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Dynamics under location uncertainty and other energy-related stochastic subgrid schemes

Valentin Resseguier, Long Li, Gabriel Jouan, Pierre Dérian, Etienne Mémin,
Bertrand Chapron

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Dynamics under location uncertainty and other energy-related stochastic subgrid schemes

V. Resseguier
L. Li, G. Jouan, P. Derian,
E. Memin, B. Chapron

Motivations

- More rigorously identified subgrid dynamics effects
- Quantification of modeling errors (UQ)



Ensemble forecasts and data assimilation

Contents

- Models under location uncertainty (LU)
- Some parameterization of the models under location uncertainty
- A new energy-budget-based stochastic scheme: WaveHyperv
- Numerical comparisons

Part I

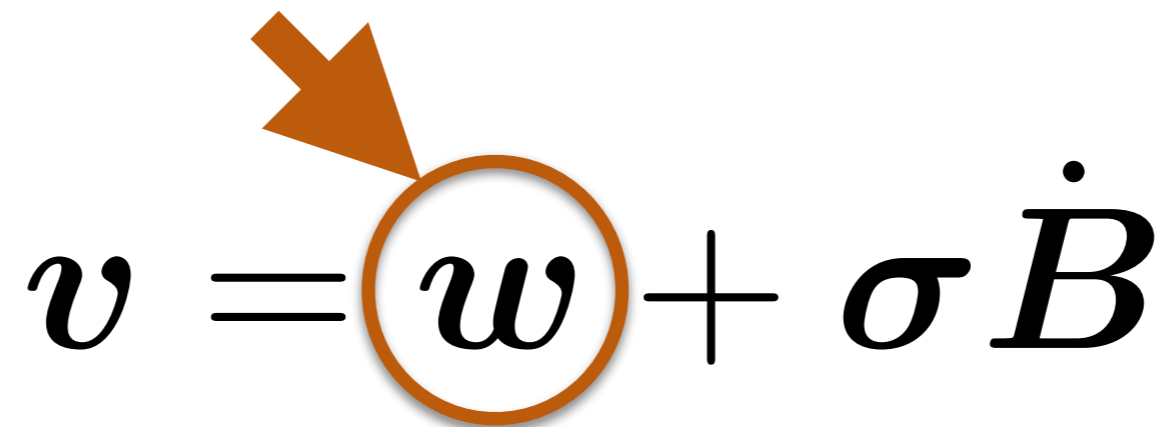
Models under location uncertainty (LU)

LU : Adding
random velocity

$$\mathbf{v} = \mathbf{w} + \sigma \dot{\mathbf{B}}$$

LU : Adding random velocity

Resolved
large scales

$$\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$$
The diagram shows the equation $\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$. The variable \boldsymbol{w} is circled in orange, and an orange arrow points from the text "Resolved large scales" to the circle.

LU : Adding random velocity

Resolved
large scales

White-in-time
small scales

$$\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$$

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

LU : Adding random velocity

Resolved
large scales

White-in-time
small scales

$$v = w + \sigma \dot{B}$$

LU : Adding random velocity

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Resolved
large scales

White-in-time
small scales

$$v = w + \sigma \dot{B}$$

References : Mikulevicius &
Rozovskii, 2004
Flandoli, 2011

Memin, 2014
Resseguier et al. 2017 a, b, c
Cai et al. 2017
Chapron et al. 2018
Yang & Memin 2019
Resseguier et al. 2019 a,b

Holm, 2015
Holm and
Tyranowski, 2016
Arnaudon et al. 2017
Cotter and al. 2017

Crisan et al., 2017
Gay-Balmaz & Holm 2017
Cotter and al. 2018 a, b
Cotter and al. 2019

LU : Adding random velocity

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Resolved large scales

White-in-time small scales

$$v = w + \sigma \dot{B}$$

LU

Memin, 2014
 Resseguier et al. 2017 a, b, c
 Cai et al. 2017
 Chapron et al. 2018
 Yang & Memin 2019
 Resseguier et al. 2019 a,b

SALT

Holm, 2015
 Holm and Tyranowski, 2016
 Arnaudon et al. 2017
 Cotter and al. 2017
 Crisan et al., 2017
 Gay-Balmaz & Holm 2017
 Cotter and al. 2018 a, b
 Cotter and al. 2019

References : Mikulevicius & Rozovskii, 2004
 Flandoli, 2011

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

$$\frac{D\Theta}{Dt} = 0$$

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

Advection of tracer Θ

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection of tracer Θ

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection of tracer Θ

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

Advection of tracer Θ

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

Drift correction

Advection of tracer Θ

Large scales:

$$w$$

Small scales:

$$\sigma \dot{B}$$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

Multiplicative random forcing

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta}_{\text{Advection}} + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)_{\text{Diffusion}}$$

Drift correction

Advection of tracer Θ

Large scales:

$$w$$

Small scales:

$$\sigma \dot{B}$$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

Multiplicative random forcing

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

The equation is presented within two colored boxes: a cyan box labeled "Advection" containing the left-hand side terms, and a green box labeled "Diffusion" containing the right-hand side term. An orange arrow points to w^* and a purple arrow points to $\sigma \dot{B}$.

Drift correction

Advection of tracer Θ

Large scales:

$$w$$

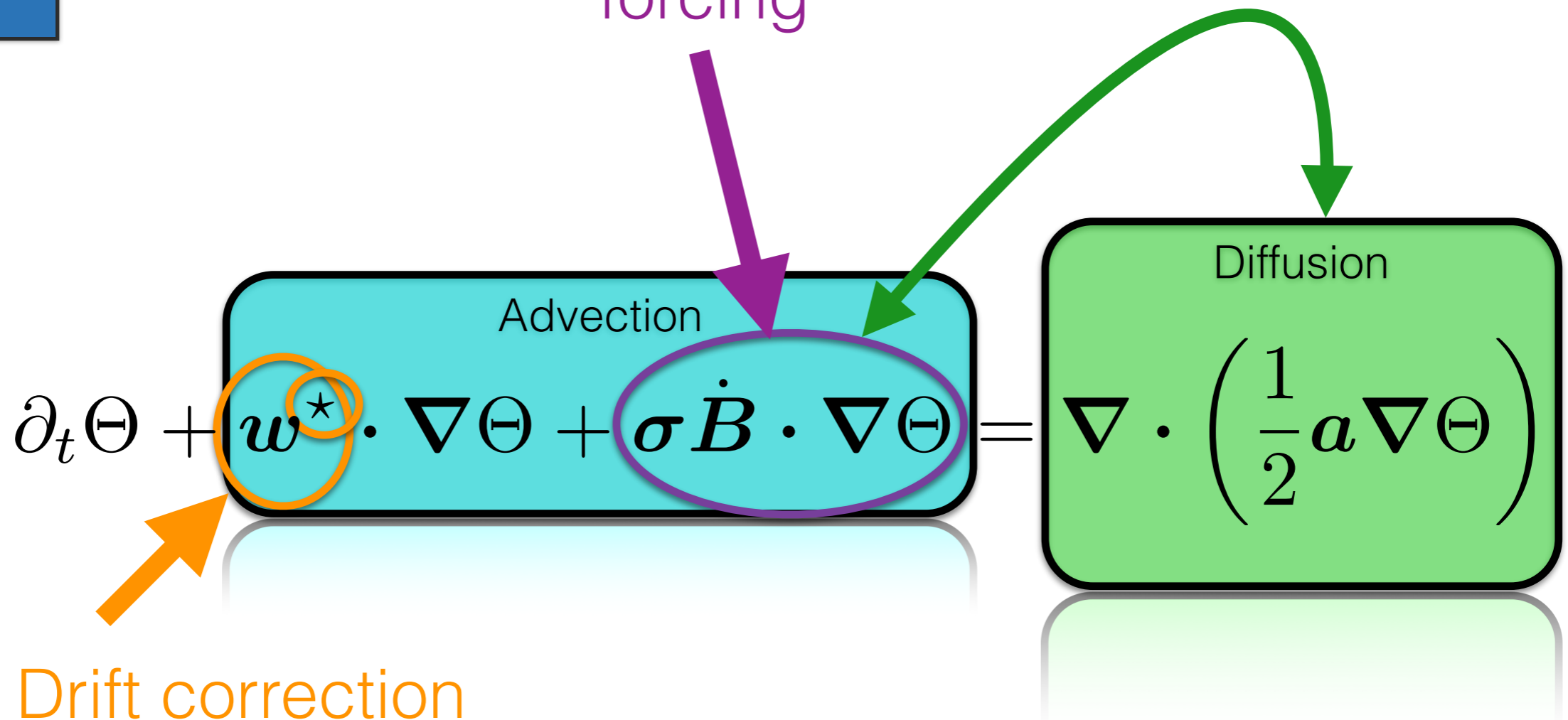
Small scales:

$$\sigma \dot{B}$$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$

Multiplicative random forcing



Advection of tracer Θ

Large scales:
 w

Small scales:
 $\sigma \dot{B}$

Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$

Multiplicative random forcing

Balanced energy exchanges

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Drift correction

Part II

Some parameterization of LU models

Part II

Some parameterization
of LU models

$\sigma \equiv ?$

Spectral model

(homogeneous and stationary $\sigma \dot{B}$)

Large scales:

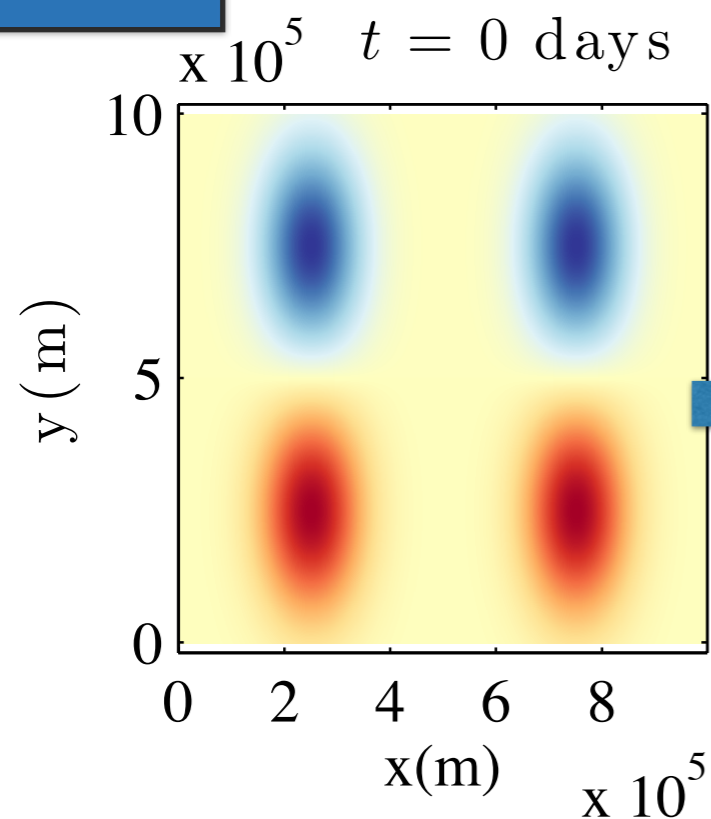
$$w$$

Small scales:

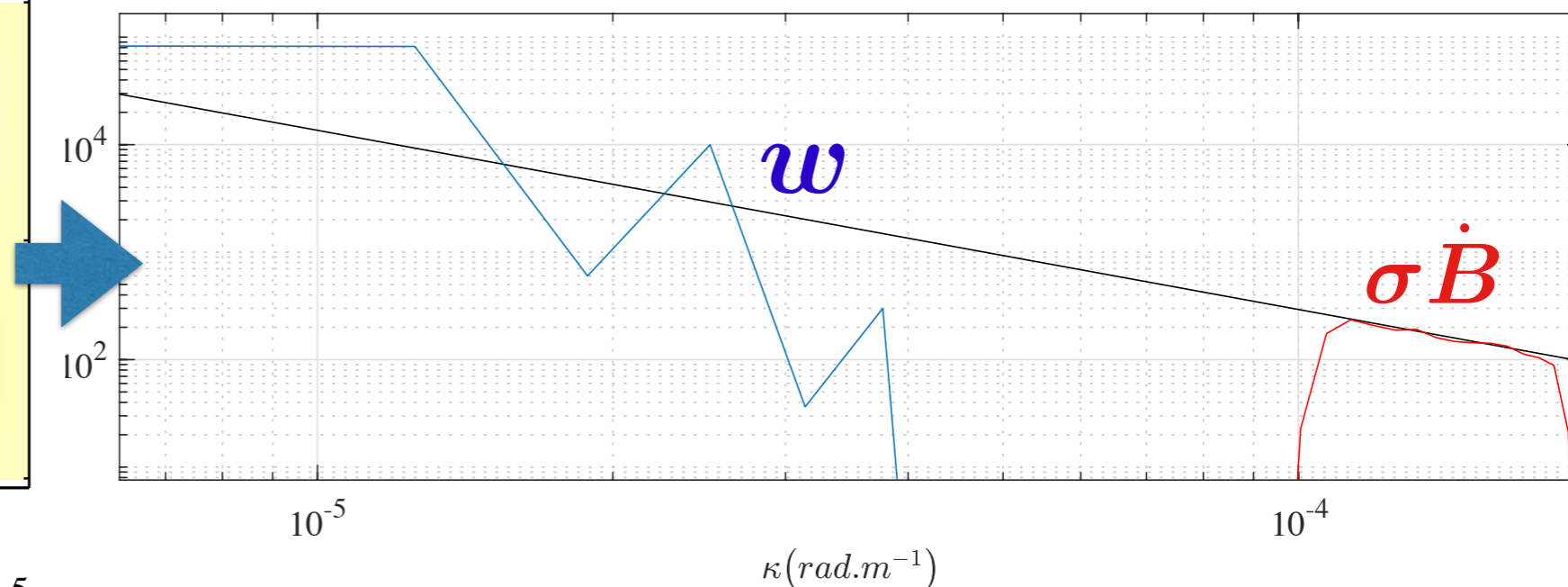
$$\sigma \dot{B}$$

Variance tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$



KE Spectrum



Code online

Spectral model

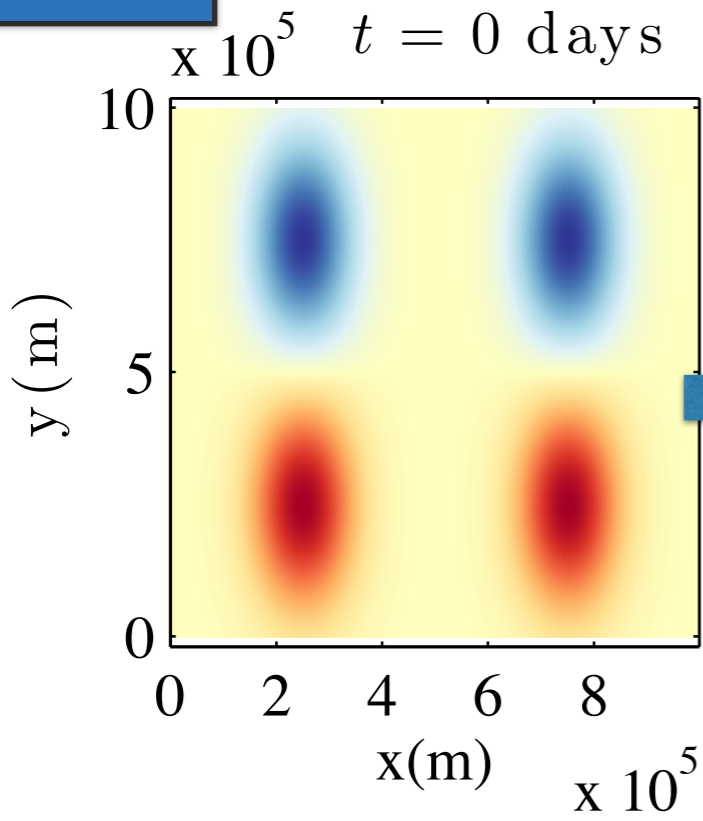
(homogeneous and stationary $\sigma \dot{B}$)

Large scales:
 w

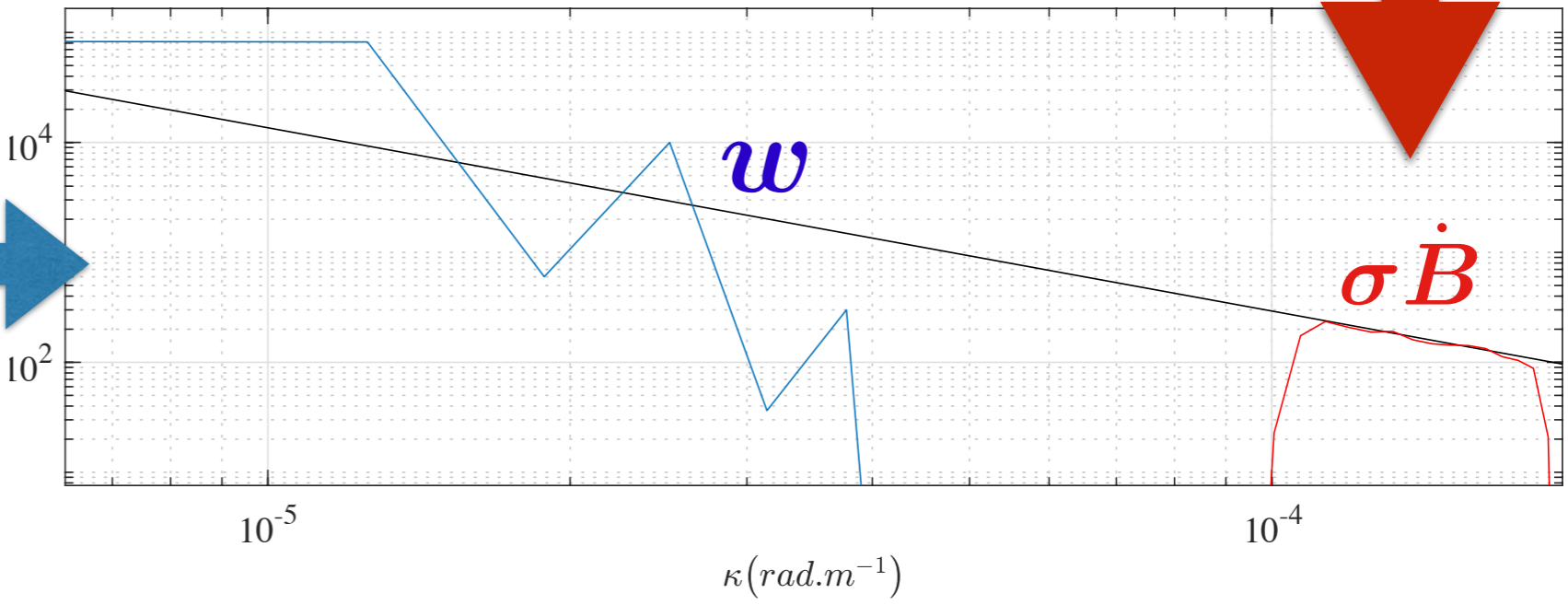
Small scales:
 $\sigma \dot{B}$

Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

Fixed spectrum at small scales



KE Spectrum



Code online

Reference:
Resseguier, Memin & Chapron 2017b

Spectral model

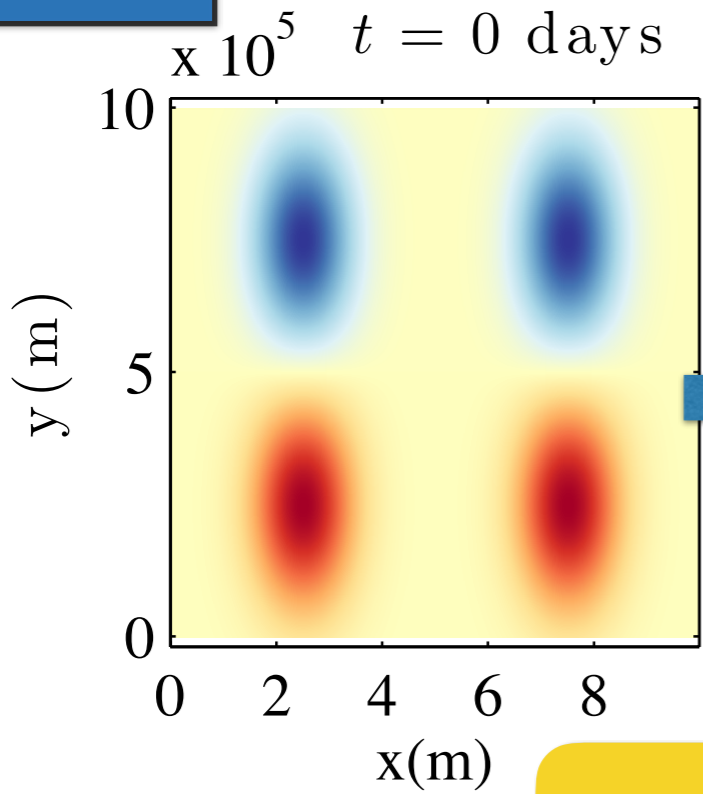
(homogeneous and stationary $\sigma \dot{B}$)

Large scales:
 w

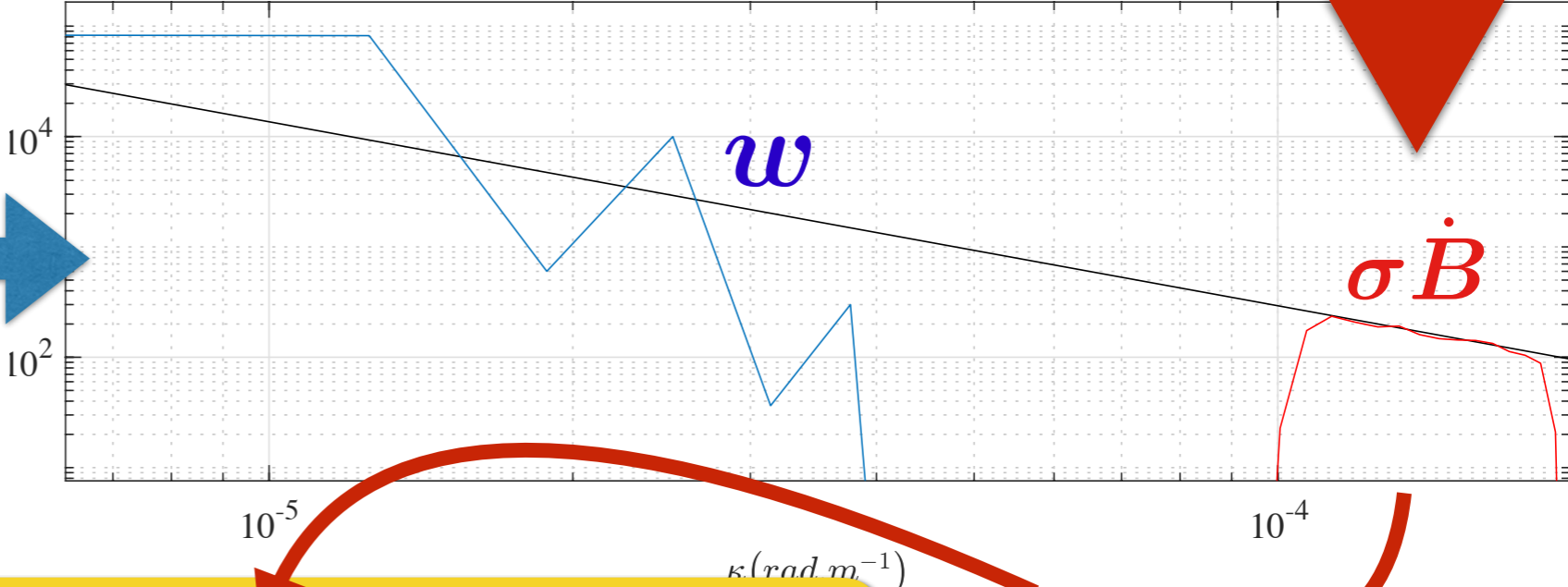
Small scales:
 $\sigma \dot{B}$

Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$

Fixed spectrum at small scales



KE Spectrum



$\sigma \dot{B} = (\text{filter}) * (\text{white noise})$

Code online

Reference:
Resseguier, Memin & Chapron 2017b

Absolute Diffusivity Spectral Density

MU ADSD

Large scales:

w

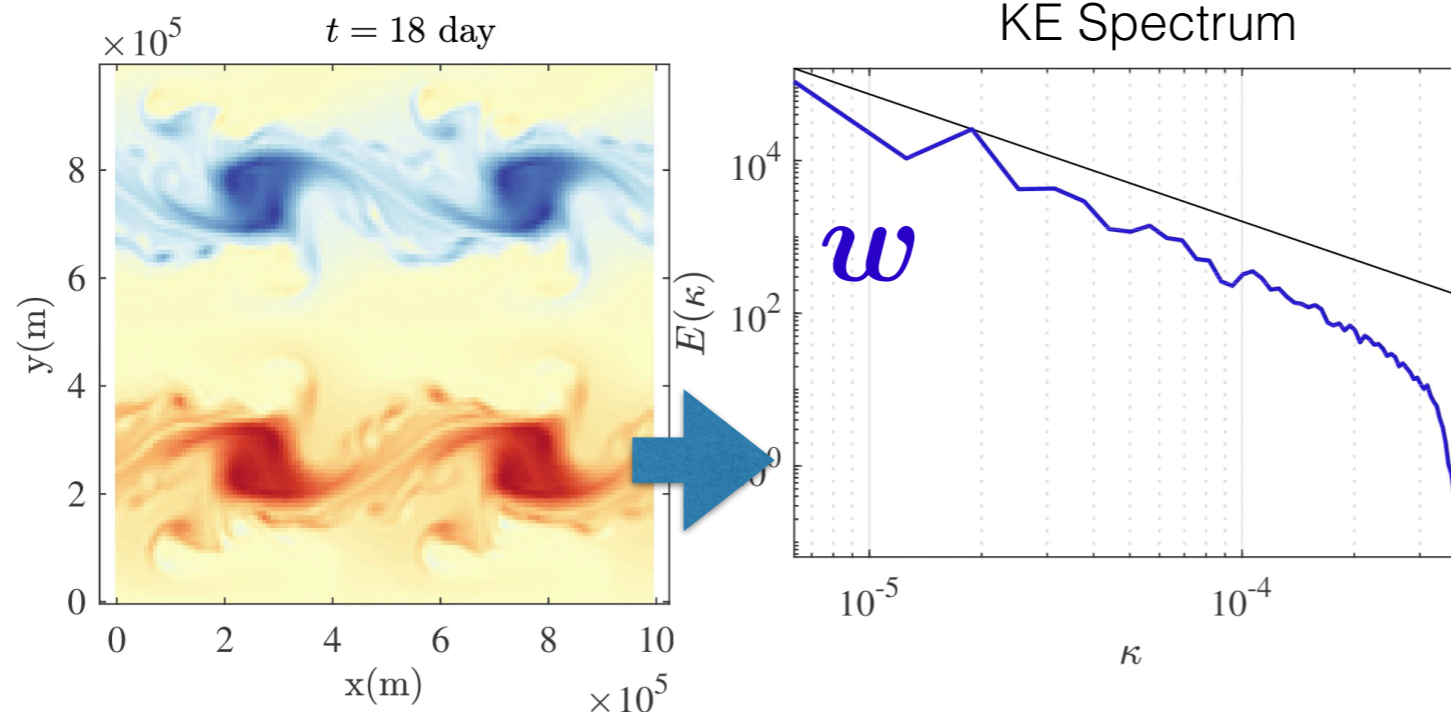
Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

(homogeneous but non-stationary and tuning-free $\sigma \dot{B}$)

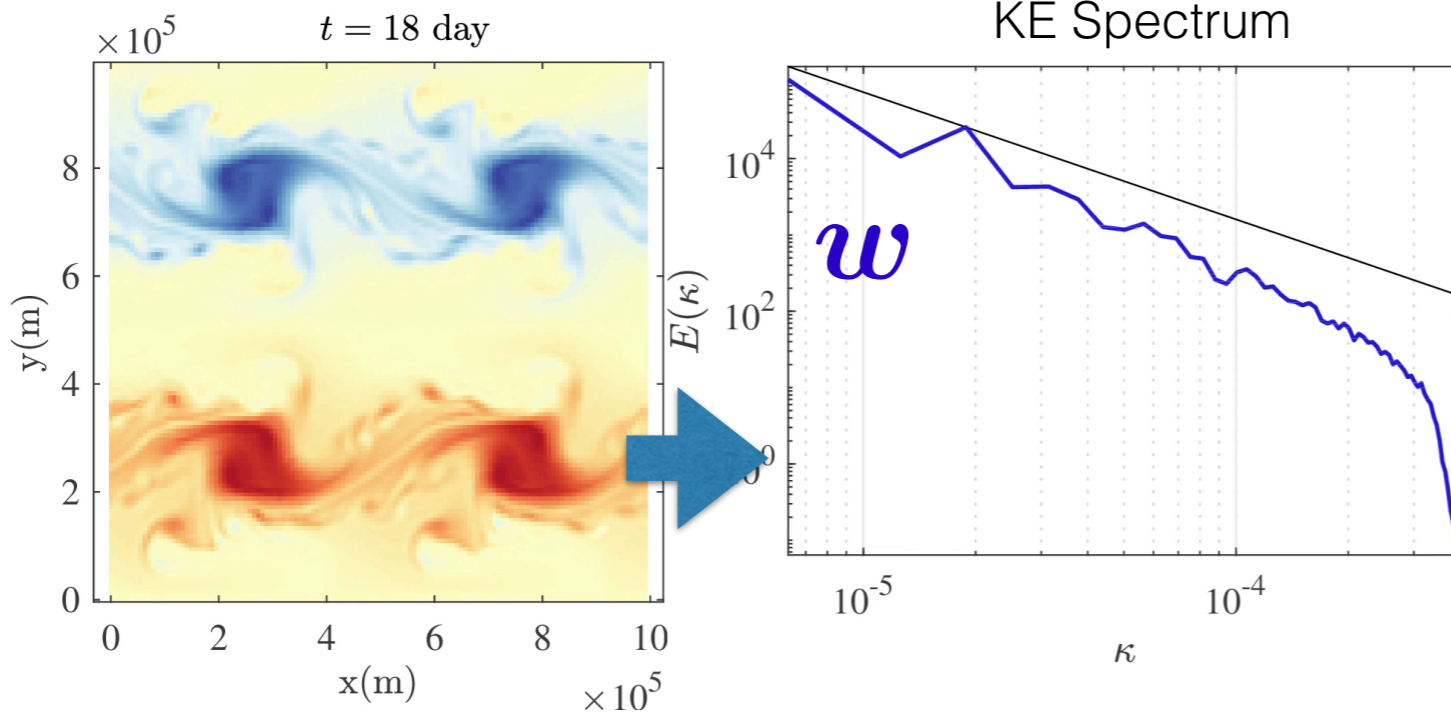


Absolute Diffusivity Spectral Density

(homogeneous but non-stationary and tuning-free $\sigma \dot{B}$)

Large scales:
 w
 Small scales:
 $\sigma \dot{B}$
 Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

Absolute Diffusivity Spectral Density $A(\kappa) = E(\kappa)\tau(\kappa)$



Reference:
Resseguier, Pan & Fox-Kemper 2019a

Absolute Diffusivity Spectral Density

MU ADSD

Large scales:

w

Small scales:

$\sigma \dot{B}$

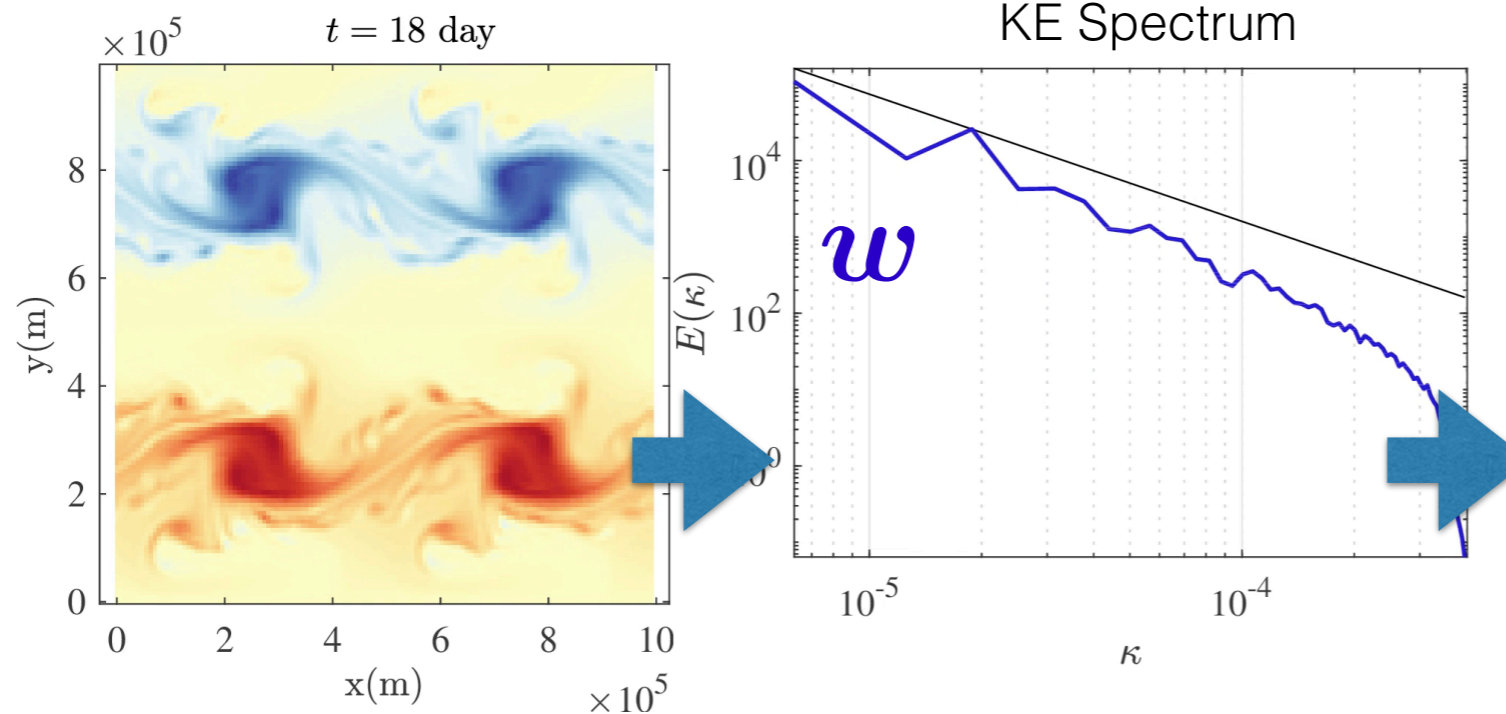
Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

(homogeneous but non-stationary and tuning-free $\sigma \dot{B}$)

Absolute Diffusivity
Spectral Density

$$A(\kappa) = E(\kappa)\tau(\kappa)$$



Reference:
Resseguier, Pan & Fox-Kemper 2019a

Absolute Diffusivity Spectral Density

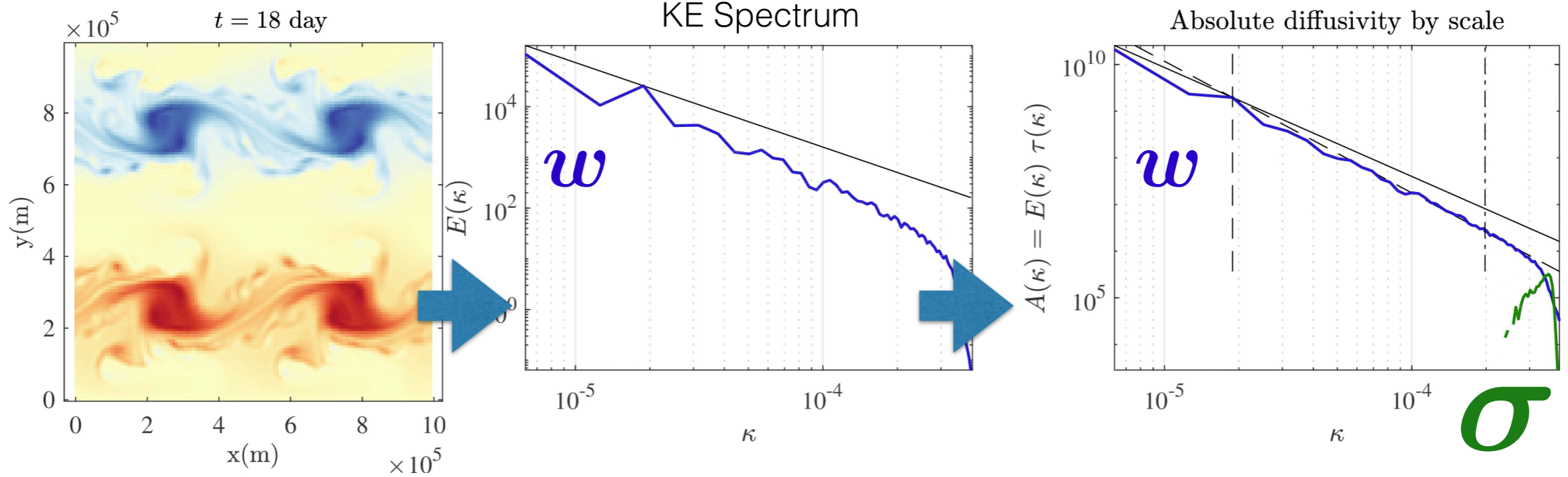
Large scales:
 w

Small scales:
 $\sigma \dot{B}$

Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

(homogeneous but non-stationary and tuning-free $\sigma \dot{B}$)

Absolute Diffusivity Spectral Density $A(\kappa) = E(\kappa)\tau(\kappa)$



Reference:
Resseguier, Pan & Fox-Kemper 2019a

Absolute Diffusivity Spectral Density

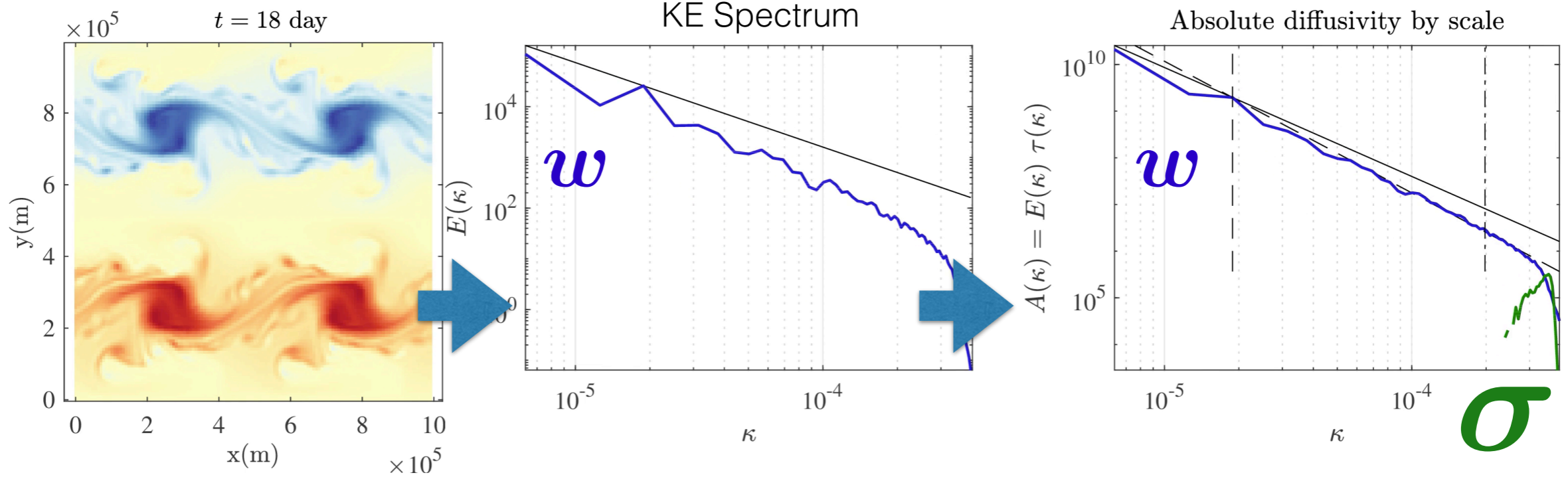
Large scales:
 w

Small scales:
 $\sigma \dot{B}$

Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

(homogeneous but non-stationary and tuning-free $\sigma \dot{B}$)

Absolute Diffusivity Spectral Density $A(\kappa) = E(\kappa)\tau(\kappa)$



Reference:
Resseguier, Pan & Fox-Kemper 2019a

Absolute Diffusivity Spectral Density

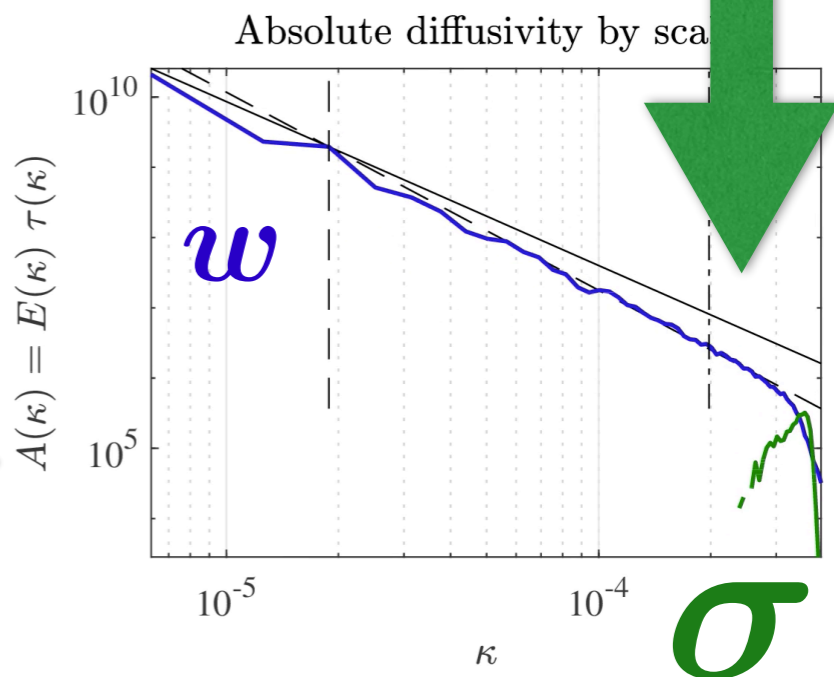
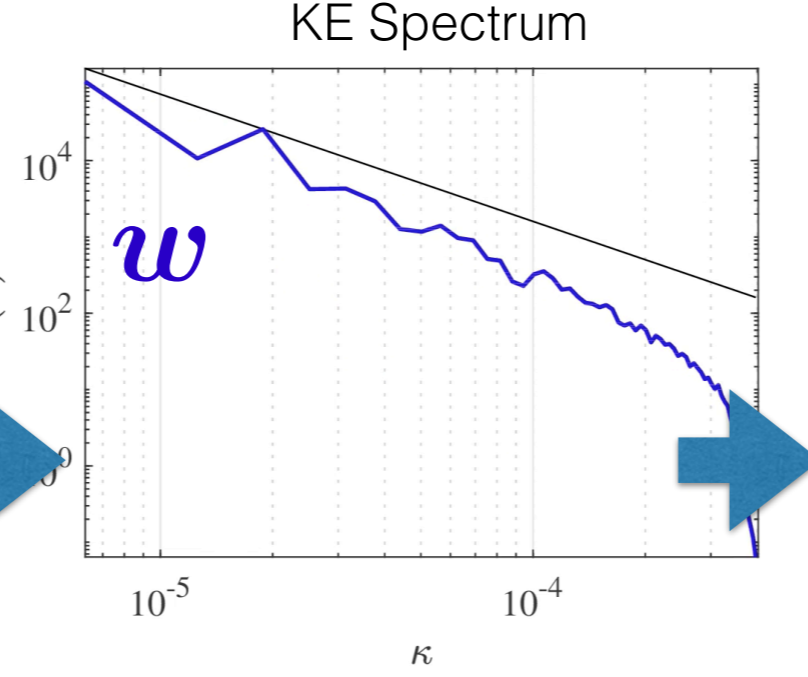
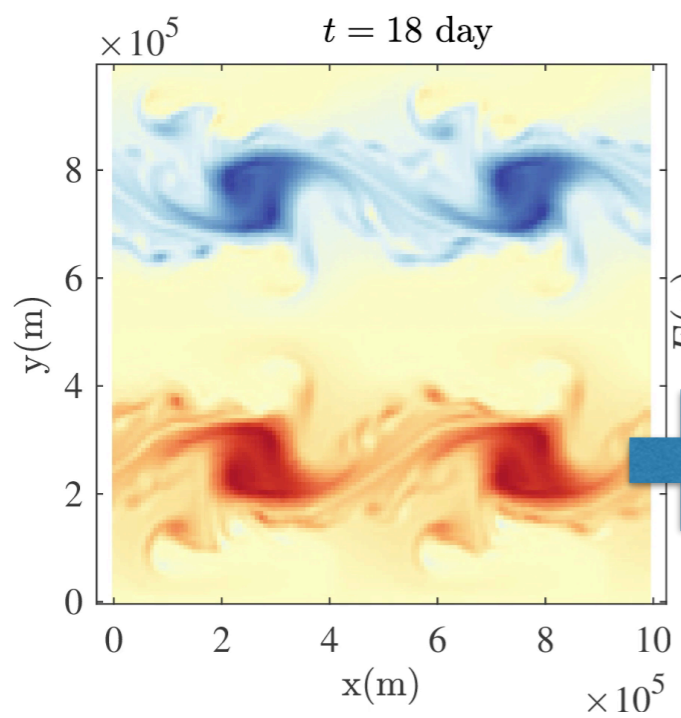
MU ADSD

Large scales:
 w
 Small scales:
 $\sigma \dot{B}$
 Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

(homogeneous but non-stationary and tuning-free $\sigma \dot{B}$)

Absolute Diffusivity Spectral Density $A(\kappa) = E(\kappa)\tau(\kappa)$

Residual non-stationary ADSD



Reference:
Resseguier, Pan & Fox-Kemper 2019a

Absolute Diffusivity Spectral Density

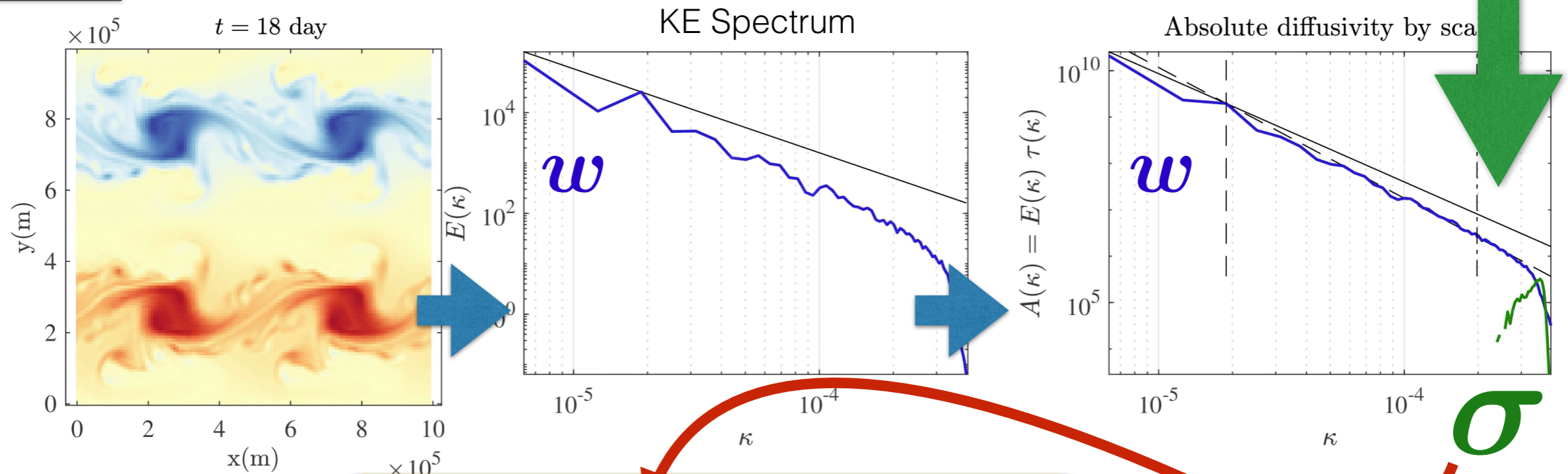
MU ADSD

Large scales:
 w
 Small scales:
 $\sigma \dot{B}$
 Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$

(homogeneous but non-stationary and tuning-free $\sigma \dot{B}$)

Absolute Diffusivity Spectral Density $A(\kappa) = E(\kappa)\tau(\kappa)$

Residual non-stationary ADSD



$\sigma \dot{B} = (\text{filter}) * (\text{white noise})$

Reference: Resseguier, Pan & Fox-Kemper 2019a

Random switching of points

(heterogeneous and non-stationary $\sigma \dot{B}$)

Large scales:

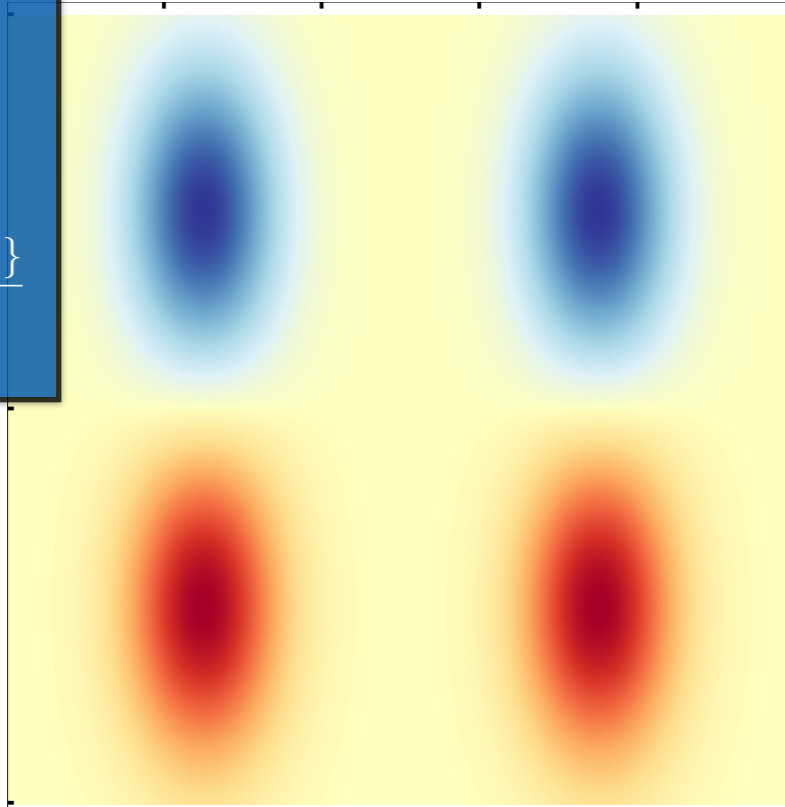
w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

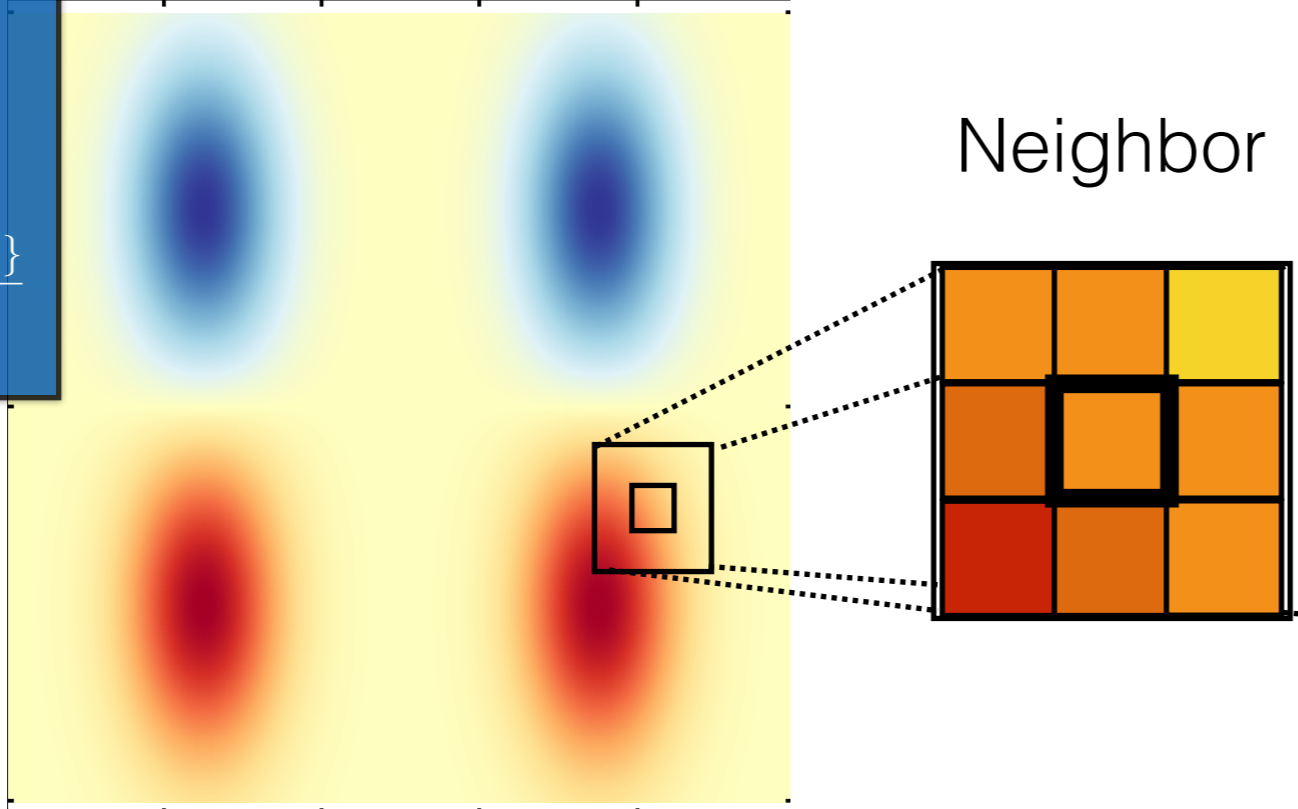
$$a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$$



Random switching of points

(heterogeneous and non-stationary $\sigma \dot{B}$)

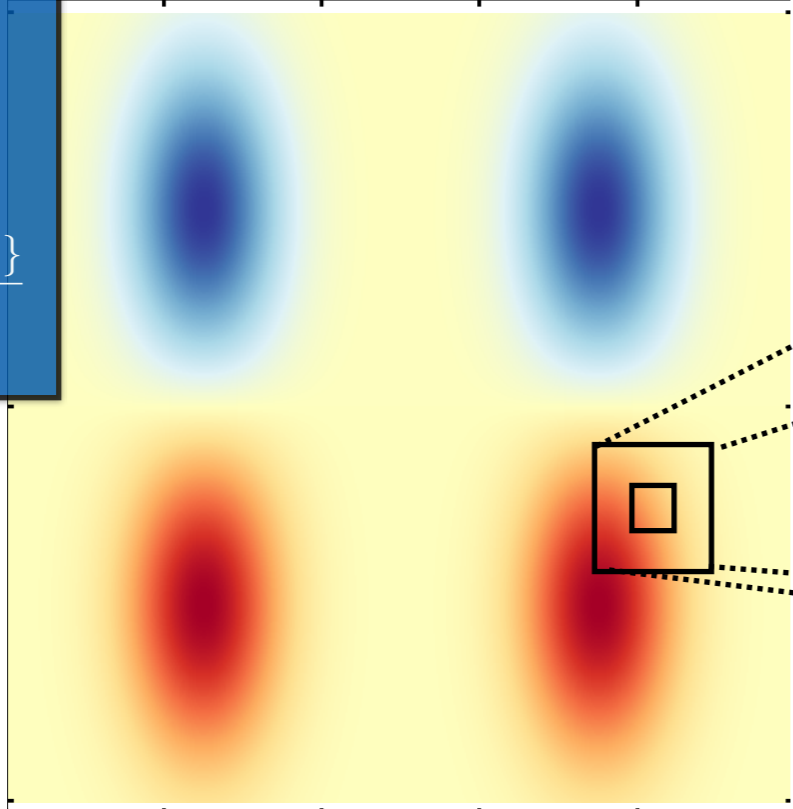
Large scales:
 w
Small scales:
 $\sigma \dot{B}$
Variance
tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



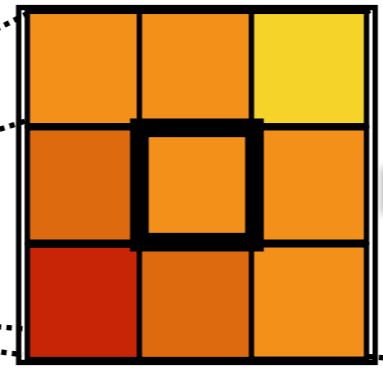
Random switching of points

(heterogeneous and non-stationary $\sigma \dot{B}$)

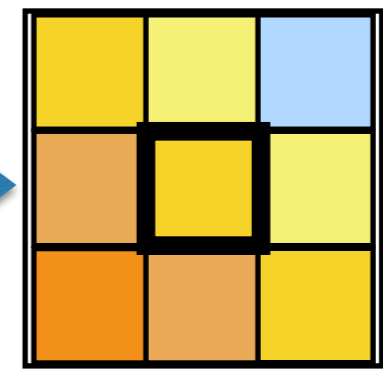
Large scales:
 w
Small scales:
 $\sigma \dot{B}$
Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



Neighbor



Centered neighbor

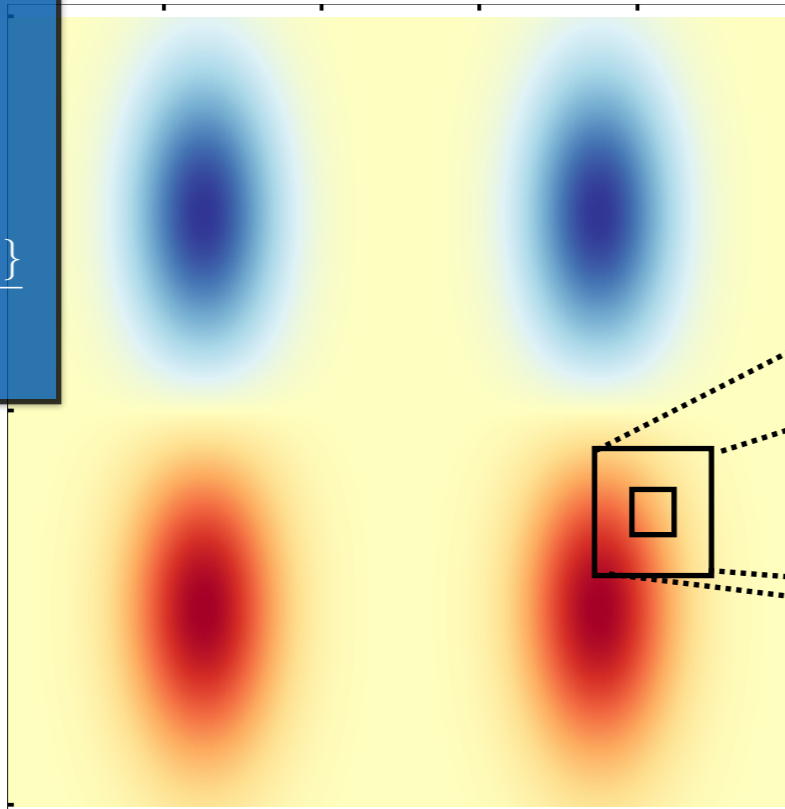


Random switching of points

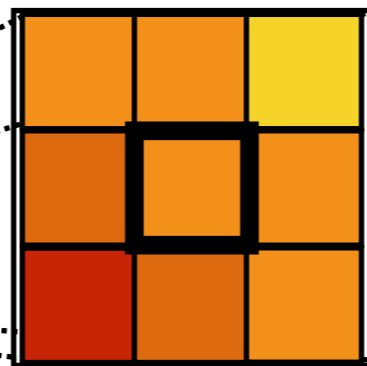
MU SVD

(heterogeneous and non-stationary $\sigma \dot{B}$)

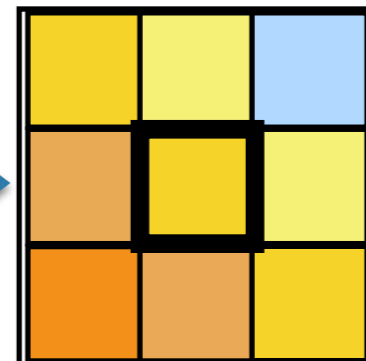
Large scales:
 w
Small scales:
 $\sigma \dot{B}$
Variance
tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



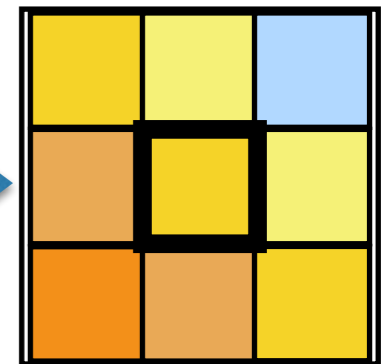
Neighbor



Centered
neighbor



Random
selection

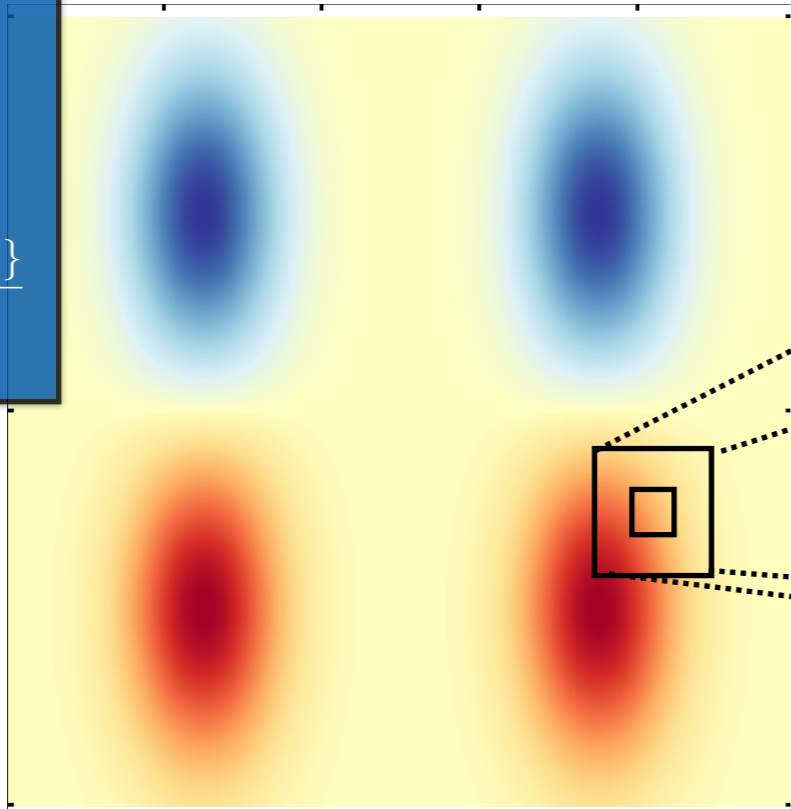


Random switching of points

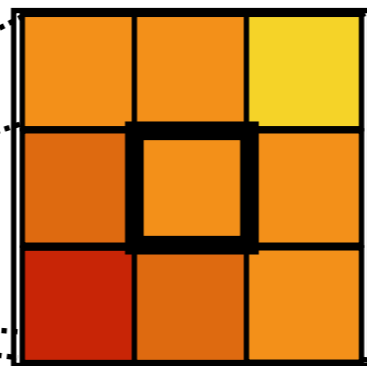
MU SVD

(heterogeneous and non-stationary $\sigma \dot{B}$)

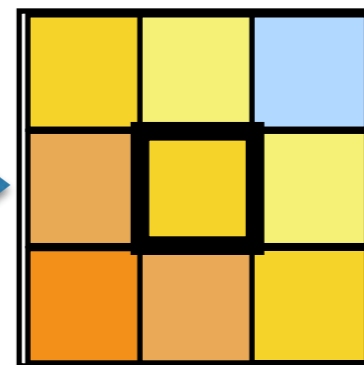
Large scales:
 w
Small scales:
 $\sigma \dot{B}$
Variance
tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



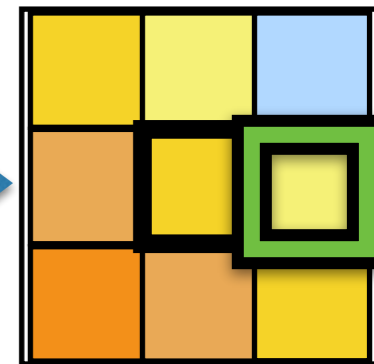
Neighbor



Centered
neighbor



Random
selection

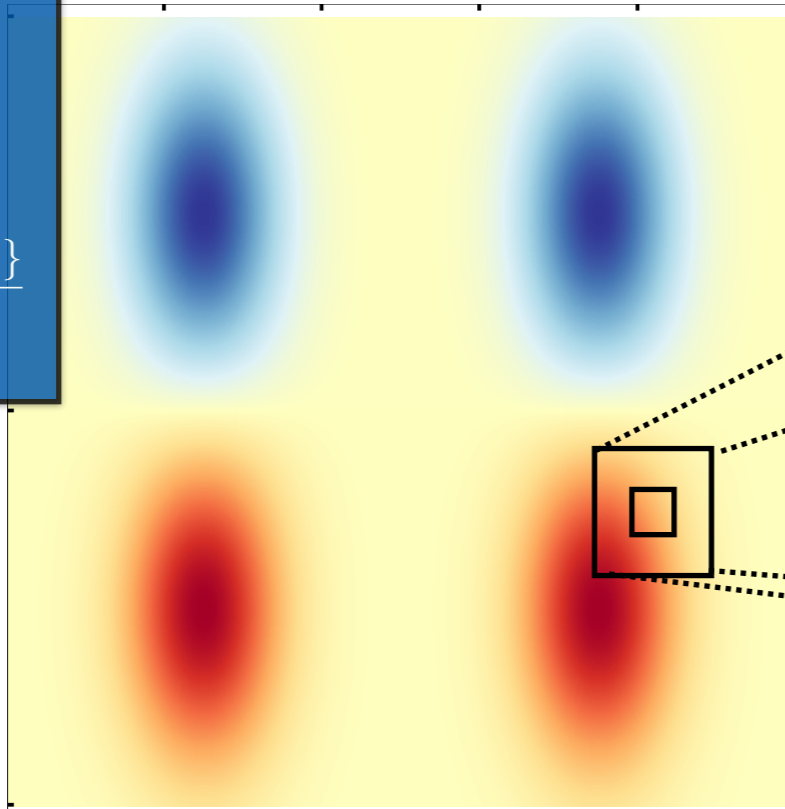


Random switching of points

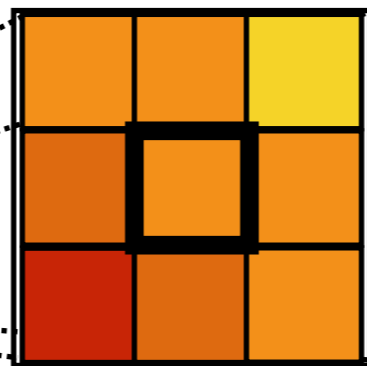
MU SVD

(heterogeneous and non-stationary $\sigma \dot{B}$)

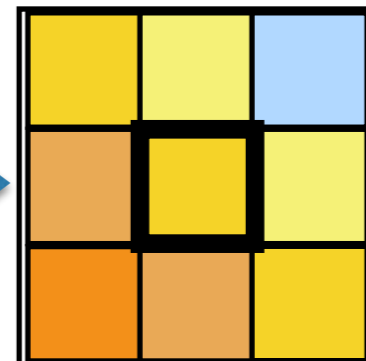
Large scales:
 w
Small scales:
 $\sigma \dot{B}$
Variance
tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



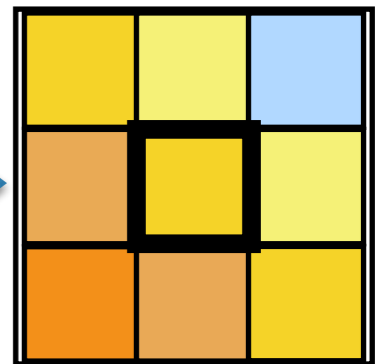
Neighbor



Centered
neighbor



Random
selection

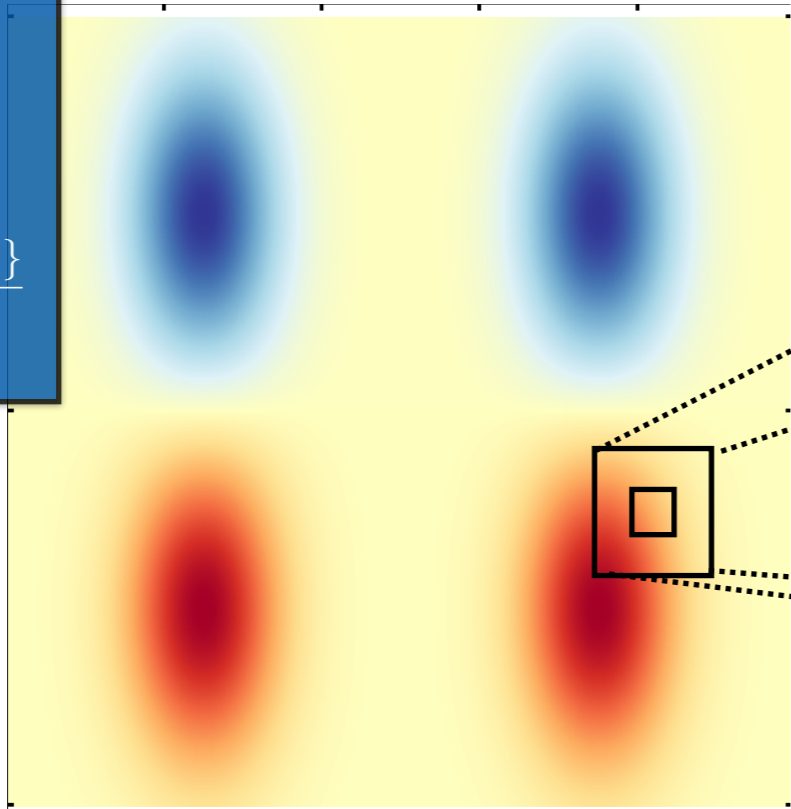


Random switching of points

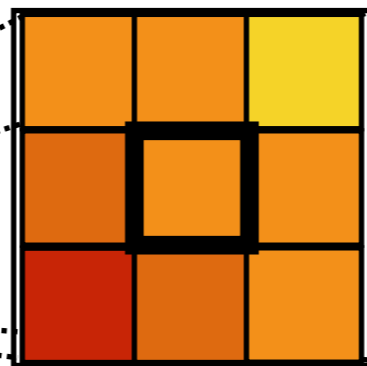
MU SVD

(heterogeneous and non-stationary $\sigma \dot{B}$)

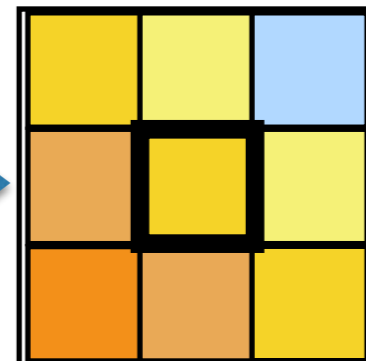
Large scales:
 w
Small scales:
 $\sigma \dot{B}$
Variance
tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



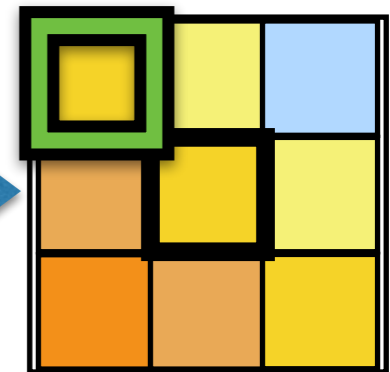
Neighbor



Centered
neighbor



Random
selection

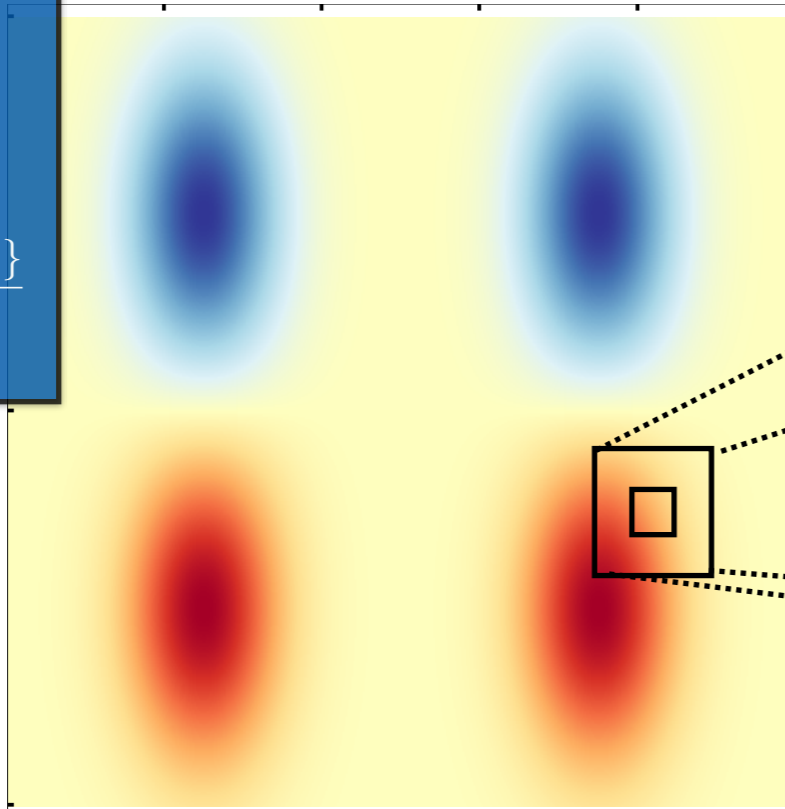


Random switching of points

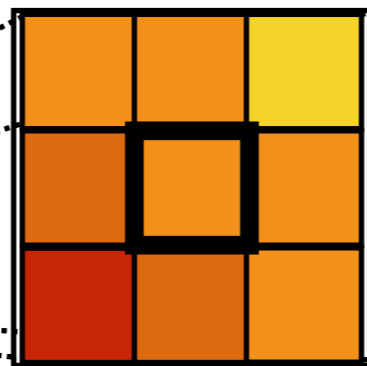
MU SVD

(heterogeneous and non-stationary $\sigma \dot{B}$)

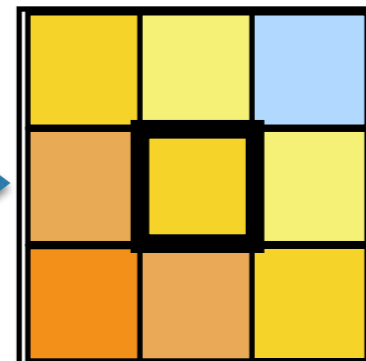
Large scales:
 w
Small scales:
 $\sigma \dot{B}$
Variance
tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



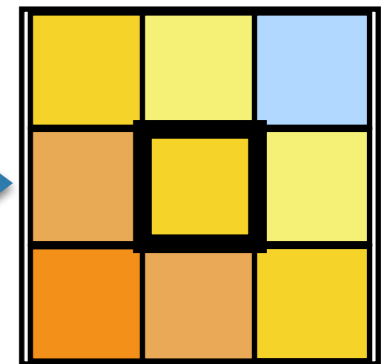
Neighbor



Centered
neighbor



Random
selection

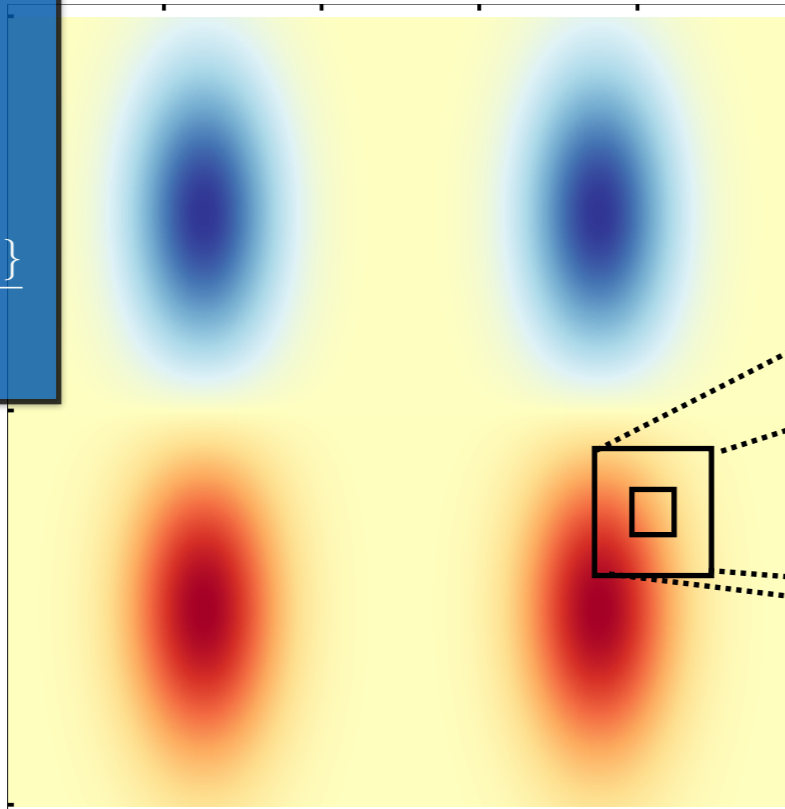


Random switching of points

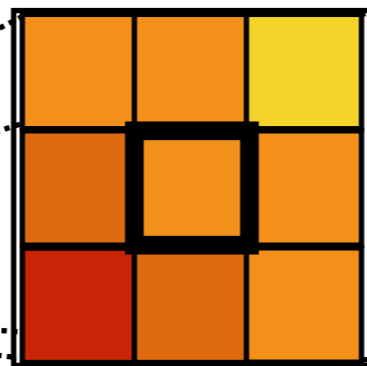
MU SVD

(heterogeneous and non-stationary $\sigma \dot{B}$)

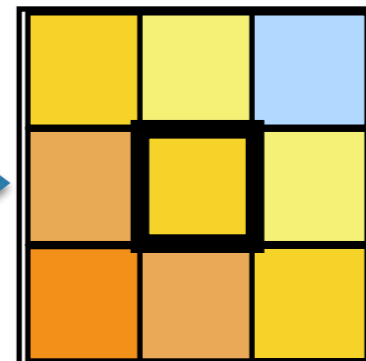
Large scales:
 w
Small scales:
 $\sigma \dot{B}$
Variance
tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



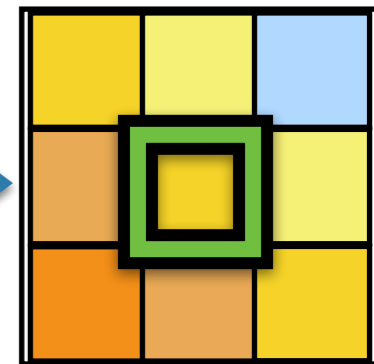
Neighbor



Centered
neighbor



Random
selection

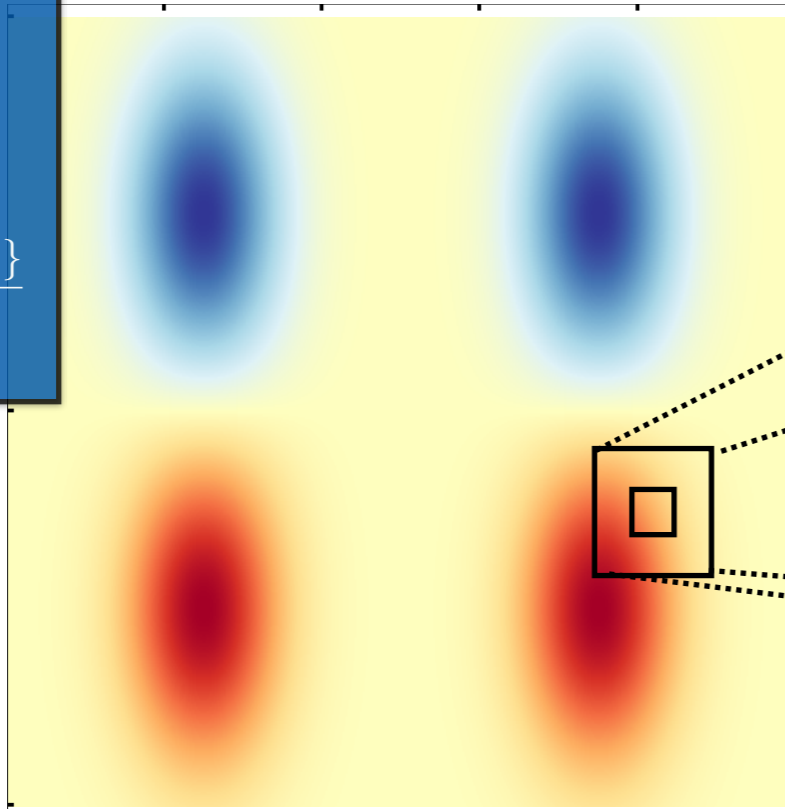


Random switching of points

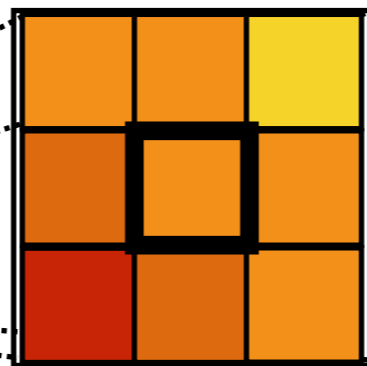
MU SVD

(heterogeneous and non-stationary $\sigma \dot{B}$)

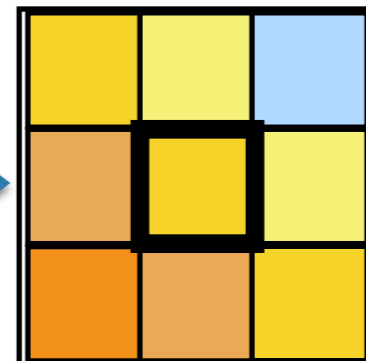
Large scales:
 w
Small scales:
 $\sigma \dot{B}$
Variance
tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



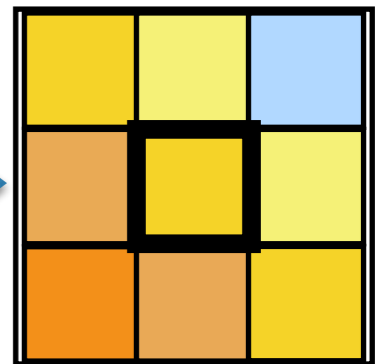
Neighbor



Centered
neighbor



Random
selection

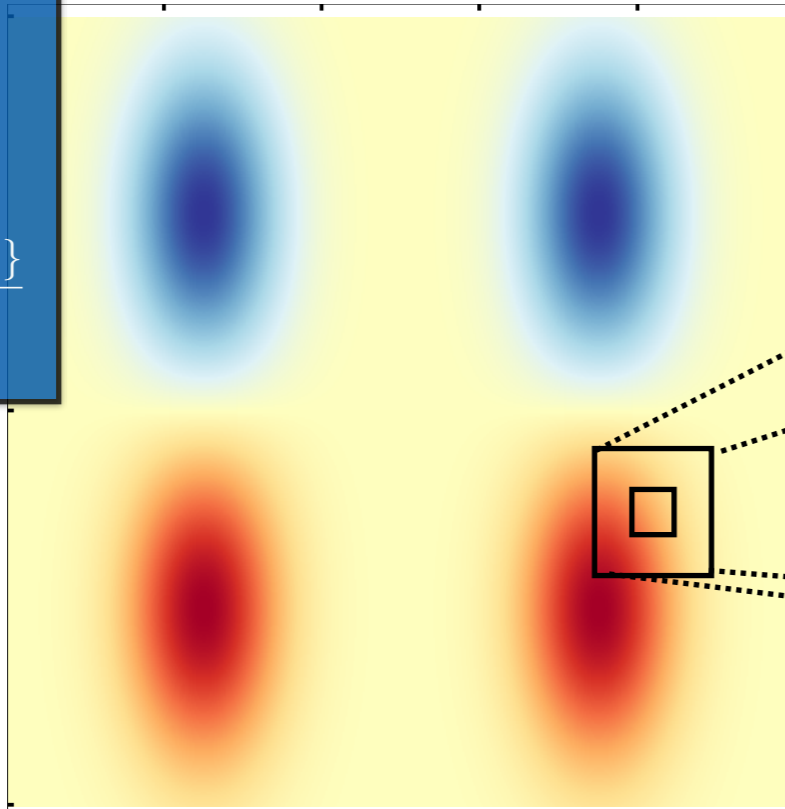


Random switching of points

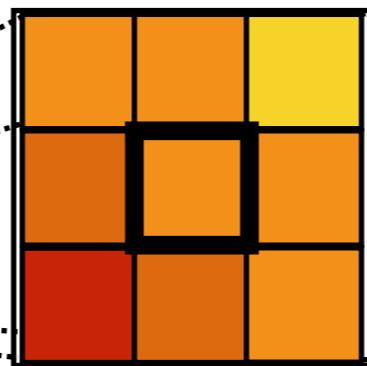
MU SVD

(heterogeneous and non-stationary $\sigma \dot{B}$)

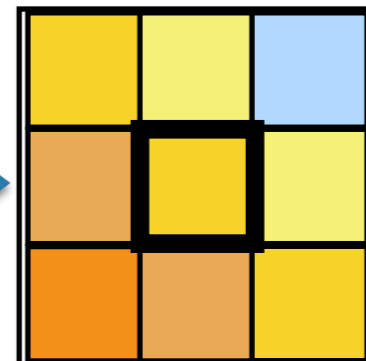
Large scales:
 w
Small scales:
 $\sigma \dot{B}$
Variance
tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



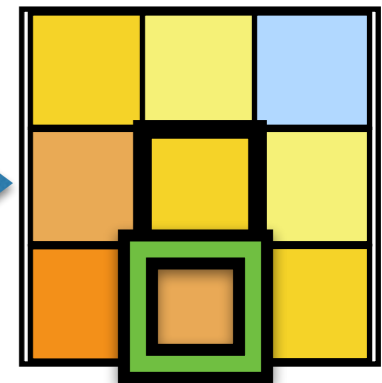
Neighbor



Centered
neighbor



Random
selection

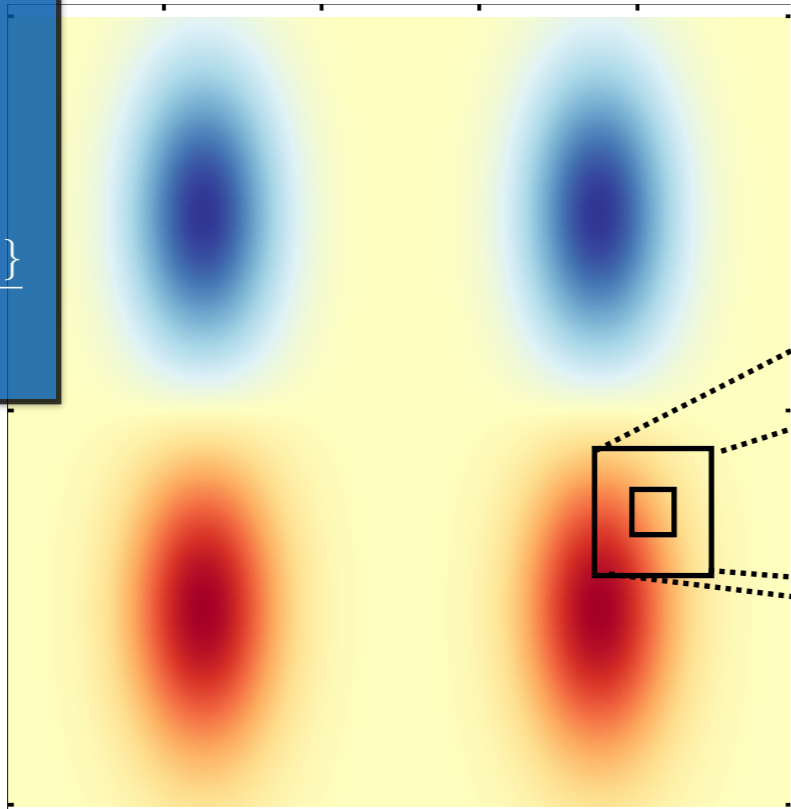


Random switching of points

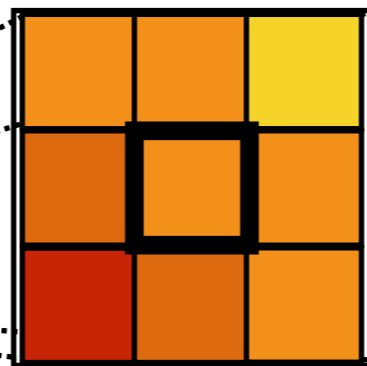
MU SVD

(heterogeneous and non-stationary $\sigma \dot{B}$)

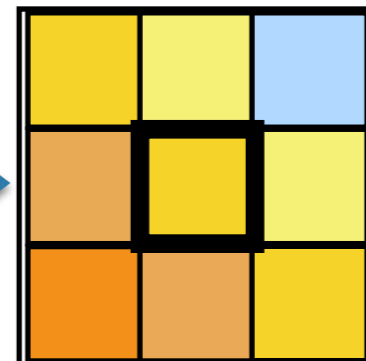
Large scales:
 w
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 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma d\mathbf{B} (\sigma d\mathbf{B})^T\}}{dt}$



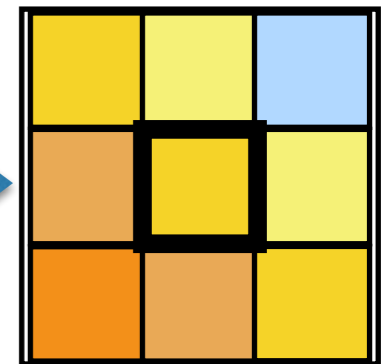
Neighbor



Centered
neighbor



Random
selection

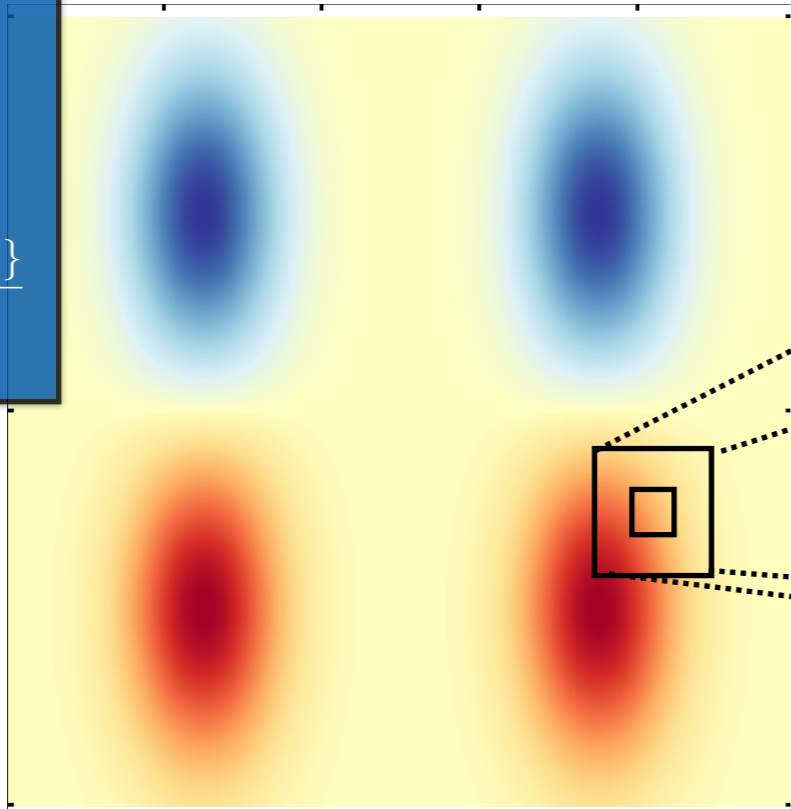


Random switching of points

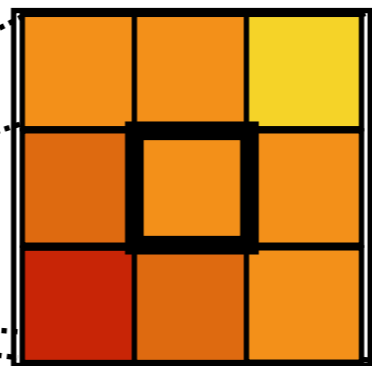
MU SVD

Large scales:
 w
 Small scales:
 $\sigma \dot{B}$
 Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

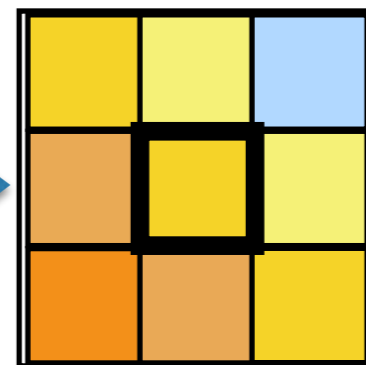
(heterogeneous and non-stationary $\sigma \dot{B}$)



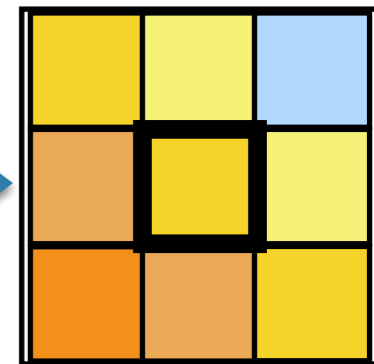
Neighbor



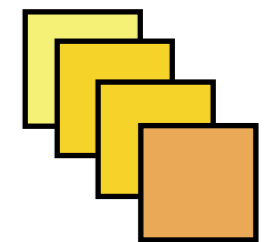
Centered neighbor



Random selection



Local ensemble :

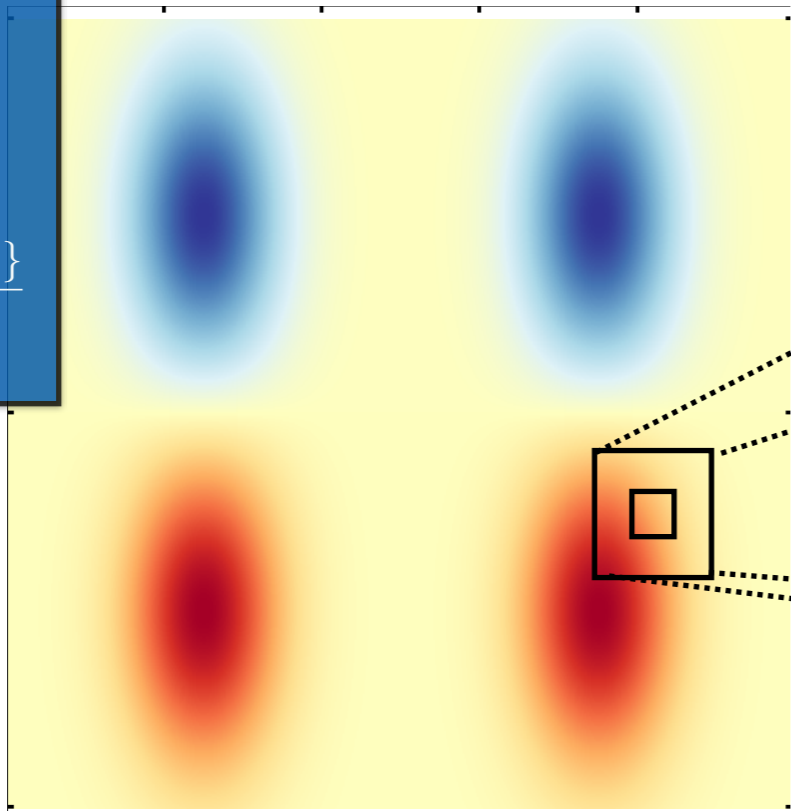


Random switching of points

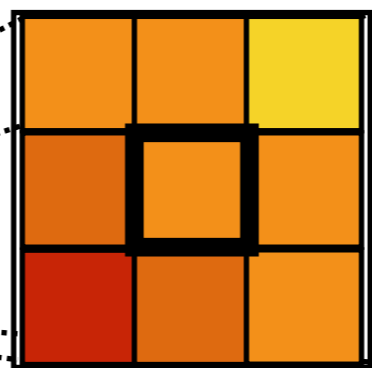
MU SVD

(heterogeneous and non-stationary $\sigma \dot{B}$)

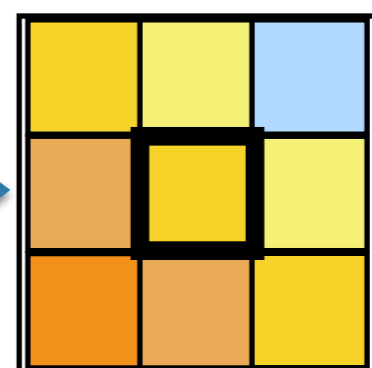
Large scales:
 w
Small scales:
 $\sigma \dot{B}$
Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$



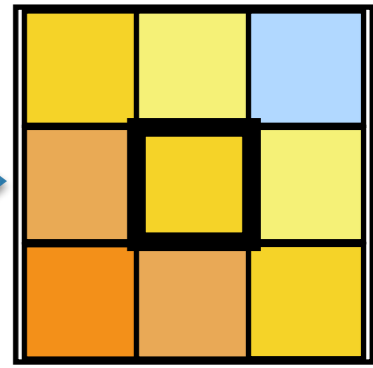
Neighbor



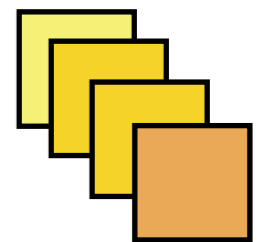
Centered neighbor



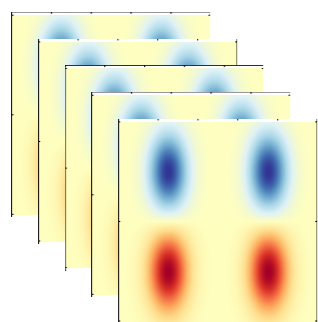
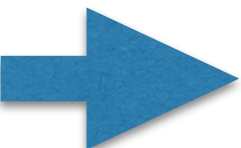
Random selection



Local ensemble :



Global ensemble

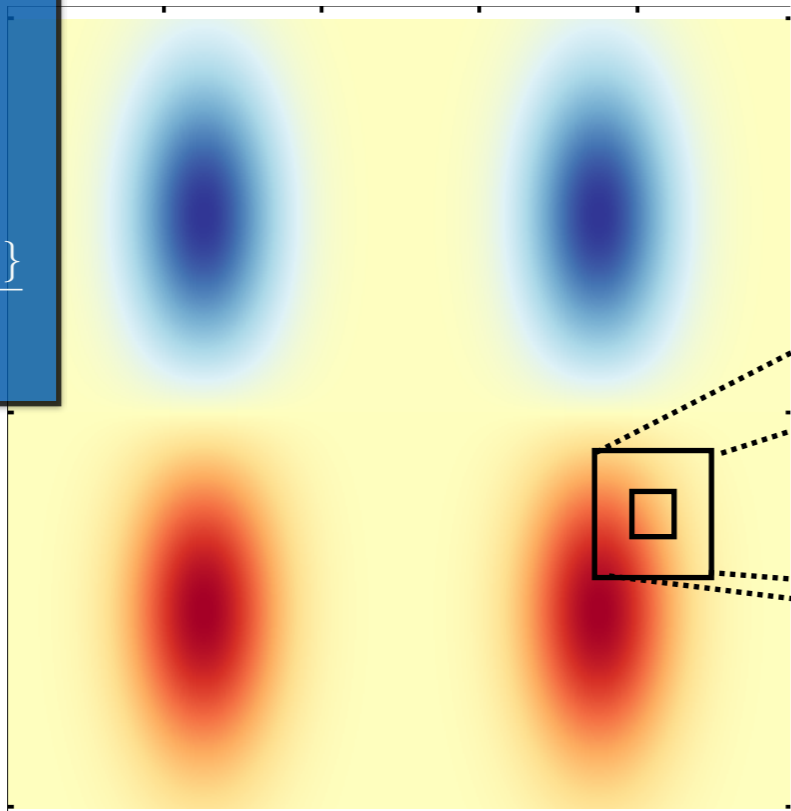


Random switching of points

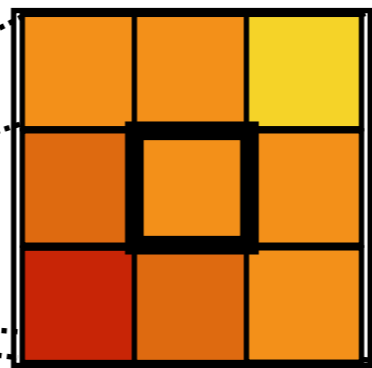
MU SVD

Large scales:
 w
Small scales:
 $\sigma \dot{B}$
Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

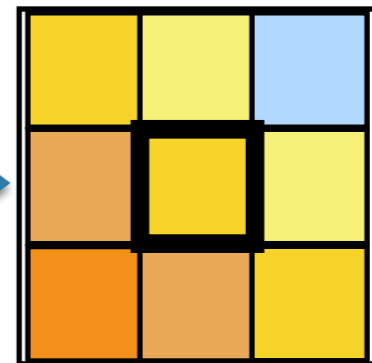
(heterogeneous and non-stationary $\sigma \dot{B}$)



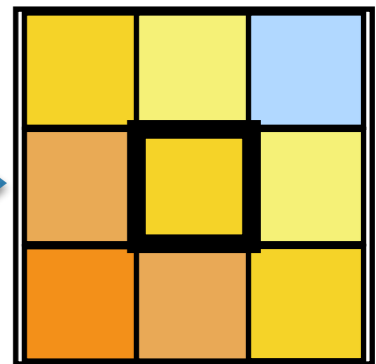
Neighbor



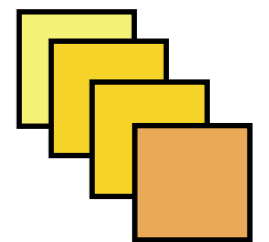
Centered neighbor



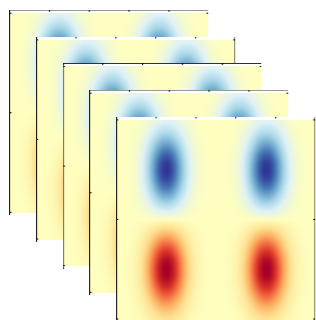
Random selection



Local ensemble :



Global ensemble



SVD



$\sigma \dot{B}$

Part III

A new energy-budget-based
stochastic scheme:

WaveHyperv


WaveHyperv

Transport
equation

$$\frac{Dq}{Dt} = \mathcal{L}[q] + \eta$$

WaveHyperv

Transport equation

$$\frac{Dq}{Dt} = \mathcal{L}[q] + \eta$$


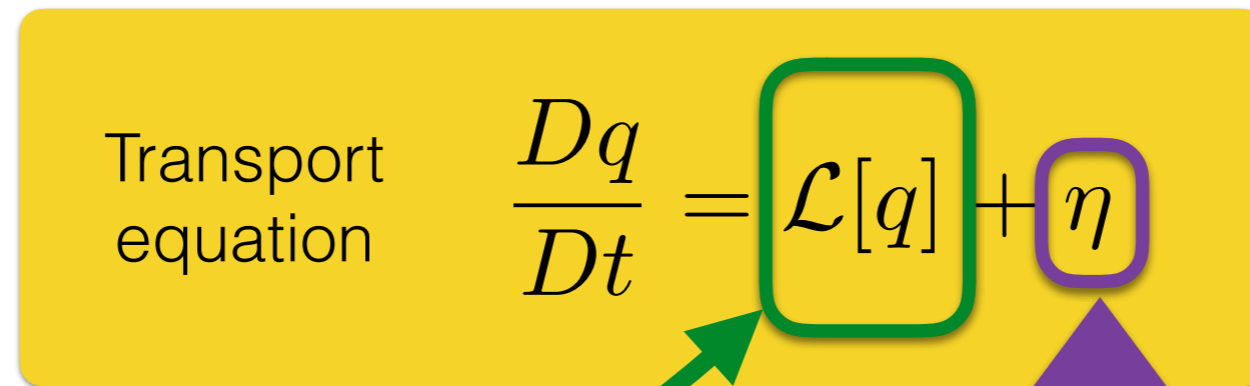
Usual
deterministic
subgrid tensor

e.g. Hyper-viscosity

$$\mathcal{L}[q] = -\nu \Delta^4 q$$

WaveHyperv

Transport equation

$$\frac{Dq}{Dt} = \mathcal{L}[q] + \eta$$


Usual
deterministic
subgrid tensor
e.g. Hyper-viscosity

$$\mathcal{L}[q] = -\nu \Delta^4 q$$

Random forcing

built to meet the energy budget:

$$(\text{Random energy intake}) = \zeta \times \text{Dissipation}$$

Summary of UQ methods

Name	Method
MU Spec	LU with homogeneous and stationary small-scale velocity
MU ADSD	LU with homogeneous, non-stationary and tuning-free small-scale velocity
MU SVD	LU with inhomogeneous and non-stationary small-scale velocity
WaveHyperv	Energy-budget-based stochastic scheme
PIC Spec	Perturbed initial conditions with homogeneous noise
PIC SVD	Perturbed initial conditions with inhomogeneous noise

Part IV

Numerical comparisons

Test case 1:

$t = 17$ days

SQG

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

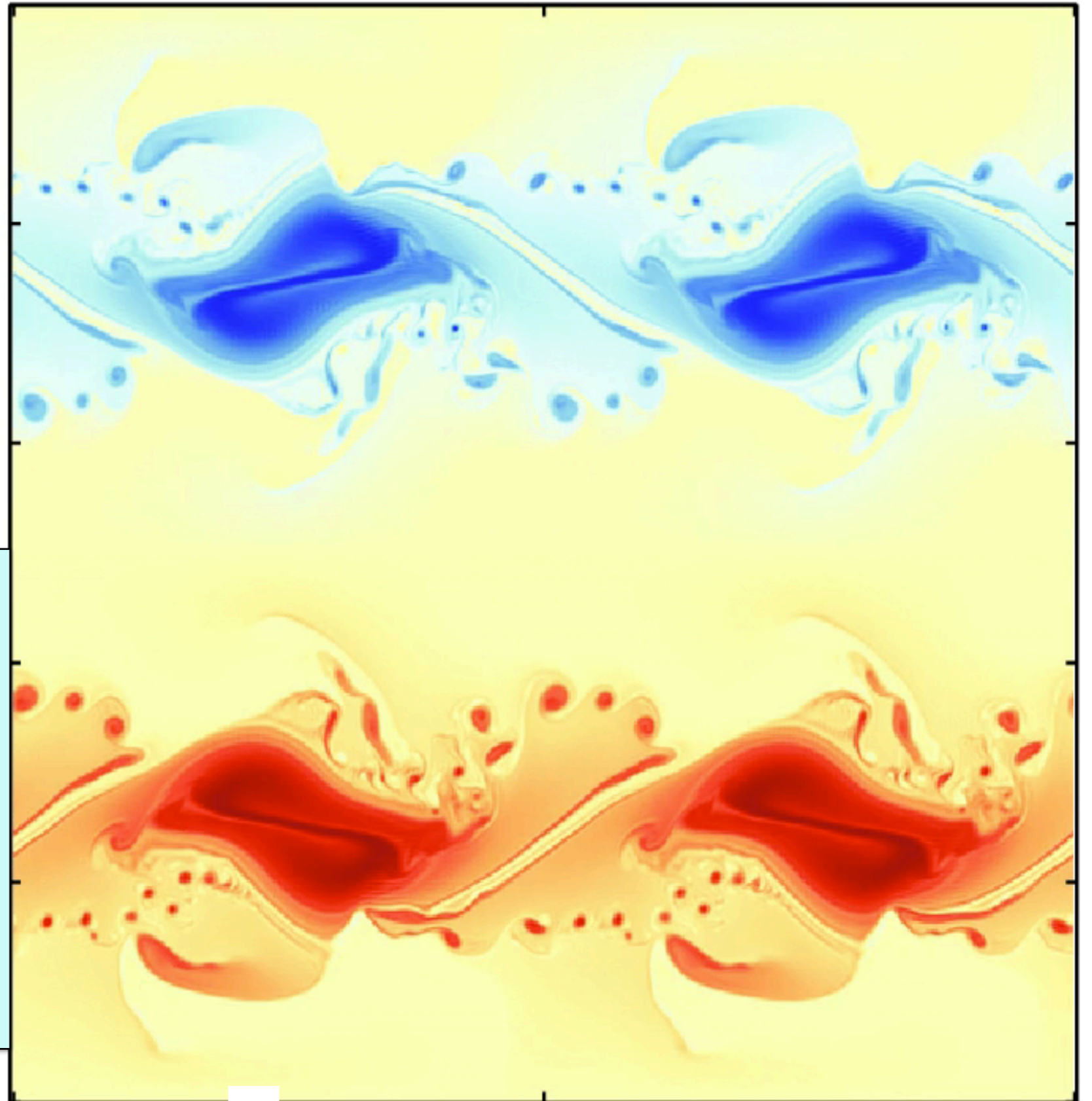
$$\mathbf{u} = \left(\text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

SQG

512 x 512



Test case 1:

$t = 17$ days

SQG

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

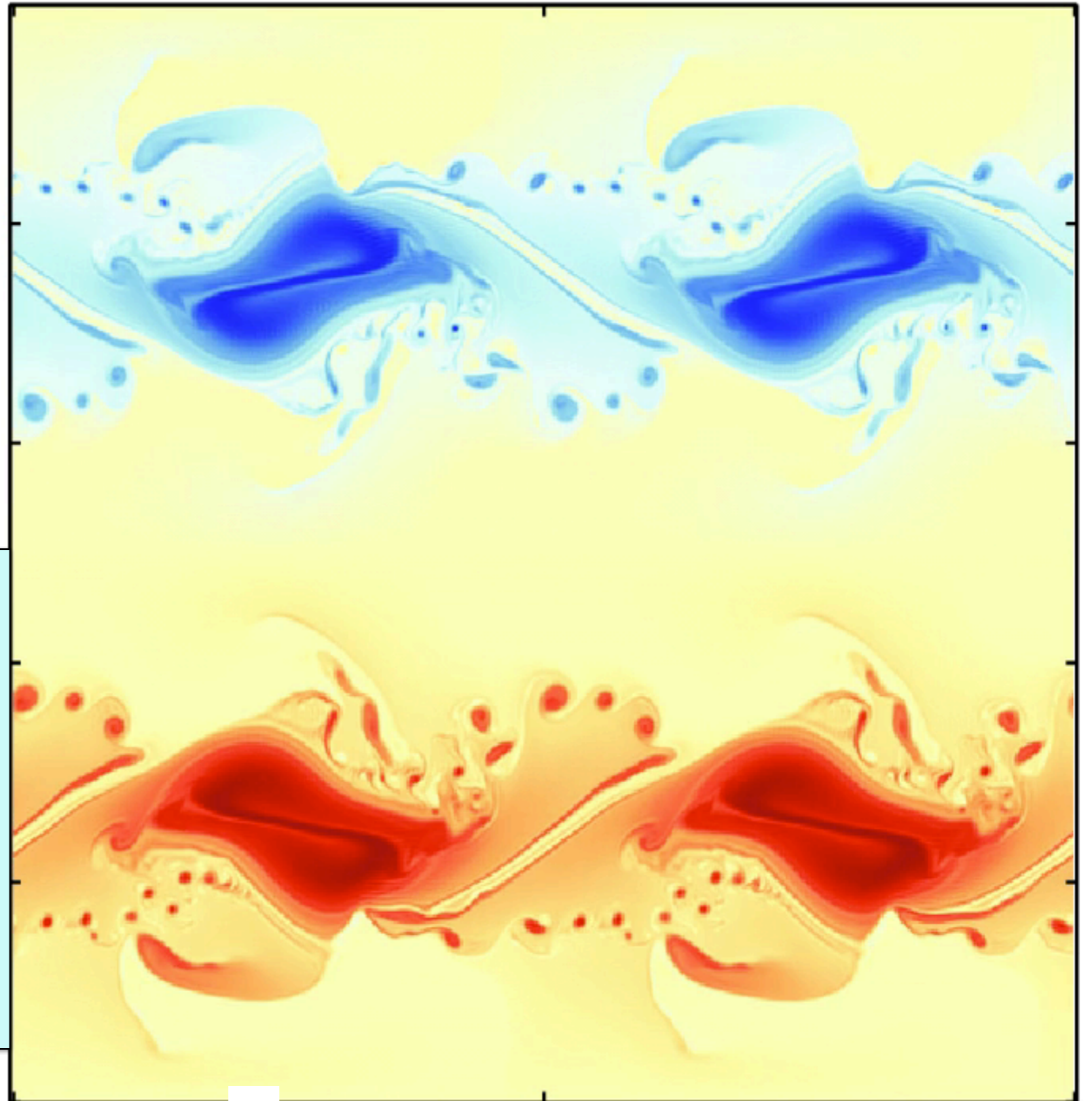
$$\mathbf{u} = \left(\text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

SQG

512 x 512



Test case 2:

$t = 10$ days

SQG

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

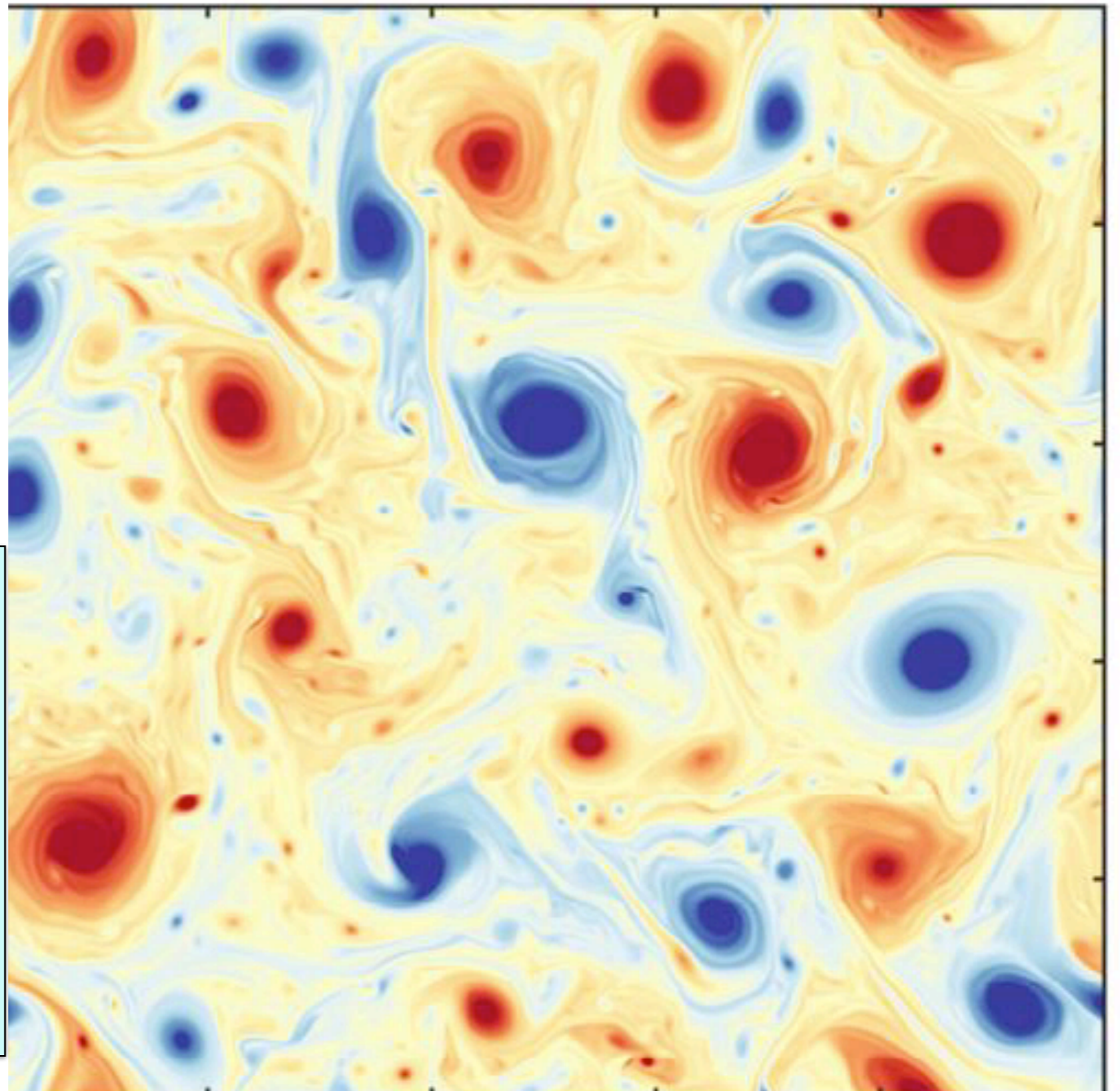
$$\mathbf{u} = \left(\text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

SQG

512 x 512



Test case 2:

$t = 10$ days

SQG

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

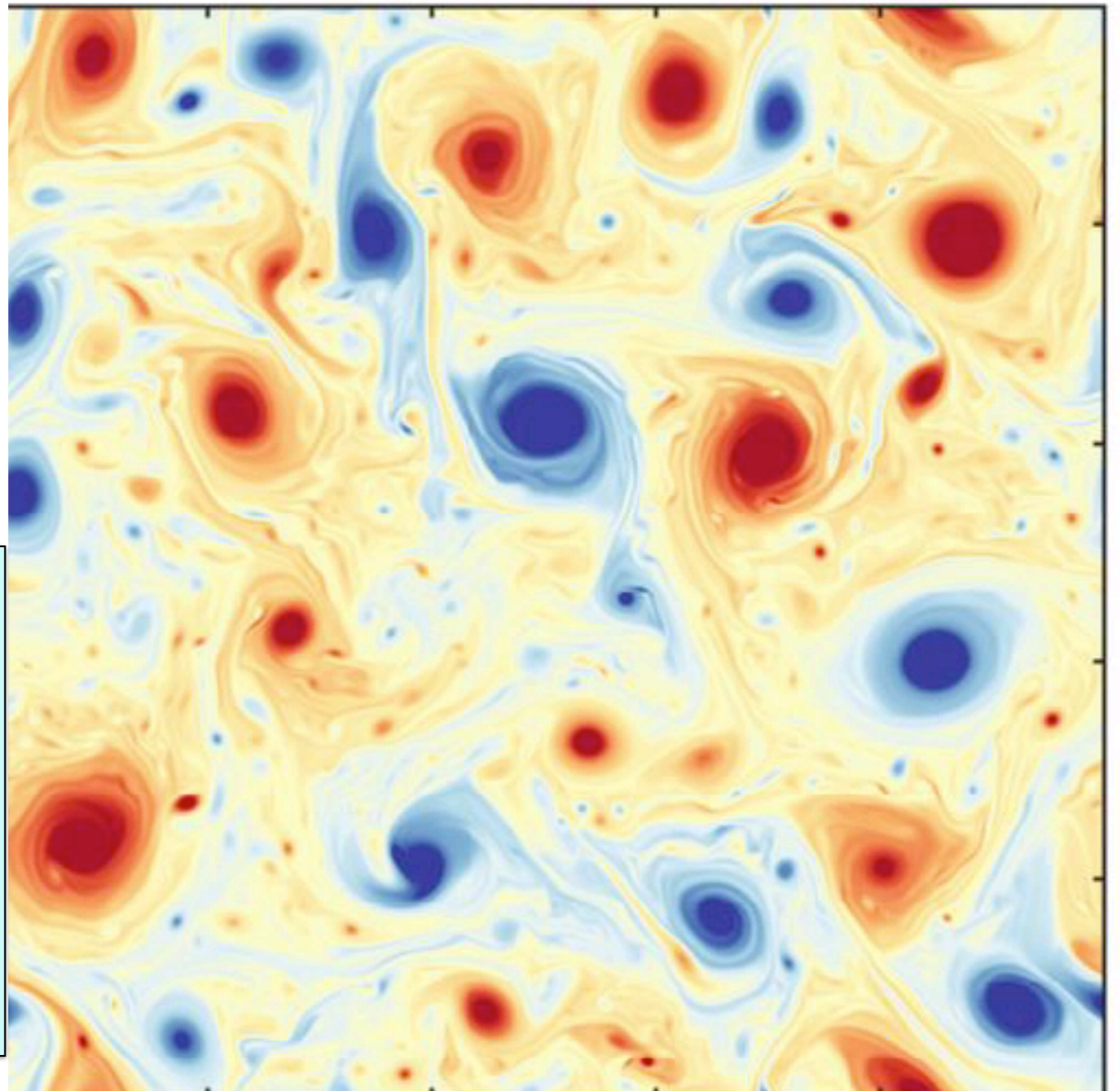
$$\mathbf{u} = \left(\text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

SQG

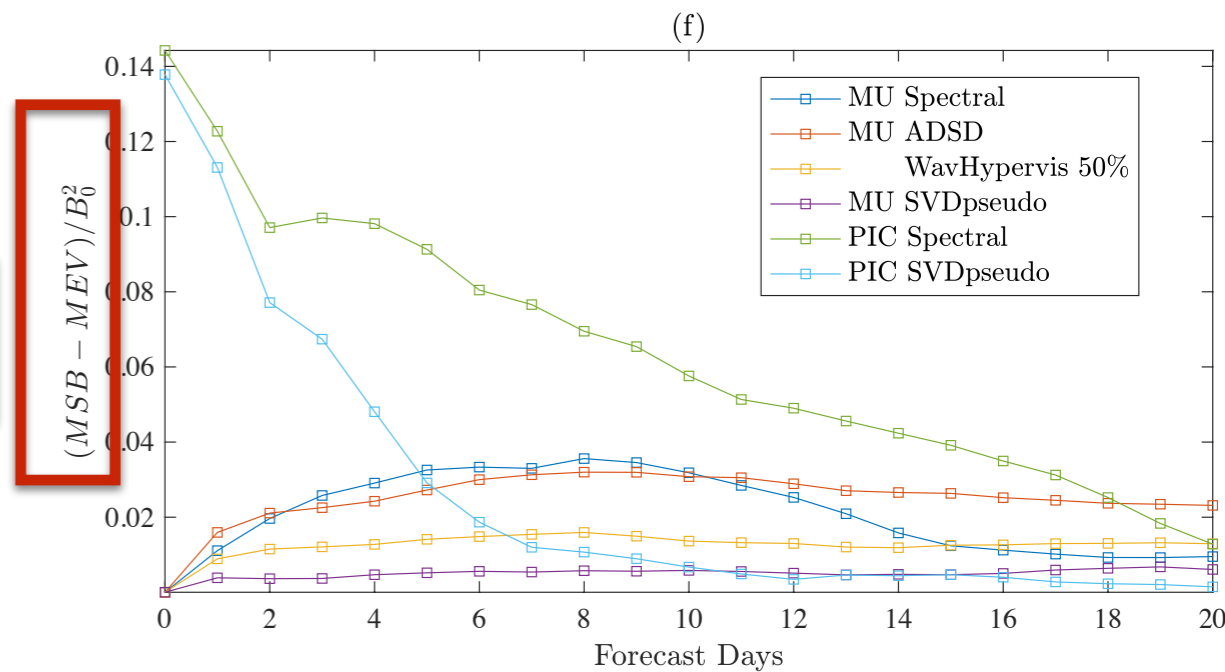
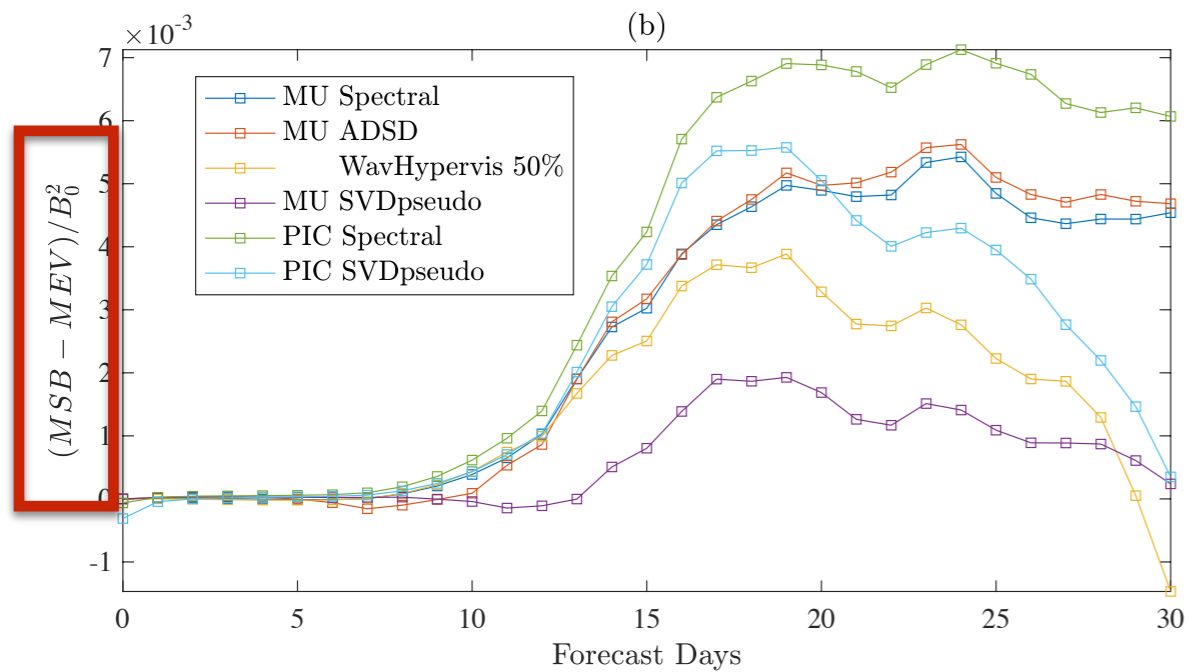
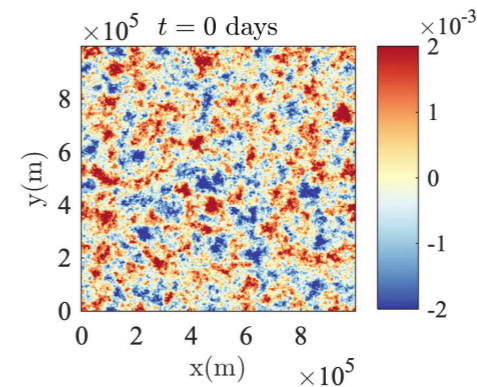
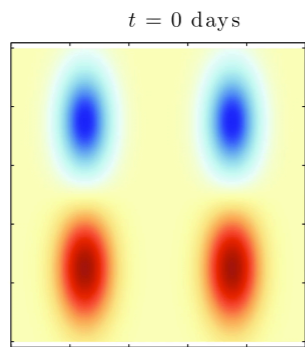
512 x 512



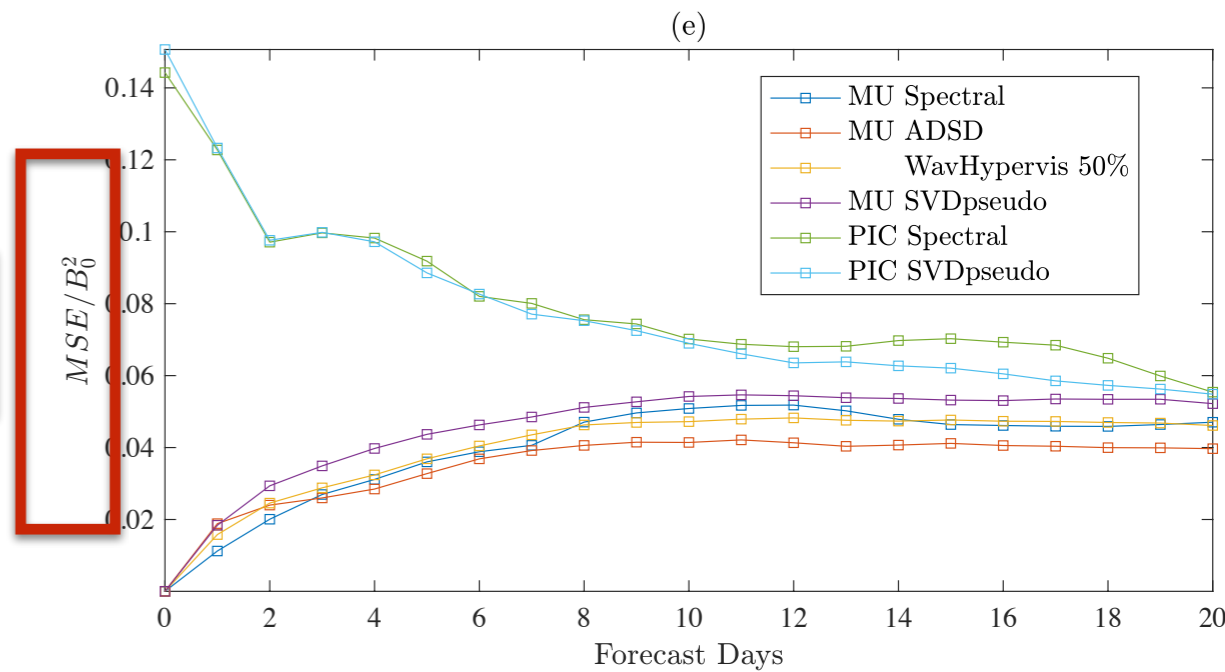
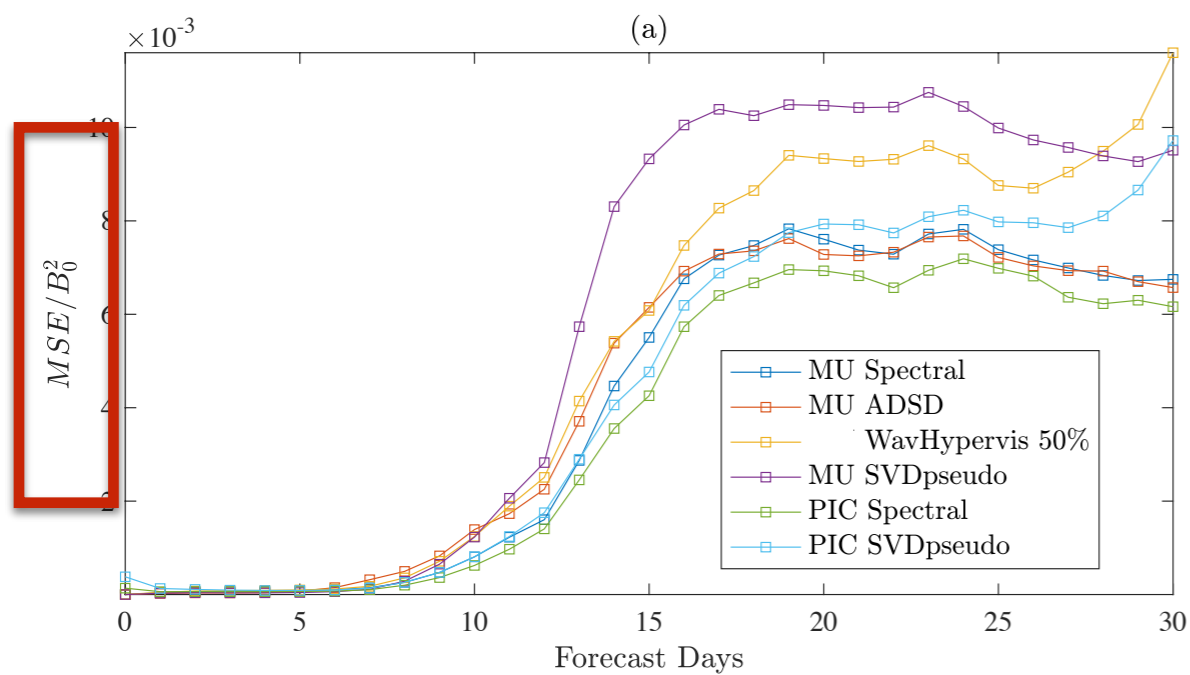
UQ metrics

Metric	Meaning
RMSE	<u>Error</u> of ensemble members
Talagrand histogram (TH)	Capacity of the ensemble to <u>explore</u> all reference possible values
Bias ² -spread	Capacity of the ensemble to <u>explore</u> all reference possible values
CRPS	Point-wise distance between the ensemble CFD and the indicator function of the event
Energy Score (ES)	Generalized CRPS for multivariate ensemble

Spreading VS Errors

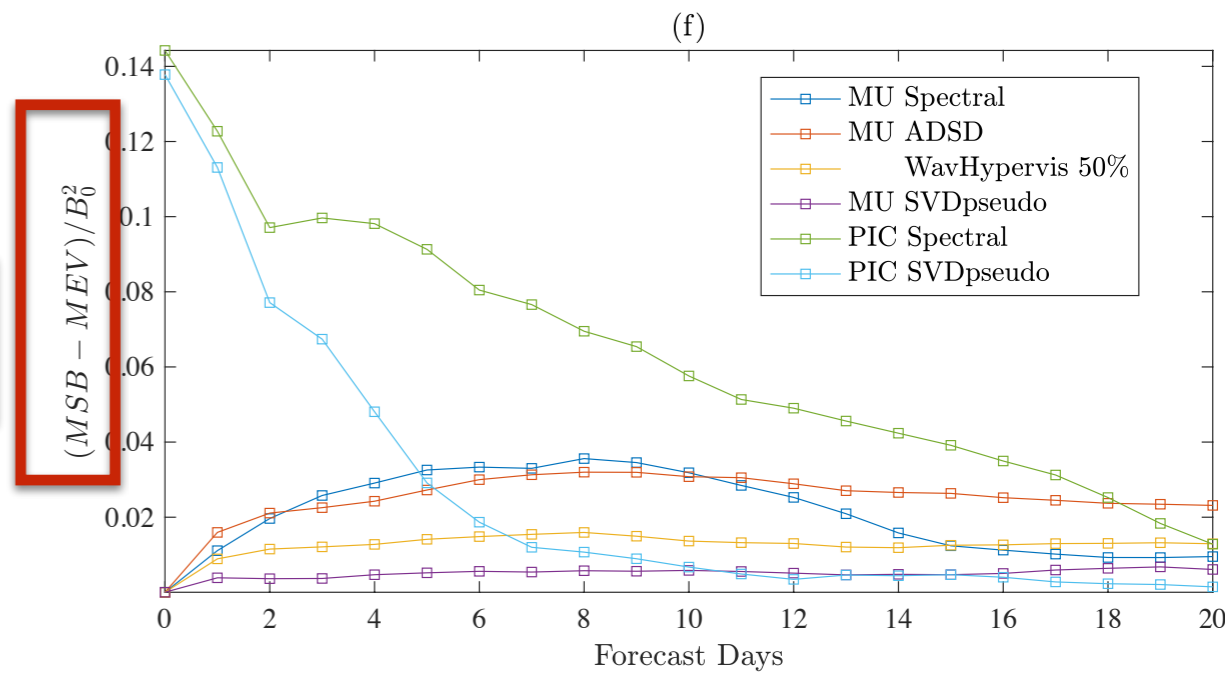
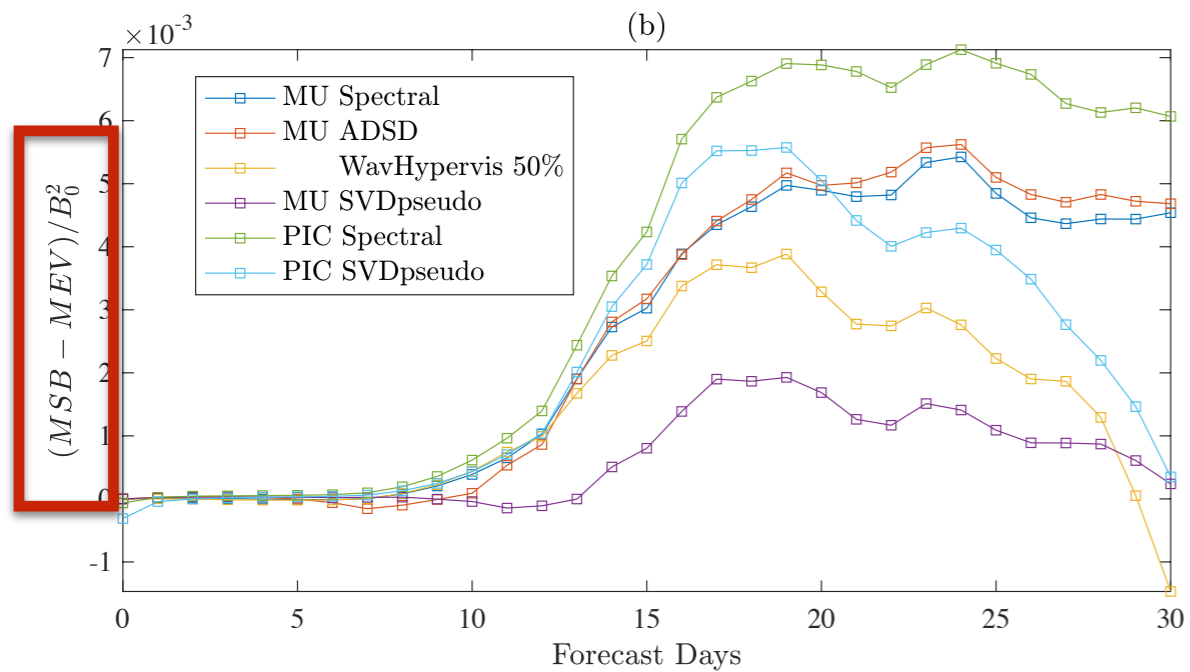
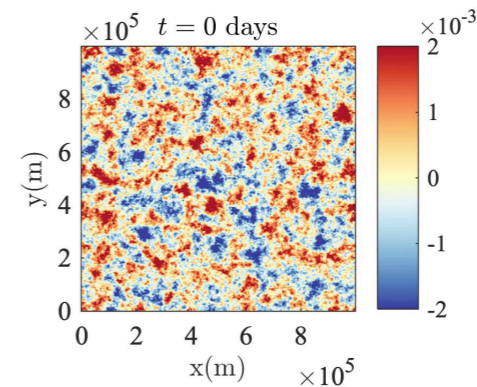
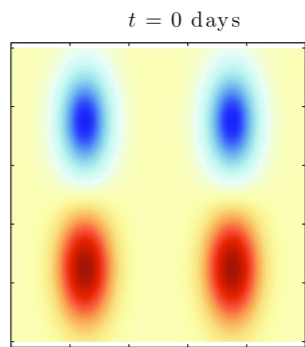


Spread

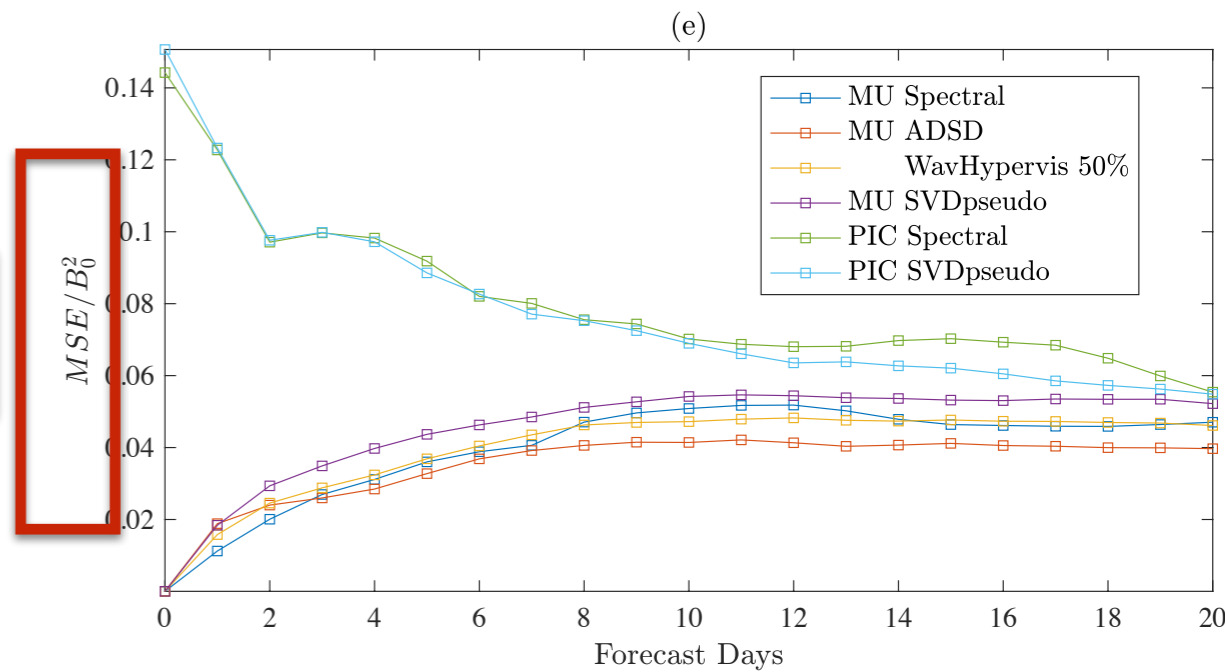
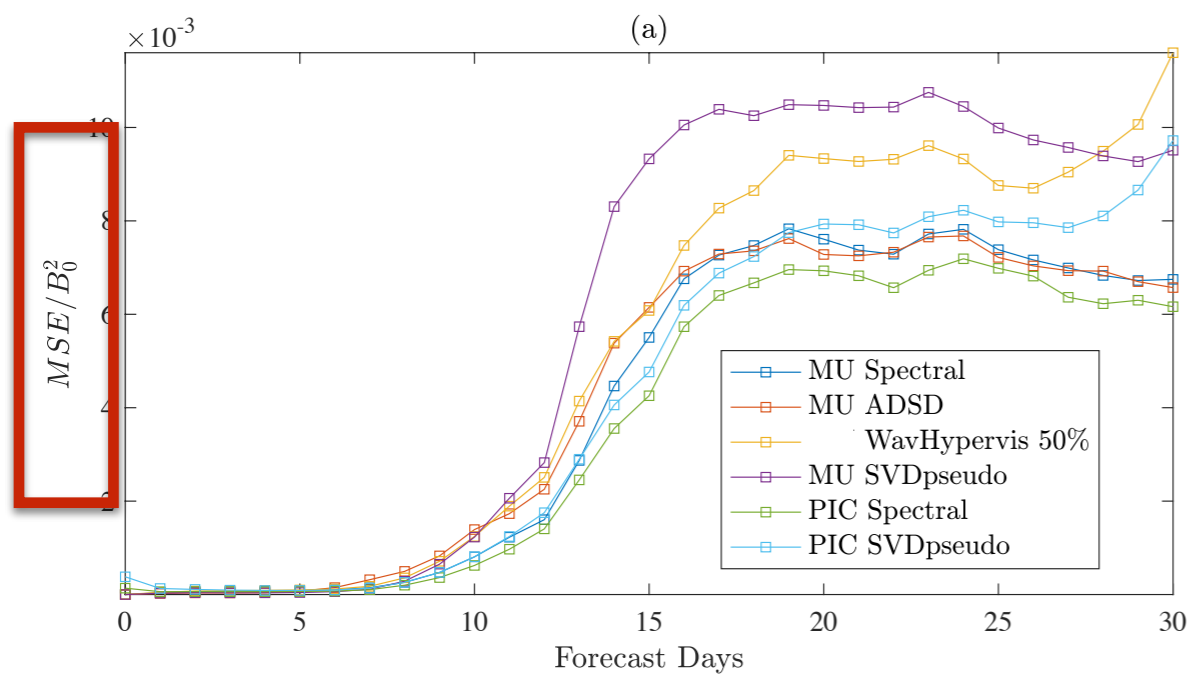


Error

Spreading VS Errors

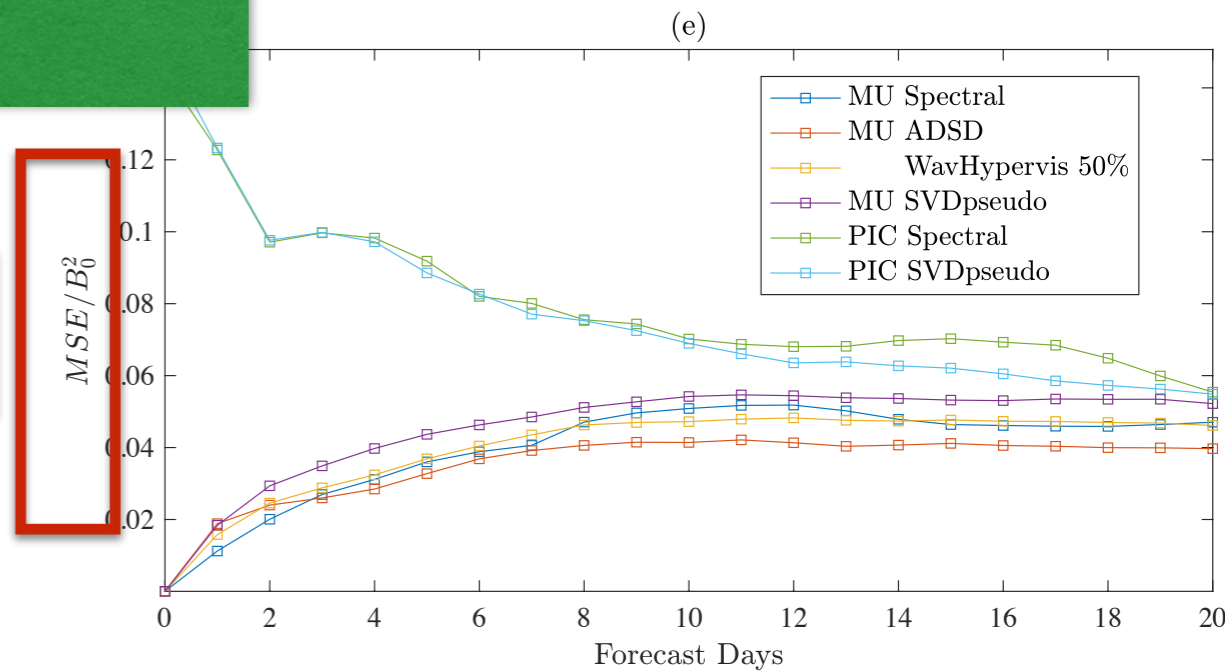
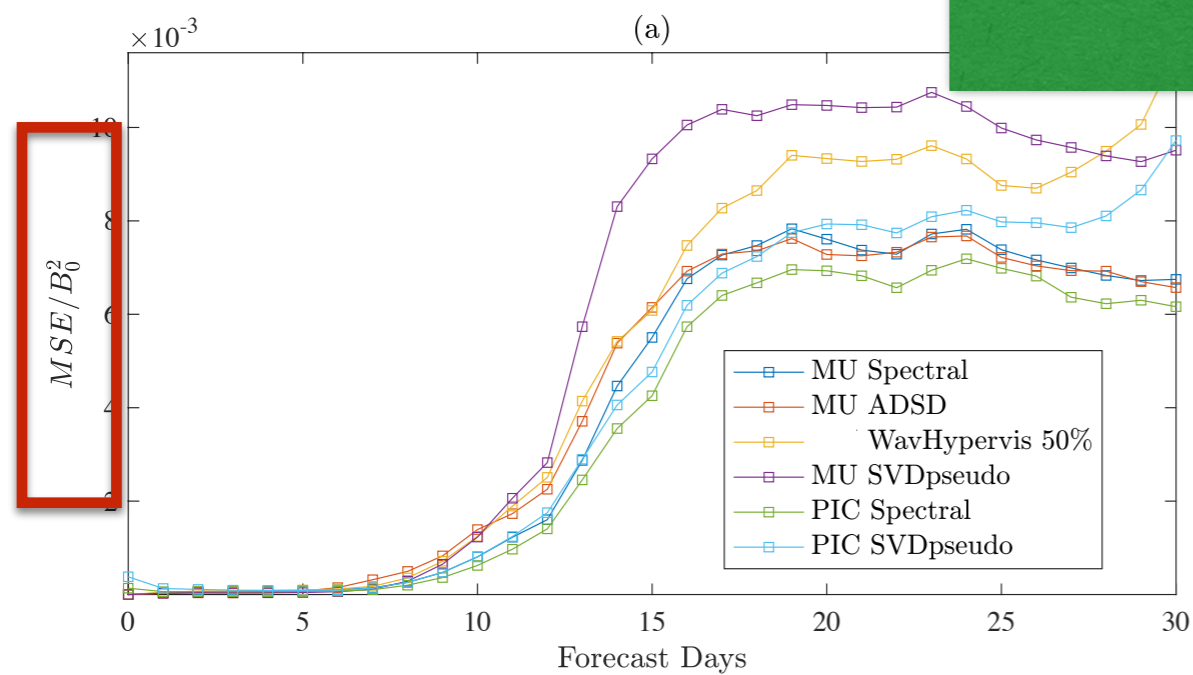
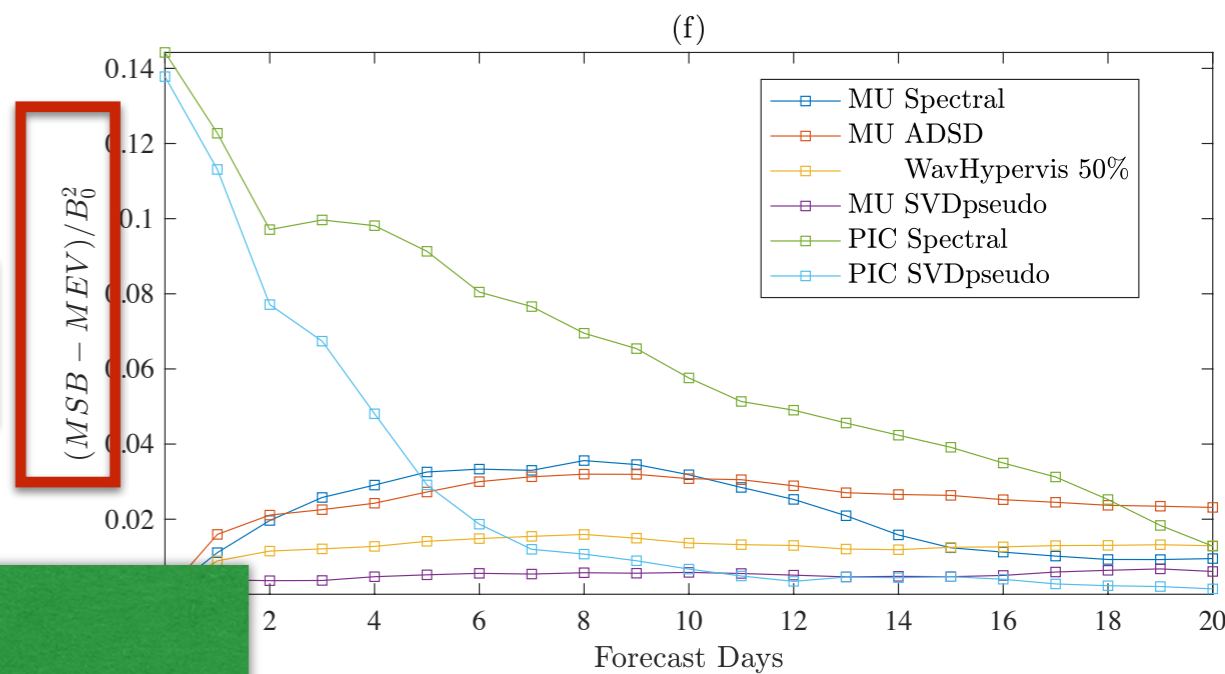
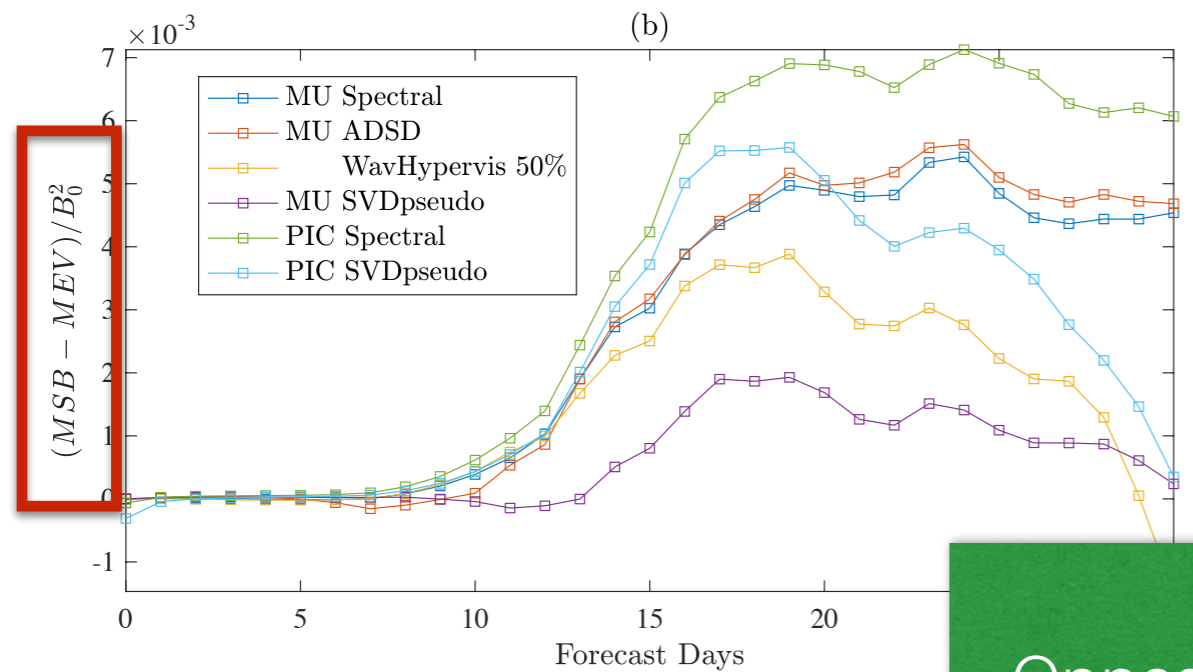
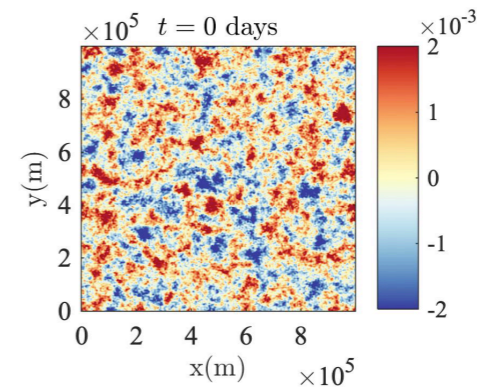
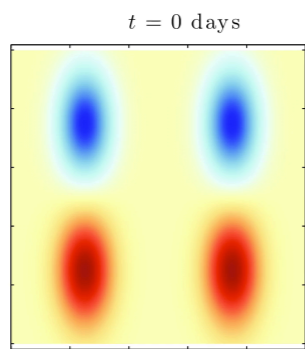


Spread



Error

Spreading VS Errors



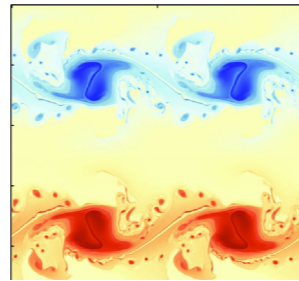
Spread

Error

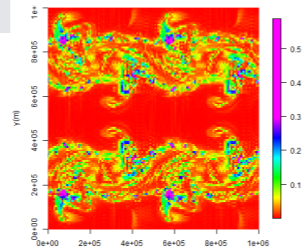
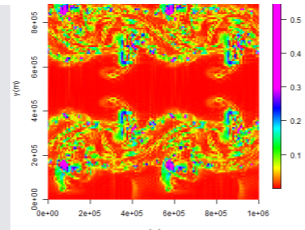
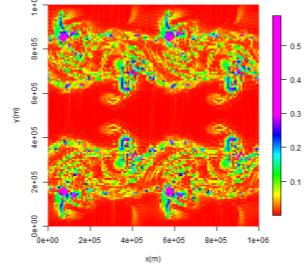
Opposite conclusions

Ensemble point-wise skills : CRPS

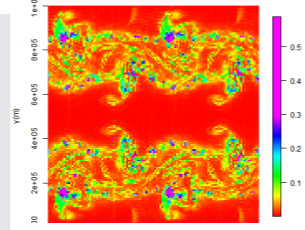
$t = 18$ days



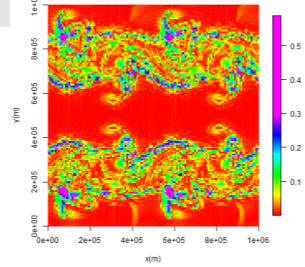
CRPS mu_spec, day 18



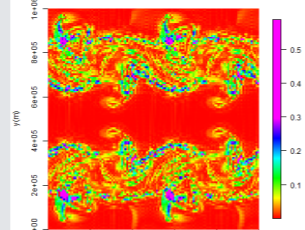
CRPS mu_wavhv50, day 18



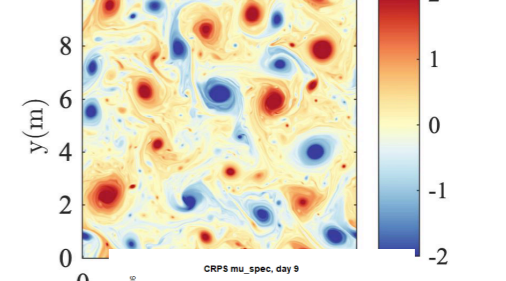
CRPS pic_spec, day 18



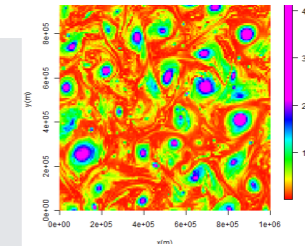
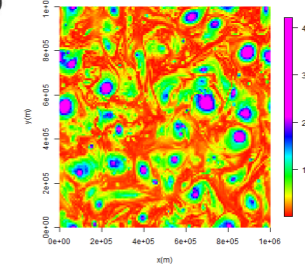
CRPS pic_svd, day 18



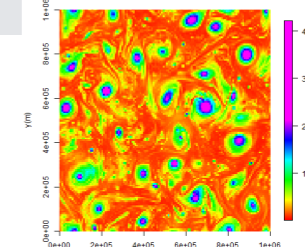
$\times 10^5$ $t = 10$ days



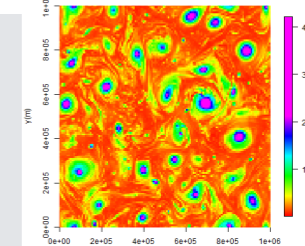
CRPS mu_spec, day 9



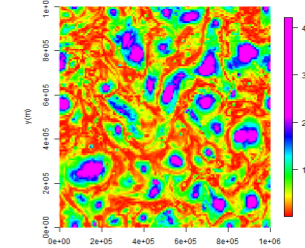
CRPS mu_svd, day 9



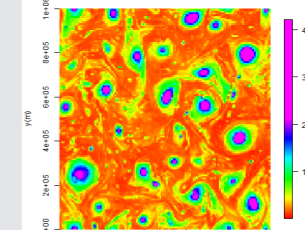
CRPS wavhv50, day 9



CRPS pic_spec, day 9



CRPS pic_svd, day 9



MU Spec

MU ADSD

MU SVD

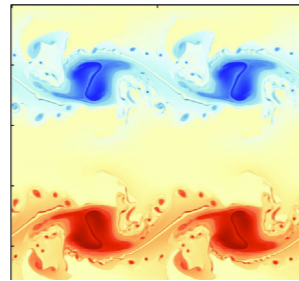
WaveHyperv

PIC Spec

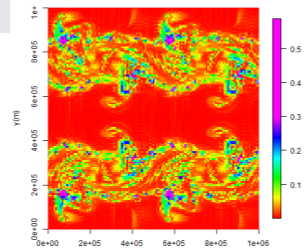
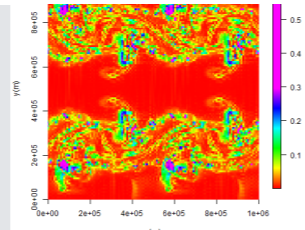
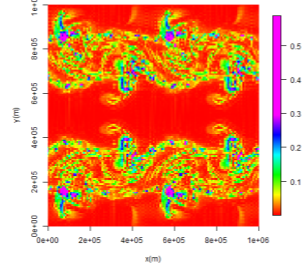
PIC SVD

Ensemble point-wise skills : CRPS

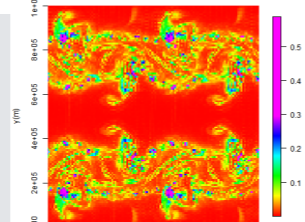
$t = 18$ days



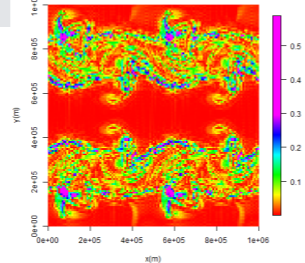
CRPS mu_spec, day 18



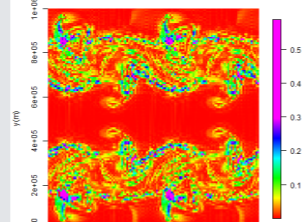
CRPS mu_wavhv50, day 18



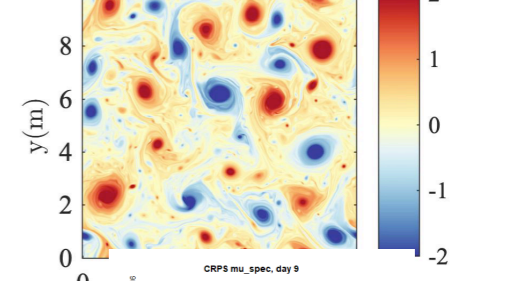
CRPS pic_spec, day 18



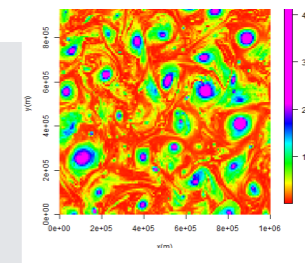
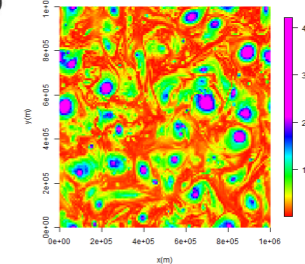
CRPS pic_svd, day 18



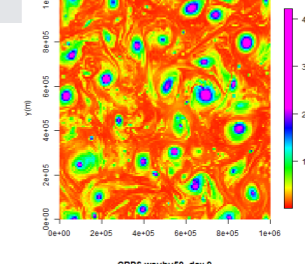
$\times 10^5$ $t = 10$ days



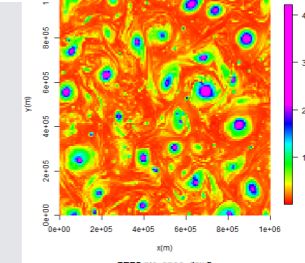
CRPS mu_spec, day 9



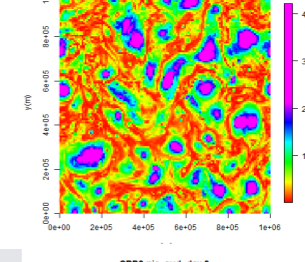
CRPS mu_svd, day 9



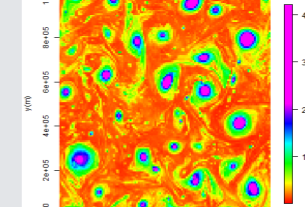
CRPS wavhv50, day 9



CRPS pic_spec, day 9



CRPS pic_svd, day 9



MU Spec

MU ADSD

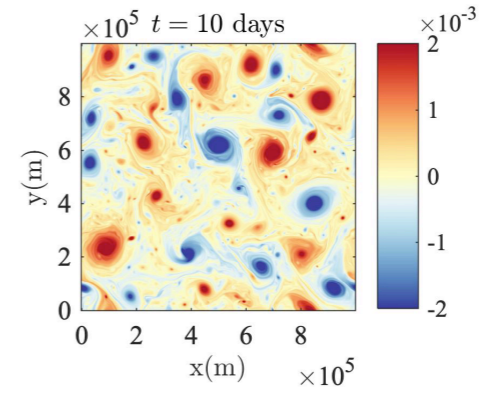
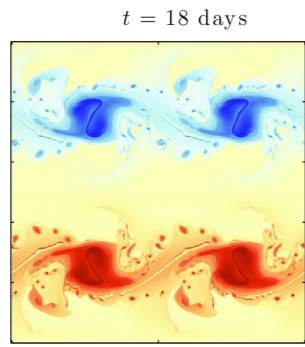
MU SVD

WaveHyperv

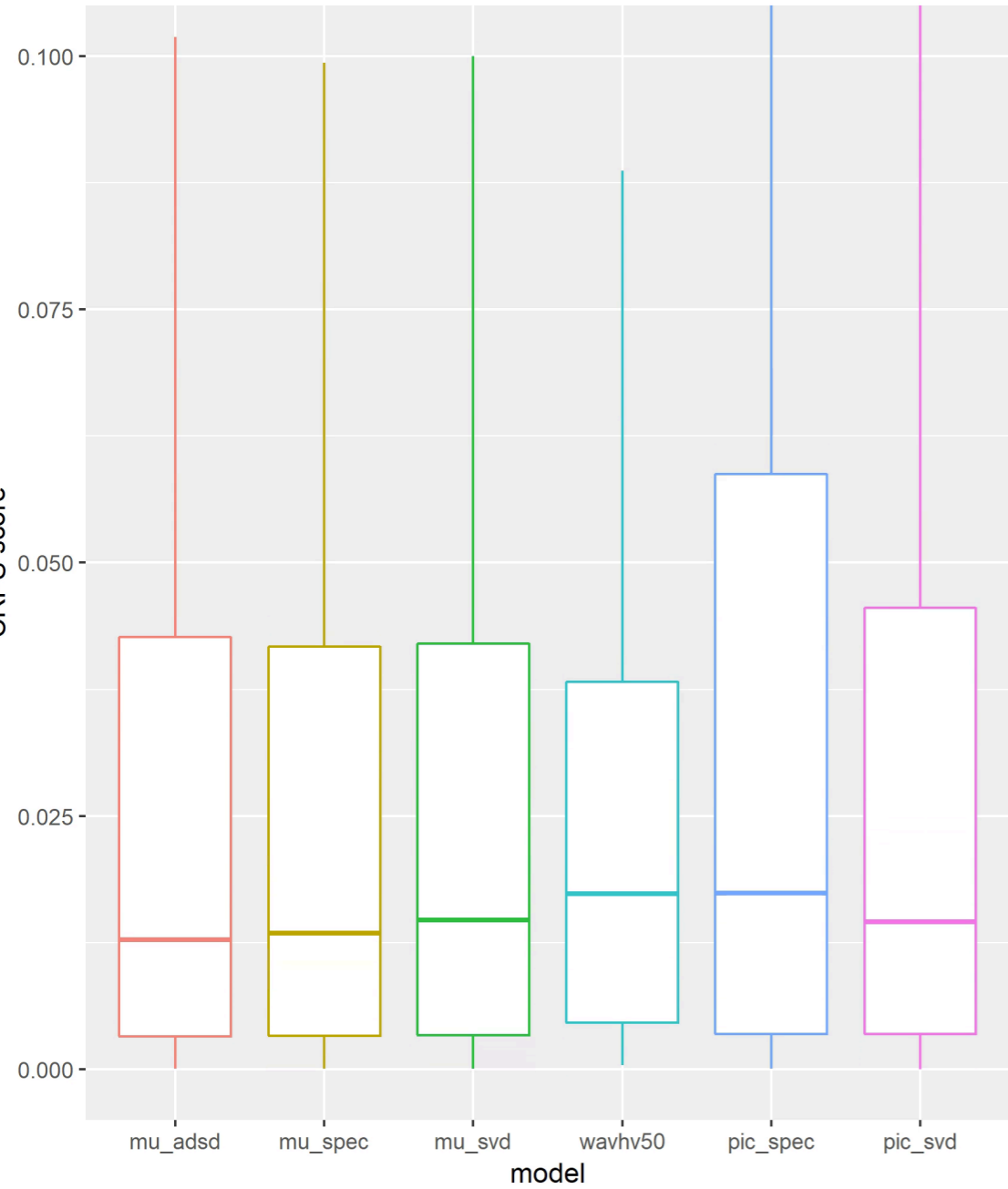
PIC Spec

PIC SVD

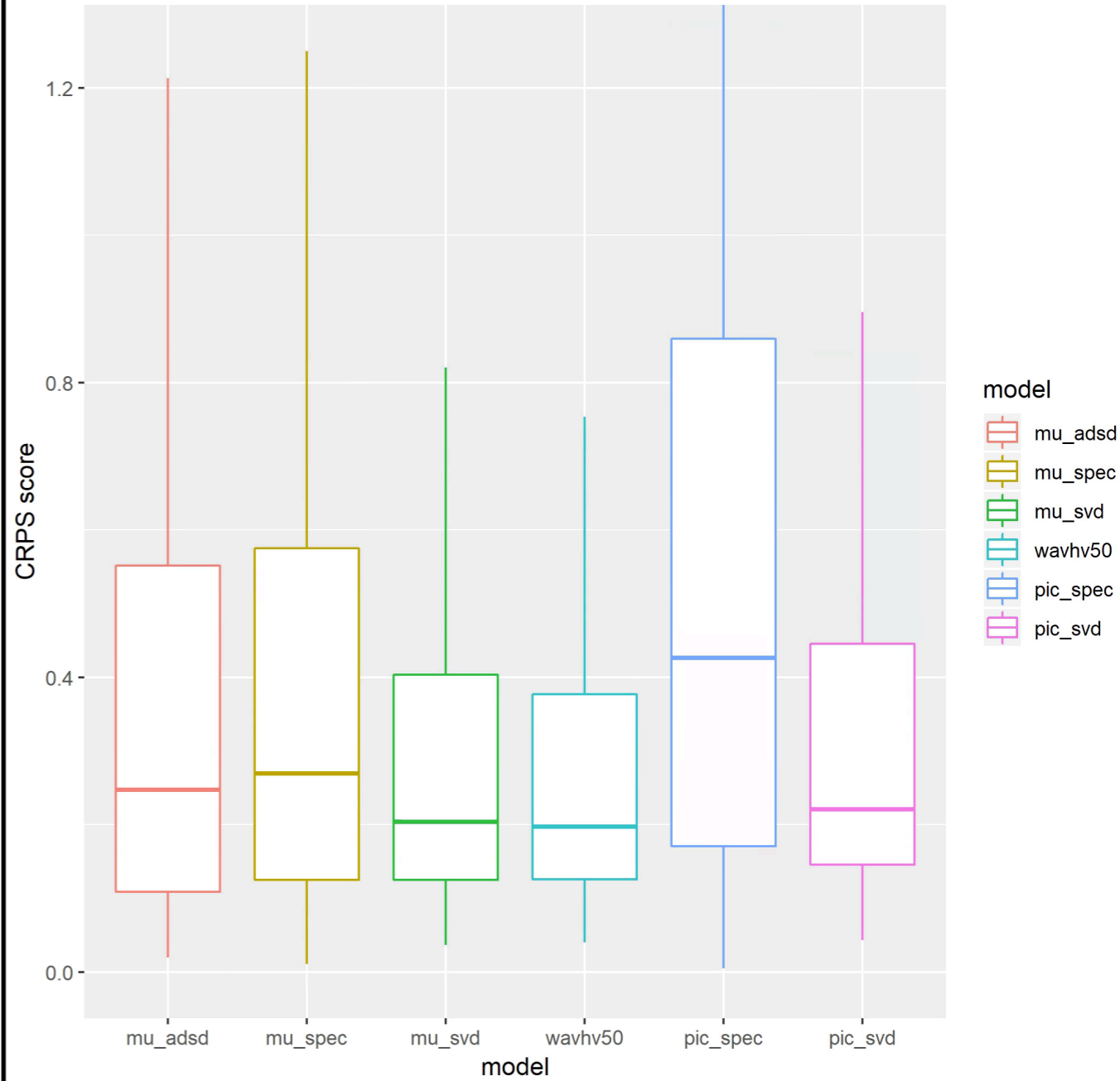
Ensemble point-wise skills : CRPS



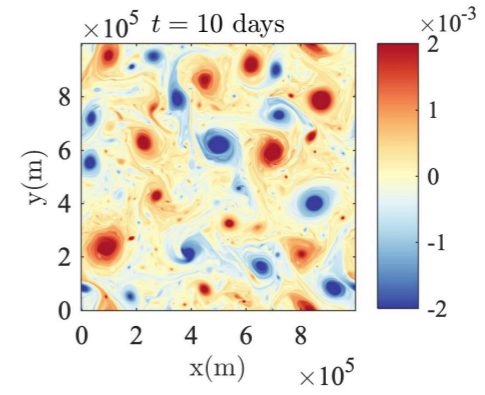
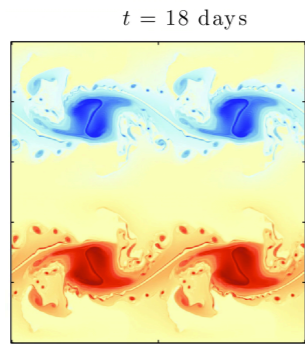
CRPS spatial of each model at day 19



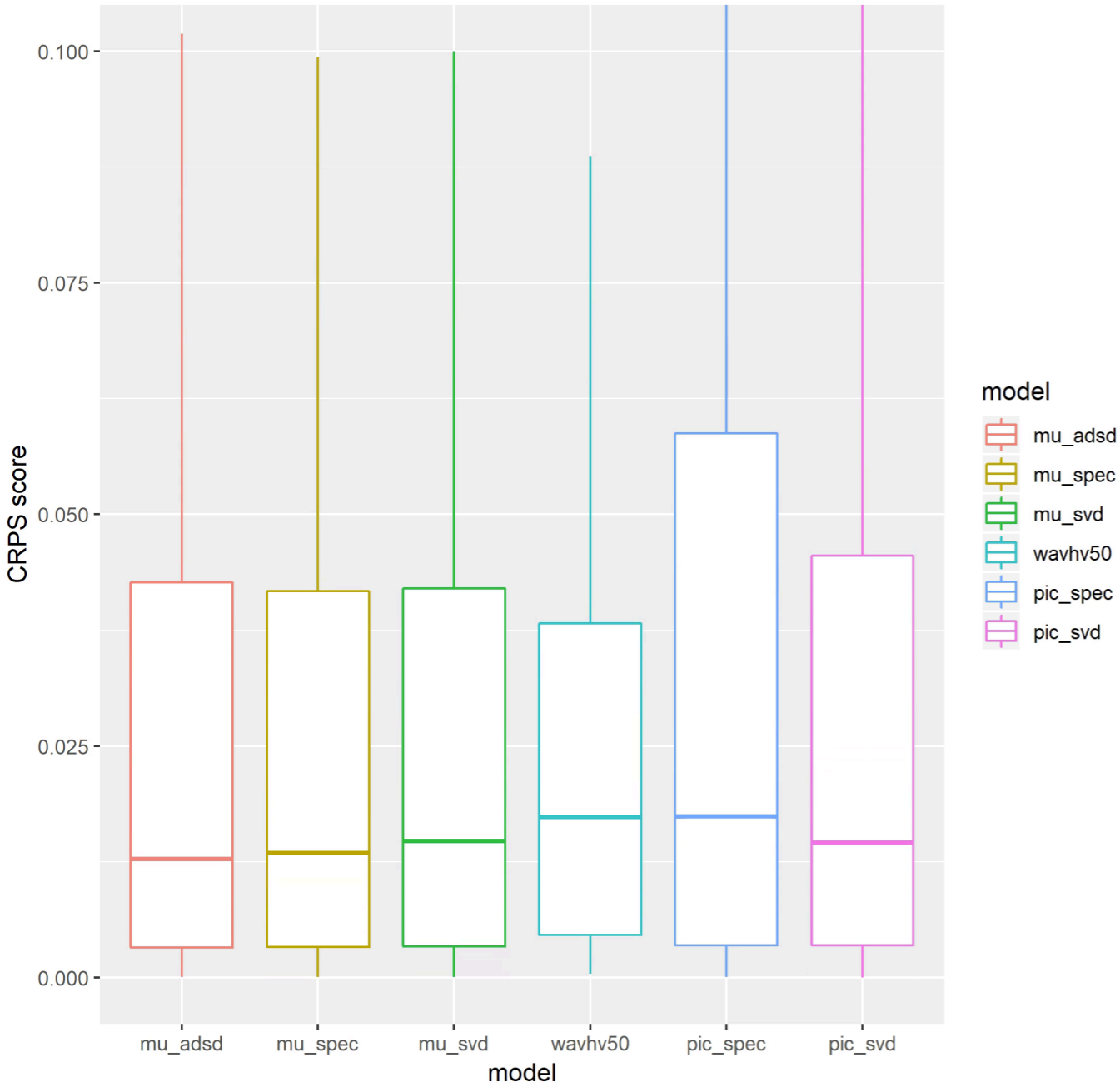
CRPS spatial of each model at day 10



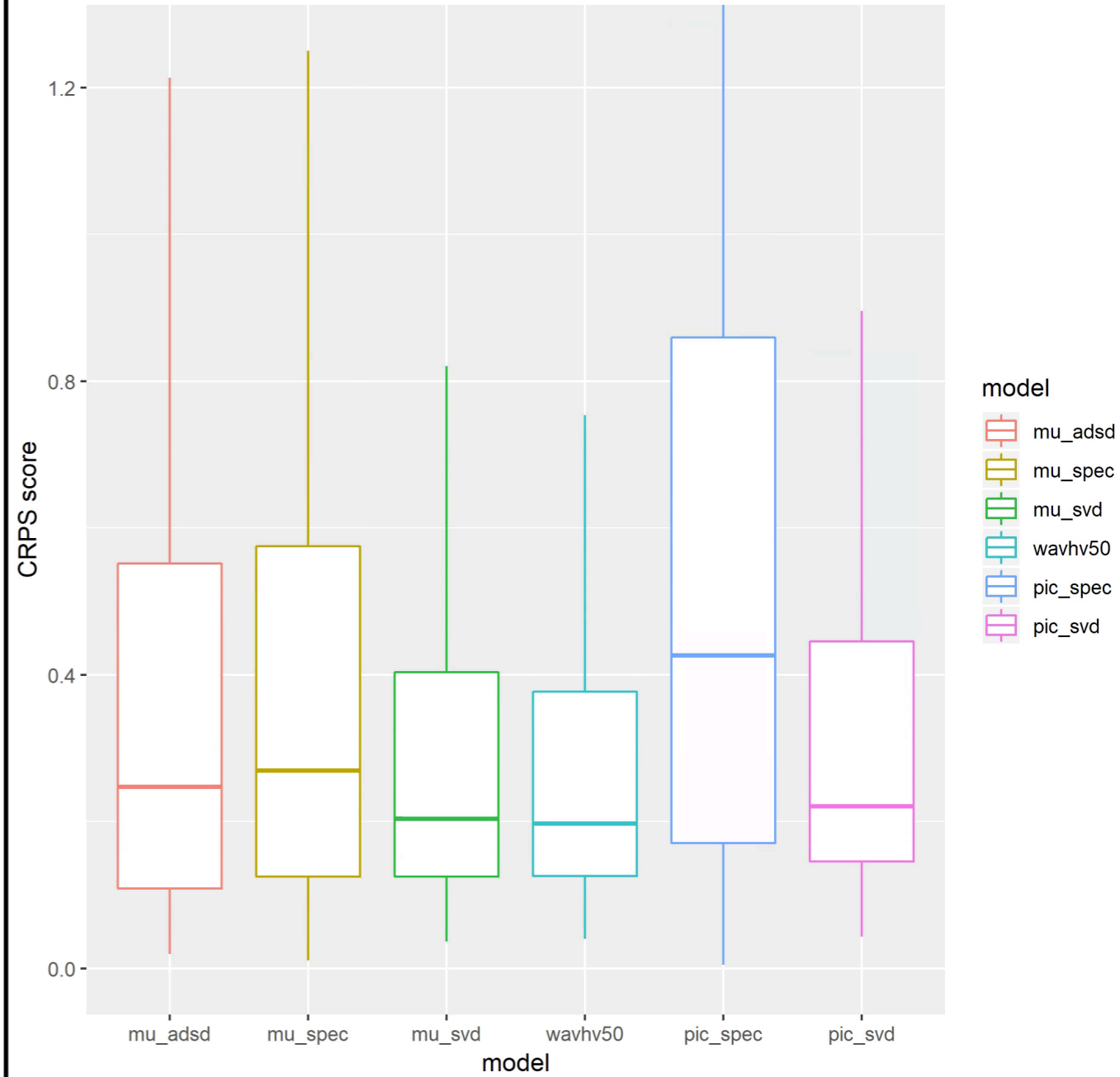
Ensemble point-wise skills : CRPS



CRPS spatial of each model at day 19



CRPS spatial of each model at day 10



Conclusion

Conclusion

	RMSE (errors)	B ² -Var (spread)	CRPS (point-wise)	ES (global)
MU Spec	+	+	+	+
MU ADSD	++	+	+	+
MU SVD	-	++	++	++
WaveHyperv	-	+	++	++
PIC Spec	-	--	--	--
PIC SVD	--	-	+	+

Conclusion

	RMSE (errors)	B ² -Var (spread)	CRPS (point-wise)	ES (global)
MU Spec	+	+	+	+
MU ADSD	++	+	+	+
MU SVD	-	++	++	++
WaveHyperv	-	+	++	++
PIC Spec	-	--	--	--
PIC SVD	--	-	+	+

Conclusion

	RMSE (errors)	B ² -Var (spread)	CRPS (point-wise)	ES (global)
MU Spec	+	+	+	+
MU ADSD	++	+	+	+
MU SVD	-	++	++	++
WaveHyperv	-	+	++	++
PIC Spec	-	--	--	--
PIC SVD	--	-	+	+

Conclusion

	RMSE (errors)	B ² -Var (spread)	CRPS (point-wise)	ES (global)
MU Spec	+	+	+	+
MU ADSD	++	+	+	+
MU SVD	-	++	++	++
WaveHyperv	-	+	++	++
PIC Spec	-	--	--	--
PIC SVD	--	-	+	+