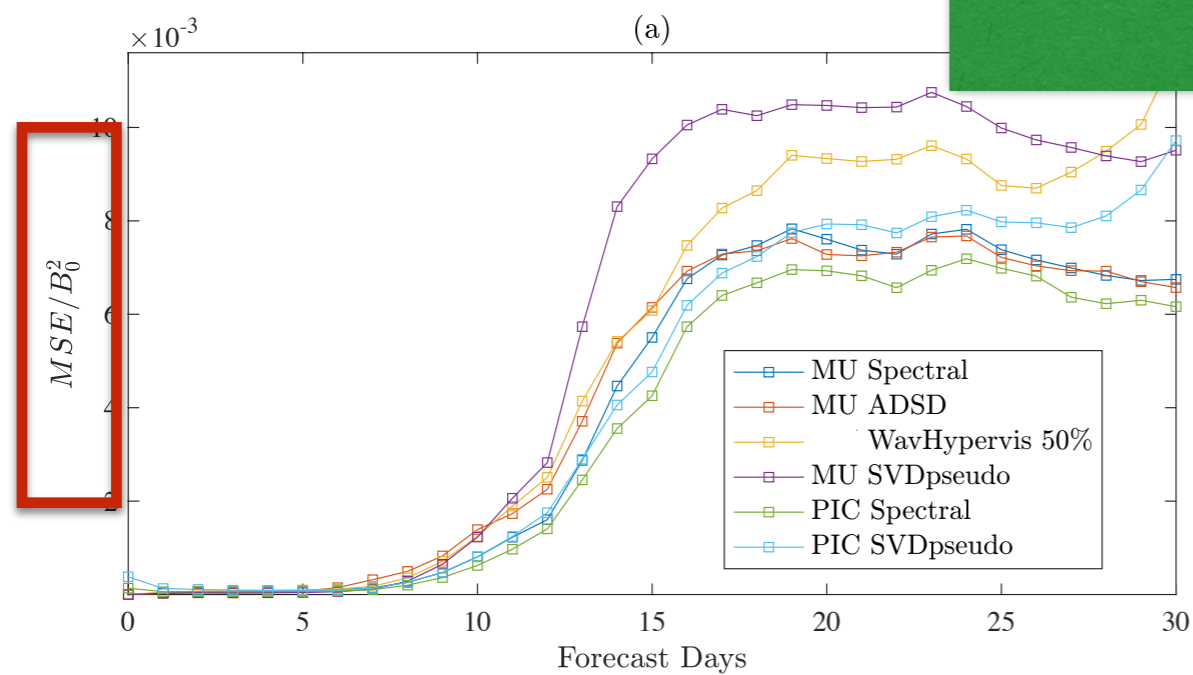
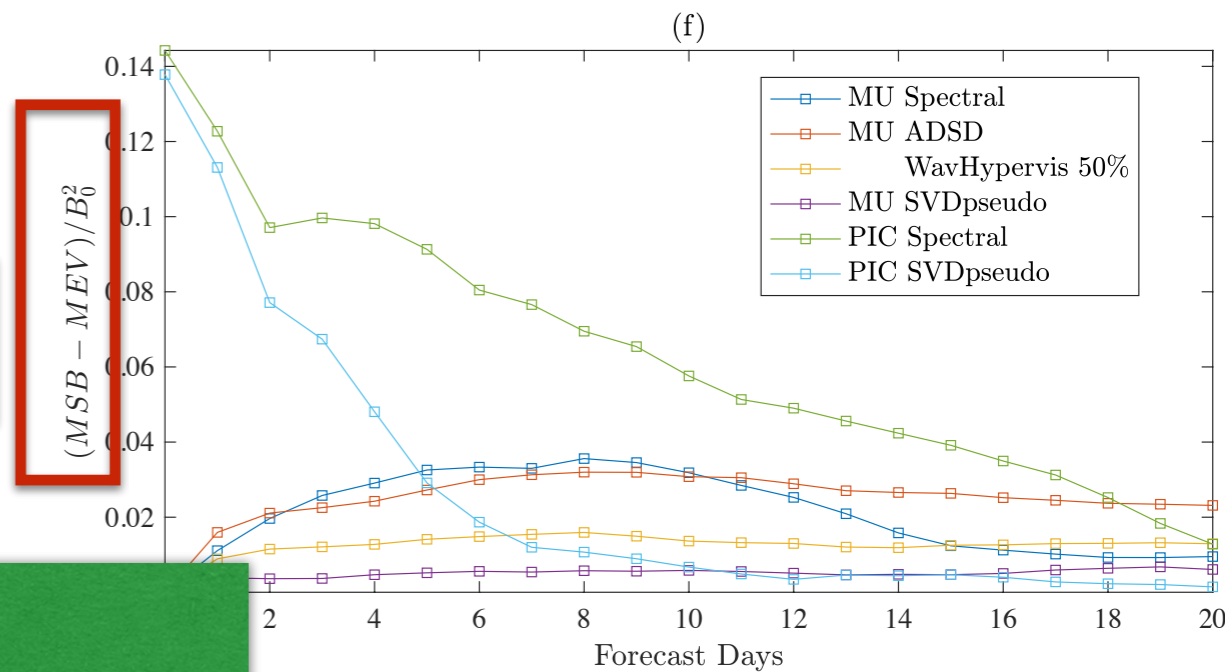
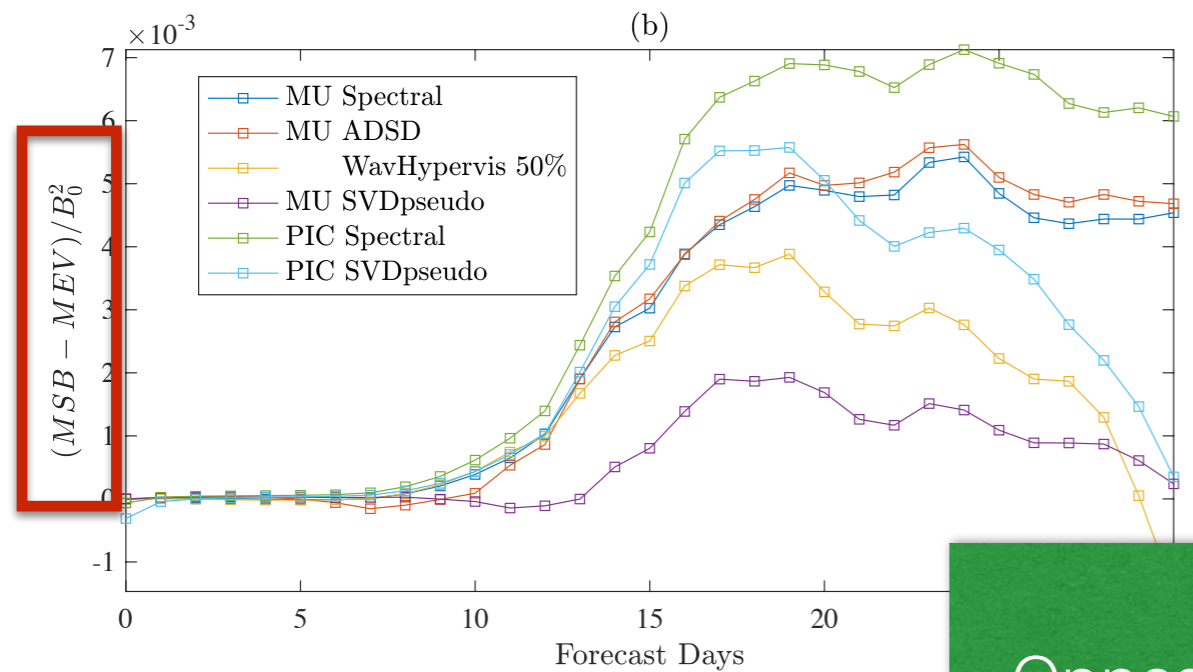
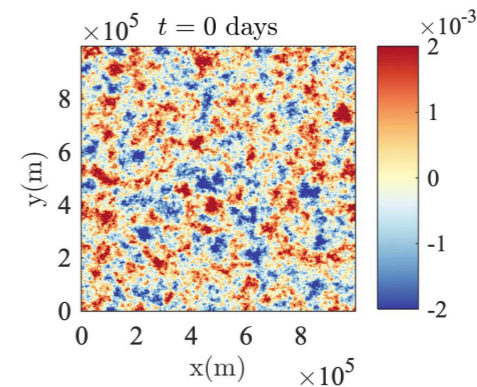
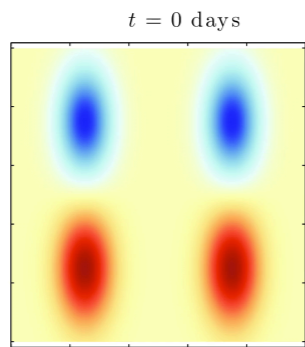


Spread



Error

# Spreading VS Errors



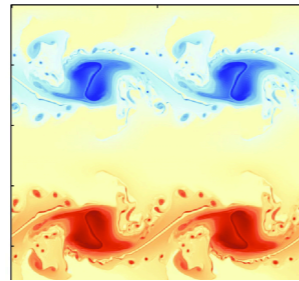
Spread

Error

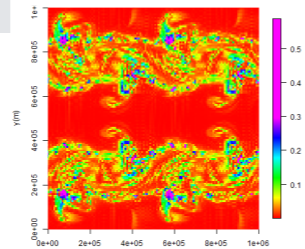
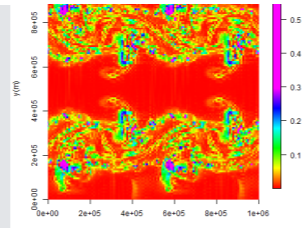
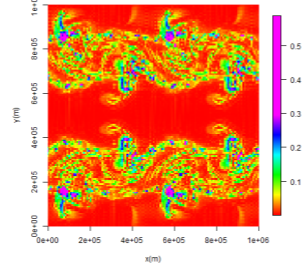
Opposite conclusions

# Ensemble point-wise skills : CRPS

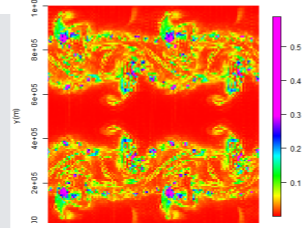
$t = 18$  days



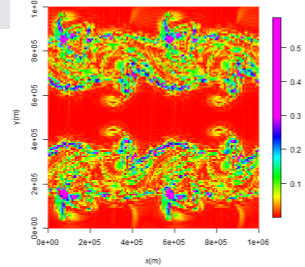
CRPS mu\_spec, day 18



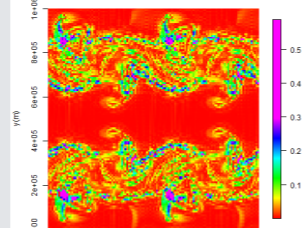
CRPS mu\_wavhv50, day 18



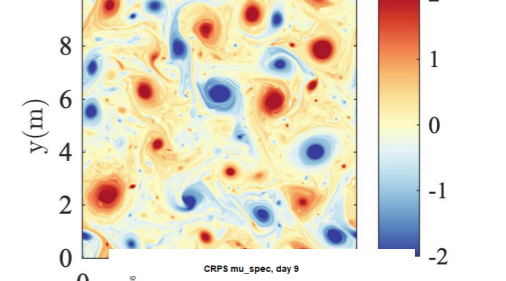
CRPS pic\_spec, day 18



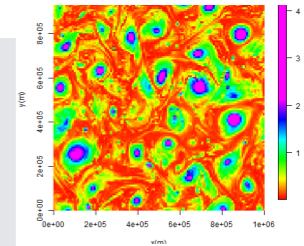
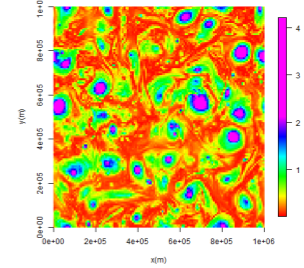
CRPS pic\_svd, day 18



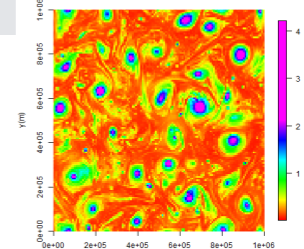
$\times 10^5$   $t = 10$  days



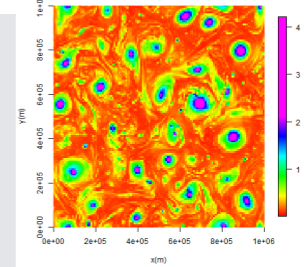
CRPS mu\_spec, day 9



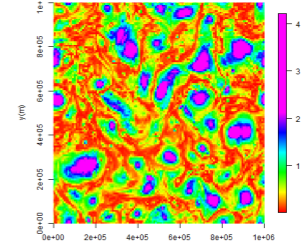
CRPS mu\_svd, day 9



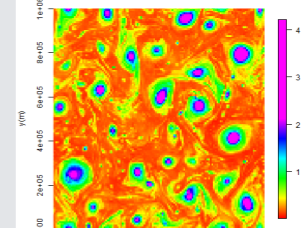
CRPS wavhv50, day 9



CRPS pic\_spec, day 9



CRPS pic\_svd, day 9



MU Spec

MU ADSD

MU SVD

WaveHyperv

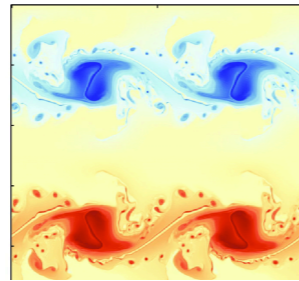
PIC Spec

PIC SVD

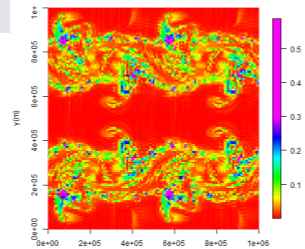
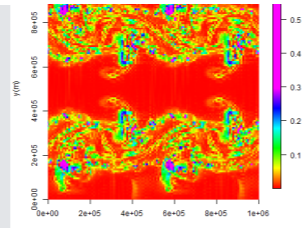
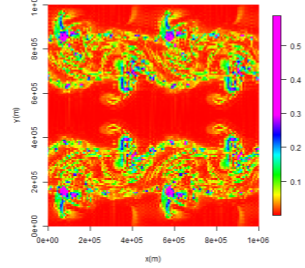


# Ensemble point-wise skills : CRPS

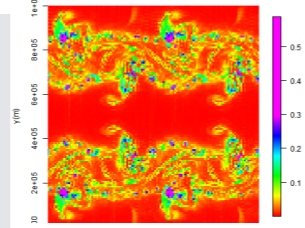
$t = 18$  days



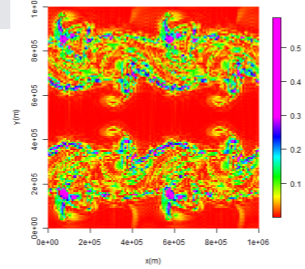
CRPS mu\_spec, day 18



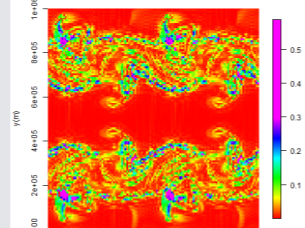
CRPS mu\_wavhv50, day 18



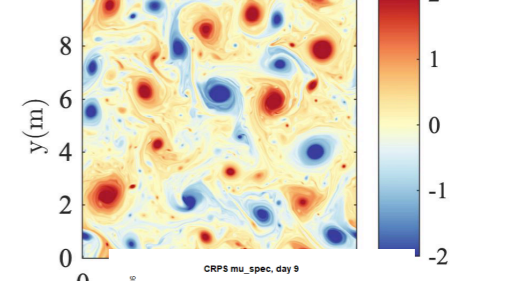
CRPS pic\_spec, day 18



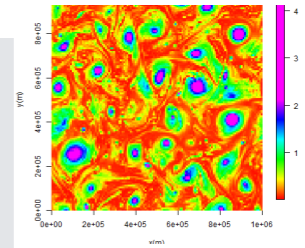
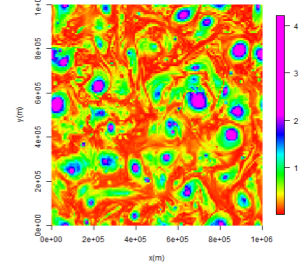
CRPS pic\_svd, day 18



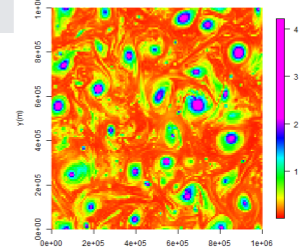
$\times 10^5$   $t = 10$  days



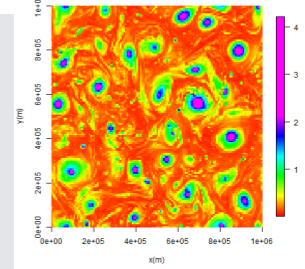
CRPS mu\_spec, day 9



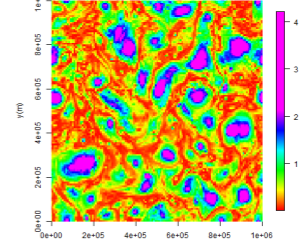
CRPS mu\_svd, day 9



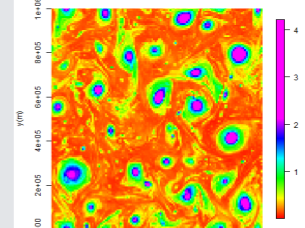
CRPS wavhv50, day 9



CRPS pic\_spec, day 9



CRPS pic\_svd, day 9



MU Spec

MU ADSD

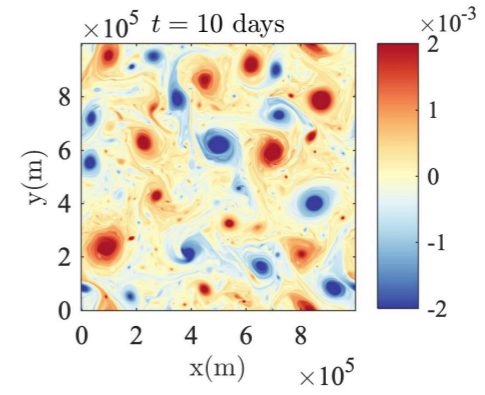
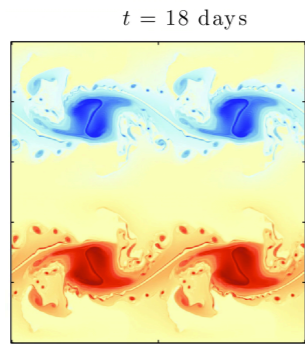
MU SVD

WaveHyperv

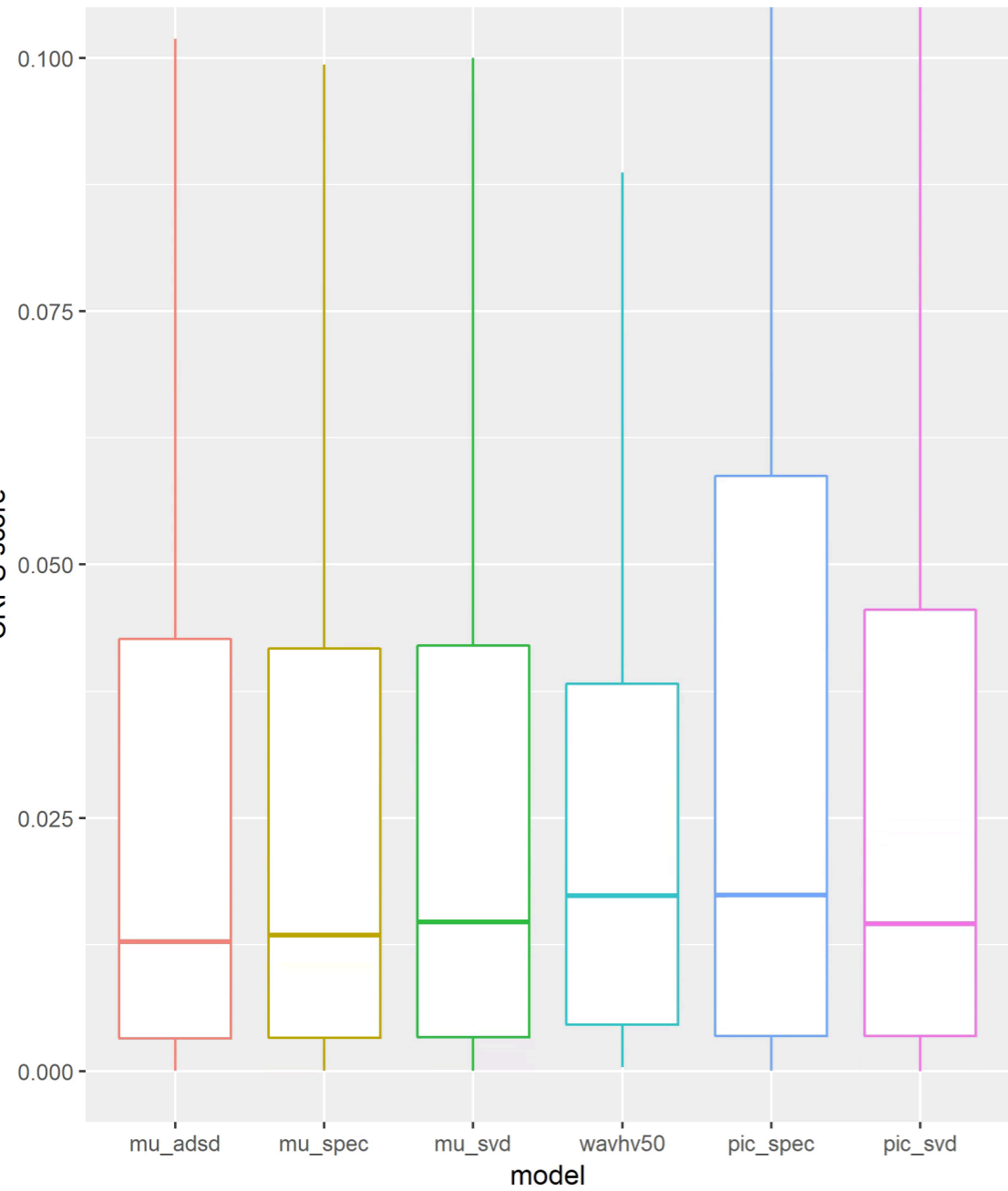
PIC Spec

PIC SVD

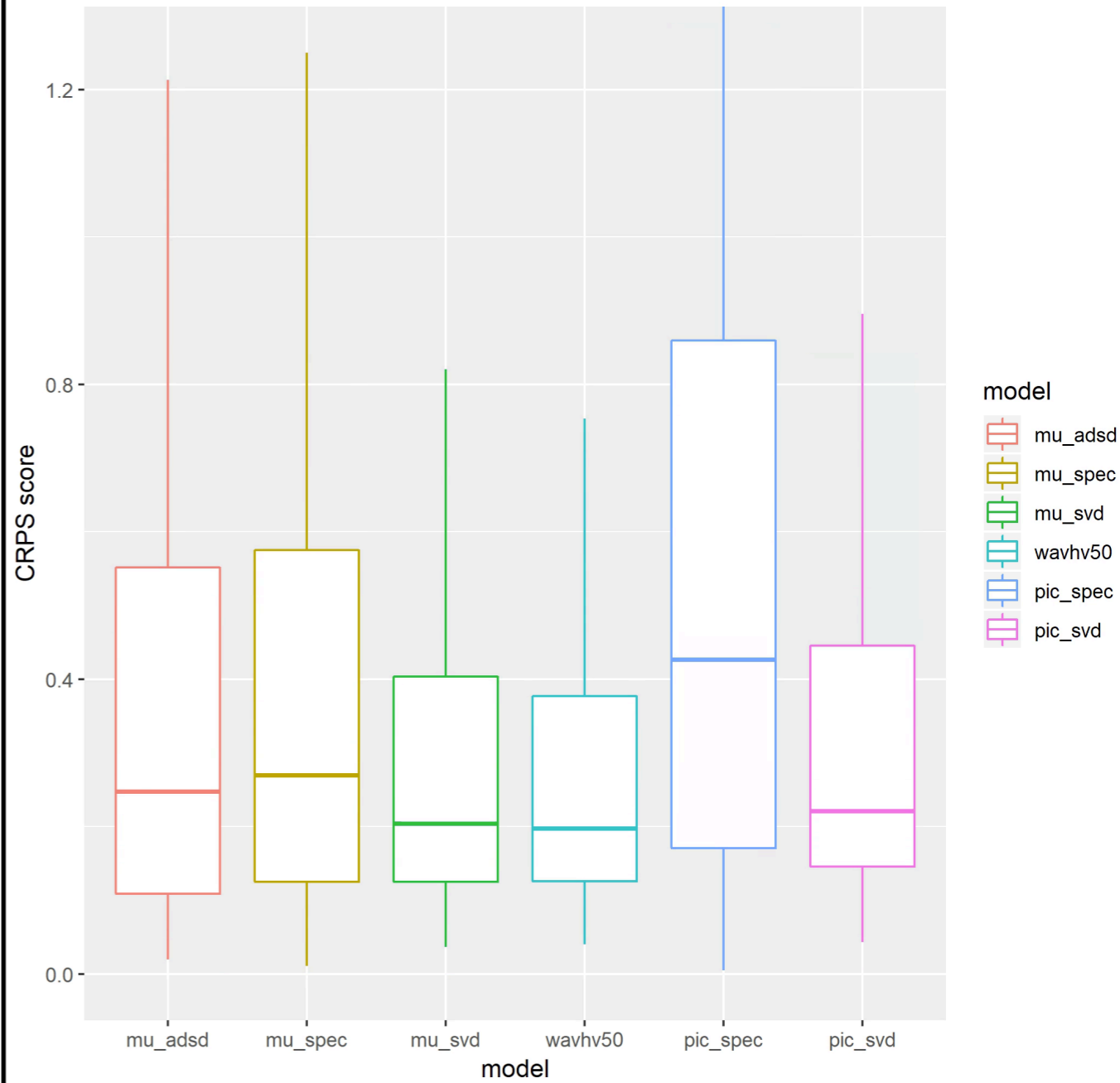
# Ensemble point-wise skills : CRPS



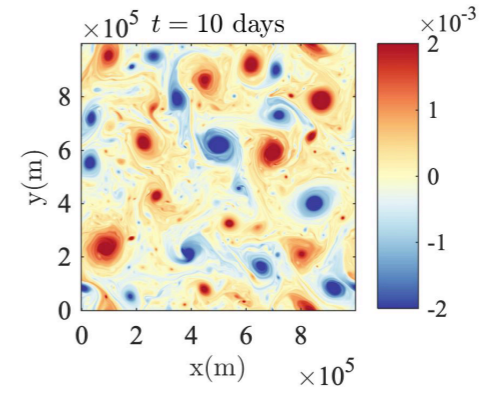
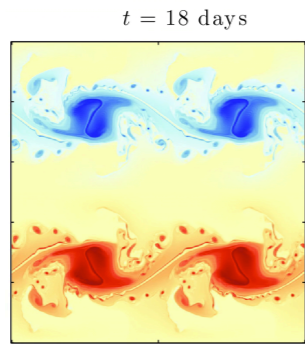
CRPS spatial of each model at day 19



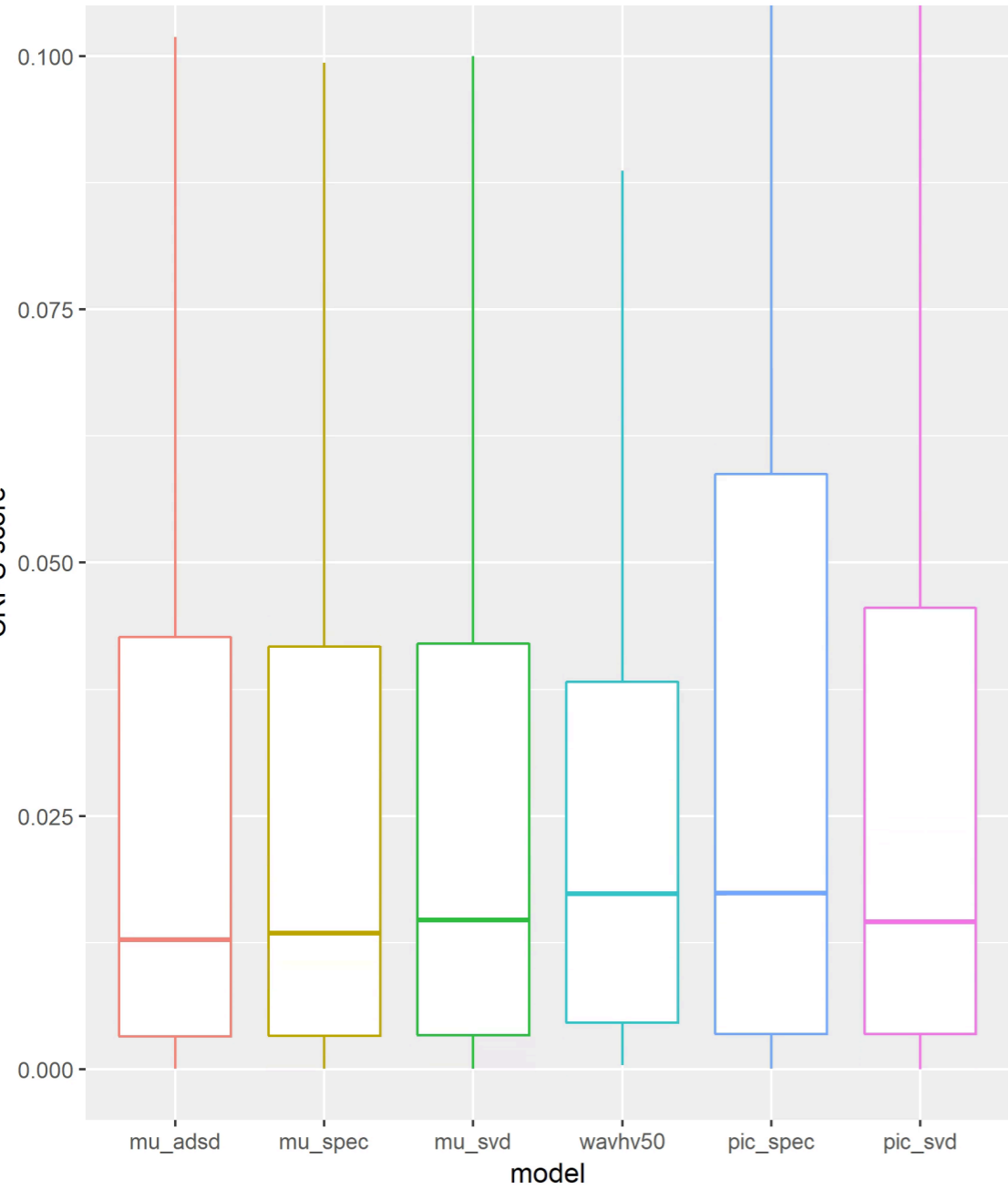
CRPS spatial of each model at day 10



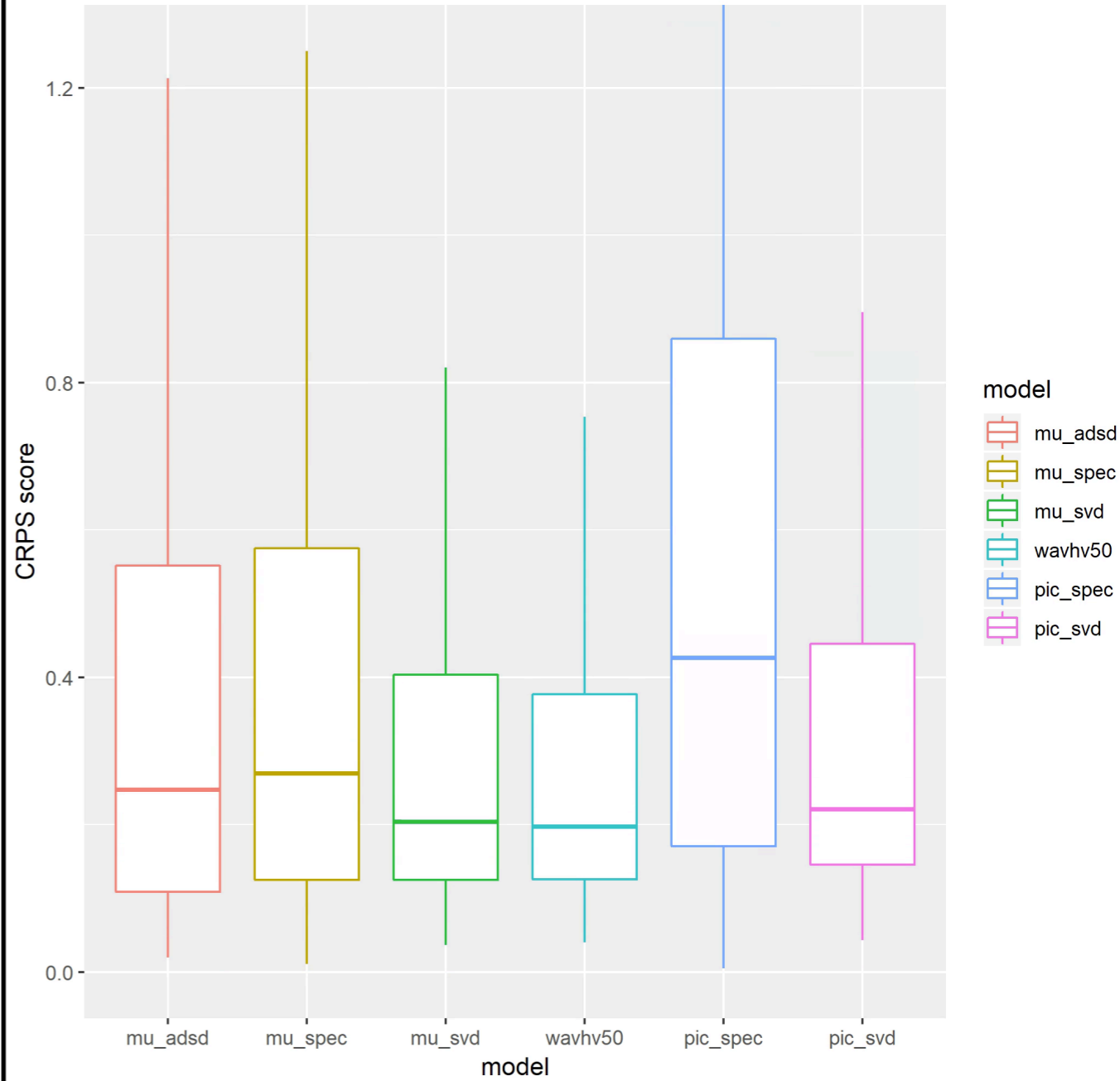
# Ensemble point-wise skills : CRPS



CRPS spatial of each model at day 19



CRPS spatial of each model at day 10



# Conclusion

# Conclusion

	RMSE (errors)	B <sup>2</sup> -Var (spread)	CRPS (point-wise)	ES (global)
MU Spec	+	+	+	+
MU ADSD	++	+	+	+
MU SVD	-	+++	++	+++
WaveHyperv	-	+	+++	++
PIC Spec	-	--	--	--
PIC SVD	--	-	+	+



# Conclusion

	RMSE (errors)	B <sup>2</sup> -Var (spread)	CRPS (point-wise)	ES (global)
MU Spec	+	+	+	+
MU ADSD	++	+	+	+
MU SVD	-	++	++	++
WaveHyperv	-	+	++	++
<del>PIC Spec</del>	<del>-</del>	<del>--</del>	<del>--</del>	<del>--</del>
<del>PIC SVD</del>	<del>--</del>	<del>-</del>	<del>+</del>	<del>+</del>

# Conclusion

	RMSE (errors)	B <sup>2</sup> -Var (spread)	CRPS (point-wise)	ES (global)
MU Spec	+	+	+	+
MU ADSD	++	+	+	+
MU SVD	-	++	++	++
WaveHyperv	-	+	++	++
<del>PIC Spec</del>	<del>-</del>	<del>--</del>	<del>--</del>	<del>--</del>
<del>PIC SVD</del>	<del>--</del>	<del>-</del>	<del>+</del>	<del>+</del>

# Conclusion

	RMSE (errors)	B <sup>2</sup> -Var (spread)	CRPS (point-wise)	ES (global)
MU Spec	+	+	+	+
MU ADSD	++	+	+	+
MU SVD	-	++	++	++
WaveHyperv	-	+	++	++
<del>PIC Spec</del>	<del>-</del>	<del>--</del>	<del>--</del>	<del>--</del>
<del>PIC SVD</del>	<del>--</del>	<del>-</del>	<del>+</del>	<del>+</del>