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On Integer Additive Set-Sequential Graphs

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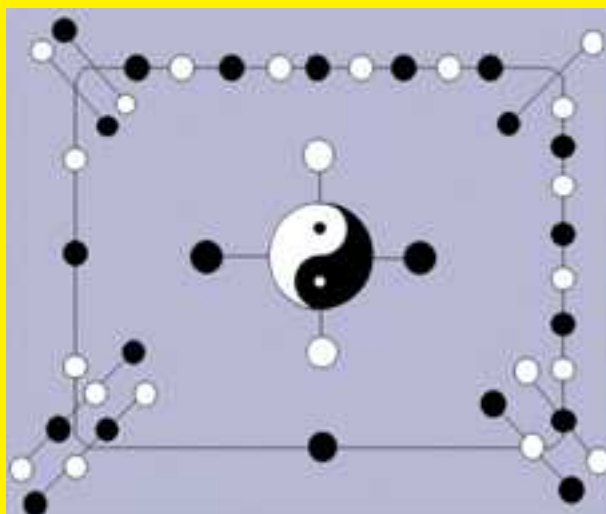
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Famous Words:

It is at our mother's knee that we acquire our noblest and truest and highest ideals, but there is seldom any money in them.

By Mark Twain, an American writer.

On Integer Additive Set-Sequential Graphs

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Abstract: A set-labeling of a graph G is an injective function $f : V(G) \rightarrow \mathcal{P}(X)$, where X is a finite set of non-negative integers and a set-indexer of G is a set-labeling such that the induced function $f^\oplus : E(G) \rightarrow \mathcal{P}(X) - \{\emptyset\}$ defined by $f^\oplus(uv) = f(u) \oplus f(v)$ for every $uv \in E(G)$ is also injective. A set-indexer $f : V(G) \rightarrow \mathcal{P}(X)$ is called a set-sequential labeling of G if $f^\oplus(V(G) \cup E(G)) = \mathcal{P}(X) - \{\emptyset\}$. A graph G which admits a set-sequential labeling is called a set-sequential graph. An integer additive set-labeling is an injective function $f : V(G) \rightarrow \mathcal{P}(\mathbb{N}_0)$, \mathbb{N}_0 is the set of all non-negative integers and an integer additive set-indexer is an integer additive set-labeling such that the induced function $f^+ : E(G) \rightarrow \mathcal{P}(\mathbb{N}_0)$ defined by $f^+(uv) = f(u) + f(v)$ is also injective. In this paper, we extend the concepts of set-sequential labeling to integer additive set-labelings of graphs and provide some results on them.

Key Words: Integer additive set-indexers, set-sequential graphs, integer additive set-labeling, integer additive set-sequential labeling, integer additive set-sequential graphs.

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§1. Introduction

For all terms and definitions, not defined specifically in this paper, we refer to [4], [5] and [9] and for more about graph labeling, we refer to [6]. Unless mentioned otherwise, all graphs considered here are simple, finite and have no isolated vertices.

All sets mentioned in this paper are finite sets of non-negative integers. We denote the cardinality of a set A by $|A|$. We denote, by X , the finite ground set of non-negative integers that is used for set-labeling the elements of G and cardinality of X by n .

The research in graph labeling commenced with the introduction of β -valuations of graphs in [10]. Analogous to the number valuations of graphs, the concepts of set-labelings and set-indexers of graphs are introduced in [1] as follows.

Let G be a (p, q) -graph. Let X, Y and Z be non-empty sets and $\mathcal{P}(X), \mathcal{P}(Y)$ and $\mathcal{P}(Z)$ be their power sets. Then, the functions $f : V(G) \rightarrow \mathcal{P}(X)$, $f : E(G) \rightarrow \mathcal{P}(Y)$ and $f : V(G) \cup E(G) \rightarrow \mathcal{P}(Z)$ are called the *set-assignments* of vertices, edges and elements of G respectively. By a set-assignment

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of a graph, we mean any one of them. A set-assignment is called a *set-labeling* or a *set-valuation* if it is injective.

A graph with a set-labeling f is denoted by (G, f) and is referred to as a *set-labeled graph* or a *set-valued graph*. For a (p, q) - graph $G = (V, E)$ and a non-empty set X of cardinality n , a *set-indexer* of G is defined as an injective set-valued function $f : V(G) \rightarrow \mathcal{P}(X)$ such that the function $f^\oplus : E(G) \rightarrow \mathcal{P}(X) - \{\emptyset\}$ defined by $f^\oplus(uv) = f(u) \oplus f(v)$ for every $uv \in E(G)$ is also injective, where $\mathcal{P}(X)$ is the set of all subsets of X and \oplus is the symmetric difference of sets.

Theorem 1.1([1]) *Every graph has a set-indexer.*

Analogous to graceful labeling of graphs, the concept of set-graceful labeling and set-sequential labeling of a graph are defined in [1] as follows.

Let G be a graph and let X be a non-empty set. A set-indexer $f : V(G) \rightarrow \mathcal{P}(X)$ is called a *set-graceful labeling* of G if $f^\oplus(E(G)) = \mathcal{P}(X) - \{\emptyset\}$. A graph G which admits a set-graceful labeling is called a *set-graceful graph*.

Let G be a graph and let X be a non-empty set. A set-indexer $f : V(G) \rightarrow \mathcal{P}(X)$ is called a *set-sequential labeling* of G if $f^\oplus(V(G) \cup E(G)) = \mathcal{P}(X) - \{\emptyset\}$. A graph G which admits a set-sequential labeling is called a *set-sequential graph*.

Let A and B be two non-empty sets. Then, their *sum set*, denoted by $A + B$, is defined to be the set $A + B = \{a + b : a \in A, b \in B\}$. If $C = A + B$, then A and B are said to be the *summands* of C . Using the concepts of sum sets of sets of non-negative integers, the notion of integer additive set-labeling of a given graph G is introduced as follows.

Let \mathbb{N}_0 be the set of all non-negative integers. An *integer additive set-labeling* (IASL, in short) of graph G is an injective function $f : V(G) \rightarrow \mathcal{P}(\mathbb{N}_0)$ such that the induced function $f^+ : E(G) \rightarrow \mathcal{P}(\mathbb{N}_0)$ is defined by $f^+(uv) = f(u) + f(v)$ for $\forall uv \in E(G)$. A graph G which admits an IASL is called an IASL graph.

An *integer additive set-labeling* f is an integer additive set-indexer (IASI, in short) if the induced function $f^+ : E(G) \rightarrow \mathcal{P}(\mathbb{N}_0)$ defined by $f^+(uv) = f(u) + f(v)$ is injective(see [7]). A graph G which admits an IASI is called an IASI graph.

The following notions are introduced in [11] and [8]. The cardinality of the set-label of an element (vertex or edge) of a graph G is called the *set-indexing number* of that element. An IASL (or an IASI) is said to be a k -uniform IASL (or k -uniform IASI) if $|f^+(e)| = k \forall e \in E(G)$. The vertex set $V(G)$ is called *l -uniformly set-indexed*, if all the vertices of G have the set-indexing number l .

Definition 1.2([13]) *Let G be a graph and let X be a non-empty set. An integer additive set-indexer $f : V(G) \rightarrow \mathcal{P}(X) - \{\emptyset\}$ is called a integer additive set-graceful labeling (IASGL, in short) of G if $f^+(E(G)) = \mathcal{P}(X) - \{\emptyset, \{0\}\}$. A graph G which admits an integer additive set-graceful labeling is called an integer additive set-graceful graph (in short, IASG-graph).*

Motivated from the studies made in [2] and [3], in this paper, we extend the concepts of set-sequential labelings of graphs to integer additive set-sequential labelings and establish some results on them.

§2. IASSL of Graphs

First, note that under an integer additive set-labeling, no element of a given graph can have \emptyset as its

set-labeling. Hence, we need to consider only non-empty subsets of X for set-labeling the elements of G .

Let f be an integer additive set-indexer of a given graph G . Define a function $f^* : V(G) \cup E(G) \rightarrow \mathcal{P}(X) - \{\emptyset\}$ as follows.

$$f^*(x) = \begin{cases} f(x) & \text{if } x \in V(G) \\ f^+(x) & \text{if } x \in E(G) \end{cases} \quad (2.1)$$

Clearly, $f^*[V(G) \cup E(G)] = f(V(G)) \cup f^+(E(G))$. By the notation, $f^*(G)$, we mean $f^*[V(G) \cup E(G)]$. Then, f^* is an extension of both f and f^+ of G . Throughout our discussions in this paper, the function f^* is as per the definition in Equation 2.1.

Using the definition of new induced function f^* of f , we introduce the following notion as a sum set analogue of set-sequential graphs.

Definition 2.1 An IASI f of G is said to be an integer additive set-sequential labeling (IASSL) if the induced function $f^*(G) = f(V(G)) \cup f^+(E(G)) = \mathcal{P}(X) - \{\emptyset\}$. A graph G which admits an IASSL may be called an integer additive set-sequential graph (IASS-graph).

Hence, an integer additive set-sequential indexer can be defined as follows.

Definition 2.2 An integer additive set-sequential labeling f of a given graph G is said to be an integer additive set-sequential indexer (IASSI) if the induced function f^* is also injective. A graph G which admits an IASSI may be called an integer additive set-sequential indexed graph (IASSI-graph).

A question that arouses much in this context is about the comparison between an IASGL and an IASSL of a given graph if they exist. The following theorem explains the relation between an IASGL and an IASSL of a given graph G .

Theorem 2.3 Every integer additive set-graceful labeling of a graph G is also an integer additive set-sequential labeling of G .

Proof Let f be an IASGL defined on a given graph G . Then, $\{0\} \in f(V(G))$ (see [13]) and $|f^+(E(G))| = \mathcal{P}(X) - \{\emptyset, \{0\}\}$. Then, $f^*(G)$ contains all non-empty subsets of X . Therefore, f is an IASSL of G . \square

Let us now verify the injectivity of the function f^* in the following proposition.

Proposition 2.4 Let G be a graph without isolated vertices. If the function f^* is an injective, then no vertex of G can have a set-label $\{0\}$.

Proof If possible let a vertex, say v , has the set-label $\{0\}$. Since G is connected, v is adjacent to at least one vertex in G . Let u be an adjacent vertex of v in G and u has a set-label $A \subset X$. Then, $f^*(u) = f(u) = A$ and $f^*(uv) = f^+(uv) = A$, which is a contradiction to the hypothesis that f^* is injective. \square

In view of Observation 2.4, we notice the following points.

Remark 2.5 Suppose that the function f^* defined in (2.1) is injective. Then, if one vertex v of G has the set label $\{0\}$, then v is an isolated vertex of G .

Remark 2.6 If the function f^* defined in (2.1) is injective, then no edge of G can also have the set

label $\{0\}$.

The following result is an immediate consequence of the addition theorem on sets in set theory and provides a relation connecting the size and order of a given IASS-graph G and the cardinality of its ground set X .

Proposition 2.7 *Let G be a graph on n vertices and m edges. If f is an IASSL of a graph G with respect to a ground set X , then $m + n = 2^{|X|} - (1 + \kappa)$, where κ is the number of subsets of X which is the set-label of both a vertex and an edge.*

Proof Let f be an IASSL defined on a given graph G . Then, $|f^*(G)| = |f(V(G)) \cup f^+(E(G))| = |\mathcal{P}(X) - \{\emptyset\}| = 2^{|X|} - 1$. But by addition theorem on sets, we have

$$\begin{aligned} |f^*(G)| &= |f(V(G)) \cup f^+(E(G))| \\ \text{That is, } 2^{|X|} - 1 &= |f(V(G))| + |f^+(E(G))| - |f(V(G)) \cap f^+(E(G))| \\ &= |V| + |E| - \kappa \\ \implies &= m + n - \kappa \\ \text{Whence } m + n &= 2^{|X|} - 1 - \kappa. \end{aligned}$$

This completes the proof. □

We say that two sets A and B are of *same parity* if their cardinalities are simultaneously odd or simultaneously even. Then, the following theorem is on the parity of the vertex set and edge set of G .

Proposition 2.8 *Let f be an IASSL of a given graph G , with respect to a ground set X . Then, if $V(G)$ and $E(G)$ are of same parity, then κ is an odd integer and if $V(G)$ and $E(G)$ are of different parity, then κ is an even integer, where κ is the number of subsets of X which are the set-labels of both vertices and edges.*

Proof Let f be a integer additive set-sequential labeling of a given graph G . Then, $f^*(G) = \mathcal{P}(X) - \{\emptyset\}$. Therefore, $|f^*(G)| = 2^{|X|} - 1$, which is an odd integer.

Case 1. Let $V(G)$ and $E(G)$ are of same parity. Then, $|V| + |E|$ is an even integer. Then, by Proposition 2.7, $2^{|X|} - 1 - \kappa$ is an even integer, which is possible only when κ is an odd integer.

Case 2. Let $V(G)$ and $E(G)$ are of different parity. Then, $|V| + |E|$ is an odd integer. Then, by Proposition 2.7, $2^{|X|} - 1 - \kappa$ is an odd integer, which is possible only when κ is an even integer. □

A relation between integer additive set-graceful labeling and an integer additive set-sequential labeling of a graph is established in the following result.

The following result determines the minimum number of vertices in a graph that admits an IASSL with respect to a finite non-empty set X .

Theorem 2.9 *Let X be a non-empty finite set of non-negative integers. Then, a graph G that admits an IASSL with respect to X have at least ρ vertices, where ρ is the number of elements in $\mathcal{P}(X)$ which are not the sum sets of any two elements of $\mathcal{P}(X)$.*

Proof Let f be an IASSL of a given graph G , with respect to a given ground set X . Let \mathcal{A} be the collection of subsets of X such that no element in \mathcal{A} is the sum sets any two subsets of X . Since f an IASL of G , all edge of G must have the set-labels which are the sum sets of the set-labels of their

end vertices. Hence, no element in \mathcal{A} can be the set-label of any edge of G . But, since f is an IASSL of G , $\mathcal{A} \subset f^*(G) = f(V(G)) \cup f^+(E(G))$. Therefore, the minimum number of vertices of G is equal to the number of elements in the set \mathcal{A} . \square

The structural properties of graphs which admit IASSLs arouse much interests. In the example of IASS-graphs, given in Figure 1, the graph G has some pendant vertices. Hence, there arises following questions in this context. Do an IASS-graph necessarily have pendant vertices? If so, what is the number of pendant vertices required for a graph G to admit an IASSL? Let us now proceed to find the solutions to these problems.

The minimum number of pendant vertices required in a given IASS-graph is explained in the following Theorem.

Theorem 2.10 *Let G admits an IASSL with respect to a ground set X and let \mathcal{B} be the collection of subsets of X which are neither the sum sets of any two subsets of X nor their sum sets are subsets of X . If \mathcal{B} is non-empty, then*

- (1) $\{0\}$ is the set-label of a vertex in G ;
- (2) the minimum number pendant vertices in G is cardinality of \mathcal{B} .

Remark 2.11 Since the ground set X of an IASS-graph must contain the element 0, every subset A_i of X sum set of $\{0\}$ and A_i itself. In this sense, each subset A_i may be considered as a *trivial sum set* of two subsets of X .

In the following discussions, by a sum set of subsets of X , we mean the non-trivial sum sets of subsets of X .

Proof Let f be an IASSL of G with respect to a ground set X . Also, let \mathcal{B} be the collection of subsets of X which are neither the sum sets of any two subsets of X nor their sum sets are subsets of X . Let $A \in \mathcal{B}$. then A must be the set-label of a vertex of G . Since $A \in \mathcal{B}$, the only set that can be adjacent to A is $\{0\}$. Therefore, since G is a connected graph, $\{0\}$ must be the set-label of a vertex of G . More over, since A is an arbitrary vertex in \mathcal{B} , the minimum number of pendant vertices in G is $|\mathcal{B}|$. \square

The following result thus establishes the existence of pendant vertices in an IASS-graph.

Theorem 2.12 *Every graph that admits an IASSL, with respect to a non-empty finite ground set X , have at least one pendant vertex.*

Proof Let the graph G admits an IASSL f with respect to a ground set X . Let \mathcal{B} be the collection of subsets of X which are neither the sum sets of any two subsets of X nor their sum sets are subsets of X .

We claim that \mathcal{B} is non-empty, which can be proved as follows. Since X is a finite set of non-negative integers, X has a smallest element, say x_1 , and a greatest element x_l . Then, the subset $\{x_1, x_l\}$ belongs to $f^*(G)$. Since it is not the sum set any sets and is not a summand of any set in $\mathcal{P}(X)$, $\{x_1, x_l\} \in \mathcal{B}$. Therefore, \mathcal{B} is non-empty.

Since \mathcal{B} is non-empty, by Theorem 2.10, G has some pendant vertices. \square

Remark 2.13 In view of the above results, we can make the following observations.

- (1) No cycle C_n can have an IASSL;
- (2) For $n \geq 2$, no complete graph K_n admits an IASSL.

(3) No complete bipartite graph $K_{m,n}$ admits an IASL.

The following result establish the existence of a graph that admits an IASSL with respect to a given ground set X .

Theorem 2.14 *For any non-empty finite set X of non-negative integers containing 0, there exists a graph G which admits an IASSL with respect to X .*

Proof Let X be a given non-empty finite set containing the element 0 and let $\mathcal{A} = \{A_i\}$, be the collection of subsets of X which are not the sum sets of any two subsets of X . Then, the set $\mathcal{A}' = \mathcal{P}(X) - \mathcal{A} \cup \{\emptyset\}$ is the set of all subsets of X which are the sum sets of any two subsets of X and hence the sum sets of two elements in \mathcal{A} .

What we need here is to construct a graph which admits an IASSL with respect to X . For this, begin with a vertex v_1 . Label the vertex v_1 by the set $A_1 = \{0\}$. For $1 \leq i \leq |\mathcal{A}|$, create a new vertex v_i corresponding to each element in \mathcal{A} and label v_i by the set $A_i \in \mathcal{A}$. Then, connect each of these vertices to V_1 as these vertices v_i can be adjacent only to the vertex v_1 . Now that all elements in \mathcal{A} are the set-labels of vertices of G , it remains the elements of \mathcal{A}' for labeling the elements of G . For any $A'_r \in \mathcal{A}'$, we have $A'_r = A_i + A_j$, where $A_i, A_j \in \mathcal{A}$. Then, draw an edge e_r between v_i and v_j so that e_r has the set-label A'_r . This process can be repeated until all the elements in \mathcal{A}' are also used for labeling the elements of G . Then, the resultant graph is an IASS-graph with respect to the ground set X . \square

Figure 1 illustrates the existence of an IASSL for a given graph G .

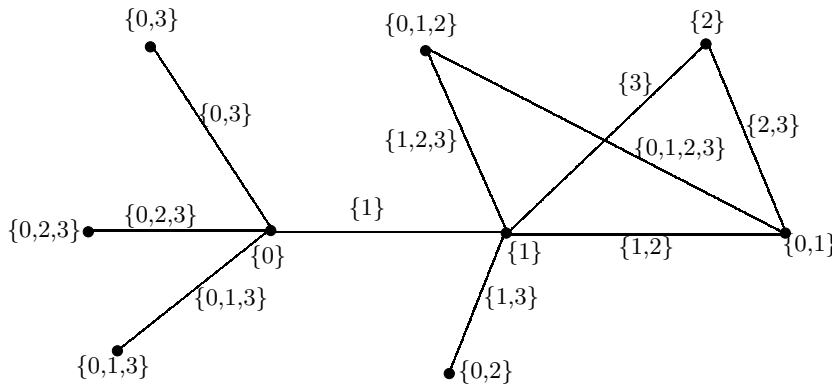


Figure 1

On the other hand, for a given graph G , the choice of a ground set X is also very important to have an integer additive set-sequential labeling. There are certain other restrictions in assigning set-labels to the elements of G . We explore the properties of a graph G that admits an IASSL with respect to a given ground set X . As a result, we have the following observations.

Proposition 2.15 *Let G be a connected integer additive set-sequential graph with respect to a ground set X . Let x_1 and x_2 be the two minimal non-zero elements of X . Then, no edges of G can have the set-labels $\{x_1\}$ and $\{x_2\}$.*

Proof In any IASL-graph G , the set-label of an edge is the sum set of the set-labels of its end vertices. Therefore, a subset A of the ground set X , that is not a sum set of any two subsets of X , can

not be the set-label of any edge of G . Since x_1 and x_2 are the minimal non-zero elements of X , $\{x_1\}$ and $\{x_2\}$ can not be the set-labels of any edge of G . \square

Proposition 2.16 *Let G be a connected integer additive set-sequential graph with respect to a ground set X . Then, any subset A of X that contains the maximal element of X can be the set-label of a vertex v of G if and only if v is a pendant vertex that is adjacent to the vertex u having the set-label $\{0\}$.*

Proof Let x_n be the maximal element in X and let A be a subset of X that contains the element x_n . If possible, let A be the set-label of a vertex, say v , in G . Since G is a connected graph, there exists at least one vertex in G that is adjacent to v . Let u be an adjacent vertex of v in G and let B be its set-label. Then, the edge uv has the set-label $A + B$. If $B \neq \{0\}$, then there exists at least one element $x_i \neq 0$ in B and hence $x_i + x_n \notin X$ and hence not in $A + B$, which is a contradiction to the fact that G is an IASS-graph. \square

Let us now discuss whether trees admit integer additive set-sequential labeling, with respect to a given ground set X .

Theorem 2.17 *A tree G admits an IASSL f with respect to a finite ground set X , then G has $2^{|X|-1}$ vertices.*

Proof Let G be a tree on n vertices. If possible, let G admits an IASSI. Then, $|E(G)| = n - 1$. Therefore, $|V(G)| + |E(G)| = n + n - 1 = 2n - 1$. But, by Theorem 2.9, $2^{|X|-1} = 2n - 1 \implies n = 2^{|X|-1}$. \square

Invoking the above results, we arrive at the following conclusion.

Theorem 2.18 *No connected graph G admits an integer additive set-sequential indexer.*

Proof Let G be a connected graph which admits an IASI f . By Proposition 2.4, if the induced function f^* is injective, then $\{0\}$ can not be the set-label of any element of G . But, by Propositions 2.15 and 2.16, every connected IASS-graph has a vertex with the set-label $\{0\}$. Hence, a connected graph G can not have an IASSI. \square

The problem of characterizing (disconnected) graphs that admit IASSIs is relevant and interesting in this situation. Hence, we have

Theorem 2.19 *A graph G admits an integer additive set-sequential indexer f with respect to a ground set X if and only if G has ρ' isolated vertices, where ρ' is the number of subsets of X which are neither the sum sets of any two subsets of X nor the summands of any subsets of X .*

Proof Let f be an IASI defined on G , with respect to a ground set X . Let \mathcal{B} be the collection of subsets of X which are neither the sum sets of any two subsets of X nor the summands of any subsets of X .

Assume that f is an IASSI of G . Then, the induced function f^* is an injective function. We have already showed that \mathcal{B} is a non-empty set. By Theorem 2.10, $\{0\}$ must be the set-label of one vertex v in G and the vertices of G with set-labels from \mathcal{B} can be adjacent only to the vertex v . By Remark 2.5, v must be an isolated vertex in G . Also note that $\{0\}$ is also an element in \mathcal{B} . Therefore, all the vertices which have set-labels from \mathcal{B} must also be isolated vertices of G . Hence G has $\rho' = |\mathcal{B}|$ isolated vertices.

Conversely, assume that G has $\rho' = |\mathcal{B}|$ isolated vertices. Then, label the isolated vertices of G by

the sets in \mathcal{B} in an injective manner. Now, label the other vertices of G in an injective manner by other non-empty subsets of X which are not the sum sets of subsets of X in such a way that the subsets of X which are the sum sets of subsets of X are the set-labels of the edges of G . Clearly, this labeling is an IASSI of G . \square

Analogous to Theorem 2.14, we can also establish the existence of an IASSI-graph with respect to a given non-empty ground set X .

Theorem 2.20 *For any non-empty finite set X of non-negative integers, there exists a graph G which admits an IASSI with respect to X .*

Figure 2 illustrates the existence of an IASSL for a given graph with isolated vertices.

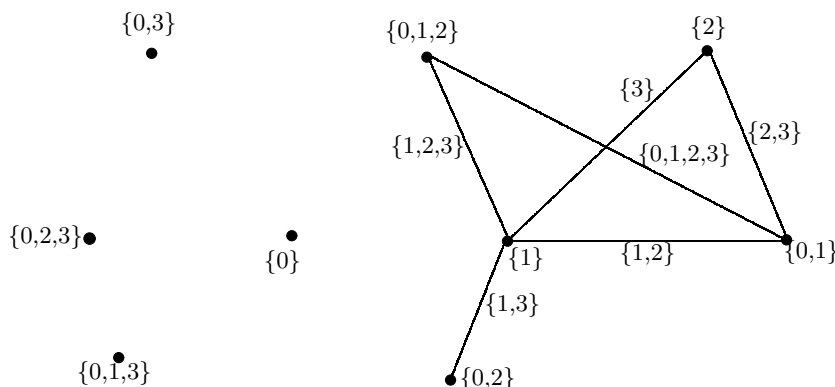


Figure 2

§3. Conclusion

In this paper, we have discussed an extension of set-sequential labeling of graphs to sum-set labelings and have studied the properties of certain graphs that admit IASSLs. Certain problems regarding the complete characterization of IASSI-graphs are still open.

We note that the admissibility of integer additive set-indexers by the graphs depends upon the nature of elements in X . A graph may admit an IASSL for some ground sets and may not admit an IASSL for some other ground sets. Hence, choosing a ground set is very important to discuss about IASSI-graphs.

There are several problems in this area which are promising for further studies. Characterization of different graph classes which admit integer additive set-sequential labelings and verification of the existence of integer additive set-sequential labelings for different graph operations, graph products and graph products are some of them. The integer additive set-indexers under which the vertices of a given graph are labeled by different standard sequences of non-negative integers, are also worth studying.

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In silence, in steadiness, in severe abstraction, let him hold by himself, add observation to observation, patient of neglect, patient of reproach , and bide his own time , happy enough if he can satisfy himself alone that the day he has seen something truly.

By Ralph Waldo Emerson, an American thinker.

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