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# Application of a stochastic version of the restoring force surface method to identify a Duffing oscillator

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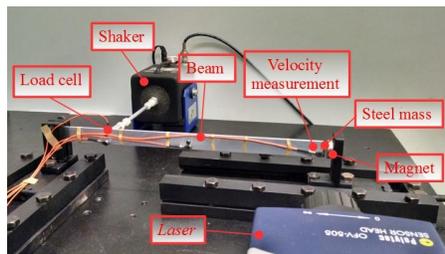
**Abstract.** This work deals with the identification of stochastic parameters in a nonlinear single degree-of-freedom system. For this purpose, a stochastic version of the restoring force surface method is proposed and used to identify the parameters of a clamped-free beam, with nonlinear effects induced by the presence of a magnet near to the free extremity. This system recalls a Duffing oscillator, which is used as mechanical-mathematical model. In order to validate the obtained stochastic model, experimental and theoretical responses are compared in time and frequency domains, taking into account a probabilistic band of confidence. The results show that obtained stochastic model adequately predict the beam's vibration responses, which ensure the robustness of identification of a stochastic model.

## Introduction

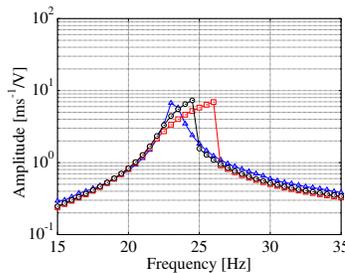
It is known that many engineering structures can present nonlinear behavior caused by geometric effects, severe operating conditions, materials with complex structure and others. So, to perform a reliable analysis of a structure, the nonlinear effects have to be taken into account [1]. Furthermore, most of the approaches used to describe the nonlinear phenomena are deterministic. Then, these models are not robust to variations in the system parameters, neither offer a confidence interval to the identified parameters. However, real systems are uncertain with respect to the project nominal values, due to material imperfections, boundary conditions, noise and others, becoming essential to consider the presence of uncertainties [2]. In this sense, the use of a stochastic version of the restoring force surface (RFS) method is proposed here, where the probability density functions (PDFs) of model parameters are identified, instead of the parameters deterministic values as obtained through the conventional RFS method [3]. In order to illustrate the results, a nonlinear system composed by a clamped-free beam is identified.

## Experimental Setup

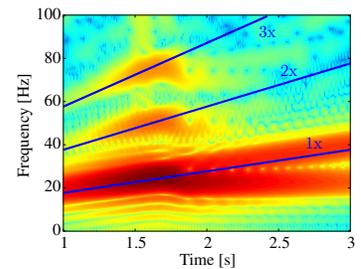
The experimental setup is composed by a clamped-free beam ( $300 \times 18 \times 3$  [mm<sup>3</sup>]) with a steel mass glued in the free extremity, which is connected to cause a magnetic interaction between the beam and a magnet (fig. 1(a)). A shaker is used to excite the structure considering different levels of voltage amplitude. A vibrometer laser is also used to measure the beam free extremity velocity. The magnetic interaction of the system generates a nonlinear hardening shown in fig. 1(b). Additionally, the spectrogram of the system response can be seen in fig. 1(c) where is observed the presence of second and third order harmonics in the response.



(a) Experimental apparatus.



(b) Stepped sine test.  $\Delta$  - 0.01 V,  $\circ$  - 0.10 V and  $\square$  - 0.15 V.



(c) Spectrogram for the system response.

Figure 1: Illustration of the experimental apparatus and its nonlinear behavior.

## Stochastic model - A random Duffing oscillator

The model parameters subjected to uncertainties are described as random variables or random processes, defined on the probability space  $(\Theta, \mathcal{F}, \mathbb{P})$ , where  $\Theta$  is sample space,  $\mathcal{F}$  is a  $\sigma$ -algebra over  $\Theta$ , and  $\mathbb{P}$  is a probability measure. For the Duffing oscillator [4], the stochastic equivalent motion equation is given by

$$\ddot{\theta}(t) + \dot{\theta}(t) + \underbrace{1 \theta(t) + 2 \theta(t)^2 + 3 \theta(t)^3}_{\mathbb{F}(\mathbf{x}(\theta,t),t) \rightarrow \text{random restoring force}} = U(t), \quad (1)$$

where the random processes  $(\theta, t)$ ,  $\dot{(\theta, t)}$ , and  $\ddot{(\theta, t)}$ , represent the displacement, velocity and acceleration, and  $(\theta)$  are the random mass and damping coefficient,  $\mathbb{F}((\theta, t), t)$  is the random process that represents the stochastic restoring force,  $k_1(\theta)$ ,  $k_2(\theta)$  and  $k_3(\theta)$  are the random linear, quadratic and cubic stiffness, and  $U(t)$  represents the input signal. The stochastic model of eq. (1) is used to describe the nonlinear random dynamics of the system. The PDFs of mass and damping are identified considering modal analysis. The linear and nonlinear stiffness are identified using the restoring force surface method and the polynomial regression based on the minimization of the squared error (least squares method), for each realization  $\theta$ .

### Parameters Estimation and Model Validation

Figure 2(a) shows the PDF of the quadratic stiffness. The mean value is equal to  $\mu_{k_2} = -30.867 \text{ N/m}^2$  and dispersion  $\delta_{k_2} = 2.72\%$ , with unimodal behavior. The PDF of the cubic stiffness (fig. 2(b)) has also unimodal distribution with mean value  $\mu_{k_3} = 39.859 \times 10^7 \text{ [N/m}^3\text{]}$  and dispersion  $\delta_{k_3} = 4.06\%$ . Figure 2(c) shows the experimental  $\mathbb{F}$  and the polynomial modeling identified with 99% of confidence bands. It is observed a satisfactory prediction of the restoring force. A validation test in the frequency domain was also performed, where the stochastic model is compared with new experimental data from the stepped sine test, considering two levels of input. Figure 3 shows that the stochastic model describes well the system behavior, both in linear as nonlinear regime of motion.

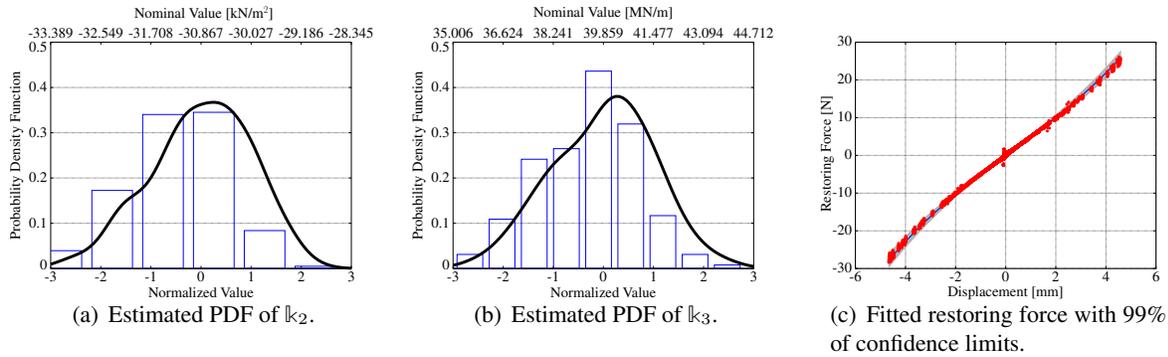


Figure 2: Estimated PDFs for mechanical system stiffnesses and the fitted restoring force.

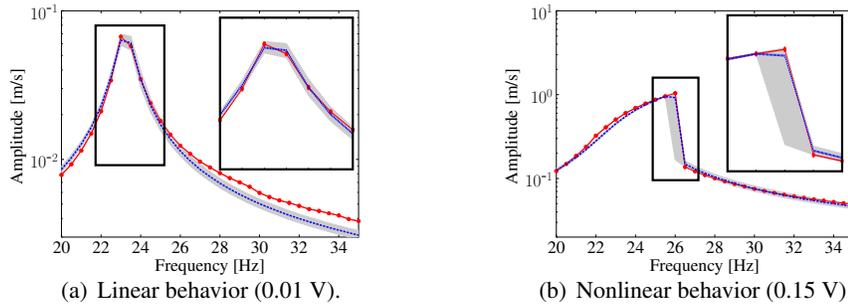


Figure 3: Comparison between experimental and theoretical beam velocity in frequency domain. The stochastic model mean value is presented as  $-\text{ }-\text{ }-$ , the 99% confidence band as grey shown, and the experimental realization as  $-\text{ } \circ \text{ } -\text{ }-$ .

### Final Remarks

The stochastic version of the Duffing oscillator identified based on a random restoring force surface method showed satisfactory performance to describe the nonlinear behavior of the system assuming uncertainties.

### Acknowledgements

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### References

- [1] Virgin LN. (2000) Introduction to experimental nonlinear dynamics: a case study in mechanical vibration. Cambridge University Press.
- [2] Soize C. (2005) A comprehensive overview of a non-parametric probabilistic approach of model uncertainties for predictive models in structural dynamics. *J. Sound Vib* **288**: 623-652.
- [3] Kerschen G, Golinval JC and Worden K. (2001) Theoretical and experimental identification of a non-linear beam. *J. Sound Vib* **244**: 597-613.
- [4] Kovacic I and Brennan M. (2011) The Duffing equation: nonlinear oscillators and their behavior. Chichester: Wiley, 2011.