

Linear Adaptive Computed Torque Control for Singularity Crossing of Parallel Robots

A. Koessler[†], N. Bouton[†], S. Briot^{*}, B. C. Bouzgarrou[†], and Y. Mezouar[†]

[†] Université Clermont Auvergne, CNRS, SIGMA, Institut Pascal, France

^{*} Laboratoire des Sciences du Numérique de Nantes (LS2N), CNRS, France

Abstract The operational workspace of parallel robots is often reduced by the presence of singularities. Recently, it has been proven that Type 2 singularities can be crossed in a way such that the dynamic model of the robot never degenerates. This discovery has been the starting point of several works on multi-model Computed Torque Control (CTC) that allow crossing of Type 2 singularities. In this paper, we propose a further improvement thanks to adaptive control. The major contribution of the paper is in the control law synthesis, which uses only linear methods, in contrast to usual approaches based on Lyapunov theory. This theoretical development will be validated both in simulation and experimentally.

1 Introduction

It is often said that parallel manipulators have the edge on their serial counterparts in terms of cycle time and rigidity. However, the size of their workspace is a serious drawback. In general, this workspace is further split by Type 2 singularities (Gosselin and Angeles, 1990). This is especially annoying as joint control usually computes divergent setpoints in the neighborhood of those singularities, meaning that crossing them is not a solution.

A lot of propositions revolve around the idea of getting rid off singularities. Optimal design allows to eliminate them or to reduce their impact (Gogu, 2004). Actuation redundancy or variable actuation modes (Rakotomanga et al., 2008) allows to cross them at a supplementary cost.

Contrary to these approaches, some researchers have proved that Type 2 singularities do not always induce the degeneracy of the inverse dynamical model of parallel robots (IDER, 2005), (Briot and Arakelian, 2008). The expression of these non-degeneracy conditions and their use in multi-model control law will be explained further on.

While being fairly reliable, this method does not ensure success in singularity crossing, mainly because of trajectory tracking error induced by the

control law. Then, in some cases, the desired trajectory is not respected and the robot is unable to cross the singularity. Therefore, more advanced control is needed in order to ensure a perfect crossing of the singularity.

Adaptive control is widely used when robot dynamic models are not perfectly known or change over time (Zhang and Wei, 2017). Our application falls into the first case, as the dynamic model has poor precision and crossing trajectories need high accelerations. To simplify implementation and tuning, we propose a control law derived from linear algebra, contrary to usual Lyapunov-based approaches.

The paper is organized as follows: previous work on singularity crossing is recalled in section 2. Main contribution lies in section 3, where the linear adaptive control law is presented. Its efficiency is proved in section 4, with simulations and experiments carried out on a planar parallel robot.

2 Previous Work on Singularity Crossing

2.1 Dynamical Non-Degeneracy Rule

It has been proven by Ider (2005) and Briot and Arakelian (2008) that parallel singularities do not always induce degeneracy of parallel robot's dynamic model. The rule for non-degenerating motion planing is the following: *in singular configurations, the wrench exerted on the end-effector (by inertia and external forces) must be orthogonal to the direction of the uncontrollable motion.*

This result can be used to generate a crossing trajectory that ensures the non-degeneracy of the dynamic model as explained in (Pagis et al., 2015).

2.2 Control Scheme for Singularity Crossing

Multi-model Computed Torque Control law is recalled as explained in (Pagis et al., 2015).

The control error to minimize is $\mathbf{e} = \mathbf{q}_d - \mathbf{q}$ where \mathbf{q}_d is the desired joint value and \mathbf{q} the measured joint value. Parallel robot IDM can be generally written as :

$$\boldsymbol{\tau} = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{h} = \mathbf{M}\mathbf{v} + \mathbf{h} \quad (1)$$

with the mass matrix of the robot $\mathbf{M}(\mathbf{q})$ and the vector of centrifugal effects $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}})$. The intermediate control input \mathbf{v} allows to perform input-output linearization and is defined by 3rd-order dynamics :

$$\mathbf{v} = \ddot{\mathbf{q}}_d + \mathbf{K}_v\dot{\mathbf{e}} + \mathbf{K}_p\mathbf{e} + \mathbf{K}_i \int \mathbf{e} \quad (2)$$

where gain matrices \mathbf{K}_v , \mathbf{K}_p , \mathbf{K}_i impose the dynamics of articular error.

However, in singular configurations, IDM (1) cannot be computed. Yet, as explained in (Pagis et al., 2015), the IDM in singularity has a new non-degenerating expression. Transition between the two models can be achieved using a logistic function σ . Consequently, a multi-model control scheme was implemented in (Pagis et al., 2015) with the following dynamic models:

- Model 1 - complete model ($\sigma = 0$)

$$\boldsymbol{\tau} = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{h} \quad (3)$$

- Model 2 - non-degenerating model around singularities ($\sigma = 1$)

$$\boldsymbol{\tau} = \mathbf{M}'\ddot{\mathbf{q}} + \mathbf{h}' \quad (4)$$

3 Linear Synthesis of Adaptive Control Law

To improve tracking performances, a more advanced control algorithm scheme is required. Adaptive control of robotic manipulators is well-known and has been the subject of many works, recently reviewed in (Zhang and Wei, 2017). Adaptation of dynamic parameters is needed to reduce modeling error, resulting in a better trajectory tracking. This could be especially useful when using the less precise non-degenerating model around the singularities. In this paper, contrary to usual approaches, the presented law for parameter adaptation is based on linear techniques.

3.1 Modeling Error

This section presents the usual computation of the error between the $(n_d \times 1)$ vector of real dynamic parameters $\boldsymbol{\chi}$ and its estimate $\hat{\boldsymbol{\chi}}$.

The real dynamic model of the robot and the one used for control are

$$\boldsymbol{\tau} = \mathbf{M}(\boldsymbol{\chi})\ddot{\mathbf{q}} + \mathbf{h}(\boldsymbol{\chi}) \quad \text{and} \quad \boldsymbol{\tau} = \hat{\mathbf{M}}(\hat{\boldsymbol{\chi}})\mathbf{v} + \hat{\mathbf{h}}(\hat{\boldsymbol{\chi}}) \quad (5)$$

with \mathbf{v} defined in (2). Modifying (5) allows to get the classical result :

$$\ddot{\mathbf{e}} + \mathbf{K}_v\dot{\mathbf{e}} + \mathbf{K}_p\mathbf{e} + \mathbf{K}_i \int \mathbf{e} = \hat{\mathbf{M}}^{-1}((\mathbf{M} - \hat{\mathbf{M}})\ddot{\mathbf{q}} + \mathbf{h} - \hat{\mathbf{h}}). \quad (6)$$

Full computation of (6) can be found in (Khalil and Dombre, 2001). Since the dynamic model is linear with regards to parameters $\boldsymbol{\chi}$, one can write

$$\boldsymbol{\tau} = \boldsymbol{\Phi}\boldsymbol{\chi} = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{h} \quad (7)$$

where $\boldsymbol{\Phi}$ is a $(n \times n_d)$ matrix. Consequently, the differential equation on articular error is linked to dynamic parameter error, that is the difference $\tilde{\boldsymbol{\chi}} = \boldsymbol{\chi} - \hat{\boldsymbol{\chi}}$ between real and estimated value:

$$\ddot{\mathbf{e}} + \mathbf{K}_v\dot{\mathbf{e}} + \mathbf{K}_p\mathbf{e} + \mathbf{K}_i \int \mathbf{e} = \hat{\mathbf{M}}^{-1}\boldsymbol{\Phi}\tilde{\boldsymbol{\chi}}. \quad (8)$$

3.2 Adaptation Law based on State Feedback

Equation (8) can be put under the form of a state-space model, where state vector \mathbf{X} is a vector of articular error, its integral and its derivative:

$$\mathbf{X} = \begin{pmatrix} \int \mathbf{e} \\ \mathbf{e} \\ \dot{\mathbf{e}} \end{pmatrix} \Rightarrow \dot{\mathbf{X}} = \begin{pmatrix} \mathbf{0}_n & \mathbf{1}_n & \mathbf{0}_n \\ \mathbf{0}_n & \mathbf{0}_n & \mathbf{1}_n \\ -\mathbf{K}_i & -\mathbf{K}_p & -\mathbf{K}_v \end{pmatrix} \mathbf{X} + \begin{pmatrix} \mathbf{0}_n \\ \mathbf{0}_n \\ \mathbf{1}_n \end{pmatrix} \hat{\mathbf{M}}^{-1} \Phi \tilde{\chi}. \quad (9)$$

The state-space model (9) can be written as:

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{w} \quad \text{with} \quad \mathbf{w} = \hat{\mathbf{M}}^{-1} \Phi \tilde{\chi} \quad (10)$$

where state matrix \mathbf{A} and input matrix \mathbf{B} are used to check the controllability of the system using Kalman criterion. Then, instead of using Lyapunov theory (Khalil and Dombre, 2001), we stick to linear control methods and use classical state feedback

$$\mathbf{w} = -\mathbf{L}_s \mathbf{X}. \quad (11)$$

where the gain matrix \mathbf{L}_s is computed by using pole placement methods in order to ensure that closed-loop state matrix $\mathbf{F} - \mathbf{G}\mathbf{L}_s$ is a Hurwitz one. Then, state vector is ensured to exponentially converge towards zero. Finally, the estimated dynamic parameters $\tilde{\chi}$ are obtained by imposing a first order dynamic evolution on the estimation error $\tilde{\chi}$:

$$\dot{\tilde{\chi}} = -\mathbf{G}\tilde{\chi} \quad , \quad \mathbf{G} > 0. \quad (12)$$

By combining (10), (11) and (12), one can obtain

$$\dot{\tilde{\chi}} = -\mathbf{G}\Phi^{-1}\hat{\mathbf{M}}\mathbf{L}_s\mathbf{X}. \quad (13)$$

The stability of the control law (13) is ensured by the linear methods used to synthesize it ((11) and (12)). Finally, dynamic parameters are estimated by numerical integration of (13), resulting in the adaptive control scheme presented in Fig. 1.

4 Case Study on DexTAR Robot

4.1 DexTAR Dynamic Modeling

To check the efficiency of the proposed approach, experiments are carried on a five-bar parallel robot, the DexTAR manufactured by *Mecademic*,

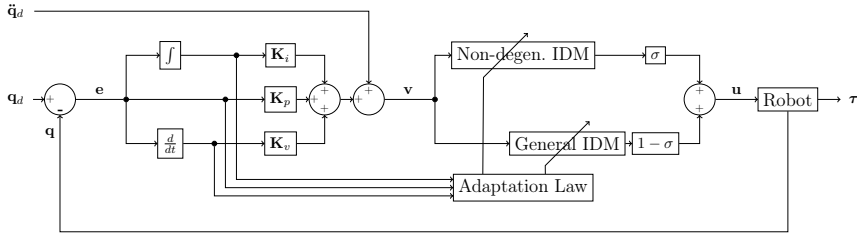


Figure 1: Proposed multi-model adaptive control scheme

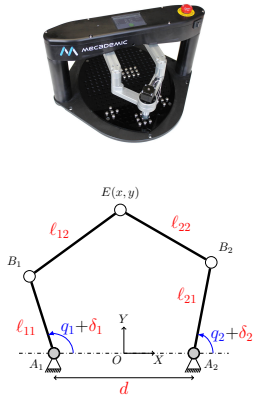


Figure 2: DexTAR geometry

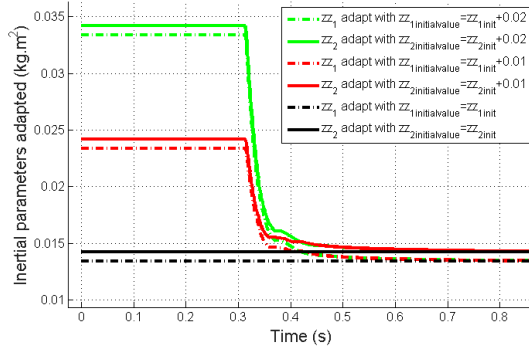


Figure 3: zz adaptation in simulation

which has two degrees of freedom. Its geometry is presented on Fig. 2. The end-effector of the robot is E , which vector of Cartesian coordinates is \mathbf{x} . The robot is in a Type 2 singularity when distal elements B_1E and B_2E are aligned.

The inverse dynamic model used in the multi-model control law (see (Pagis et al., 2015)) is :

$$\boldsymbol{\tau} = \mathbf{ZZ}\dot{\mathbf{q}} + \mathbf{F}_s \text{sign}(\dot{\mathbf{q}}) + \mathbf{f}_f + (1 - \sigma)\mathbf{J}^T m\ddot{\mathbf{x}} \quad (14)$$

It is well known that parameters to adapt have to be chosen carefully. The choice can be further helped by a sensitivity analysis (see (Pagis, 2015)). Based on this analysis, the two parameters zz_{11} and zz_{21} of the matrix \mathbf{ZZ} (see (14)) are adapted. They are expected to have a strong impact on torque setpoint computation, since high articular accelerations are needed to cross the singularities.

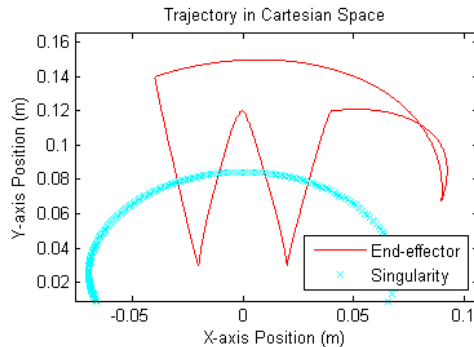


Figure 4: Testing trajectory

4.2 Simulation Results

In order to verify that the proposed adaptive control scheme works as intended, wrong initial values of zz_{11} and zz_{21} are fed to the algorithm. Simulations on Matlab/Simulink have been carried out on a standard trajectory that does not cross singularities. As a result, it can be seen in Fig. 3 that estimates of inertia converge back to the real values. This means that the proposed control law is stable and robust. Moreover, articular error is drastically reduced by the proposed control law.

4.3 Experimental Results

Now that theoretical stability and performance are verified on simulation, we want to see if adaptive control can help singularity crossing. For this purpose, a trajectory where the singularity is crossed four times was generated, obviously abiding to the non-degeneracy rule (see Fig. 4).

The difficulty induced by singularity crossing is that the robot has to find a configuration where distal legs are fully extended in order to cross the singularity. Articular errors can either lead to crossing failure (not extending enough) or peaks in torque setpoint (trying to extend more than robot geometry allows to).

The adaptive scheme proposed will be judged on several criteria: articular error, setpoint torque values and setpoint torque smoothness.

Experiments are run with unbiased initial values, which correspond to the identified ones ($\hat{z}z_{11} = 0.0134$ and $\hat{z}z_{21} = 0.0142 \text{ kgm}^2$). Afterwards, initial values are changed by adding or subtracting 0.01 kgm^2 . Logically, introducing biased parameters is expected to favor adaptive control. The improvement obtained with adaptive control is summarized in Table 1.

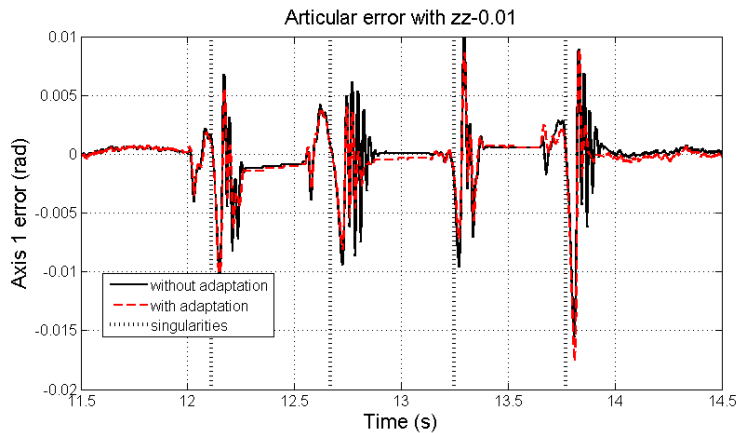


Figure 5: Articular error comparison

Table 1: Experimental results

data	$\hat{z}z = zz$		$\hat{z}z = zz + 0.01$		$\hat{z}z = zz - 0.01$	
	no adapt	adapt	no adapt	adapt	no adapt	adapt
RMS error (mrad)	2.82	3.70	3.26	3.07	14.20	5.53
peak error (mrad)	9.03	10.63	10.24	8.91	46.99	17.05
RMS torque (V)	1.37	0.96	2.26	1.71	1.61	0.89
peak torque (V)	8	3.18	8	8	5.87	3.18
peak t. derivative (V/s)	1680	715	2209	1949	1079	302

It can be seen that in all cases, adaptive control reduces input torque peaks and improves smoothness, which is our first goal. Articular error is reduced only if initial parameters are biased. With initial parameters set as $zz - 0.01$, error reduction is particularly impressive, as witnessed on Fig 5.

Overall, the proposed control scheme works as intended and is a good alternative for smoothing of torque setpoints, even when the initial values of parameters are precise. Articular error is kept low before reaching the singularity, which is crucial for the success of the crossing.

5 Conclusion

Crossing singularities is a potential improvement for the use of parallel robots. It allows to increase the size of operational workspace, which is one of their main downsides. An approach based only on optimal trajectory generation and dedicated control was proposed to achieve singularity cross-

ing. For the crossing to succeed, it is crucial that the trajectory is precisely tracked around singularities. To do so, an advanced control solution was developed using adaptive control algorithm. The originality of our work is the synthesis of the adaptation law based solely on linear methods.

The proposed control law proved to be stable and robust. It was shown experimentally that it is beneficial in terms of trajectory tracking, torque values and smoothness. Future work consists in transposing the proposed control scheme to predictive control, in order to improve the estimation of dynamic parameters and expected to yield better anticipation of singularity crossing.

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