

Measurement of kinematic fields via DIC for impact engineering applications

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Abstract

Two dynamic Brazilian tests are analyzed via digital image correlation for acquisition rates equal to 5 and 10 million frames per second. Displacements of the order of 1 mm, velocities of the order of ± 250 m/s, and accelerations as high as $\pm 4 \times 10^7$ m/s² are measured with spacetime DIC. Uncertainty quantifications enable spacetime DIC to be compared with instantaneous analyses. The gains provided by the temporal regularization are very significant for the acceleration fields. The observed levels are consistent with a priori estimates.

Key words: Digital Image Correlation, Full-field measurements, Spatial registration, Temporal registration, uncertainty quantification.

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1 **1 Introduction**

2 In impact engineering, as in other areas of solid mechanics, the understanding
3 and quantification of the loading conditions is one key point to be addressed.
4 One of the most used loading devices to characterize the material behavior at
5 high-rate loadings is split Hopkinson pressure bars [1,2]. For many years the
6 analysis of such tests relied on the use of strain gauge data and thus requires
7 long bars to be used. These strain data then enabled the loading conditions to
8 be inferred on the interfaces between the bars and the studied sample. This
9 situation may eventually change with the use of full-field measurements that
10 enable the experimentalist to focus more on the sample itself.

11 Among various optical techniques, Digital image correlation (DIC) and its 3D
12 extension for measuring 3D deformations via stereocorrelation [3] is gradually
13 emerging as suitable and reliable for high rate deformation and shock stud-
14 ies [4,5]. DIC, which was introduced in the early 1980s [6,7], was very quickly
15 applied to the study of cracking under dynamic loading. As early as 1985,
16 a first series of results was reported for a 3-point flexural test on Araldite B
17 samples [8]. The authors used a Crazz-Shardin camera, measured displacement
18 fields and compared them to the static solution for the estimation of stress
19 intensity factors. A good agreement was found when these results were com-
20 pared with those given by the analysis of caustics. This type of analysis was
21 continued with improved experimental conditions as well as post-processing
22 procedures [9].

23 With the advent of digital high speed cameras [10], the number of studies
24 have broadened and are no longer exclusively devoted to defense applications
25 (*e.g.*, see Refs. [11–19]). In the International Journal of Impact Engineering,

26 the first (and as of today [20], the most cited) paper on image correlation was
27 published by Tiwary et al. [21]. Displacement, velocity, acceleration, strain
28 and strain rate fields were reported. All of them were based on instantaneous
29 stereocorrelations with a commercial code. The measured displacement fields
30 were subsequently processed in order to compute all differentiated fields. The
31 subsequent publications [22–26] reported only strain fields, which is a stan-
32 dard output of instantaneous DIC codes. In the following papers [27–29], the
33 displacement fields were always reported, and sometimes the strain and veloc-
34 ity fields. The uncertainty were evaluated by Besnard et al. [30] for deformed
35 shape measurements and strain rates. Strain fields were then the primary
36 kinematic data that were discussed in the following publications [31–34].

37 The year 2014 saw no papers using DIC being published in the Journal (Fig-
38 ure 1). Over the previous five years a total of 14 papers was published. Over
39 the next four years (2018 included), a total of 39 papers was published. Of
40 those, only a few [35–41] did not only report displacement and/or strain fields,
41 which required additional processing when instantaneous analyses were per-
42 formed. It is worth noting that uncertainty quantifications were not reported
43 for any of the last 39 papers.

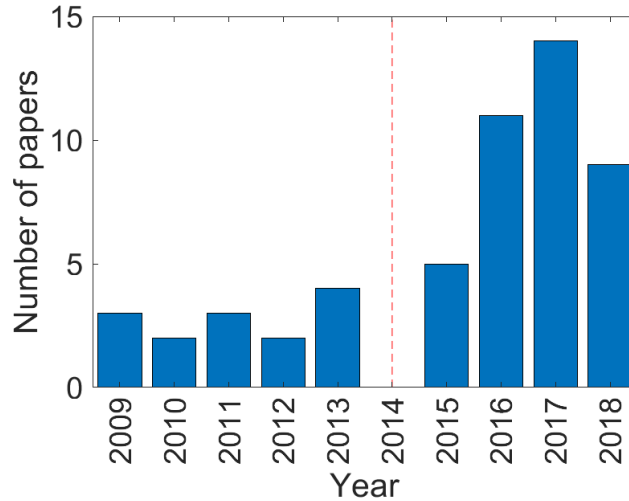


Fig. 1. Number of papers dealing with DIC and published yearly in the International Journal of Impact Engineering [20]

44 From this short literature review, it is concluded that virtually all studies
 45 used existing (and mostly commercial) DIC codes. It follows that most of the
 46 reported kinematic fields were those directly provided by the latter ones (*i.e.*,
 47 displacement and strain fields). Further, all reported DIC analyses were based
 48 on *instantaneous* registration, namely, the registration of image(s) of one single
 49 deformed configuration with respect to the reference configuration. As already
 50 mentioned, when the analyses required strain rate, velocity or acceleration
 51 fields, additional post-processing had to be performed [21,30,35–41].

52 An alternative route to instantaneous analyses is to perform local time in-
 53 terpolations in order to measure directly strain rate fields [42]. Global spa-
 54 tiotemporal analyses were also introduced [43,44] in which displacement and
 55 velocity fields were a priori interpolated along the time axis in the registration
 56 procedure, not a posteriori as would be performed with instantaneous DIC.
 57 Such techniques were applied to videos acquired with a camcorder [43] or high
 58 speed cameras [44]. Because the reference image played a central role in such

59 analyses, the gain in displacement uncertainty was not shown to be as high as
60 expected with noiseless references [44]. A denoising procedure was introduced
61 very recently to address this particular point [45].

62 In the following, instantaneous and spacetime DIC analyses will be compared.
63 Two dynamic Brazilian tests performed in split Hopkinson pressure bars and
64 monitored with an ultra-high speed camera will be analyzed. The acquisition
65 rate was set to 5 and 10 million frames per second. Five different kinematic
66 fields will be reported namely, displacement, velocity and acceleration fields on
67 the one hand. From these fields, the mean crack opening displacement, velocity
68 and accelerations will be discussed. On the other hand, strain and strain rate
69 fields will also be studied. In order to understand the differences between
70 instantaneous and spacetime DIC, the uncertainties on the five kinematic data
71 will be analyzed thanks to the fact that movies with no motions were available
72 for both experiments.

73 **2 Instantaneous and spacetime DIC**

74 Two different approaches will be compared herein, namely, instantaneous and
75 spacetime DIC. The first one consists in registering pairs of images whereas the
76 second one performs the registration over whole videos (*i.e.*, series of pictures).

77 *2.1 Instantaneous DIC*

In the following, a global approach will be used for the spatial registration.
The displacement field $\mathbf{u}(\mathbf{x}, t)$ is discretized by using finite element shape

functions [46] $\boldsymbol{\varphi}_i(\boldsymbol{x})$

$$\boldsymbol{u}(\boldsymbol{x}, t) = v_i(t)\boldsymbol{\varphi}_i(\boldsymbol{x}) \quad (1)$$

where $v_i(t)$ are the nodal displacements to be determined by minimizing the global residuals over the considered region of interest (ROI)

$$R(t) = \sum_{ROI} \rho^2(\boldsymbol{x}, t) \quad (2)$$

with respect to the unknown amplitudes $v_i(t)$, which are gathered the column vector $\{\boldsymbol{v}(t)\}$, where

$$\rho(\boldsymbol{x}, t) = f(\boldsymbol{x}, t) - f_0(\boldsymbol{x}) \quad (3)$$

78 denotes the gray level residual computed for any pixel \boldsymbol{x} of the ROI, $f(\boldsymbol{x}, t)$
 79 the gray level picture of the current configuration (at time t), and $f_0(\boldsymbol{x})$ the
 80 reference picture.

A priori estimates of the displacement and strain uncertainties for instantaneous analyses have been proposed [47]. Since the outputs of DIC analyses are displacement *fields*, the covariance matrix of the measured degrees of freedom has to be evaluated for characterizing measurement uncertainties [48]. In global approaches, each degree of freedom is not determined independently of the other ones. Consequently, they are correlated (in a statistical sense) and the covariance matrix is required to fully characterize their variances *and* covariances. For local (*i.e.*, subset-based) approaches, as soon as interrogation windows overlap, the displacement measurements are no longer independent (*i.e.*, some correlations exist since part of the pixels are shared by neighboring windows). It was shown that the covariance matrix $[\boldsymbol{C}_v]$ of the measured degrees of freedom $\{\boldsymbol{v}(t)\}$ is related to the Hessian at convergence of the minimization scheme $[\boldsymbol{M}]$

$$[\boldsymbol{C}_v] = 2\sigma_f^2[\boldsymbol{M}]^{-1} \quad (4)$$

81 where σ_f^2 is the variance of acquisition noise.

82 When a time series of pictures is analyzed, instantaneous DIC is usually run [3].
83 For each new image, the initial displacement estimate is that of the converged
84 solution of the previous image pair. In the following analyses, three-noded fi-
85 nite elements will be considered (*i.e.*, linear displacement interpolations within
86 each triangular element). Further, the reference image will not be the first
87 image of the time series but will be denoised [45,49]. Once the nodal displace-
88 ments are determined, the strain fields are evaluated by exact differentiation
89 of the spatial shape functions. The nodal velocities are determined by forward
90 finite differences of the nodal displacements, and the nodal accelerations by
91 forward finite differences of the nodal velocities. The strain rates are obtained
92 by using the nodal velocities and the exact derivatives of the shape functions.

93 2.2 Spatiotemporal DIC

An alternative route to the previous approach is to consider global analyses in time [43–45,49]. Instead of performing a series of instantaneous minimizations of instantaneous residuals, a *single* spatiotemporal minimization is run, and the following gray level residual

$$\mathcal{R} = \sum_t R(t) \quad (5)$$

is minimized with respect to the kinematic parameters. Different parameterizations of the displacement fields have been discussed [44]. In the following, space/time separations are assumed

$$\mathbf{u}(\mathbf{x}, t) = v_{ij} \boldsymbol{\varphi}_i(\mathbf{x}) \phi_j(t) \quad (6)$$

94 where v_{ij} denote the spatiotemporal unknowns to be determined. In the present
 95 study finite element discretizations will also be used for the temporal shape
 96 functions $\phi_j(t)$. The modal decomposition introduced in Ref. [45], which en-
 97 ables instantaneous DIC codes to be extended to spacetime analyses in an
 98 non-intrusive way, was implemented.

The a priori uncertainties can also be estimated by extending the previous
 results to spatiotemporal analyses [44]. It is first based on the spatiotempo-
 ral Hessian $[\mathbf{N}]$, which is constructed with the spatial Hessian $[\mathbf{M}]$, and the
 temporal matrix $[\Phi]$ such that

$$N_{ijkl} = M_{ik} \Phi_{jl} \quad (7)$$

with

$$\Phi_{ij} = \sum_t \varphi_i(t) \varphi_j(t) \quad (8)$$

It follows that the covariance matrix of spatiotemporal degrees of freedom
 becomes

$$C(v_{ij}, v_{kl}) = \sigma_f^2 \left(\Phi_{jp}^{-1} \Phi_p \Phi_{lq}^{-1} \Phi_q + \Phi_{jl}^{-1} \right) M_{ik}^{-1} \quad (9)$$

with

$$\Phi_k = \sum_t \varphi_k(t) \quad (10)$$

99 2.3 Reference picture f_0

100 In the sequel, high speed experiments will be analyzed. With the ultra-high
 101 speed camera used herein, a full video will be acquired prior to the experiment
 102 per se. Consequently, there exists a set of images with no motions. This series
 103 of images is averaged time-wise for each pixel to form a denoised reference
 104 picture f_0 . There is no need to follow the denoising procedure proposed in

105 Refs. [45,49].

106 When considering a denoised reference image, the factor 2 in Equation (4)
107 no longer appears. The direct consequence is that the standard displacement
108 uncertainties are divided by a factor of 2 for instantaneous analyses. Further,
109 the first term in Equation (9) vanishes for denoised reference pictures [44].
110 Consequently, the covariance matrix $C(v_{ij}, v_{kl})$ is equal to $\sigma_f^2 \Phi_{jl}^{-1} M_{ik}^{-1}$. It fol-
111 lows that the displacement uncertainties will be inversely proportional to the
112 square root number of pictures when linear interpolations are used.

113 3 Uncertainty quantifications

114 Before reporting any measurement, it is important to evaluate the correspond-
115 ing uncertainties [50]. This type of analysis was carried out on time series for
116 which no motion occurred.

117 3.1 *Dynamic Brazilian tests*

118 Brazilian tests enable the tensile strength of brittle materials such as concrete
119 and rocks to be assessed [51,52]. They consist in performing compression tests
120 on disks or cylinders. In each reported test, a 72 mm in diameter and 10 mm
121 thick disk made of Ductal[®] concrete was loaded up to failure under impact.
122 The dynamic tests were carried out via Hopkinson bars [2] made of aluminum
123 alloy. The loading system consisted of a pressurized air gun, a 60 mm in
124 diameter and 80 mm long projectile, an input bar with the same diameter
125 and a length of 4.5 m, and an output bar with a length of 2 m. The speed of
126 the input bar was equal to about 6 m/s. In both tests, the observed sample

127 surface was speckled by random spraying of black and white paints.

128 One spot light (Dedolight[®], maximum power: 400 W) and two light heads
129 (Dedocool[®], maximum power: 250 W) were switched on just before starting
130 the tests to avoid heating. Such lighting power was needed because of the very
131 high acquisition rates used herein. It enabled for enhanced contrast in the ac-
132 quired videos, which allows the measurement uncertainties to be lowered [47].
133 A Shimadzu[®] HPV-X ultra-high speed camera was used to record the defor-
134 mation of the specimen surface during the tests. Image videos with definition
135 of 400×250 pixels were acquired. The lens used with the camera was a
136 50 mm Nikon[®] F-Mount. The physical size of one pixel was set to $360 \mu\text{m}$,
137 which maximized the number of pixels of the monitored sample surfaces.

138 3.2 5 Mfps video

139 First, an acquisition rate of 5 million frames per second (fps) was selected.
140 The (fixed) exposure time was equal to 110 ns. A video of 256 images was
141 recorded. The reference image was then constructed as the temporal average
142 for any pixel \mathbf{x} . From this picture, the root mean square (RMS) difference was
143 computed for each time. This quantity evaluates the noise level of the camera.
144 Figure 2 shows the change of the RMS difference. The average level is equal to
145 147 gray levels, which is 0.2 % of the dynamic range (*i.e.*, 65500 gray levels).
146 This level is extremely low in comparison with high speed or even low speed
147 cameras [5].

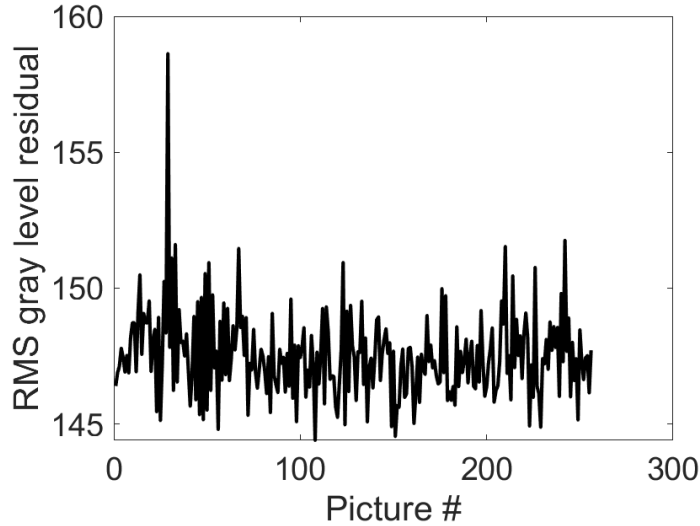


Fig. 2. RMS gray level difference between the acquired pictures and their temporal average for the 5 Mfps experiment

148 Instantaneous and spatiotemporal analyses were then run with this set of
 149 pictures. A finite element mesh was adapted to the actual sample geometry
 150 (Figure 3). The mean element edge was equal to 3 pixels (or 1.1 mm). In order
 151 to filter out spatial fluctuations of displacement fields, a regularization length
 152 equal to 10 pixels was selected [53]. For spatiotemporal analyses, linear inter-
 153 polations are considered along the temporal axis. Because of acquisition noise,
 154 each nodal displacement is a random variable with variances and covariances.
 155 For the sake of simplicity, the temporal fluctuations of measured nodal dis-
 156 placements are only characterized by the corresponding standard deviation in
 157 time. The measurement uncertainty is then *defined* in the following analyses
 158 as the spatial average of all standard deviations.

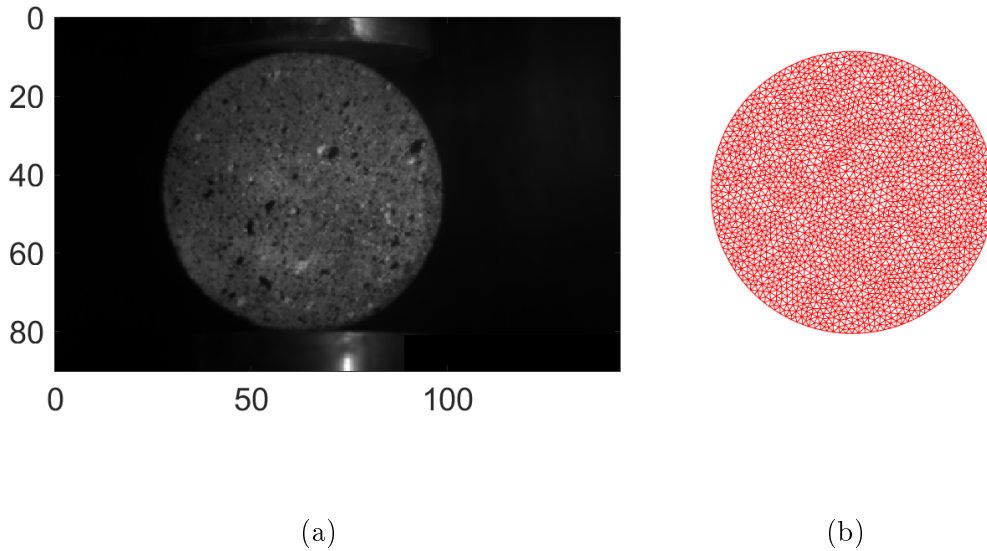


Fig. 3. (a) Denoised reference image. (b) Corresponding mesh made of 3-pixel elements for the 5 Mfps experiment. The dimensions are expressed in mm

159 In Figure 4, the measurement uncertainties are reported for displacements,
 160 velocities, principal strains and their rates. All these quantities are plotted
 161 as functions of the time interval over which the DIC analyses are performed.
 162 The lower level (*i.e.*, 0.2 μs) corresponds to instantaneous DIC. For all the
 163 reported quantities, the larger the time interval of the interpolation, the lower
 164 the uncertainties. This trend is to be expected since it corresponds to stronger
 165 temporal regularization. When the reference picture is denoised, it is anti-
 166 cipated that the standard displacement uncertainty σ_u is inversely proportional
 167 to the square root of the number of pictures per interval [44], or equivalently
 168 to the time interval ℓ_t . This trend is depicted by the green dashed line in Fig-
 169 ure 4(a) as the best log-fit. This assumption is not totally satisfied. It can be
 170 understood by noting that there are correlated temporal fluctuations in ac-
 171 quisition noise (see Figure 2). The blue dashed line correspond to the a priori
 172 estimates constructed with the Hessian of instantaneous DIC. The diagonal

173 terms correspond to the variances of each measured nodal displacement. Its
 174 average is then computed, from which the a priori estimate corresponds to its
 175 square root, namely, the a priori standard displacement uncertainty. In the
 176 present case, the a priori estimate virtually coincides with the log-fit.

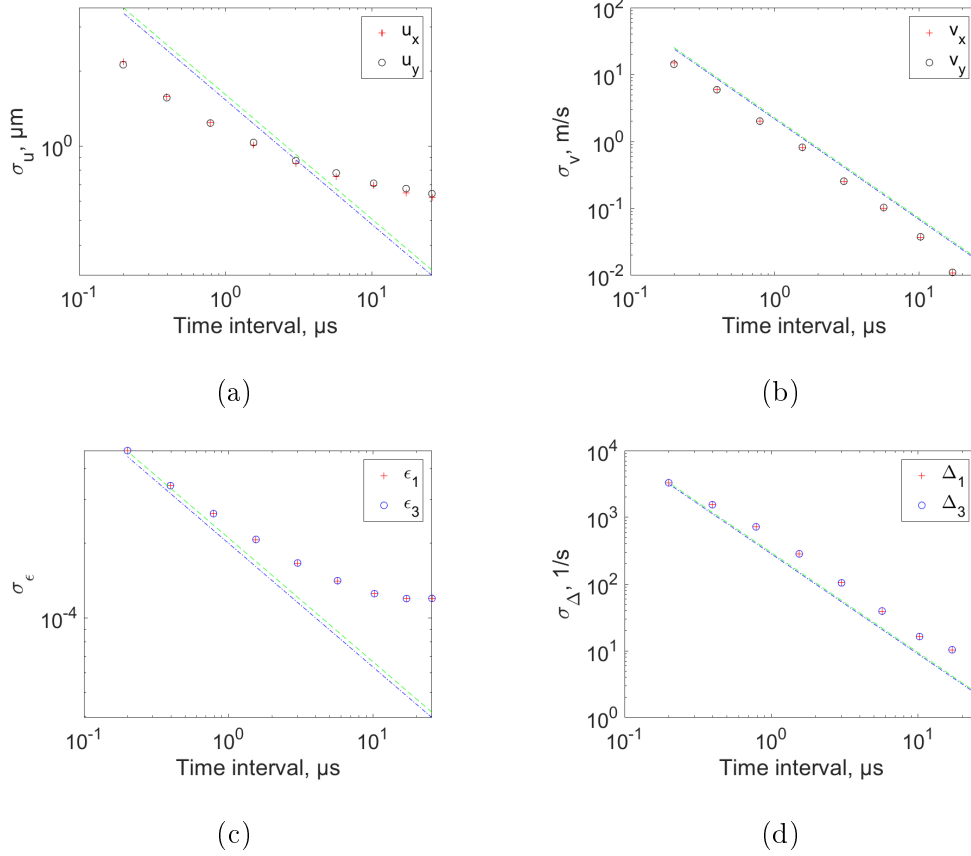


Fig. 4. Standard displacement (a), velocity (b), strain (c) and strain rate (d) uncertainties as functions of the time interval ℓ_t for the 5 Mfps experiment

177 The velocity uncertainty σ_v is equal to $\sqrt{2}\sigma_u/\ell_t$. From the previous analysis,
 178 this leads to the green dashed line shown in Figure 4(b), which slightly over-
 179 estimates the actual levels. However, the general trend is very well captured.
 180 This is also true for the a priori estimate (blue dashed line). The strain uncer-
 181 tainty, which depends on the displacement interpolation [47], is proportional
 182 to the displacement uncertainty divided by the regularization length ℓ_r [53]. As

183 for the displacements, this interpolation underestimates the standard strain
184 uncertainties (for the same reasons, see Figure 4(c)). Last, the strain rate un-
185 certainty σ_{Δ} is equal to $\sqrt{2}\sigma_{\epsilon}/\ell_t$, which is depicted by the green dashed line in
186 Figure 4(d). Again a very good agreement is observed. The same observations
187 apply for the a priori estimates.

188 Since acceleration fields will be sought, linear shape functions were not suf-
189 ficient. Six cubic B-splines were selected instead to model the displacement
190 fields for the first 150 pictures of the video. With the chosen parameterization
191 (see below), there were 6 degrees of freedom to be compared with 150 for
192 instantaneous DIC. Consequently, it is expected that the measurement uncer-
193 tainties will be lower for spacetime DIC when compared with instantaneous
194 analyses (Table 1). The gain in uncertainty is rather small (*i.e.*, ≈ 4 on av-
195 erage) for displacements and strains, becomes more important (*i.e.*, $\approx 4^3$) for
196 velocities and strain rates, and very significant for accelerations (*i.e.*, $\approx 4^5$).
197 For the latter, the cubic interpolation and the continuity of the accelerations
198 yield very significant gains (*i.e.*, three orders of magnitude).

Table 1

Standard uncertainties for instantaneous DIC (*i.e.*, $\ell_t = 0.1 \mu\text{s}$) and spline interpolation with six degrees of freedom

Standard uncertainty	instantaneous DIC	spatiotemporal DIC	ratio
σ_u (μm)	2.1	0.6	3.5
σ_v (m/s)	15	0.2	65
σ_a (m/s ²)	1.3×10^8	1.1×10^5	1200
σ_ϵ	4.7×10^{-4}	1.3×10^{-4}	3.7
σ_Δ (1/s)	3.3×10^3	36	91

199 The results of Table 1 show that significant gains in terms of measurement
 200 uncertainties can be achieved thanks to spatiotemporal analyses. This is par-
 201 ticularly true for acceleration fields (*i.e.*, three orders of magnitude in the
 202 present case), and to a lesser degree for velocity and strain rate fields (*i.e.*,
 203 two orders of magnitude). Such performances are made possible thanks to
 204 higher order interpolations in the time domain.

205 3.3 10 Mfps series

206 The second series was a sequence of 128 frames, which were acquired at a
 207 rate of 10 million fps. The (fixed) exposure time was equal to 50 ns. Once the
 208 reference picture was constructed, the noise level of the camera was estimated.
 209 Figure 5 shows the change of the RMS difference with time. The average level
 210 is equal to 192 gray levels, which is 0.3 % of the dynamic range. Even though
 211 higher than in the previous case, this level is very low given the very high

212 acquisition rate. As for the previous case, the temporal fluctuations are not
213 totally uncorrelated.

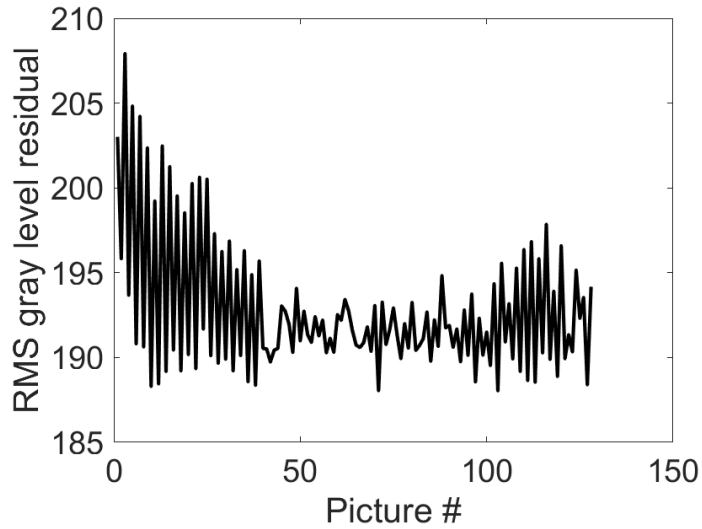


Fig. 5. RMS gray level difference between the acquired pictures and their temporal average for the 10 Mfps experiment

214 A finite element mesh (average element edge equal to 3 pixels or 1 mm) was
215 also adapted to the actual sample geometry (Figure 6). For spatiotemporal
216 analyses, only linear interpolations were considered along the temporal axis
217 (since no acceleration fields will be analyzed hereafter).

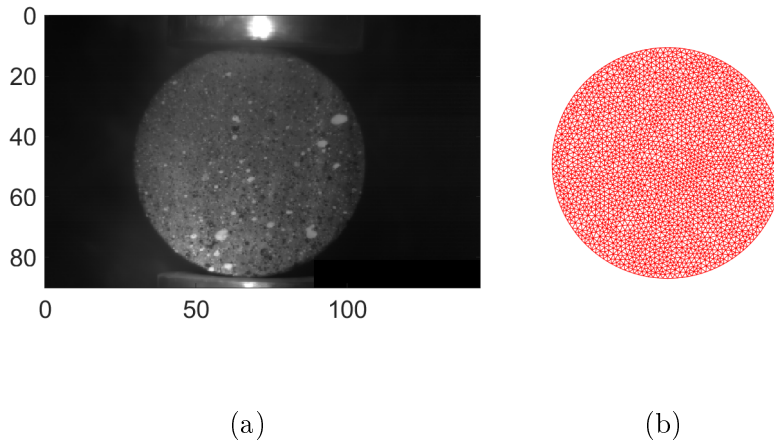


Fig. 6. (a) Denoised reference image. (b) Corresponding mesh made of 3-pixel elements for the 10 Mfps experiment. The dimensions are expressed in mm

218 For comparison purposes with the previous results, the measurement uncer-
 219 tainties are reported for linear interpolations in Figure 7. When compared to
 220 Figure 4 the overall levels are higher. This is to be expected since the acquisi-
 221 tion rate is higher and the associated noise level as well [5]. The displacement
 222 and strain uncertainties also decrease with ℓ_t but not with a $-1/2$ slope in a
 223 log-log plot. Conversely, the velocity and strain rate uncertainties have a vari-
 224 ation with ℓ_t described by a power law of exponent $-3/2$. These new results
 225 are consistent with those reported for the 5 Mfps video.¹

¹ The acquisitions at 5 and 10 Mfps show temporal fluctuations (Figures 2 and 5) that explain the deviations observed in Figures 4(a,c) and 7(a,c) with respect to the expected trends. It is worth noting that these effects are less pronounced when temporal derivatives are applied (see Figures 4(b,d) and 7(b,d)).

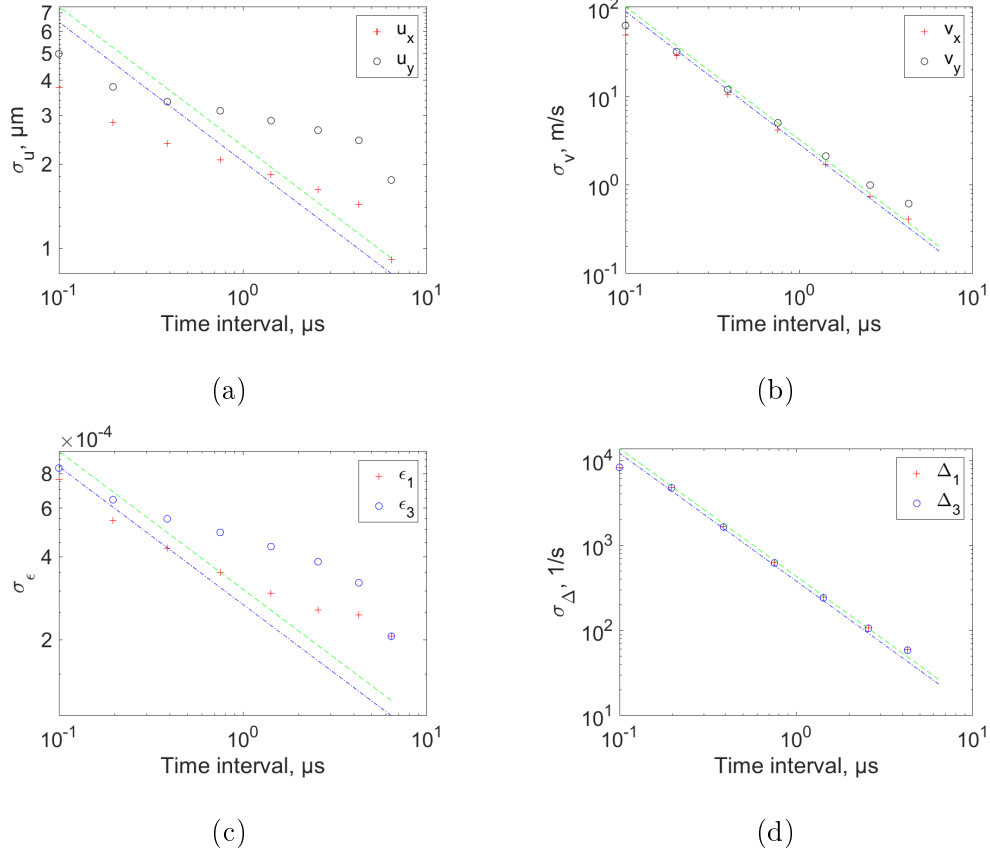


Fig. 7. Standard displacement (a), velocity (b), strain (c) and strain rate (d) uncertainties as functions of the time interval ℓ_t for the 10 Mfps experiment

226 All these uncertainty quantifications showed that the levels of the investigated
 227 kinematic variables can be made very small even though the acquisition rate
 228 was very high. All the general trends to be expected by time regularization
 229 were observed. Last, the a priori estimates were very close to the power law fits
 230 of the raw uncertainties. When comparing the uncertainties corresponding to
 231 the two acquisition rates, the difference in level is directly related to acquisition
 232 noise (Figures 2 and 5), namely, the higher the acquisition rate, the higher
 233 the noise level, and the higher the measurement uncertainties. Conversely,
 234 for acquisition rates less than 5 Mfps, it was shown that the measurement
 235 uncertainties could be further decreased [5].

236 It is worth remembering that the present uncertainty quantifications were
237 performed with videos that were acquired under the very same conditions of
238 the actual experiment (*i.e.*, surface of interest, optical setup, illumination,
239 acquisition rate) but with a motionless sample. Thus, such quantifications do
240 not account for blur induced by the motions of the sample during the time
241 exposure (*e.g.*, 50 ns for an acquisition rate of 10 Mfps). As will be discussed
242 farther down, the displacement amplitude for a maximum velocity of 20 m/s
243 is about 3×10^{-3} pixel. For the lower acquisition rate, the maximum velocity is
244 higher, 200 m/s, and the exposure time longer, hence the displacement during
245 the acquisition time is of order 0.06 pixel at most. The effect of motion blur
246 is thus expected not to be a limiting factor in the present experiments.

247 3.4 10 Mfps series

248 The first analyzed video consisted of 128 frames acquired at a rate of 10 Mfps.
249 The acquisition was started after the specimen had already been fractured
250 (Figure 8). It is therefore expected that most of the kinematics is associated
251 with a free flight condition (*i.e.*, constant velocity).

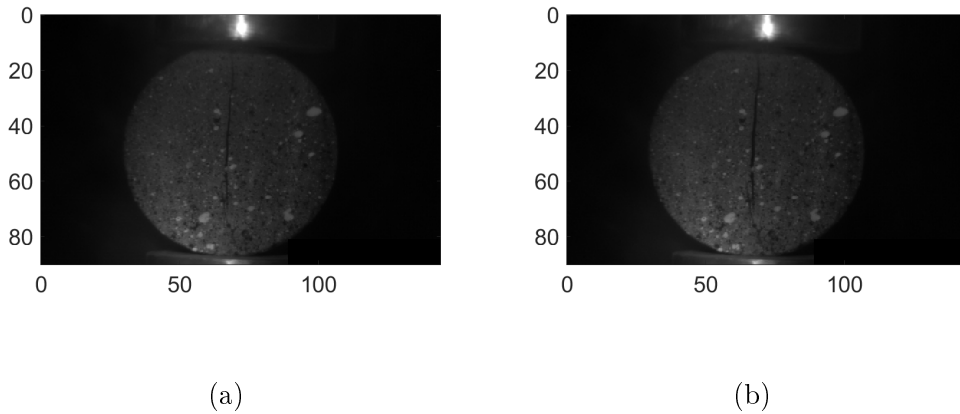


Fig. 8. (a) Initial and (b) final frame of the 10 Mfps video. The dimensions are expressed in mm

252 In order to account for the presence of the dominant crack, a node splitting
 253 technique was followed to enable displacement discontinuities to occur. The
 254 following steps were considered:

- 255 • a first instantaneous DIC analysis was run for the first frame of the video,
 256 namely, that for which only the dominant crack was present. The initial
 257 mesh was considered in which no displacement discontinuity was accounted
 258 for (Figure 6)
- 259 • From the gray level residual map, the crack path was easily selected (Fig-
 260 ure 9) since the crack opening is significant (Figure 8). The advantage of
 261 using the residuals instead of the deformed image was that the crack was po-
 262 sitioned in the reference frame (consistent with the Lagrangian description
 263 used herein).
- 264 • The mesh was then adapted to the crack path so that displacement discon-
 265 tinuities were enabled. Given the fact that a node splitting technique was
 266 used, the nodal crack opening displacements and velocities could be directly
 267 computed as differences of twin nodes.

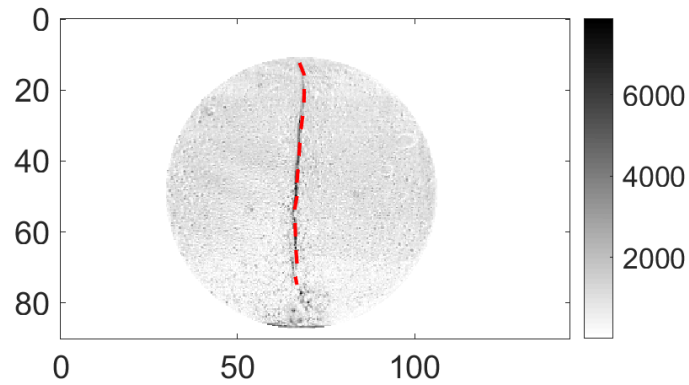


Fig. 9. Absolute gray level residual of the instantaneous registration of the first image of 10 Mfps video. The dashed line corresponds to the selected crack path. The dimensions are expressed in mm

268 Figure 10 shows displacement fields for the last frame of the video when dif-
 269 ferent temporal discretizations were considered. There is virtually no effect
 270 of the discretization for this field. Since one of the DIC calculation was per-
 271 formed with a single (linear) temporal element, it proves that the hypothesis
 272 of vanishing accelerations was satisfied (*i.e.*, free flight of fragments). Further,
 273 the displacement discontinuity is clearly visible for all displacement fields.

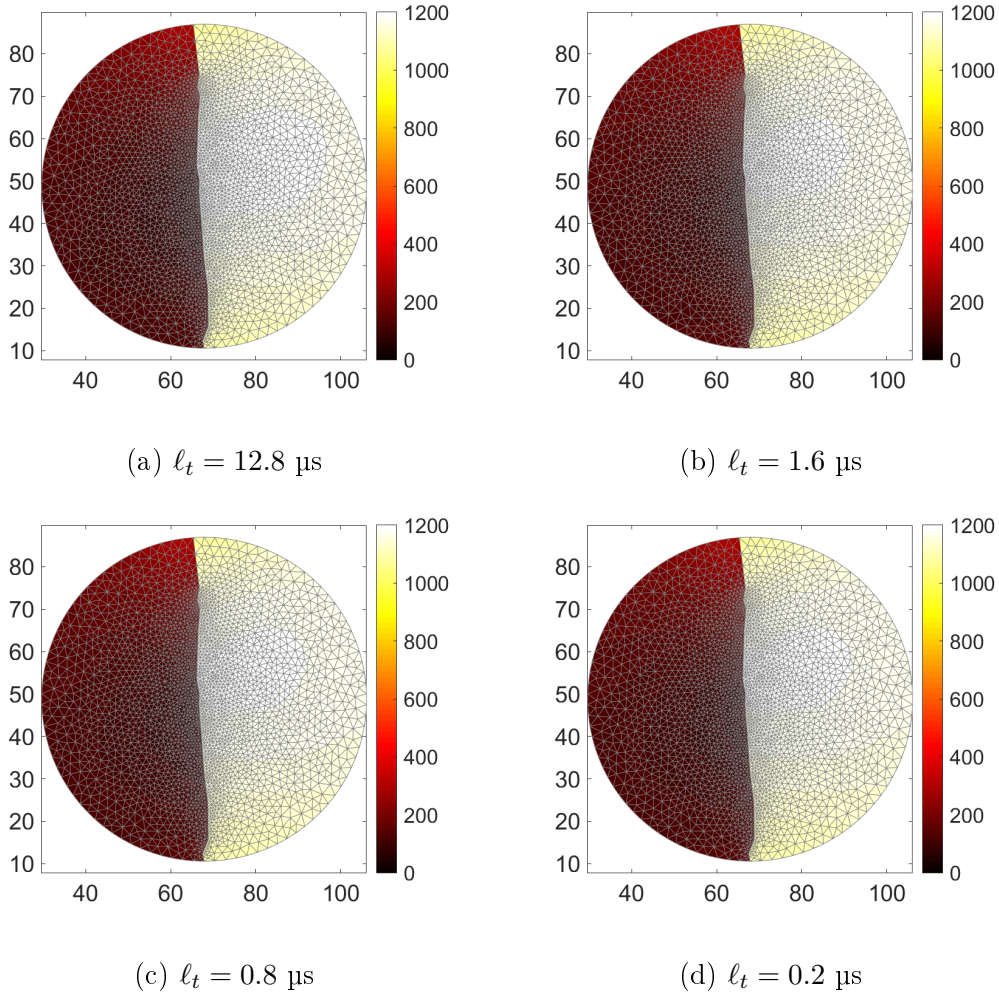


Fig. 10. Horizontal displacement field (expressed in μm) at the end of the video for different time discretizations

274 This observation was further confirmed by analyzing the mean RMS residuals
 275 reported in Figure 11. The fact that they were independent of the discretiza-
 276 tion level proves that the use of a single temporal element was sufficient to
 277 describe the temporal history of the test. Conversely, their mean level was 4
 278 times higher than that associated with noise, which indicates that the spa-
 279 tial discretization was not fully consistent with the experiment (*i.e.*, only one
 280 crack was explicitly accounted for in the present case whereas a more complex
 281 crack pattern was taking place).

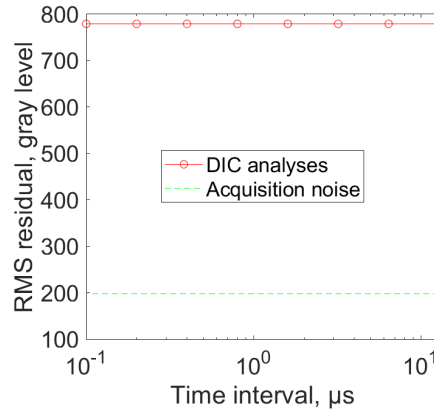


Fig. 11. RMS gray level residual for different time discretizations.

282 In Figure 12, the horizontal velocity fields are reported for the last frame and
 283 for different temporal discretizations. Contrary to the previous case, velocity
 284 fluctuations are observed. Their origin can be understood by the fact that
 285 rigid body rotations occurred at the very end of the video, which were not
 286 fully captured by the spacetime interpolations.

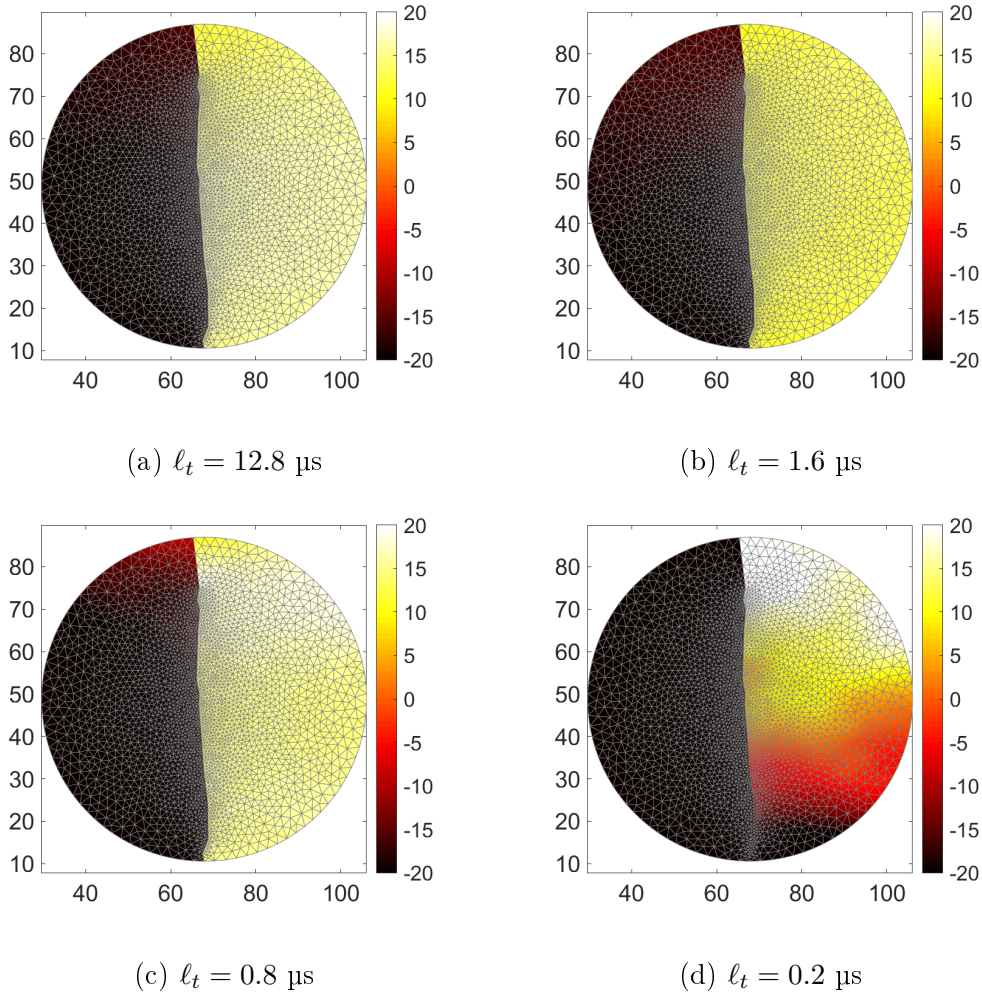


Fig. 12. Horizontal velocity field (expressed in m/s) at the end of the video for different time discretizations

287 From the measured displacement fields and corresponding velocity fields, the
 288 crack opening displacement and velocities were obtained as mean averages of
 289 the displacements (or velocities) of the two halves of the sample. Figure 13
 290 shows their temporal history for different discretization levels. For the crack
 291 opening displacements (Figure 13(a)), the eight temporal discretizations lead
 292 to very similar results, which was expected from the gray level residuals (Fig-
 293 ure 11). This is no longer the case of the crack opening velocities (Figure 13(b))
 294 for which only the last three discretizations yield consistent results of 37 m/s.

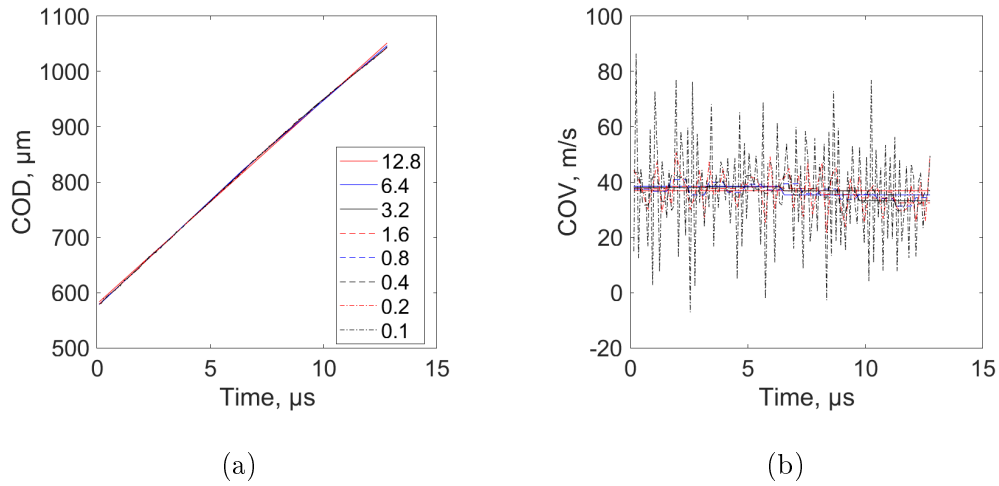


Fig. 13. Mean crack opening displacement (a) and velocity (b) for different time discretizations

295 The major principal strain field is shown in Figure 14 for the last frame and
 296 different discretizations. These results are in line with displacement fields.
 297 This is to be expected since they were obtained by spatial differentiation only.
 298 Their levels being significantly higher than the corresponding uncertainties
 299 (Figure 7(c)), no significant difference is expected provided the captured kine-
 300 matics was consistent with the experiment (Figure 11).

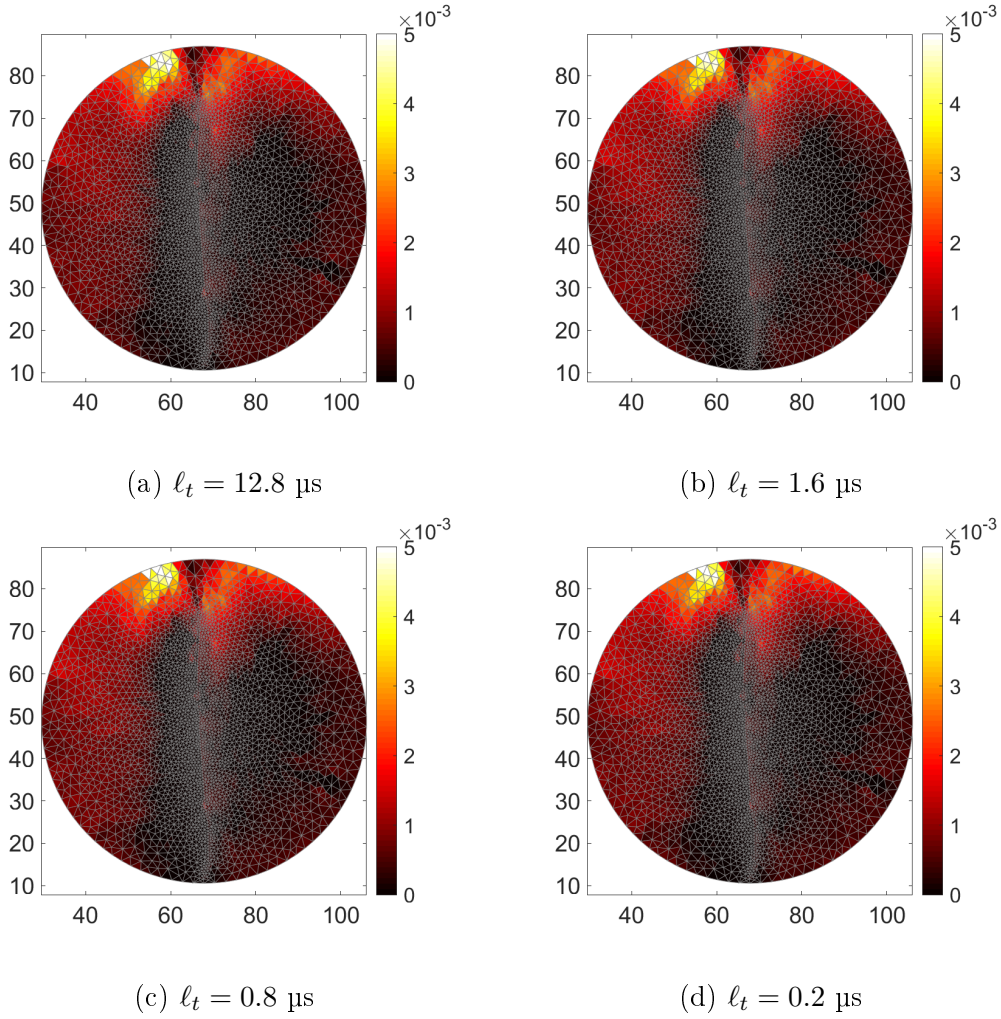


Fig. 14. Major principal strain field at the end of the video for different time discretizations

301 Contrary to the previous case, the major principal strain rate fields reported
 302 in Figure 15 are very different. This is due to the measurement uncertainties
 303 (Figure 7(d)). The effect of the temporal regularization is illustrated thanks to
 304 this last field. If consistent strain rates are sought, either spatiotemporal DIC
 305 is required or instantaneous DIC results would have to be filtered. Further, the
 306 strain rate field (Figure 15(a)) is consistent with the fact that rotations may
 307 be induced by the development of secondary cracks at the end of the video.

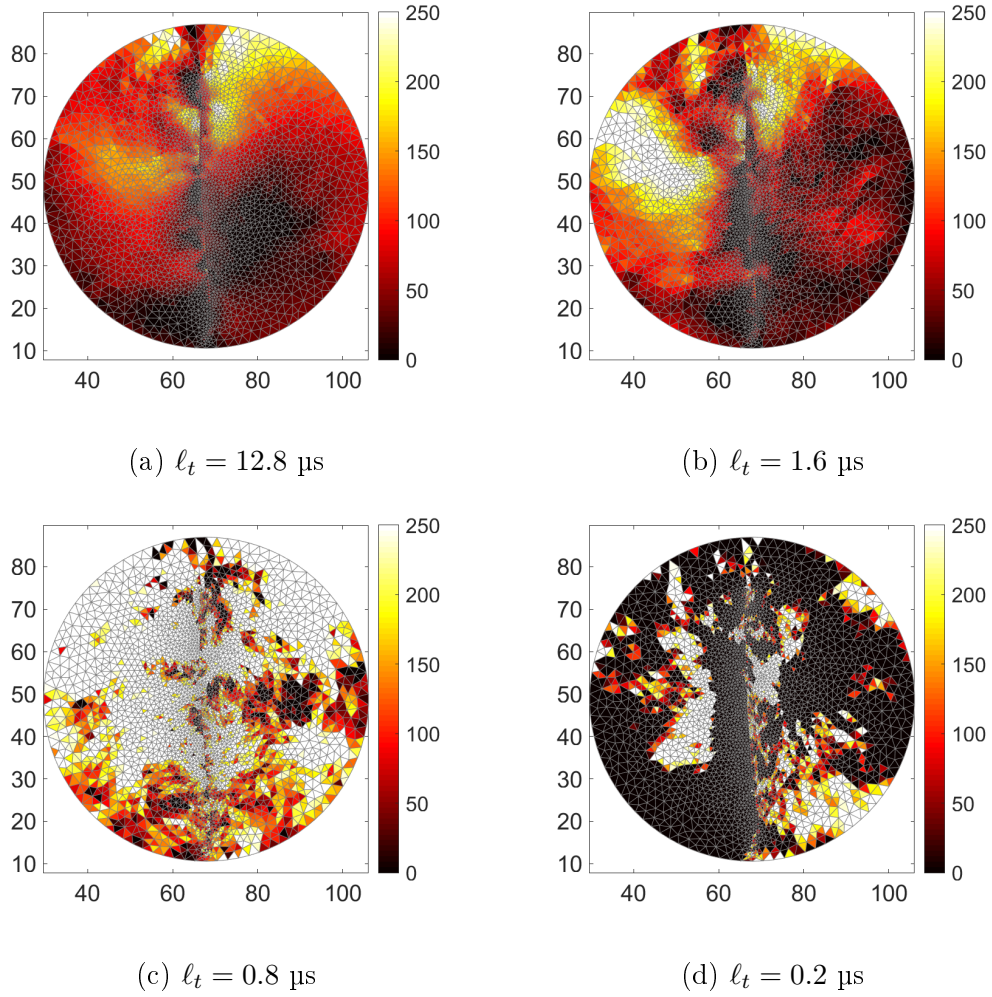


Fig. 15. Major principal strain rate field (expressed in 1/s) at the end of the video for different time discretizations

308 *3.5 5 Mfps video*

309 The acquisition rate was reduced to 5 Mfps to make triggering easier. The
 310 specimen remained motionless for about the first 50 frames. Then a complex
 311 multiple cracking pattern occurs (Figure 16). The cracks do not initiate si-
 312 multaneously, namely, a first pair emanating from the bottom contact zone
 313 initiates after the 50th frame, while a third one between the first two appears
 314 at about the 120th frame. A fourth crack again vertical but located on the

315 right side emerges from the free surface.

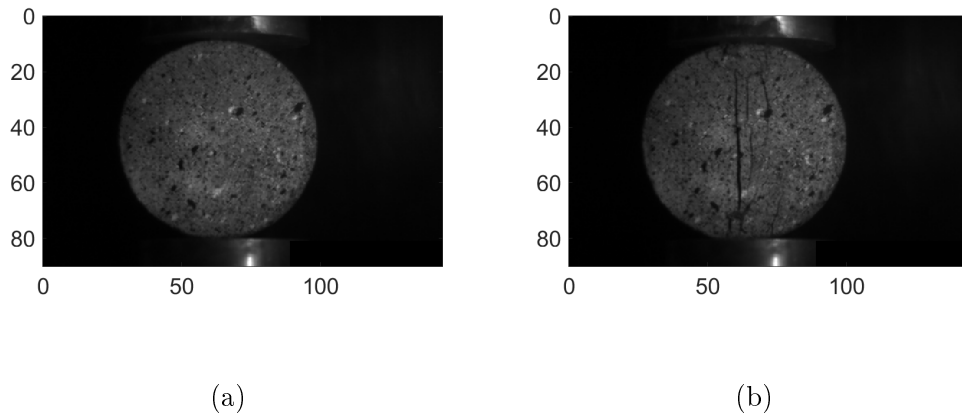


Fig. 16. (a) Initial and (b) final frame of the 5 Mfps video. The dimensions are expressed in mm

316 As for the previous case, one of the major cracks was explicitly accounted for
317 by following the above-described node splitting procedure. Given the temporal
318 complexity of the cracking phenomena, it was expected that the temporal dis-
319 cretization would have a much more important effect. It was indeed observed
320 that the mean gray level residuals depend on the discretization, namely, the
321 smaller the time interval, the lower the overall residuals (Figure 17).

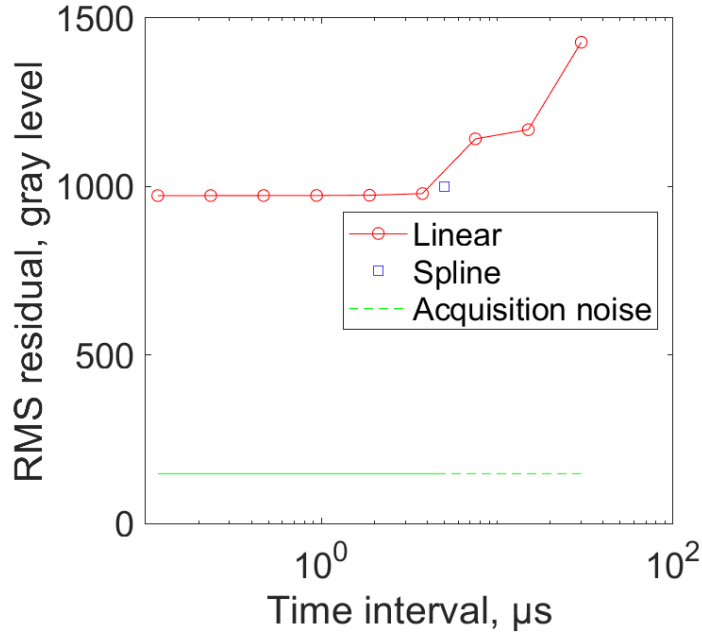


Fig. 17. RMS gray level residual for different time discretizations

322 This trend can be understood by analyzing the crack opening kinematics
 323 shown in Figure 18. A rather fine discretization was required in order to
 324 capture all the temporal details of the cracking phenomenon. This is particu-
 325 larly visible on the crack opening velocities (Figure 18(b)) that show different
 326 regimes and levels of velocities. However, with piece-wise linear interpolations,
 327 the acceleration fields cannot be estimated (without posterior smoothing pro-
 328 cess that was not performed herein). The measurement uncertainties are too
 329 high to provide any meaningful result (Figure 18(c)). Consequently, higher
 330 order interpolations are required.

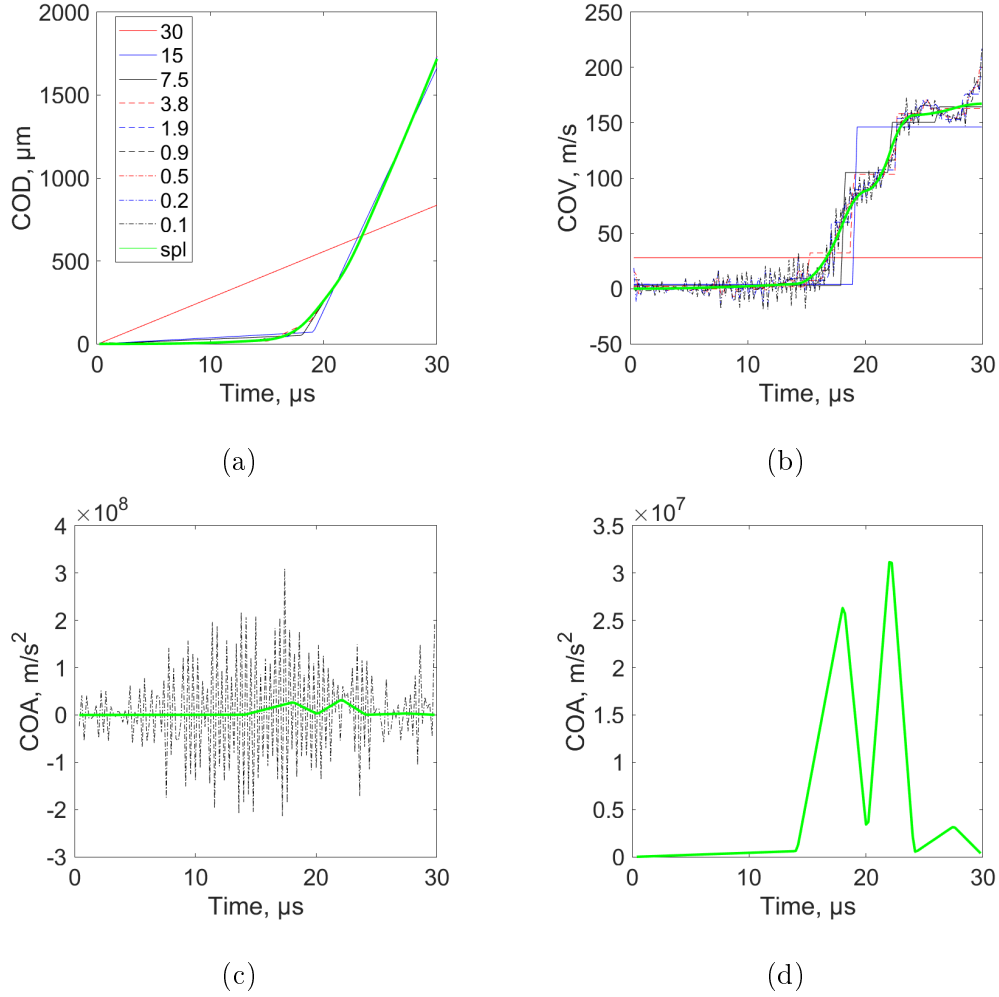
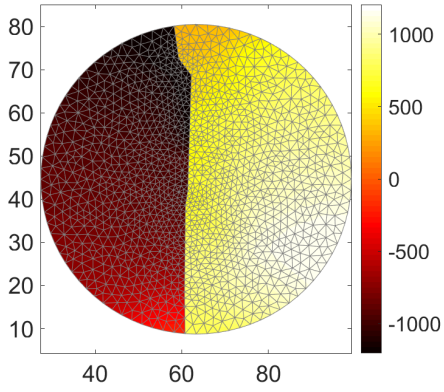


Fig. 18. Mean crack opening displacement (a), velocity (b), and acceleration (c) for different temporal discretizations. (d) Crack opening acceleration when a B-spline interpolation is considered

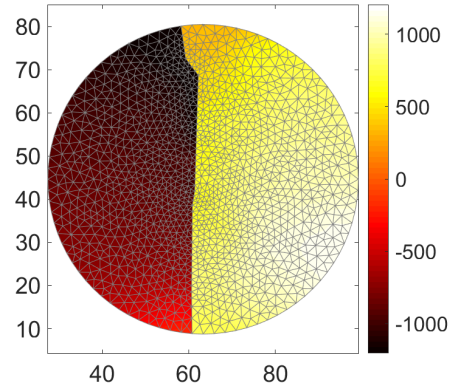
331 In the following, cubic B-spline interpolations were used. This choice allowed
 332 displacement and velocity fields to be continuous by construction. The continu-
 333 ity of the acceleration was also assumed during the whole analyzed sequence.
 334 Further, for the first temporal element the medium was assumed to be ini-
 335 tially motionless (*i.e.*, the displacement, velocity and acceleration vanish at
 336 the initial time). For the last element, a natural spline was also implemented
 337 (*i.e.*, the acceleration was assumed to vanish at the last frame). Six elements

338 were considered. When all the previous conditions were accounted for, only 6
339 temporal degrees of freedom needed to be measured. With such parameteri-
340 zation, the gray level residuals were very close to those observed with a larger
341 number of degrees of freedom (Figure 17). The crack opening displacements
342 (Figure 18(a)), and more importantly crack opening velocities (Figure 18(b))
343 are in good agreement with the other discretizations except for the final time
344 where the natural spline condition prescribes a constant velocity (which is
345 probably not valid here). Further, the crack opening acceleration has a com-
346 plex history (Figure 18(d)), which is consistent with the crack opening velocity
347 history. Levels as high as 3×10^7 m/s² were reached.

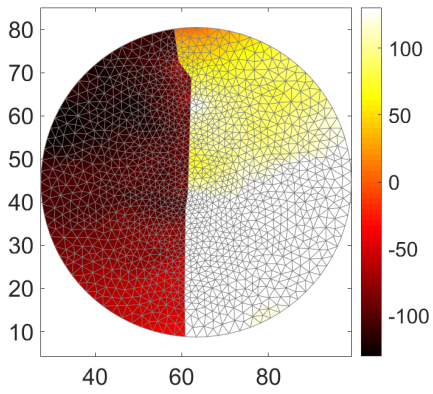
348 Figure 19 compares two kinematic fields at the end of the video obtained
349 via instantaneous DIC and when the B-spline interpolation is selected in the
350 spacetime analyses. The displacement fields are very similar in both cases
351 (Figure 19(a-b)). For the velocity fields, the differences are more pronounced
352 (Figure 19(c-d)). Interestingly, spatial fluctuations appear in the velocity field
353 measured via spacetime DIC. They are less resolved with instantaneous DIC.
354 The acceleration fields, which correspond to the time for highest crack open-
355 ing acceleration) are significantly different for both methods (Figure 19(e-f)).
356 Only spacetime DIC provided acceleration fields that were meaningful (*i.e.*,
357 their levels were significantly higher than the measurement uncertainties). The
358 results of instantaneous DIC would have to be filtered to compute acceleration
359 fields. Levels as high as $\pm 4 \times 10^7$ m/s² were reached in the reported field.



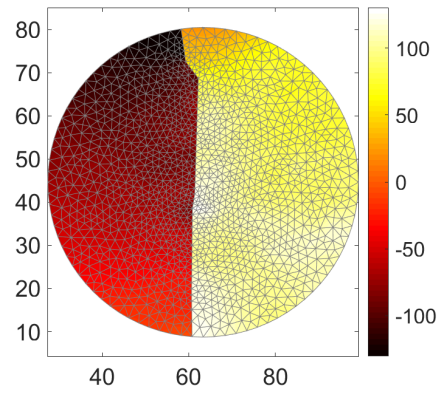
(a) $t = 30 \mu\text{s}$



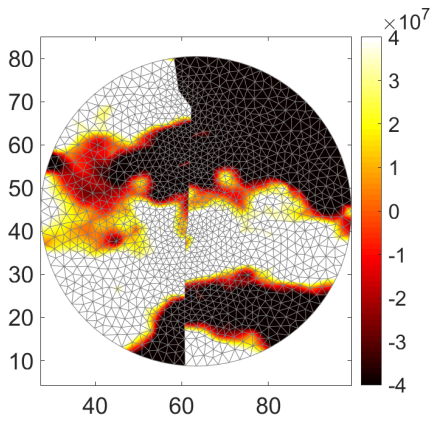
(b) $t = 30 \mu\text{s}$



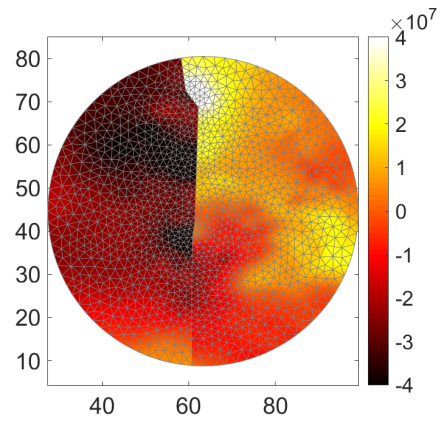
(c) $t = 30 \mu\text{s}$



(d) $t = 30 \mu\text{s}$



(e) $t = 24 \mu\text{s}$



(f) $t = 24 \mu\text{s}$

Fig. 19. (a,b) Horizontal displacement fields (expressed in μm). (c,d) Horizontal velocity fields (expressed in m/s). (e,f) Horizontal acceleration fields (expressed in m/s^2). The left column corresponds to instantaneous DIC and the right column to spacetime DIC with B-spline interpolation

360 4 Conclusion

361 Digital image correlation was used to measure displacement, velocity, and ac-
362 celeration fields via instantaneous and spacetime approaches of videos acquired
363 with an ultra-high speed camera of two dynamic Brazilian tests. Major prin-
364 cipal strains and strain rates were also evaluated with the previous technique.
365 Maximum displacements of the order of 1 mm, velocities up to ± 200 m/s,
366 and acceleration levels of ± 5 million g's were obtained. Similar levels were
367 observed from the mean crack opening kinematics.

368 Uncertainty quantifications were performed with videos acquired prior to each
369 experiment. With this information at hand, a denoised reference image could
370 be constructed for each analyzed test. Further, the standard uncertainties
371 were evaluated for the five kinematic quantities. With the selected camera, the
372 acquisition noise was observed to be very small for 5 and 10 million frames per
373 second videos. As a result, very low uncertainties were achieved even though
374 ultra-high acquisition rates were used. Very significant gains were observed
375 when spacetime discretizations could be used in comparison with standard
376 (*i.e.*, instantaneous) analyses, especially for the acceleration fields.

377 It is worth noting that such uncertainty quantifications are very useful not only
378 when analyzing impact tests as was illustrated herein but also for comparing
379 or selecting a camera [5]. This approach is very generic and complies with the
380 ASD-STAN prEN 4861 procedure for metrological assessment of kinematic
381 fields measured by digital image correlation [48].

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385 References

- 386 [1] D. Hopkinson, A method of measuring the pressure produced in the detonation
387 of high explosives or by the impact of bullets, *Phil. Trans. Roy. Soc. A* 213
388 (1914) 437.
- 389 [2] H. Kolsky, An investigation of the mechanical properties of materials at very
390 high rates of loading, *Proc. Phys. Soc. London* 62b (1949) 676–700.
- 391 [3] M. Sutton, J. Orteu, H. Schreier, *Image correlation for shape, motion and*
392 *deformation measurements: Basic Concepts, Theory and Applications*, Springer,
393 New York, NY (USA), 2009.
- 394 [4] J. Field, S. Walley, W. Proud, H. Goldrein, C. Siviour, Review of experimental
395 techniques for high rate deformation and shock studies, *Int. J. Impact Eng.* 30
396 (2004) 725–775.
- 397 [5] F. Hild, A. Bouterf, P. Forquin, S. Roux, On the Use of Digital Image Correlation
398 for the Analysis of the Dynamic Behavior of Materials, in: *The Micro-World*
399 *Observed by Ultra High-Speed Cameras*, 2018, pp. 185–206.
- 400 [6] W. Peters, W. Ranson, Digital imaging techniques in experimental stress
401 analysis, *Opt. Eng.* 21 (1982) 427–431.
- 402 [7] M. Sutton, W. Wolters, W. Peters, W. Ranson, S. McNeill, Determination of
403 displacements using an improved digital correlation method, *Im. Vis. Comp.*
404 1 (3) (1983) 133–139.

- 405 [8] W. Peters, W. Ranson, J. Kalthoff, S. Winkler, A study of dynamic near-crack-
406 tip fracture parameters by digital image analysis, *J. Phys. Coll.* 46 (C5) (1985)
407 631–638.
- 408 [9] Y. Chao, P. Luo, J. Kalthoff, An experimental study of the deformation fields
409 around a propagating crack tip, *Exp. Mech.* 38 (2) (1998) 79–85.
- 410 [10] T. Etoh, Q. Nguyen, *Evolution of High-Speed Image Sensors*, Springer
411 International Publishing, 2018, pp. 81–101.
- 412 [11] F. Barthelat, Z. Wu, B. Prorok, H. Espinosa, Dynamic torsion testing of
413 nanocrystalline coatings using high-speed photography and digital image
414 correlation, *Exp. Mech.* 43 (3) (2003) 331–340.
- 415 [12] J. Kajberg, M. Sjödaahl, Optical method to study material behaviour at
416 high strain rates, in: P. Stähle, K. Sundin (Eds.), *IUTAM Symposium on
417 Field Analyses for Determination of Material Parameters – Experimental and
418 Numerical Aspects*, Vol. 109 of *Solid Mechanics and its Applications*, Springer
419 (the Netherlands), 2003, pp. 37–49.
- 420 [13] A. Gilat, T. Schmidt, J. Tyson, Full field measurement during a tensile split
421 hopkinson bar experiment, *J. Phys. IV* 134 (2006) 687–692.
- 422 [14] I. Elnasri, S. Patoatto, H. Zhao, H. Tsitsiris, F. Hild, Y. Girard, Shock
423 enhancement of cellular structures under impact loading: Part i experiments,
424 *J. Mech. Phys. Solids* 55 (2007) 2652–2671.
- 425 [15] J. Kajberg, B. Wikman, Viscoplastic parameter estimation by high strain-
426 rate experiments and inverse modelling – speckle measurements and high-speed
427 photography, *Int. J. Solids Struct.* 44 (1) (2007) 145–164.
- 428 [16] P. Reu, T. Miller, The application of high-speed digital image correlation, *J.*
429 *Strain Anal.* 43 (8) (2008) 673–688.

- 430 [17] V. Tarigopula, O. Hopperstad, M. Langseth, A. Clausen, F. Hild, A study of
431 localisation in dual phase high-strength steels under dynamic loading using
432 digital image correlation and fe analysis, *Int. J. Solids Struct.* 45 (2) (2008)
433 601–619.
- 434 [18] F. Pierron, M. Sutton, V. Tiwari, Ultra high speed dic and virtual fields method
435 analysis of a three point bending impact test on an aluminium bar, *Exp. Mech.*
436 51 (4) (2011) 537–563.
- 437 [19] D. Saletti, S. Pattofatto, H. Zhao, Measurement of phase transformation
438 properties under moderate impact tensile loading in a niti alloy, *Mech. Mat.*
439 65 (2013) 1–11.
- 440 [20] Search with PUBLICATION NAME: (international journal of impact
441 engineering) & TOPIC: (image correlation), [accessed September 5, 2018],
442 www.webofknowledge.com.
- 443 [21] V. Tiwari, M. Sutton, S. McNeill, S. Xu, X. Deng, W. Fourney, D. Bretall,
444 Application of 3d image correlation for full-field transient plate deformation
445 measurements during blast loading, *International Journal of Impact Engineering*
446 36 (6) (2009) 862–874.
- 447 [22] D. Dubois, H. Zellmer, E. Markiewicz, Experimental and numerical analysis of
448 seat belt bunching phenomenon, *International Journal of Impact Engineering*
449 36 (6) (2009) 763–774.
- 450 [23] P. Hogström, J. Ringsberg, E. Johnson, An experimental and numerical study of
451 the effects of length scale and strain state on the necking and fracture behaviours
452 in sheet metals, *International Journal of Impact Engineering* 36 (10) (2009)
453 1194–1203.
- 454 [24] F. Lauro, B. Bennani, D. Morin, A. Epee, The SEĖ-method for determination
455 of behaviour laws for strain rate dependent material: Application to polymer
456 material, *International Journal of Impact Engineering* 37 (6) (2010) 715–722.

- 457 [25] R. Moura, A. Clausen, E. Fagerholt, M. Alves, M. Langseth, Impact on hdpe
458 and pvc plates - experimental tests and numerical simulations, *International*
459 *Journal of Impact Engineering* 37 (6) (2010) 580–598.
- 460 [26] J. Peirs, P. Verleysen, W. V. Paepegem, J. Degrieck, Determining the stress-
461 strain behaviour at large strains from high strain rate tensile and shear
462 experiments, *International Journal of Impact Engineering* 38 (5) (2011) 406–
463 415.
- 464 [27] A. Collins, J. Addiss, S. Walley, K. Promratana, F. Bobaru, W. Proud,
465 D. Williamson, The effect of rod nose shape on the internal flow fields during
466 the ballistic penetration of sand, *International Journal of Impact Engineering*
467 38 (12) (2011) 951–963.
- 468 [28] J. LeBlanc, A. Shukla, Response of e-glass/vinyl ester composite panels to
469 underwater explosive loading: Effects of laminate modifications, *International*
470 *Journal of Impact Engineering* 38 (10) (2011) 796–803.
- 471 [29] P. Kumar, J. LeBlanc, D. Stargel, A. Shukla, Effect of plate curvature on blast
472 response of aluminum panels, *International Journal of Impact Engineering* 46
473 (2012) 74–85.
- 474 [30] G. Besnard, F. Hild, J. Lagrange, P. Martinuzzi, S. Roux, Analysis of necking
475 in high speed experiments by stereocorrelation, *Int. J. Impact Eng.* 49 (2012)
476 179–191.
- 477 [31] K. Spranghers, I. Vasilakos, D. Lecompte, H. Sol, J. Vantomme, Numerical
478 simulation and experimental validation of the dynamic response of aluminum
479 plates under free air explosions, *International Journal of Impact Engineering* 54
480 (2013) 83–95.
- 481 [32] R. Ambriz, C. Froustey, G. Mesmacque, Determination of the tensile behavior
482 at middle strain rate of aa6061-t6 aluminum alloy welds, *International Journal*
483 *of Impact Engineering* 60 (2013) 107–119.

- 484 [33] J. Seidt, J. Pereira, A. Gilat, D. Revilock, K.Nandwana, Ballistic impact of
485 anisotropic 2024 aluminum sheet and plate, *International Journal of Impact*
486 *Engineering* 62 (2013) 27–34.
- 487 [34] L. Wang, Y. Ding, L. Yang, Experimental investigation on dynamic constitutive
488 behavior of aluminum foams by new inverse methods from wave propagation
489 measurements, *International Journal of Impact Engineering* 62 (2013) 48–59.
- 490 [35] M. O’Masta, B. Compton, E. Gamble, F. Zok, V. Deshpande, H. Wadley,
491 Ballistic impact response of an uhmwpe fiber reinforced laminate encasing of an
492 aluminum-alumina hybrid panel, *International Journal of Impact Engineering*
493 86 (2015) 131–144.
- 494 [36] M. Louar, B. Belkassem, H. Ousji, K. Spranghers, D. Kakogiannis, L. Pyl,
495 J. Vantomme, Explosive driven shock tube loading of aluminium plates:
496 experimental study, *International Journal of Impact Engineering* 86 (2015) 111–
497 123.
- 498 [37] G. Gruben, M. Langseth, E. Fagerholt, O. Hopperstad, Low-velocity impact on
499 high-strength steel sheets: An experimental and numerical study, *International*
500 *Journal of Impact Engineering* 88 (2016) 153–171.
- 501 [38] B. Koohbor, A. Kidane, W.-Y. Lu, M. Sutton, Investigation of the dynamic
502 stress-strain response of compressible polymeric foam using a non-parametric
503 analysis, *International Journal of Impact Engineering* 91 (2016) 170–182.
- 504 [39] M. Omidvar, M. Iskander, S. Bless, Soil-projectile interactions during low
505 velocity penetration, *International Journal of Impact Engineering* 93 (2016)
506 211–221.
- 507 [40] B. Koohbor, A. Kidane, M. Sutton, X. Zhao, S. Mallon, Analysis of dynamic
508 bending test using ultra high speed dic and the virtual fields method,
509 *International Journal of Impact Engineering* 110 (2017) 299–310.

- 510 [41] H. Xing, Q. Zhang, D. Ruan, S. Dehkoda, G. Lu, J. Zhao, Full-field
511 measurement and fracture characterisations of rocks under dynamic loads using
512 high-speed three-dimensional digital image correlation, *International Journal of*
513 *Impact Engineering* 113 (2018) 61–72.
- 514 [42] G. Broggiato, L. Casarotto, Z. Del Prete, D. Maccarrone, Full-field strain rate
515 measurement by white-light speckle image correlation, *Strain* 45 (4) (2009) 364–
516 372.
- 517 [43] G. Besnard, S. Guérard, S. Roux, F. Hild, A space-time approach in digital
518 image correlation: Movie-DIC, *Optics Lasers Eng.* 49 (2011) 71–81.
- 519 [44] G. Besnard, H. Leclerc, S. Roux, F. Hild, Analysis of image series through digital
520 image correlation, *J. Strain Analysis* 47 (4) (2012) 214–228.
- 521 [45] M. Berny, T. Archer, A. Mavel, P. Beauchêne, S. Roux, F. Hild, On the analysis
522 of heat haze effects with spacetime DIC, *Opt. Lasers Eng.* 40 (2018) 544–556.
- 523 [46] G. Besnard, F. Hild, S. Roux, “Finite-element” displacement fields analysis
524 from digital images: Application to Portevin-Le Chatelier bands, *Exp. Mech.*
525 46 (2006) 789–803.
- 526 [47] F. Hild, S. Roux, Comparison of local and global approaches to digital image
527 correlation, *Exp. Mech.* 52 (9) (2012) 1503–1519.
- 528 [48] ASD-STAN prEN 4861 P1 Aerospace series-Metrological assessment procedure
529 for kinematic fields measured by digital image correlation, [accessed September
530 5, 2018], www.asd-stan.org/downloads/asd-stan-pren-4861-p1/.
- 531 [49] M. Berny, C. Jailin, A. Bouterf, F. Hild, S. Roux, Model Reduction for Space-
532 Time DIC, *Mea. Sci. Technol.* 29 (12) (2018) 125008.
- 533 [50] ISO, Guide to the Expression of Uncertainty in Measurements (GUM),
534 International Organization for Standardization, Geneva (Switzerland), 1995.

- 535 [51] F. Lobo Carneiro, Um novo método para determinação da resistência à tração
536 dos concretos, in: Anais 5a reunião da Associação Brasileira de Normas Técnicas
537 (ABNT) em São Paulo, 1943, pp. 127–129.
- 538 [52] F. Lobo Carneiro, Une nouvelle méthode pour la détermination de la résistance
539 à la traction des bétons, Bull. RILEM 13 (1953) 103–108.
- 540 [53] Z. Tomičević, F. Hild, S. Roux, Mechanics-aided digital image correlation, J.
541 Strain Analysis 48 (2013) 330–343.