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Supplement to "Market Selection with Differential Financial Constraints"*

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Abstract

In this Supplement we provide foundations for the asset structures used in the main part of the paper, as well as in Appendix A. We use results by Choquet (1966), Kendall (1962) and Polyakis (1999) to demonstrate how these asset structures can be generated from a general set of assets available in the economy and a general set of financial constraints. A sufficient condition called "internal completeness" is for the set of assets to contain an appropriate set of put and call options so that the implied set of payoffs is a sublattice of the Euclidean space.

1 Foundations for the Partition-Based Structure of Financial Constraints

1.1 The Case of Period-0 Trade

In the main part of the paper, we consider the set of assets available to an individual trader to be the set of generalized unit securities paying on a partition of the state space. In this Supplement, we show how such a structure can be derived from a more general structure of incomplete markets and financial constraints.

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For a given period $t \geq 1$ (and a corresponding set of nodes Ω_t), we consider a set of assets A_t^0 , with typical element $a : \Omega_t \rightarrow \mathbb{R}_+$ defined as a state-contingent vector of payoffs. We will assume that A_t^0 consists of $|A_t^0|$ linearly independent assets a_t , denoted $a_{t,1} \dots a_{t,|A_t^0|}$ and that asset $a_{t,1}$ is a bond that pays 1 at every $\sigma_t \in \Omega_t$. However, we will not assume that A_t^0 is complete relative to Ω_t . $A^0 = \cup_{t=1}^{\infty} A_t^0$ is the set of all assets in the economy.

A vector $\theta = (\theta_1 \dots \theta_{|A_t^0|}) \in \mathbb{R}^{|A_t^0|}$ corresponds to a portfolio¹ or to a "marketed security" with payoffs given by $T_t(\theta)(\sigma_t) = \sum_{j=1}^{|A_t^0|} \theta_j a_{t,j}(\sigma_t)$. Let Θ_t^0 stand for the set of all portfolios at time t . The range of $T_t : \Theta_t^0 \rightarrow \mathbb{R}^{|\Omega_t|}$ is the asset span of A_t^0 . Note that $T_t(\theta + \theta') = T_t(\theta) + T_t(\theta')$.

We will endow the set of marketed securities with the following structure: for any θ and $\theta' \in \Theta_t^0$, there exist portfolios $\theta \vee \theta'$ and $\theta \wedge \theta'$ with payoffs:

$$T_t(\theta \vee \theta') = \sup \{T_t(\theta); T_t(\theta')\} = (\max \{T_t(\theta)(\sigma_t); T_t(\theta')(\sigma_t)\})_{\sigma_t \in \Omega_t}$$

$$T_t(\theta \wedge \theta') = \inf \{T_t(\theta); T_t(\theta')\} = (\min \{T_t(\theta)(\sigma_t); T_t(\theta')(\sigma_t)\})_{\sigma_t \in \Omega_t}$$

In other words, the set of marketed securities generates a payoff space which is a sublattice of $\mathbb{R}^{|\Omega_t|}$, denoted M_t^0 . This space is linear and, as shown by Polyrakis (1999), a minimal such space exists. Hence, M_t^0 is uniquely specified. We will call such a payoff space "internally complete", (see Assumption: Internal Completeness below for a formal definition).

A set of marketed securities with a sublattice structure can be generated by an arbitrary set of assets enriched by a set of options on the marketed securities. To understand this, consider θ and $\theta' \in \Theta_t^0$. The portfolio $T_t(\theta) - T_t(\theta')$ is generated by buying 1 unit of θ and selling 1 unit of θ' and is thus also a marketed security with payoff $T_t(\theta) - T_t(\theta')$. In order to obtain the payoff structure $T_t(\theta \vee \theta')$, the agent would have to buy θ' and a call option on the portfolio $\theta - \theta'$ with an exercise price of 0, so that for each $\sigma_t \in \Omega_t$,

$$T_t(\theta'(\sigma_t)) + \max \{T_t(\theta(\sigma_t) - \theta'(\sigma_t)); 0\} = \max \{T_t(\theta(\sigma_t)); T_t(\theta'(\sigma_t))\}.$$

Similarly, in order to obtain the payoff structure $T_t(\theta \wedge \theta')$, the agent would have to buy θ' and a put option on the portfolio $\theta - \theta'$ with an exercise price of 0, so that:

$$T_t(\theta'(\sigma_t)) + \min \{T_t(\theta(\sigma_t) - \theta'(\sigma_t)); 0\} = \min \{T_t(\theta(\sigma_t)); T_t(\theta'(\sigma_t))\}.$$

Hence, if such options can be written with respect to any two traded securities, the asset span is a sublattice of $\mathbb{R}^{|\Omega_t|}$.

Each agent i has access to a nonempty set $A_t^i \subseteq A_t$ of assets with cardinality $|A_t^i|$. $A^i = \cup_{t=1}^{\infty} A_t^i$ is the set of all assets available to $i \in I$. Let Θ_t^i stand for the set of all portfolios available to i at time t . We assume:

¹Portfolio holdings can be negative and thus can include short sales. However, the existence of a positive basis for the set of positive payoffs spanned by the set of assets as shown below ensures that every feasible consumption stream can be obtained without recurring to short sales.

Assumption (Internal Completeness) For all $i \in \{0; 1 \dots n\}$ and all $t \geq 1$,

- (i) $a_{t,1} \in A_t^i$. That is, the economy, as well as each agent has access to the bond at each period;
- (ii) the payoff space generated by A_t^i , M_t^i is internally complete, that is, a sublattice of $\mathbb{R}^{|\Omega_t|}$.

Thus, every agent can be thought of as having access to options written on those marketed securities he can trade in. Furthermore, for each agent $i \in \{1 \dots n\}$, M_t^i is a sublattice of M_t^0 .

We now make use of results by Choquet (1956), Kendall (1962) and Polyraakis(1999), who show:

Theorem 1 (Choquet–Kendall, Polyraakis) *A finite dimensional ordered vector space is a vector lattice if and only if it has a positive basis.*

Since for $i \in \{0; 1 \dots n\}$, and any $t \geq 1$, M_t^i are finite dimensional vector spaces ordered by \geq and also, by definition, vector lattices, we conclude that each of them has a positive basis, that is, sets of linearly independent vectors $B_t^i = \left(b_{t,1}^i \dots b_{t,|A_t^i|}^i \right) \subset M_t^i$ such that the positive cone of M_t^i , $M_{t+}^i = \mathbb{R}_+^{|\Omega_t|} \cap M_t^i$ are given by $M_{t+}^i = \left\{ \sum_{k=1}^{|A_t^i|} \lambda_k b_k^i \mid \lambda_k \in \mathbb{R}_+ \text{ for all } k \right\}$. That is, a basis for M_t^i is positive if any positive element of M_t^i , has positive coefficients² for base B_t^i .

Furthermore, the positive basis of M_t^i ($i \in \{0; 1 \dots n\}$, $t \geq 1$) is unique up to a multiplication of each of the basis vectors by a strictly positive number (Kountzakis and Polyraakis 2006, pp. 8-9). In general, $|A_t^0| \neq |A_t^i|$ for $i \in \{1 \dots n\}$. Further, $|A_t^0|$ (and thus $|A_t^i|$) will in general be smaller than the number of states, $|\Omega_t|$. That is, in general neither the original set of assets, nor that assigned to each agent need be complete. Proposition 4 in Kountzakis and Polyraakis (2006, p. 9) further demonstrates that for any two vectors in B_t^i , the supports of the vectors are disjoint, that is, for any two $b_{t,k}^i$, $b_{t,k'}^i$ with $k \neq k'$, $\text{supp} \left(b_{t,k}^i \right) \cap \text{supp} \left(b_{t,k'}^i \right) = \emptyset$. Finally, by Proposition 6 in Kountzakis and Polyraakis (2006, p. 9), since the bond $a_{t,1} \in A_t^i$, the supports of the vectors in B_t^i form a partition of the state space Ω_t and can be chosen such that $b_{t,k}^i(\sigma_t) = 1$ whenever $\sigma_t \in \text{supp} \left(b_{t,k}^i \right)$ and $b_{t,k}^i(\sigma_t) = 0$, else. We will refer to the partition of Ω_t generated by the vectors in B_t^i , $i \in \{0; 1 \dots n\}$ as Ω_t^i . We will show below that if $A_t^i \subset A_t^j$, then Ω_t^i is a coarsening of Ω_t^j .

Finally, Theorem 9 in Kountzakis and Polyraakis (2006, p. 10) provides an explicit formula for deriving the set of vectors B_t^i for a given set of assets A_t^i .

We summarize this discussion in the following Proposition

²Hence, replacing the original set of assets A_t^i by the basis B_t^i allows obtaining any non-negative consumption profile without resorting to short sales.

Proposition 1 (based on Kountzakis and Polyrakis, 2006) *Suppose that Assumption Internal Completeness holds. The space of payoffs spanned by the assets in A_t^i for some $i \in \{0; 1 \dots n\}$ and $t \geq 1$ is spanned by a set of $|A_t^i|$ vectors $B_t^i = \left\{ \left(b_{t,k}^i \right)_{k=1}^{|A_t^i|} \right\}$. There is a partition Ω_t^i of Ω_t with representative element ω_t^i such that $|\Omega_t^i| = |A_t^i|$ and $b_{t,k}^i = \mathbf{1}_{\omega_{t,k}^i}$ for all $k \in \{1 \dots |A_t^i|\}$. Finally, if $A_t^i \subset A_t^j$, the partition Ω_t^j is a refinement of Ω_t^i .*

Proof of Proposition 1:

The proofs of all statements but the last are in Polyrakis (1999). We will now prove that if $A_t^i \subset A_t^j$, then Ω_t^j is a refinement of Ω_t^i . To do so, start by determining the partition corresponding to A_t^j , Ω_t^j using the algorithm suggested by Polyrakis (1999). Since the payoffs of all assets in A_t^j are nonnegative, and since the base for M_t^j is given by B_t^j , there exist coefficients $\left(\lambda_1(a_{t,k}) \dots \lambda_{|A_t^j|}(a_{t,k}) \right)$ such that

$$a_{t,k}(s) = \sum_{l=1}^{|A_t^j|} \lambda_l(a_{t,k}) b_{t,l}^j(\sigma_t) \text{ for all } \sigma_t \in \Omega_t \text{ and all } a_{t,k} \in A_t^j.$$

Note, however, that each $b_{t,l}^j$ is measurable with respect to Ω_t^j . Hence, if $\sigma_t, \sigma_t' \in \omega_t^j \in \Omega_t^j$, then

$$a_{t,k}(\sigma_t) = \sum_{l=1}^{|A_t^j|} \lambda_l(a_{t,k}) b_{t,l}^j(\sigma_t) = a_{t,k}(\sigma_t') = \sum_{l=1}^{|A_t^j|} \lambda_l(a_{t,k}) b_{t,l}^j(\sigma_t') \text{ for all } a_{t,k} \in A_t^j$$

and hence, the payoffs of the assets in A_t^j are also measurable with respect to Ω_t^j . Hence, define a state space given by Ω_t^j and consider the construction of Ω_t^i with respect to this state space. By the previous part of the Proposition, we have that for B^i defined with respect to Ω_t^j , there is a partition of Ω_t^j , Ω_t^i with representative element $\omega_{t,k}^i$ such that $b_{t,k}^i = \mathbf{1}_{\omega_{t,k}^i}$ for $k \in \{1 \dots |A_t^i|\}$. We have thus shown that Ω_t^i is a coarsening of Ω_t^j and hence, Ω_t^j is a refinement of Ω_t^i . Since $|A_t^i| < |A_t^j|$, these relations are strict.

Given the result of Proposition 1, we can restrict attention to what we will call generalized unit securities, that is, securities that pay 1 unit on a subset of the state space $\omega_t \subset \Omega_t$ and nothing else. Since for each $t \geq 1$, Ω_t is finite, so is the maximal number of all such possible securities, given by $2^{|\Omega_t|} - 1$. Hence, we can now define the set of period t securities in the economy to be \tilde{A}_t^0 and that available to an agent $i \in \{1 \dots n\}$ as \tilde{A}_t^i given by:

$$\tilde{A}_t^i = \left\{ a_{\omega_t^i} \text{ with } a_{\omega_t^i}(\sigma_t) = \mathbf{1}_{\omega_t^i} \text{ for all } \omega_t^i \in \Omega_t^i \right\},$$

for $i \in \{0; 1 \dots n\}$ where Ω_t^i is the partition of Ω_t identified in Proposition 1.

Note that while in general $\tilde{A}_t^i \not\subset \tilde{A}_t^0$, we have $\tilde{M}_t^i \subseteq \tilde{M}_t$ for all $i \in I$ and, similarly, $\tilde{M}_t^j \subset \tilde{M}_t^i$, whenever Ω_t^j is coarser than Ω_t^i , which is true in particular when $A_t^j \subset A_t^i$.

We next make some assumptions about the sets of assets available in the economy and to the individual investors at different time periods. Note that for $i \in \{0; 1 \dots n\}$ and $t \geq 1$, $\theta \in \Theta_t^i$ iff $T_t(\theta) \in M_t^i$.

Assumption (Intertemporal Asset Structure) For all $t \geq 1$, and any $i \in \{0; 1 \dots n\}$

- (i) if for some $\theta \in \Theta_t^i$, then there is a $\theta' \in \Theta_{t+1}^i$ such that $T_{t+1}(\theta')(\sigma_t; s) = T_t(\theta)(\sigma_t)$ holds for all $\sigma_t \in \Omega_t$ and all $s \in S$;
- (ii) for any $\sigma_{t-1} \in \Omega_{t-1}$ and $s' \in S$, if for some $\theta \in \Theta_t^i$, then there is a $\theta' \in \Theta_{t+1}^i$ such that $T_{t+1}(\theta')(\sigma_{t-1}; s'; s) = T_t(\theta)(\sigma_{t-1}; s)$ holds for all $s \in S$;
- (iii) if for some $s, s' \in S$, $a(s) = a(s')$ for all $a \in A_1^i$, then $a(\sigma_{t-1}; s) = a(\sigma_{t-1}; s')$ holds for all $\sigma_{t-1} \in \Omega_{t-1}$ and all $a \in A_t^i$;
- (iv) if for some $\sigma_t \in \Omega$, $a(\sigma_t) = a(\sigma'_t)$ for all $a \in A_t^i$, then $a(\sigma_t; s) = a(\sigma'_t; s)$ holds for all $s \in S$ and all $a \in A_{t+1}^i$.

Conditions (i) and (ii) say that the set of payoffs that can be generated is (weakly) expanding with time. This appears natural given the tree-event structure of the uncertainty, that is, the fact that the number of contingencies on which trade is potentially possible grows over time. In particular, according to (i), if a specific payoff structure could be obtained over the nodes in Ω_t , then the same payoff structure can also be obtained over the nodes in Ω_{t+1} . Furthermore, according to (ii), if the market (or an agent i) can trade across two states s and s' at time t , he can also do so at time $t + 1$. Assumption (iii) establishes that if the market (or an agent) cannot trade between two states at time 1, then neither can they do so at any future time period. For example, if states s and s' relate to uncertainty in a foreign country, then the fact that an agent does not have access to foreign equity, precludes trades between s and s' in all periods of time. Finally, assumption (iv) says that, if it was impossible to trade on certain contingencies, the realization of which was revealed at time t , then no assets permitting trade on these contingencies will be available after period t , either. These assumptions apply to an economy, in which financial constraints do not change over time in the sense made precise in Proposition 2 below and clearly do not hold when financial constraints are relaxed over time, as, e.g., in Section 6 of the main paper, or for the case of sequential trade as in Section 2 of this Supplement.

Proposition 2 *Suppose that the Assumptions Internal Completeness and Intertemporal Asset Structure hold. Then for $i \in \{0; 1 \dots n\}$ there exist partitions of S , W^i such that for each $t \geq 1$, $\Omega_t^i = \prod_{\tau=1}^t W^i$.*

Proof of Proposition 2:

We will make the argument for Ω_t^0 . The argument for Ω_t^i , $i \in I$ is analogous.

We proceed by induction on $t \geq 1$. Note that for $t = 1$, we can set the partition derived in Proposition 1 $\Omega_1^0 = W^0$. Consider $t > 1$ and assume that $\Omega_{\tilde{t}}^0 = \prod_{\tau=1}^{\tilde{t}} W^0$ for all $\tilde{t} \leq t$. We will show that $\Omega_{t+1}^0 = \prod_{\tau=1}^{t+1} W^0$. Indeed, by assumption we have that for all $w, s, s' \in w$, and any $\theta' \in \Theta_1^0$, $T_1(\theta'(s)) = T_1(\theta'(s'))$. Furthermore, for every $\theta'' \in \Theta_t^0$ any $\omega_t \in \Omega_t^0$, all $\sigma_t, \sigma'_t \in \omega_t$, $T_t(\theta''(\sigma_t)) = T_t(\theta''(\sigma'_t))$. Thus, combining (iii) and (iv) of Assumption Intertemporal Asset Structure, we conclude that for all $\theta \in \Theta_{t+1}^0$, any $w \in W^0$, any $\omega_t \in \Omega_t^0$, all $\sigma_t, \sigma'_t \in \omega_t$ and all $s, s' \in w$,

$$T_{t+1}(\theta)(\sigma_t; s) = T_{t+1}(\theta)(\sigma_t; s') = T_{t+1}(\theta)(\sigma'_t; s) = T_{t+1}(\theta)(\sigma'_t; s').$$

It follows that the finest partition that is spanned by M_{t+1}^0 is $\prod_{\tau=1}^{t+1} W^0$. Now suppose that there are two distinct elements of $\prod_{\tau=1}^{t+1} W^0$, call them $(\omega_t; w)$ and $(\omega'_t; w')$ such that for all $\theta' \in \Theta_{t+1}^0$, $T_{t+1}(\theta')(\omega_t; w) = T_{t+1}(\theta')(\omega'_t; w')$. If $\omega_t \neq \omega'_t$, this contradicts (i), there is a $\theta \in \Theta_t^0$ with payoffs $T_t(\theta)(\omega_t) \neq T_t(\theta)(\omega'_t)$. If, $\omega_t = \omega'_t$ and $w \neq w'$, this contradicts (ii), since then for all $\sigma_t \in \omega_t$ and some $s \in w, s' \in w'$, $T_{t+1}(\theta')(\sigma_t; s) = T_{t+1}(\theta')(\sigma_t; s')$ for all $\theta' \in \Theta_{t+1}^0$, whereas there is a portfolio $\theta \in \Theta_t^0$ such that $T_t(\theta)(\sigma_{t-1}; s) \neq T_t(\theta)(\sigma_{t-1}; s')$. Hence, $\prod_{\tau=1}^{t+1} W^0$ is also the coarsest partition with respect to which all elements of M_{t+1}^0 are measurable and $\Omega_t^0 = \prod_{\tau=1}^{t+1} W^0$.

1.2 The Case of Sequential Trade

1.2.1 Partition Structures with Short Lived Assets

For a given node $\sigma_t \in \Omega_t$, we consider a set of assets independent of σ_t , $= A^0$, with typical element $a_{\sigma_t} : S \rightarrow \mathbb{R}_+$. I.e., an asset a_{σ_t} is born and traded at σ_t , lives for a single period and pays a nonnegative amount at each successor of σ_t , $(\sigma_t; s)$, $s \in S$. We will assume that at each σ_t , $A_{\sigma_t}^0$ consists of $|A^0|$ linearly independent assets a , denoted $a_{\sigma_t;1} \dots a_{\sigma_t;|A^0|}$ and that asset $a_{\sigma_t;1}$ is a bond that pays 1 at every $(\sigma_t; s) \in \Omega_{t+1}$. However, we will not assume that A^0 (and thus, $A_{\sigma_t}^0$) is complete relative to S .

A vector $\theta = (\theta_1 \dots \theta_{|A^0|}) \in \mathbb{R}^{|A^0|}$ corresponds to a portfolio or to a "marketed security" with payoffs given by $T_{\sigma_t}(\theta)(s) = \sum_{j=1}^{|A^0|} \theta_j a_{\sigma_t;j}(s)$. Let $\Theta_{\sigma_t}^0 = \Theta^0$ stand for the set of all portfolios at node σ_t . The range of $T_{\sigma_t} : \Theta_{\sigma_t}^0 \rightarrow \mathbb{R}^{|S|}$ is the asset span of A^0 .

We will endow the set of marketed securities with the following structure: for any θ and $\theta' \in \Theta^0$, there exist portfolios $\theta \vee \theta'$ and $\theta \wedge \theta'$ with payoffs:

$$T_{\sigma_t}(\theta \vee \theta') = \sup \{T_{\sigma_t}(\theta); T_{\sigma_t}(\theta')\} = (\max \{T_{\sigma_t}(\theta)(s); T_{\sigma_t}(\theta')(s)\})_{s \in S}$$

$$T_{\sigma_t}(\theta \wedge \theta') = \inf \{T_{\sigma_t}(\theta); T_{\sigma_t}(\theta')\} = (\min \{T_{\sigma_t}(\theta)(s); T_{\sigma_t}(\theta')(s)\})_{s \in S}$$

In other words, the set of marketed securities generates a payoff space which is a sublattice of $\mathbb{R}^{|S|}$, denoted M^0 . This space is linear and, as shown by Polyrakis (1999), a minimal such space exists. Hence, M^0 is uniquely specified and the corresponding payoff space is "internally complete".

As above, the set of agents is given by I and has cardinality n . At each σ_t , agent i has access to a nonempty set $A_{\sigma_t}^i \equiv A^i$, $A_{\sigma_t}^i \subseteq A_{\sigma_t}^0$ of assets with cardinality $|A^i|$. As above, we assume:

Assumption (Internal Completeness) For all $i \in \{0; 1 \dots n\}$ and all $\sigma_t \in \Omega$,

- (i) $a_{\sigma_t;1} \in A_{\sigma_t}^i$, i.e., the economy, as well as each agent has access to the bond at each period;
- (ii) the payoff space generated by $A_{\sigma_t}^i = A^i$, M^i , is internally complete, that is, a sublattice of $\mathbb{R}^{|S|}$.

Thus, every agent can be thought of as having access to options written on those marketed securities he can trade in. Furthermore, for each agent i , M^i is a sublattice of M^0 .

The discussion in Section 1 of this Supplement, as well as the results of Theorem 1 apply in this case and we can restate Proposition 2 as:

Proposition 3 (based on Kountzakis and Polyrakis, 2006) Suppose that Assumption Internal Completeness holds. For $i \in \{0; 1 \dots n\}$, there exist partitions of S , W^i with elements $w_1^i \dots w_{|A^i|}^i$ such that the space of payoffs spanned by the assets in $A_{\sigma_t}^i = A^i$ is spanned by $B_{\sigma_t}^i = B^i = \left\{ b_{\sigma(t);k}^i = \mathbf{1}_{w^i}, k = 1 \dots |A^i| \right\}$. Finally, if $A^i \subset A^j$, the partition W^j is a refinement of W^i .

1.2.2 Asset Structure with Nested Partitions

We construct the sets $\tilde{A}_{\sigma_t}^i$ for a given σ_t . To do so, we proceed by induction on the set of agents, endowing agent n with assets corresponding to the first $|W^n| - 1$ elements of n 's partition and the bond, i.e.,

$$\tilde{A}_{\sigma_t}^n = \left\{ \mathbf{1}_{w_1^n} \dots \mathbf{1}_{w_{|W^n|-1}^n}; \mathbf{1}_S \right\}.$$

For agent $n - 1$ and for an element of the partition of agent n , $w_k^n \in W^n$, let $W_{k,n}^{n-1} \subseteq W^{n-1}$ denote the set of elements of the partition of agent $n - 1$ which are subsets of w_k^n , i.e., for every $w^{n-1} \in W_{k,n}^{n-1}$, $w^{n-1} \subseteq w_k^n$. For every $k \in \{1 \dots |W^n|\}$, choose arbitrarily an element of $W_{k,n}^{n-1}$, $\tilde{w}_{k,n}^{n-1}$. Then, the set of assets of agent $n - 1$ is given by:

$$\tilde{A}_{\sigma_t}^{n-1} = \tilde{A}_{\sigma_t}^n \cup_{k=1}^{|W^n|} \left\{ \mathbf{1}_{w^{n-1}} \right\}_{w^{n-1} \in W_{k,n}^{n-1} \setminus \{\tilde{w}_{k,n}^{n-1}\}}.$$

By induction, let the set of assets available to agent i be $\tilde{A}_{\sigma_t}^i$. For agent $i - 1$ and element of the partition of agent i , $w_k^i \in W^i$, let $W_{k,i}^{i-1} \subseteq W^{i-1}$ denote the

set of elements of the partition of agent $i - 1$ which are subsets of w_k^i , i.e., for every $w^{i-1} \in W_{k,i}^{i-1}$, $w^{i-1} \subseteq w_k^i$. For every $k \in \{1 \dots |W^i|\}$, choose arbitrarily an element of $W_{k,i}^{i-1}$, $\tilde{w}_{k,i}^{i-1}$. Then, the set of assets of agent $i - 1$ is given by:

$$\tilde{A}_{\sigma_t}^{i-1} = \tilde{A}_{\sigma_t}^i \cup_{k=1}^{|W^i|} \{\mathbf{1}_{w^{i-1}}\}_{w^{i-1} \in W_{k,i}^{i-1} \setminus \{w_{k,i}^{i-1}\}}.$$

Finally, since $W^1 = W^0$ set $\tilde{A}_{\sigma_t}^0 = \tilde{A}_{\sigma_t}^1$. The obtained asset structure satisfies requirements (i), (ii) and (iii) imposed in Section 8.1 of the main part of the paper.

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