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1 Spatiotemporal properties of solitons excited on the surface 2 of shallow water in a hydrodynamic resonator

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9 We have investigated the spatiotemporal properties of solitons generated on the shallow water
10 surface over a background of a large-scale mode in a hydrodynamic resonator when it is forced near
11 the second frequency mode. We have used the space-time diagrams to highlight the spatiotemporal
12 dynamics of nonlinear fields for two solitons colliding in a resonator and compared them to those
13 of solitons occurring in an unbounded system. A state diagram of experimentally observed modes
14 for different values of the excitation parameters has been obtained. In particular, we have evidenced
15 period doubling and the multistability of nonlinear waves excited in the resonator. For a theoretical
16 description of these experimental results, we have developed a phenomenological model, which
17 leads to amplitude and phase equations of a soliton propagating over the background of a harmonic
18 wave. To reproduce experimental results on the multistability, we have supplemented our analysis
19 with a numerical simulation of a modified system of Boussinesq equations for shallow water, taking
20 into account the dissipation effect © 2006 American Institute of Physics. [DOI: 10.1063/1.2204968]
21

22 I. INTRODUCTION

23 Solitons observed in many systems such as the shallow
24 water channel in nonlinear electrical lines or in protein
25 chains, exhibit astonishing stability properties, the investiga-
26 tion of which remains a very active research area.^{1,2} The
27 excitation of solitons on the surface of shallow water and
28 their dynamics have been addressed mostly for infinitely ex-
29 tended systems.^{1,2} In an unbounded medium, solitons arising
30 from an initial perturbation propagate in opposite directions
31 and diverge in a finite time, hence their interaction can be
32 neglected. Therefore, the main attention of the researchers
33 was focused on the behavior of the solitons propagating in
34 one direction. Experimentally, such solitons were excited by
35 a single displacement of a piston pushing the liquid in the
36 horizontal direction^{3,4} or by producing an initial elevation in
37 the level of liquid in a finite region.^{5,6} The dynamics of such
38 solitons is described by the well-known Korteweg-de Vries
39 (KdV) equation that is derived under the assumption of wave
40 propagation in one direction. Feng⁷ investigated traveling
41 solitary wave solutions in the Boussinesq equations, which
42 describe counterpropagating waves, but he presented only
43 waves propagating in one direction. Besides the generation
44 of a soliton or a sequence of solitons of surface waves from
45 initial perturbations,⁸ individual properties of interaction
46 have been investigated thoroughly, namely, the interaction of
47 two solitons, the reflection of a soliton at a vertical wall,^{3,9} an
48 uneven bottom,¹⁰ and so on. It was found, in particular, that
49 the soliton amplitude at the instant of reflection at the wall
50 increases more than twice, and the reflected soliton lags be-

hind the time calculated under the assumption of the constant 51
velocity of soliton propagation.⁹ When two solitons interact, 52
similar effects are observed: the time lag and the amplitude 53
of surface displacement larger than the sum of the individual 54
soliton amplitudes. It is worthy of notice that the majority of 55
experiments agree well with basic assumptions of the 56
theory;¹¹⁻¹⁴ specifically, solitons come from one infinity, in- 57
teract with a wall or with each other, and go to the opposite 58
infinity.¹⁵ However, solitons can be excited, not only in sys- 59
tems of infinite length, but in resonators too. One expects 60
that the spatiotemporal dynamics of solitons will be different 61
in resonators, where the interaction of nonlinear waves 62
propagating in opposite directions can no longer be ne- 63
glected. Experimental works on surface wave solitons in 64
resonators are rather scanty. The parametric excitation of 65
nonlinear waves, including soliton-like pulses, was described 66
in Refs. 16 and 17 for experiments in a 60 cm long cuvette, 67
performing horizontal oscillations. That work was published 68
38 years ago, when the properties of solitons and even the 69
term «soliton» itself were not known to the broad public. 70
The authors of Refs. 16 and 17 called the observed waves 71
«resonant oscillations», as they were excited near the first 72
resonant frequency of the tank. They studied the time series 73
of free surface displacements at several points of a resonator 74
and did not present any information about the spatiotemporal 75
dynamics of the perturbations. This work has stimulated 76
a large number of theoretical and numerical studies on 77
solitons, most of which were concerned with asymptotic 78
methods applied to the Korteweg-de Vries equation, with 79
forcing near the fundamental¹⁸⁻²⁰ or half the fundamental 80
frequency.²¹ Modes of the parametric excitation of one or 81
several nonpropagating oscillating solitons were described in 82

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83 Ref. 22, where it was found that the displacement amplitude
84 of the free surface in such nonlinear waves is close to the
85 classical soliton, with the fields decreasing exponentially at
86 large distances. Soliton excitation in wave resonators has
87 been performed in systems whose physical nature is abso-
88 lutely different from that of surface waves, but that, never-
89 theless, possess similar dispersive and nonlinear characteris-
90 tics as surface waves on shallow water. For instance, solitons
91 of electromagnetic waves were observed on a section of an
92 LC line excited by sinusoidal voltage,²³ and solitons of trans-
93 verse waves were observed in a rubber strip at parametric
94 excitation by a harmonic external force.²⁴

95 When solitons are excited in resonators, we face prob-
96 lems that are quite different from those in spatially infinite
97 systems, because of the multiple interactions of solitons
98 propagating in a resonator, as well as the interaction between
99 solitons and resonator modes. In this paper, we have inves-
100 tigated these phenomena excited near the second resonant
101 frequency of a hydrodynamic resonator of a larger length.
102 We have determined a state diagram of different excited
103 modes. We have used the space-time diagrams technique that
104 allows for a better visualization of spatiotemporal properties
105 of wave phenomena and a better comparison between experi-
106 mental and numerical results. In particular, we have thor-
107 oughly characterized the soliton collision by measuring resi-
108 stance time and merging length. We have evidenced the
109 existence of soliton bound state generation. In order to repro-
110 duce some states observed in experiments, we have solved
111 numerically the Boussinesq equations with dissipation terms.

112 The paper is organized as follows. The experimental pro-
113 cedure on soliton excitation in a water wave resonator is
114 described in Sec. II. Results are presented in Sec. III, where,
115 in particular, a state diagram is presented in the space of the
116 excitation parameters (amplitude and frequency). In Sec. IV
117 we compare some of our experimental results with those ob-
118 tained in experiments on soliton reflection from a vertical
119 wall carried out by Maxworthy.³ We introduce a simplified
120 model for the amplitude and phase of the soliton interacting
121 with the monochromatic wave. The section ends with the
122 presentation of a numerical solution of a modified system of
123 the Boussinesq equations for shallow water waves. In Sec. V
124 we draw some concluding remarks.

125 II. EXPERIMENTAL SETUP

126 The experiment was carried out in a water tank of length
127 $L=10$ m and width $D=0.5$ m, with a liquid depth H
128 $=0.26$ m. Hence, the ratio $\mu=H/L=0.026 \ll 1$ lies in the
129 allowed range of the shallow water approximation.² Surface
130 waves were excited by a wavemaker (Fig. 1), which consists
131 of a flat plate driven into a periodic motion by an electric
132 motor. The plate rotated around the point O_1 (Fig. 1), the
133 amplitude of its horizontal displacement, depends on depth;
134 that is why we have used the average value of this amplitude,
135 defined as

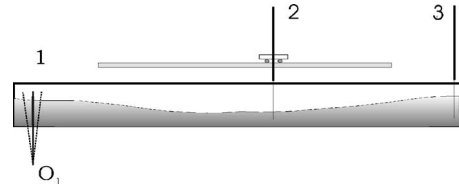


FIG. 1. Experimental setup: 1 – wave maker, O_1 – the rotation axis of a flat plate; 2 – movable resistive sensor; 3 – stationary resistive sensor.

$$a_{\text{ex}} = \frac{1}{H} \int_0^H a(z) dz.$$

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Displacement of the free surface was measured by two
resistance-wire wave gauges (Fig. 1). One of them was
moved at distances up to 5.6 m in the central part of the tank,
while the second one was fixed at the wall reflecting the
surface waves. The hydrodynamic resonator has eigenmodes
with frequencies given in Ref. 25:

$$f_n = \frac{1}{2\pi} \sqrt{gk_n \tanh z(k_n H)},$$

143

where g is the gravity acceleration and $k_n = n\pi/L$, n
 $= 1, 2, 3, \dots$. For shallow water $k_n H \ll 1$ and the frequency
becomes $f_n \approx k_n \sqrt{gH}$. The frequency of the wavemaker in
the experiment was chosen to be close to that of the mode whose
wavelength was equal to the resonator length $\lambda=L$, i.e.,
 $n=2$ and $f_2 \approx L^{-1} \sqrt{gh}$, which gives for our resonator dimen-
sions $f_2=0.1645$ Hz. At this frequency, standing harmonic
surface waves of maximal amplitude were excited in the
resonator at a very small amplitude of external forcing.
When the external force frequency was detuned by a small
value from f_2 , the amplitude of the standing waves in the
resonator decreased. We measured the mismatch from exact
resonance $\delta f_2 = f_{\text{ex}} - f_2$ at which the amplitude of oscillations
in the resonator decreased by $\sqrt{2}$ times. We have found that
this mode had a rather high Q factor (defined as $Q=f_2/\delta f_2$
 ≥ 50) induced by the dissipation in the system. The value
 $Q=50$ means that the mode oscillations at frequency f_2 damp
 $e=2.71\dots$ times over 50 periods, or approximately during
5 min. The hydrodynamic resonator has two main control
parameters: the average excitation amplitude a_{ex} and the ex-
citation frequency f_{ex} (or the detuning parameter δf_2).

III. RESULTS

A. Space-time diagrams

Signals from the two sensors were used to construct
space-time diagrams. We registered the time series at 56
points by moving the sensor along the channel and, simulta-
neously, registered the signal from the stationary sensor for
each of these points. The signal from the stationary sensor
was used as a reference signal, and phase averaging was
made. This procedure of constructing space-time diagrams is
valid for regimes that are strictly periodic in time. If the
regime has no time period, many sensors may be needed
from which signals must be registered simultaneously.
Space-time diagrams obtained in an experiment depend sig-
nificantly on the control parameters a_{ex} and δf_2 .

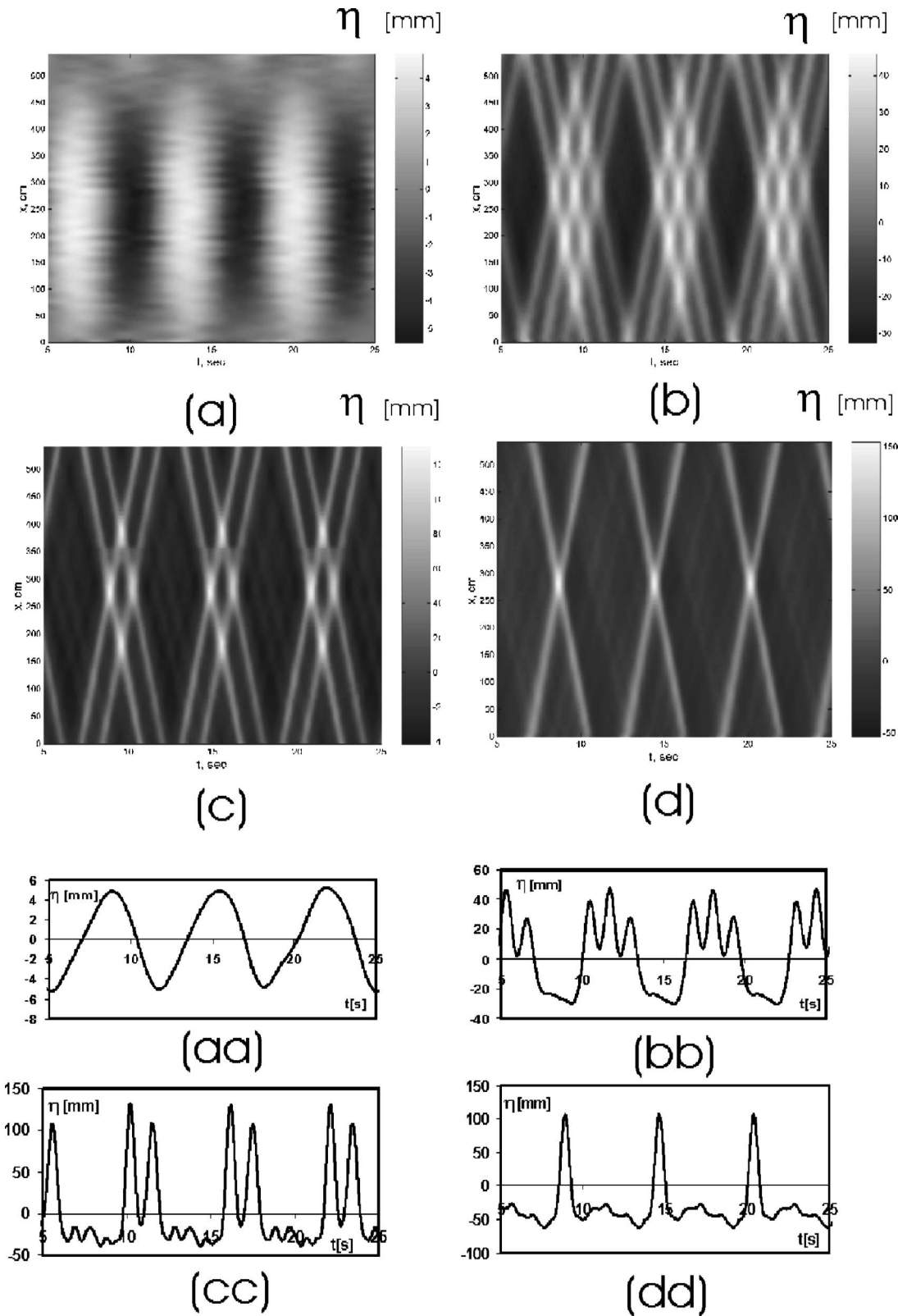


FIG. 2. Space-time diagrams and time series from stationary sensor: (a), (aa) $a_{ex}=2.3$ cm, $f_{ex}=0.151$ Hz, $Q^{-1}=(f_{ex}-f_2)/f_2=-0.087$; (b), (bb) $a_{ex}=8.2$ cm, $f_{ex}=0.158$ Hz, $Q^{-1}=-0.045$; (c), (cc) $a_{ex}=8.2$ cm, $f_{ex}=0.170$ Hz, $Q^{-1}=0.028$; (d), (dd) $a_{ex}=8.2$ cm, $f_{ex}=0.177$ Hz, $Q^{-1}=0.07$.

179 Typical diagrams for different parameters of external
 180 forcing, as well as the time series obtained by means of the
 181 stationary sensor located at the wall, are given in Fig. 2. It is
 182 clear from these diagrams that, for a small amplitude of the

external forcing, standing surface waves are excited in the 183
 resonator [Fig. 2(a)], which correspond to the time series 184
 (aa). For larger amplitudes, propagating pulses arise in the 185
 system [Figs. 2(b)–2(d)] and the time series are seen in (bb)– 186

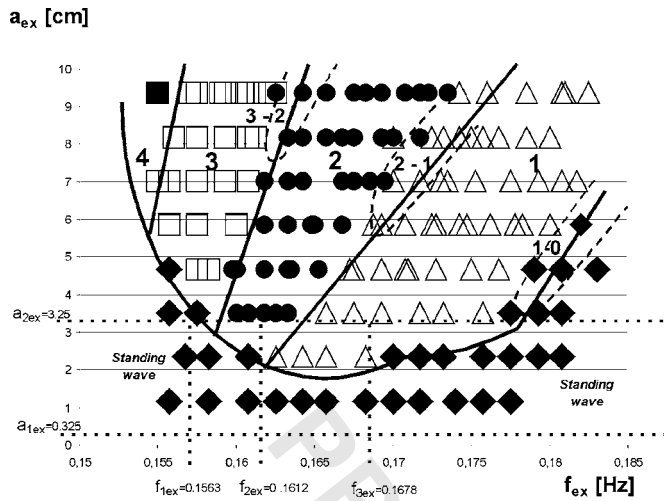


FIG. 3. Partitioning of the (f_{ex}, a_{ex}) plane into regions with different regimes of generation of nonlinear waves. Numbers 1, 2, 3, 4 mean the number of impulses generated on the background of the time period of harmonic wave. Regions isolated by the dashed curves correspond to the parameters under which generation regimes (generation of 3 or 2, 2 or 1, 1 or 0 impulses) depend on the initial conditions.

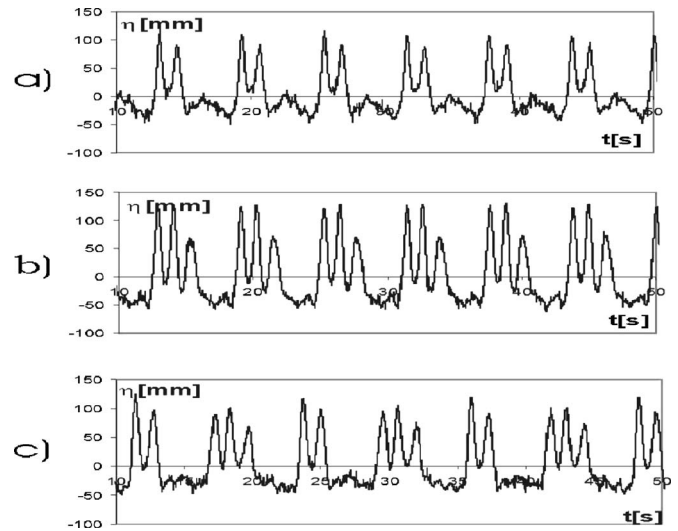


FIG. 4. Multistability observed at $a_{ex}=9.4$ cm, $f_{ex}=0.165$ Hz (region 2-3 in Fig. 3).

B. Extraction of solitons

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A detailed analysis of the experimental data revealed that, within a definite interval of control parameters, characteristics of the pulses excited in the resonator are close to those of solitons. To verify this statement, we have assumed that pulses in the resonator are excited against the background of a harmonic wave. The problem is how to separate pulses and harmonic oscillations. Obviously, this cannot be done by linear filters, as the repetition rate of pulses is equal exactly to the frequency of harmonic oscillations. Consequently, we determined the position of pulse maxima [Fig. 5(a)], and then the resulting data were replaced in the interval $\tau_1 + \tau_2$ by the linear dependence of the free surface displacement on time. In this fashion we were able to separate the pulses. Further, all the harmonics with the frequency larger than the excitation frequency were separated from the obtained signal by filtering. The resulting signal as well as the initial time series are depicted in Fig. 5(b). Figure 5(c) illustrates separately the harmonic mode of the resonator and the sequence of pulses. Apparently, the results of filtering depend significantly on the way we choose τ_1 and τ_2 . Different trials for quantities τ_1 and τ_2 showed that, with a reasonable choice of these parameters ($\tau_{1,2} < T/6$, where T is the period of external force), the scatter in the characteristics of harmonic oscillations and pulses is insignificant. Following this algorithm the time series were processed by the LabView 4.1 software. It was revealed that the shape of the pulses is close to classical solitons: $A = A_s \cosh^{-2}(t/\tau_s)$ (A_s stands for the amplitude of the soliton), everywhere except in regions where the field amplitude is small, for instance, in the oscillating tails of solitons (Fig. 6). The pulse duration τ_s defined at the level $A/A_s = \cosh^{-2}(1)/0.4199$, as a function of the pulse amplitude, is plotted in Fig. 7. We found that the pulse duration decreases with the increasing amplitude of the pulse, A_s . Pulses observed in the experiment are localized in a finite region of space and their characteristics are close to those of the theoretical soliton; therefore we have called them *solitons*.

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187 (dd). The pulses are visualized as bright strips in the space-time diagrams. The pulse propagation velocity given by the slope of the strips was close to the velocity of surface waves on shallow water in our experimental conditions. If we represent a standing wave as a superposition of two traveling waves propagating in opposite directions, pulses propagate synchronously with each of these waves. The number of pulses (from 1 to 4 in most of our experiments) registered by a sensor is different at different points of the resonator, and it coincides with the number of pulses for each traveling wave only in the center of the resonator and near its side boundaries. The number of pulses depends on the excitation frequency and on the amplitude. For a frequency lower than the resonance frequency of the mode, three pulses are excited on the wave period. Two intersecting triples of such tracks are illustrated in Fig. 2(b). For the excitation frequency higher than the resonance frequency, only one pulse arises, hence, two intersecting tracks only are seen in Fig. 2(d).

In Fig. 3 we have represented the t wave states of qualitatively different dynamics in the plane of parameters (f_{ex}, a_{ex}) . The labels 1, 2, 3, or 4 in the regions of the state diagram indicate the number of pulses excited on the wave period. When the amplitude is large enough, there exists a small region where four pulses can be excited. For the control parameters within the zones marked by a dashed line, multistability is observed, i.e., different regimes may occur at the same values of frequency and wavemaker oscillation amplitude, but under different initial conditions. For example, the excitation of two [Fig. 4(a)] or three [Fig. 4(b)] pulses, or the alternate generation of two or three pulses on a period [Fig. 4(c)] occur in zones 2–3. The regime in Fig. 4(c) corresponds to period doubling: nonlinear waves excited in the system have a period twice as large as the period of wavemaker oscillations.

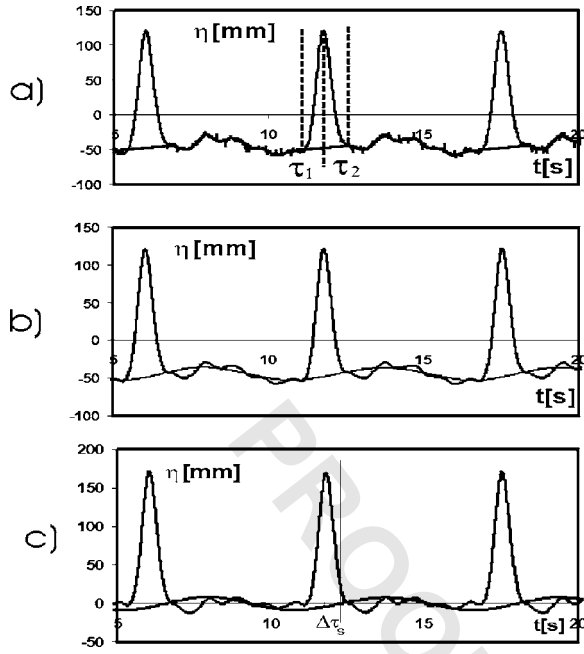


FIG. 5. Extraction of solitons and harmonic wave from the time series: (a) initial time series and series with solitons cut off on the interval $\tau_1 + \tau_2$; (b) harmonic wave and "harmonic wave+soliton;" (c) a sequence of solitons and harmonic wave. The phase shift of soliton and harmonic wave $\varphi_s = \Delta\tau_s \omega$, where $\omega = 2\pi f_{ex}$.

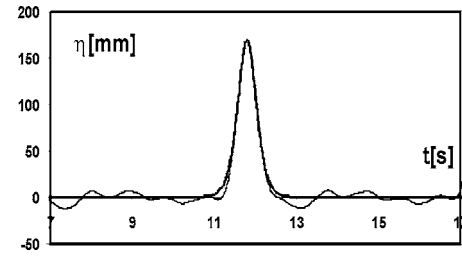


FIG. 6. A comparison of the shape of pulse with the theoretical dependence $\eta = A_s \cosh^{-2}[(t - t_0)/\tau_s]$, where $t_0 = x/V_s$.

$$\eta(t, x) = \eta_s(x - V_s t) + \eta_0 \sin(\omega t - kx - \varphi_s) + \eta_s(x + V_s t) + \eta_0 \sin(\omega t + kx - \varphi_s), \quad (3)$$

where ω, k are the frequency and wavenumber of harmonic waves, η_0 is the wave amplitude, and φ_s is the phase shift between the soliton and the harmonic wave [Fig. 5(c)]. Near the reflecting endwall ($x=0$), where the stationary sensor is located, the displacement of the free surface can be represented in the form

$$\eta = 2\eta_0 \sin(\omega t - \varphi_s) + 2\eta_s(t).$$

This relation allows for a comparison of our experimental results with analytical theory for the soliton-harmonics interaction developed in Ref. 23 and, in particular, for measuring the phase shift φ_s . The results of the measuring phase shift are presented in Fig. 8. For a small amplitude of external force, the phase shift was close to zero. As the amplitude increased, the phase shift increased up to $40^\circ - 60^\circ$. The amplitude of the soliton depended on the frequency and amplitude of the wavemaker. For the same amplitude of external forcing, the soliton amplitude and phase depended on the frequency of the external force. By measuring the time at which the soliton amplitude reaches its maximum at different positions of a sensor of free surface displacement, we have determined the trajectory of soliton motion (Fig. 9). From these trajectories, we have measured the velocity of soliton motion on different sections of the trajectory, except in the region of the merging of two maxima. A linear fit of the data

259 C. Properties of the observed solitons

The extraction of solitons and a harmonic wave from a time series allows one to compare the properties of nonlinear waves observed in the experiment with the well-known solutions of the Korteweg-de Vries (KdV) equation (2) that governs the displacement of free surface of shallow water:

$$\frac{\partial \eta}{\partial t} + V_0 \frac{\partial \eta}{\partial x} + \frac{3V_0}{2H} \eta \frac{\partial \eta}{\partial x} + \frac{1}{6} V_0 H^2 \frac{\partial^3 \eta}{\partial x^3} = 0, \quad (1)$$

where $V_0 = \sqrt{gH}$ is the velocity of surface waves of infinitely small amplitude in shallow water. The KdV equation has solutions in the form of a soliton²:

$$\eta_s(x - V_s t) = A_s \cosh^{-2} \left(\sqrt{\frac{3A_s}{4H^3}} (x - V_s t) \right), \quad (2)$$

$$V_s = V_0 \left(1 + \frac{A_s}{2H} \right).$$

Clearly, the duration of the soliton is proportional to $A_s^{-1/2}$ (the solid line in Fig. 7); its velocity depends linearly on its amplitude A_s , and the soliton size decreases with an increase of its amplitude.

In order to measure the phase shift between the soliton and background wave, we have assumed that nonlinear waves excited in a resonator consist of two solitons propagating toward each other, and each of them is propagating against the background of a harmonic wave. We have neglected the interaction of counterpropagating waves and represented the nonlinear field of surface displacement as a superposition of four components:

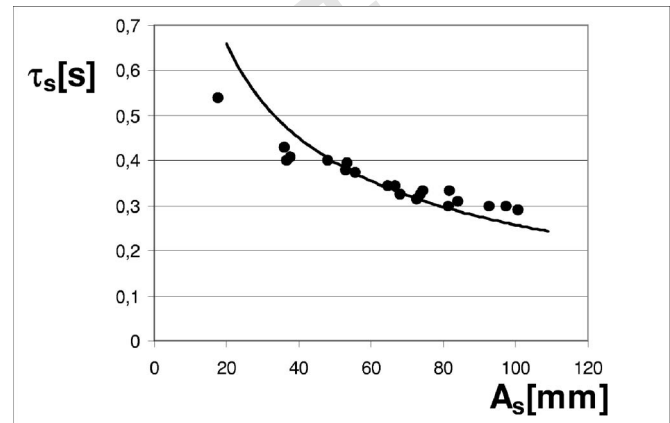


FIG. 7. The soliton duration versus the soliton amplitude a_s obtained in experiment (dots). The theoretical dependence $\tau_s = (1/V_s) \sqrt{4H^3/3A_s}$, where $V_s = V_0(1 + A_s/2H)$ is also plotted for comparison (curve).

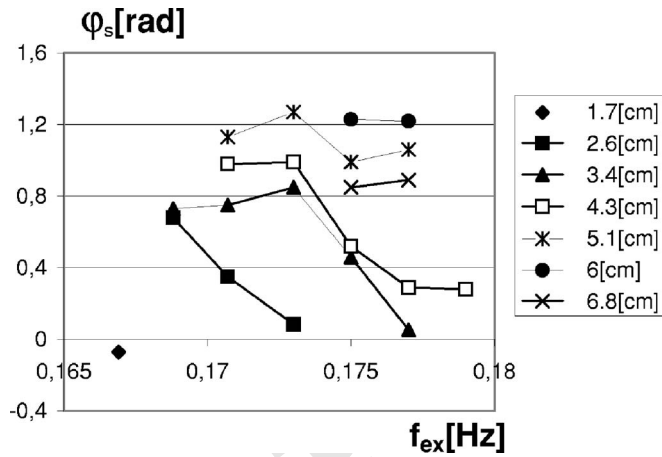
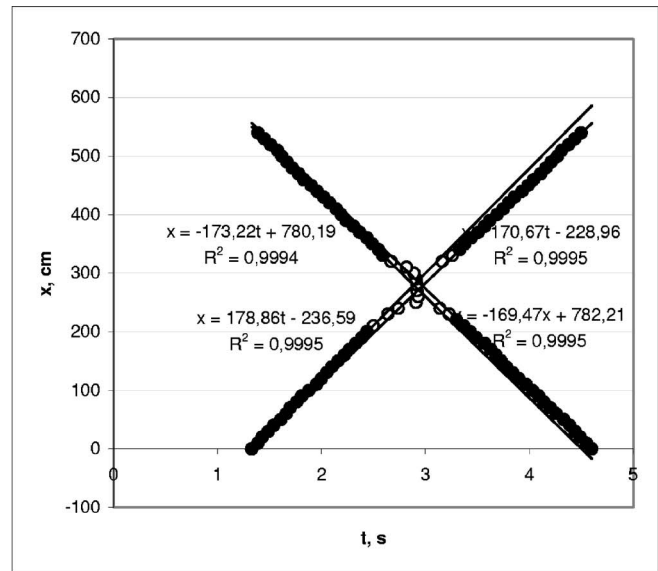
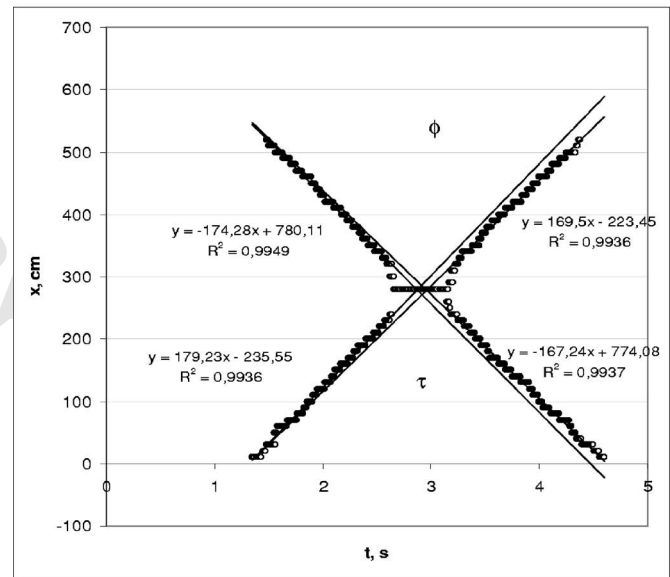


FIG. 8. The soliton phase φ_s versus the external force frequency f_{ex} . Data for different external force amplitudes a_{ex} are marked by different symbols.



a)



b)

FIG. 9. Soliton trajectories in a resonator for $a_{ex}=8.2$ cm, $f_{ex}=0.177$ Hz. Arrows indicate directions of soliton propagation. (a) Soliton trajectories determined by snapshots; (b) soliton trajectories determined by time series; ϕ is the soliton lag.

310 showed that the soliton has approximately constant velocity
311 $V_s \approx f_{ex}L$ on all the sections. The solitons propagate synchro-
312 nously with the harmonic wave.

313 The space-time diagrams illustrate how in the central
314 part of the resonator solitons propagating in opposite direc-
315 tions collide and merge. The size L_m of the merging region
316 depends on the pulse amplitude, as is clearly seen in Fig. 10.
317 The larger is the pulse amplitude, the less extended is the
318 region of merging. The pulse interaction leads to a significant
319 change in their velocity.

320 IV. DISCUSSION OF RESULTS

321 A. Merging length and residence time

322 The method of characteristics has been used by
323 Maxworthy³ to investigate the soliton interaction with a wall;
324 we may compare our results on soliton interaction in resona-
325 tors with his results. Maxworthy showed that the collision of
326 two counterpropagating surface wave solitons of equal am-
327 plitude in an unbounded medium is equivalent to the reflec-
328 tion of one soliton from a vertical wall. Indeed, on reflection
329 from a vertical wall, the horizontal velocity of perturbations
330 is zero, and when two solitons of the same amplitude collide
331 there is a symmetry axis along which the horizontal velocity
332 is also zero. Maxworthy made a film in his experiment and
333 processed its individual shots—instantaneous snapshots of
334 surface elevation from which he constructed soliton trajec-
335 tories. In our experiments we constructed an (X-T) diagram
336 using a time series of surface displacement measured at dif-
337 ferent points. These are, actually, diagram sections at a fixed
338 coordinate. Thus, the two problems are identical. By calcu-
339 lating the position of the maximum of surface displacement
340 for each section, one can construct a trajectory of soliton
341 motion. An example of such a trajectory is given in Fig. 9(a).
342 The arrows show the direction of soliton motion. The best
343 linear fit of the points is shown for different parts of the
344 trajectories. Velocities of the solitons are different in the four
345 parts. Our diagrams can also be used to determine a trajec-
346 tory like it was done by Maxworthy. Toward this end, one
347 must determine in the (X-T) diagram sections at fixed mo-
348 ments of time. Note that a trajectory will be different if one

first finds sections at a fixed time and then calculates the
position of the maximum along the coordinate [Fig. 9(b)]. In
the first case [Fig. 9(a)], there exists a length L_m that can be
referred to as the merging length. In the second case, there is
time τ , which is referred to as residence time [Fig. 9(b)]
during which the maximum stays at one point. The residence
time was determined for different amplitudes of colliding
solitons. For example, for the case depicted in Fig. 9(b), the
dimensionless residence time is $\tau' = \tau\sqrt{g/H} = 2.9$ for the
dimensionless soliton amplitude $\varepsilon = A_s/H = 0.28$. These values
are in good agreement with the data obtained by Cooker *et al.*⁹
from Maxworthy's film,³ as well as with the results of
numerical calculations also reported in Ref. 9. For instance,
numerical calculations⁹ give $\tau = 3.2$ for $\varepsilon = 0.28$, which al-

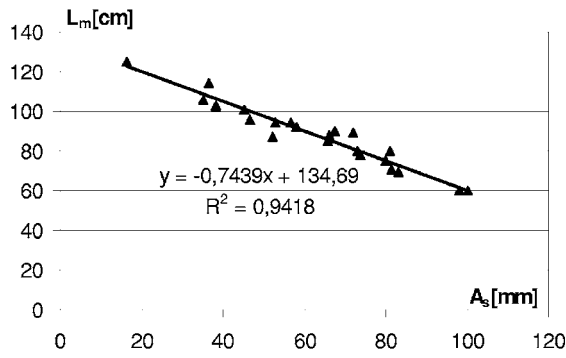


FIG. 10. The merging length L_m versus the soliton amplitude A_s .

$$\frac{dE_s}{dt} = \frac{3}{2} \frac{\omega \eta_0}{H} E_s \cos \varphi_s - \alpha E_s, \quad (4a) \quad 405$$

$$\frac{d\varphi_s}{dt} = \frac{kV_0}{2H} [A_s - 3\eta_0 \sin \varphi_s] - \Delta. \quad (4b) \quad 406$$

Here, E_s stands for soliton energy, 407

$$E_s = \int_{-\infty}^{\infty} \eta_s^2 dx \approx A_s^2 \Delta x_s \propto A_s^{3/2}, \quad 408$$

363 most coincides with the data of Maxworthy's film. For
 364 smaller amplitudes of colliding solitons, the agreement of
 365 our results with data of the works^{3,9} is worse. This is evi-
 366 dently caused by the influence of monochromatic resonator
 367 modes on the collision of small-amplitude solitons in our
 368 experiments. Measurements at different amplitudes of exter-
 369 nal force showed that the difference between soliton veloci-
 370 ties before and after collision increases with increasing soli-
 371 ton amplitude.

372 Maxworthy³ noted that the amplitude of a soliton re-
 373 flected from a wall is slightly smaller than before reflection,
 374 and attributed this difference to dissipation. When solitons
 375 are excited in a resonator, in addition to dissipation, the soli-
 376 ton interaction with a monochromatic mode occurs, and this
 377 may lead to an increase in the soliton amplitude. Conse-
 378 quently, for experiments in a resonator, we cannot state that
 379 the soliton amplitude before an interaction must be larger
 380 than after it. In addition to the effect of velocity change on a
 381 soliton collision, we have revealed a small phase shift, soli-
 382 ton lag ϕ caused by their interaction [Fig. 9(b)]. Such a time
 383 lag when solitons were reflected at the wall was also reported
 384 by Maxworthy.³

385 B. Analytical model of soliton generation

386 In this section we have compared the characteristics of
 387 surface wave solitons measured in the experiment and the
 388 results of semiphenomenological theory. A theoretical inves-
 389 tigation of nonlinear fields excited by the harmonic force in a
 390 resonator was developed for surface waves on shallow water
 391 by Chester¹⁷ and for electromagnetic waves in LC lines by
 392 Gorshkov *et al.*,²³ where perturbations propagating in reso-
 393 nators were represented as a sum of monochromatic waves
 394 and nonlinear perturbations. As the paper¹⁷ appeared before
 395 intense investigations into solitons in different fields of sci-
 396 ence, it did not contain any reference to solitons; however,
 397 Gorshkov *et al.*²³ postulated the existence of a soliton on the
 398 background of a monochromatic wave. For a description of
 399 solitons of electromagnetic waves Gorshkov *et al.* derived
 400 equations for the amplitude and the phase of a soliton propa-
 401 gating in a resonator against the background of a monochro-
 402 matic wave. We have established analogous equations for
 403 surface waves on shallow water, and they are written in the
 404 form

Δx_s is the width of the soliton, φ_s is the soliton phase that can 409
 be understood as the dimensionless time between the maxi- 410
 mum of the soliton and the zero of the monochromatic mode 411
 [Fig. 5(c)], $\Delta = 2\pi\delta f_2$ is the frequency mismatch, and α is the 412
 coefficient describing the exponential damping of the soliton. 413
 The first equation in the system (4) is the equation of energy 414
 balance. Changes in the soliton amplitude are due to energy 415
 transfer from a harmonic wave [the first term in (4); its de- 416
 tailed derivation can be found in the Appendix] and to en- 417
 ergy dissipation [the second term in (4a)]. It is worthy of 418
 notice that the coefficient α describes a type of dissipation 419
 that is independent of the scale of the perturbations. Such 420
 dissipation, indeed, is observed in different systems such as 421
 the ones considered in Refs. 26 and 27. However, in contrast 422
 to other parameters entering the system (4), we did not de- 423
 termine this parameter in our experiment, and in what fol- 424
 lows we have chosen it from the requirement that the theo- 425
 retically predicted parameters of solitons should be as close 426
 as possible to the experimental data. The second equation in 427
 (4) is a kinematic condition. The soliton velocity in the re- 428
 ference system moving with the velocity of the harmonic 429
 wave ($V_{phh} = \omega/k$) depends on the soliton amplitude A_s [the 430
 first term in (4b)]. The harmonic wave transports the soliton 431
 as a particle with corresponding velocity $u = -\sqrt{g/H}\eta_0 \sin \varphi_s$, 432
 depending on its phase. As the soliton is superposed on the 433
 background of the surface displacement of the harmonic 434
 wave, the latter modifies the velocity of soliton propagation 435
 by $\Delta V_s = \sqrt{g(H - \eta_0 \sin \varphi_s)} - \sqrt{gH} \approx -(1/2)\sqrt{g/H}\eta_0 \sin \varphi_s$. 436
 The total contribution of these two effects ($u + \Delta V_s$) gives the 437
 second term on the right side of Eq. (4b). Changes in the 438
 phase are also conditioned by the mismatch between external 439
 wave frequency and resonance frequency (the third term). 440
 We should keep in mind that the system (4) was obtained for 441
 a soliton and a monochromatic wave propagating in the same 442
 direction. For a soliton and a wave propagating in opposite 443
 directions the same equations are valid, as long as interaction 444
 of counterpropagating perturbations is not taken into consid- 445
 eration. Equations (4) may be obtained also from equations 446
 describing soliton excitation by traveling perturbations. This 447
 approach was developed, for example, by Sleath.²⁸ The 448
 qualitative analysis of solutions to the system (4) showed 449
 that stable steady states for φ_s and amplitude A_s satisfy the 450
 relations²³ 451

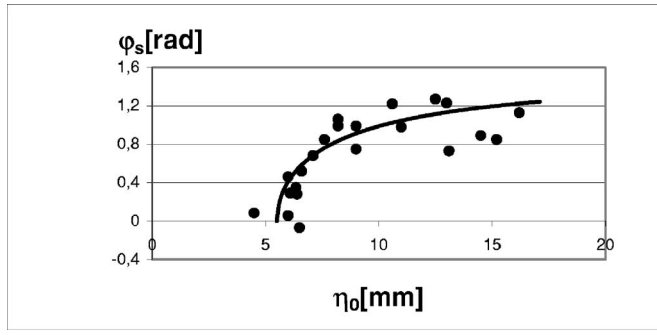


FIG. 11. The soliton phase φ_s versus the amplitude η_0 of a harmonic wave (dots), and the theoretical dependence (5a) for $\alpha=0.015$ Hz (curve).

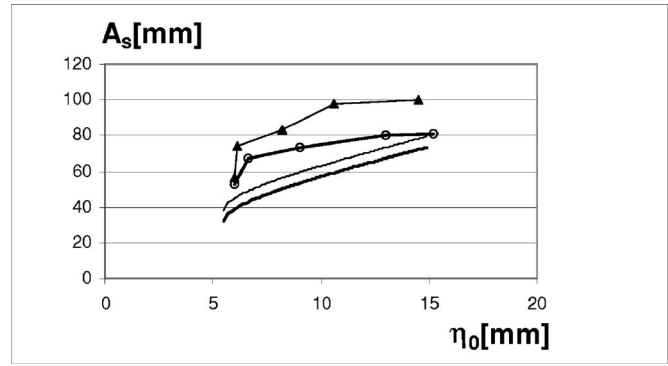


FIG. 12. Soliton amplitude A_s versus amplitude η_0 of a harmonic wave: the dots connected by broken curves are for experimental data; solid curves are for the theoretical dependence (5b). The thick curves correspond to mismatch $\Delta=0.063$ rad/s; the thin curves to $\Delta=0.075$ rad/s.

$$\varphi_s = \arccos\left(\frac{2H\alpha}{3\omega\eta_0}\right), \quad (5a)$$

$$A_s = \frac{2\Delta}{k} \sqrt{\frac{H}{g}} + 2\eta_0 \sqrt{1 - \left(\frac{2H\alpha}{3\omega\eta_0}\right)^2}. \quad (5b)$$

Using the procedure of soliton amplitude and phase extraction described in Sec. III B we have compared experimental data and theoretical predictions. It follows from the theoretical model that soliton phase does not depend on the frequency mismatch. Therefore, we represent phase φ_s as a function of the harmonic wave amplitude η_0 for all the excitation frequencies at which one soliton was observed in an experiment (Fig. 11). The plot φ_s versus frequency f_{ex} for different amplitudes of excitation (Fig. 8) exhibits points scattered over the plane. However, if we plot the same points in new variables (η_0, φ_s) , all points group in the vicinity of the theoretical curve [Eq. (5a)]. The soliton amplitude A_s versus the harmonic wave amplitude is plotted in Fig. 12 for two different excitation frequencies. For the theoretical curves plotted in Figs. 11 and 12 by solid lines, the value of damping was chosen to be $\alpha=0.015$ Hz. The plots show sufficiently good agreement between the experimental data and theoretical predictions. We have observed that the value of the coefficient α taken for the theoretical calculations correlates well with changes in the Q factor of the harmonic wave having frequency f_2 on the resonance curve width. Measurements gave $Q^{-1} \approx 0.02$. If we assume that the damping of the mode with frequency f_2 is the same as that of the soliton, then calculations give the Q factor: $Q^{-1} \approx 0.023$, which is very close to the measured value.

C. Numerical resolution of the modified Boussinesq equations

The KdV equation cannot reproduce all the spatiotemporal properties of the nonlinear waves observed in the hydrodynamic resonator for different parameters of the external forcing. For an explanation of our experimental results, we have made a numerical simulation of one-dimensional Boussinesq equations describing the propagation of waves on shallow water^{1,29} and to which we have added small damping terms:

$$\hat{\eta}_\tau + \hat{u}_\xi + \varepsilon(\hat{u}\hat{\eta})_\xi = \frac{1}{6}\mu^2\hat{u}_{\xi\xi\xi} + \gamma_1\hat{\eta}_{\xi\xi} - \gamma\hat{\eta}, \quad (6a)$$

$$\hat{u}_\tau + \hat{\eta}_\xi + \varepsilon(\hat{u}\hat{u})_\xi = \frac{1}{6}\mu^2\hat{u}_{\xi\xi\xi} + \gamma_1\hat{u}_{\xi\xi} - \gamma\hat{u}. \quad (6b)$$

These equations (6) are written in dimensionless variables: $\hat{\eta}$ is the displacement of free surface $\hat{\eta} = \eta/\varepsilon H$, hereinafter an overcaret denotes dimensionless variables; $\hat{u} = u/\varepsilon H\omega_{ex}$, where u is the velocity taken at the bottom, τ is the time $\tau = t\omega_{ex}$; ξ is the coordinate along the resonator $\xi = x/L$; $\mu = H/L$; ε is the small parameter of asymptotic expansion introduced in Ref. 1. Note that for a constant depth shallow water, the Boussinesq equations can be written in different systems:²⁹ the depth-mean velocity basis, the bottom variables basis, and the surface variable basis. In our paper, we have preferred the use of the bottom variables basis, which, in the long-wave approximation, is equivalent to the depth-mean velocity basis. We have supplemented the Boussinesq equation by phenomenological terms describing dissipation. The first type of losses (terms with coefficient γ_1) is associated with viscous damping caused by shear stresses; the intensity of this damping increases in proportion to the square of the wavenumber. The second type (terms with coefficient γ) is related to losses in the boundary layer of the surface wave at the bottom. Losses of this type were discussed in Refs. 26 and 27, where it was shown that they are caused by wave scattering at small-scale turbulence. In our experiment such turbulence exists in a boundary layer at the bottom. Measurements of velocity pulsations in the boundary layer by means of a laser Doppler anemometer revealed a high enough intensity of velocity pulsations at a distance of about 1 mm from the bottom. The corresponding value of Reynolds number defined using the velocity in the wave near the bottom is $Re = 1.6 \cdot 10^5$, which lies in the range of a turbulent boundary layer for oscillating flows.²⁸ The consideration of both kinds of dissipation is very important in our case, as the finite amplitude of nonlinear waves in a resonator depends on the balance of energy supply to the system due to external forcing and energy losses in the system.

For numerical calculations we used the following boundary conditions at $\xi=0$ and $\xi=1$:

527
$$\frac{\partial \hat{\eta}}{\partial \xi}(\tau, \xi = 0) = 0, \hat{u}(\tau, \xi = 0) = U_0 \sin(2\pi f t), \quad (7a)$$

528
$$\frac{\partial \hat{\eta}}{\partial \xi}(\tau, \xi = 1) = 0, \hat{u}(\tau, \xi = 1) = 0. \quad (7b)$$

529 We have modeled the action of the wavemaker by a periodically varying horizontal velocity at one resonator end. Numerical resolution of the system of equations (6) was made with the implicit finite-difference scheme realized using the MATLAB5.3 software:

534
$$\frac{\eta_j^{n+1} - \eta_j^{n-1}}{2 \Delta \tau} + \frac{u_{j+1}^n - u_{j-1}^n}{2 \Delta \xi} + \varepsilon u_j^n \frac{\eta_{j+1}^n - \eta_{j-1}^n}{2 \Delta \tau}$$

535
$$+ \varepsilon \eta_j^n \frac{u_{j+1}^n - u_{j-1}^n}{2 \Delta \xi} = \frac{1}{6} \mu^2 \frac{u_{j+2}^n - 2u_{j+1}^n + 2u_{j-1}^n - u_{j-2}^n}{2 \Delta \xi^3}$$

536
$$+ \gamma_1 \frac{\eta_{j+1}^{n+1} - 2\eta_j^{n+1} + \eta_{j-1}^{n+1}}{\Delta \xi^2} - \gamma \eta_j^{n+1},$$

537
$$\frac{u_j^{n+1} - u_j^{n-1}}{2 \Delta \tau} + \frac{\eta_{j+1}^n - \eta_{j-1}^n}{2 \Delta \xi} + \varepsilon u_j^n \frac{u_{j+1}^n - u_{j-1}^n}{2 \Delta \xi}$$

538
$$= \frac{\mu^2}{4 \Delta \tau} \left(\frac{u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}}{\Delta \xi^2} - \frac{u_{j+1}^{n-1} - 2u_j^{n-1} + u_{j-1}^{n-1}}{\Delta \xi^2} \right)$$

539
$$+ \gamma_1 \frac{u_{j+1}^{n+1} - u_j^{n+1} + u_{j-1}^{n+1}}{\Delta \xi^2} - \gamma u_j^{n+1}.$$

540 Here $f_j^n = f(\tau_n, \xi_j)$. These equations were used in the inner nodes of the spatial grid, and boundary conditions (7) were written in difference form.

543 Let us consider results of numerical computations obtained for different parameters. In the first series of experiments we set the frequency independent damping to be equal to zero ($\gamma=0$) and investigated the dependence of the time series and space-time diagrams on the amplitude and phase of the forcing. Results for the zero initial conditions $\hat{u}(\tau=0, \xi)=0, \hat{\eta}(\tau=0, \xi)=0$ are presented in Fig. 13. For a small amplitude of external forcing $U_0=0.05$ (the dimensional amplitude of wavemaker displacement is $a_{1ex}=0.325$ cm), standing waves are excited in the system. For a larger amplitude $U_0=0.5$ ($a_{2ex}=3.25$ cm), the shape of the nonlinear waves excited in the system depends on the ratio of the external forcing frequency f_{ex} to the resonance frequency f_2 . For $f_{ex}^{(1)}=0.95f_2=0.1563$ Hz, three pulses are observed on the period; for $f_{ex}^{(2)}=0.98f_2=0.1612$ Hz we have two pulses, and for $f_{ex}^{(3)}=1.02f_2=0.1678$ Hz, one pulse. These states are similar to those observed in the experiment for the corresponding parameter values of external forcing (Fig. 3, where corresponding dimensional parameters are indicated).

562 Both numerical computations and experiments have demonstrated multistability and period doubling. The regime of soliton generation depends on initial conditions. For example, under zero initial conditions, one soliton is generated on a wave period; whereas under the initial conditions corresponding to the generation of two solitons on a period, given the same parameters of external forcing, the regime of two-soliton generation proves to be stable. Alternating gen-

eration of two or three pulses or one and two pulses is observed in a definite space of the parameters, i.e., the regime where period doubling occurs. Several pulses on a wave period were also obtained in analytical calculations of surface wave excitation in a cuvette^{17,30} when waves were excited by horizontal oscillations of the cuvette. Chester¹⁷ and Ockendon *et al.*³⁰ sought solutions in the form of counterpropagating waves traveling with constant velocity, whereas our numerical calculation showed that wave velocity may be different on different sections of interaction.

In the second series of experiments, we have varied the ratio of the coefficients of frequency-dependent and frequency-independent dissipation. Under the assumption that the dissipation is largely due to frequency-independent losses, e.g., $\gamma_1=10^{-4}, \gamma=15 \cdot 10^{-4}$, an oscillating tail component appears on the intervals between solitons (Fig. 14; compare with Fig. 5 for experimental data). This phenomenon can be explained as follows: when the damping is proportional to $\sim k^2$, energy losses at higher harmonics increase with the number of harmonics, and high-frequency oscillations are suppressed. Consequently, the soliton changes smoothly at the periphery. In the case of frequency-independent damping, higher harmonics have a relatively large amplitude and manifest themselves in the time series obtained in numerical experiment.

It is likely that colliding solitons form oscillating tails by a different mechanism. It was demonstrated in Ref. 31 that the allowance for higher-order approximations in the Boussinesq equation as compared to that used in Eq. (6) results in appearance, on soliton collision, of a radiation field that manifests itself as an oscillating tail. Calculations in Ref. 31 were done in the absence of viscosity and external force. We have not yet investigated in numerical simulation the action of viscous forces on an ‘‘oscillating tail’’ arising when higher-order approximations are taken into account.

D. The difference between solitons observed in experiment and «theoretical» solitons

We are aware that the nonlinear waves we have investigated in this paper may not satisfy the strict theoretical definition of solitons. It is more correct to refer to them as ‘‘slowly varying solitary waves’’ because of the energy input from the harmonic wave and energy dissipation, as it was shown in Sec. III C, their speeds are not conserved through their interaction.

Experiments revealed that solitons of surface waves are excited in a hydrodynamic resonator against the background of a harmonic mode. We focused special attention on the case when one soliton is excited on a period of the harmonic wave. This regime is realized at the external forcing frequencies larger than the resonance frequency, which is clear from the following considerations. At exact resonance, the period of excited oscillations is $T_r=L/V_0$. It takes for the soliton to propagate from one resonator end to the other with the velocity $V_s > V_0$, the time $T_s=L/V_s < T_r$. Thus, a steady-state regime of single soliton excitation can exist for frequencies larger than the resonance frequency and cannot exist at frequencies lower than the resonance one. At frequencies lower

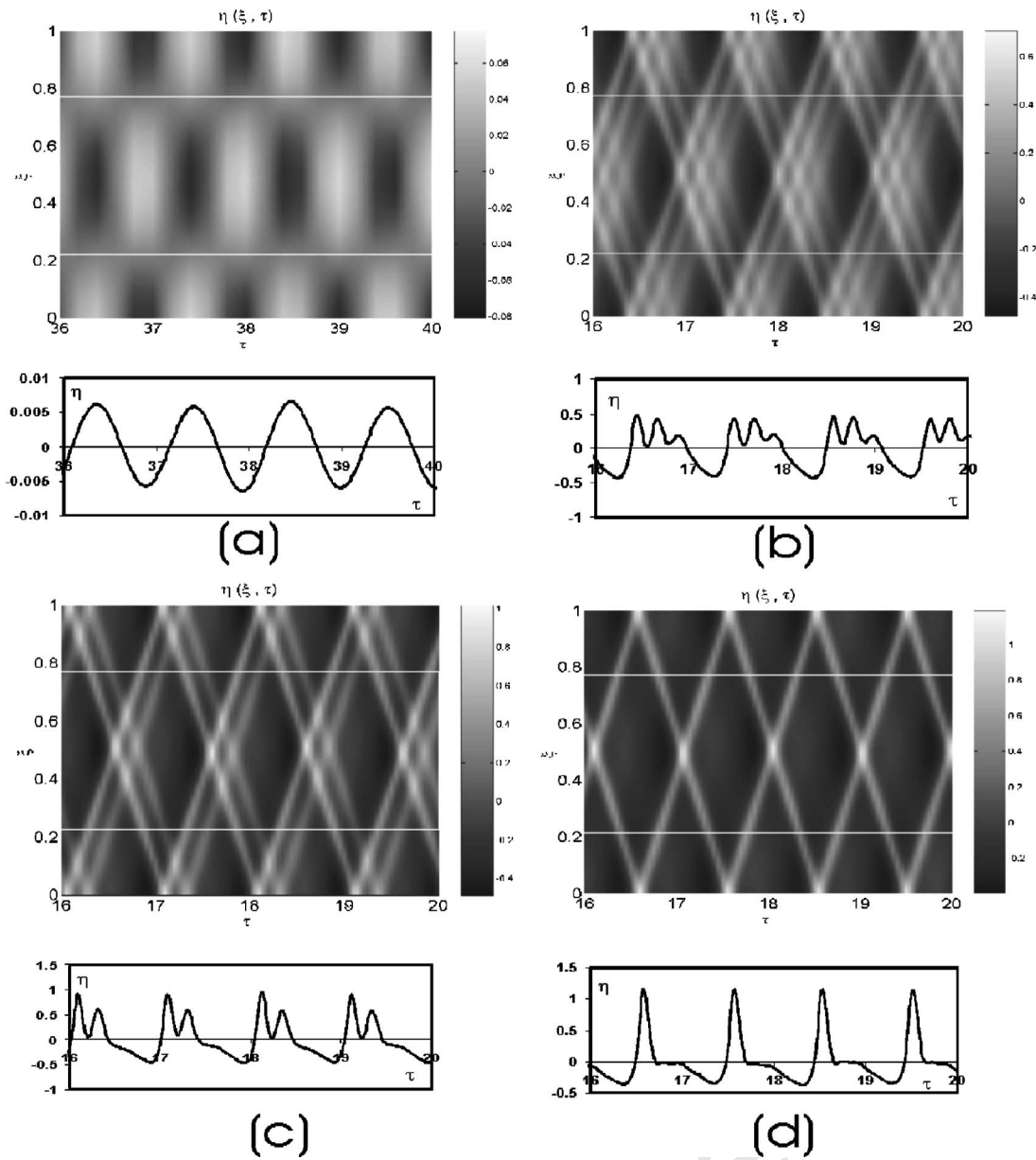


FIG. 13. Results of numerical modeling of the system of Eqs. (6): (a) $U_0=0.05$ for any frequency f_{ex} ; (b) $U_0=0.5$, $f_{ex}=0.95$; (c) $U_0=0.5$, $f_{ex}=0.98$; (d) $U_0=0.5$, $f_{ex}=1.02$. White horizontal lines mark the regions corresponding to the space-time diagrams constructed in the experiment (Fig. 2).

627 than the resonance frequency, bound states of solitons
 628 are excited in a resonator in order to achieve the condition
 629 $V_s > V_0$ for an individual soliton. The velocity of these bound
 630 states differs from that of a single soliton, and they are ex-
 631 cited under different conditions. This is exactly that we ob-
 632 served in experiment.

633 Our experiment demonstrated that the velocity of soliton
 634 propagation in the hydrodynamic resonator is a little smaller
 635 than that estimated by the formula (2). For example, for the
 636 regime illustrated in Fig. 9, soliton velocity lies in the inter-
 637 val $167 \text{ cm/s} < V_s < 173 \text{ cm/s}$. If we estimate V_s by the for-
 638 mula (2) for amplitude $A_s=8 \text{ cm}$, which corresponds to the
 639 conditions of soliton excitation in the experiment for the data
 640 given in Fig. 9, we obtain $V_s=184 \text{ cm/s}$ (the velocity of
 641 waves of infinitely small amplitude is $V_0=\sqrt{gH}=160 \text{ cm/s}$).
 642 How can we explain the discrepancy between the values of

velocities obtained theoretically and in experiments? In our
 643 opinion two effects should be taken into consideration. First,
 644 the soliton is propagating on the background of a harmonic
 645 wave. Second, a nonlinear periodic wave (cnoidal wave)
 646 rather than a solitary wave is excited in the experiment.
 647

Let us estimate, using the exact solution as a reference
 648 one, the difference between the velocities of a soliton and a
 649 cnoidal wave. The velocity of a cnoidal wave is known to
 650 depend on the so-called elliptic parameter m and is de-
 651 scribed by the formula⁹
 652

$$V_{cn} = \left[1 - \frac{1 A_s}{2 H} + \frac{1 A_s}{m H} \left(1 - \frac{3 E(m)}{2 K(m)} \right) \right] \sqrt{gH}, \quad (8) \quad 653$$

where $K(m), E(m)$ are complete elliptic functions of the first
 654 and second kind. The parameter m (which depends upon the
 655

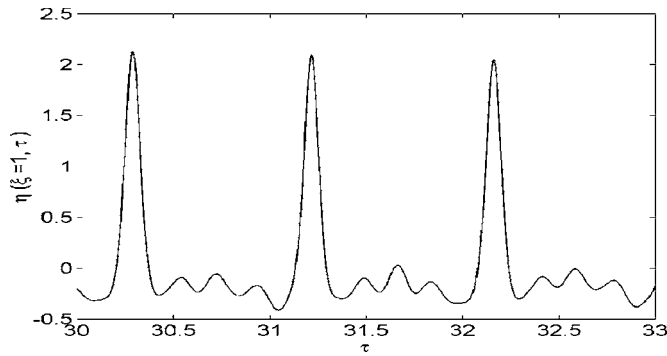


FIG. 14. Solitons with oscillating tails observed in numerical computations for the following parameters: $f_0=1.05$, $V_0=0.5$, $\gamma=1.5 \times 10^{-3}$, $\gamma_1=10^{-4}$.

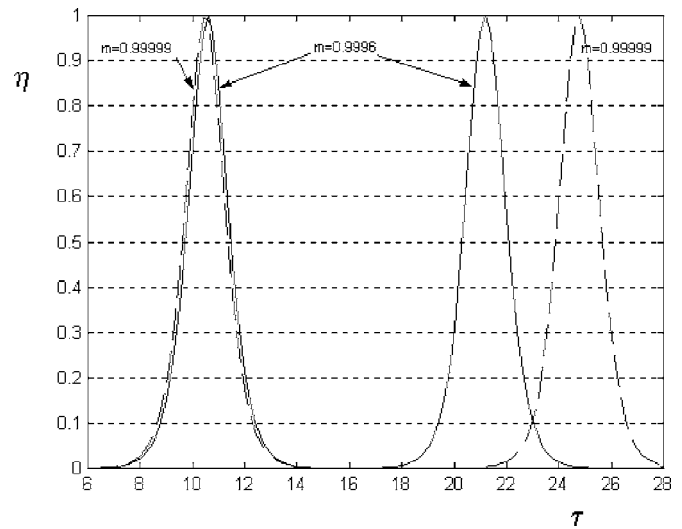


FIG. 15. The shape of cnoidal waves for the values of elliptic parameter close to 1.

ratio of the amplitude to the width of the wave) is responsible for the shape of the wave with $m=0$ corresponding to the harmonic wave, and $m=1$ to the soliton. Using the formula (8), one can estimate the elliptic parameters such that for $A_s=8$ cm and layer depth $H=26$ cm, the propagation velocity should be $V_{cn}=170$ cm/s instead of 184 cm/s, as is given by (2). Estimates from experimental data show that the elliptic parameter for these data is $m=0.9996$, and it differs a little from 1. What are the characteristic features of waves for m close to 1? Let us compare, for instance, cnoidal waves for $m=0.9996$ and $m=0.9999$ (Fig. 15). The shapes of pulses in these two cases are almost indistinguishable, whereas the pulse repetition rates differ appreciably. As $m \rightarrow 1$, the period tends to infinity, and the propagation velocity differs from the velocity of linear waves on shallow water by a finite value described by the formula (2). For the characterization of cnoidal waves we can introduce quantity $n=T/\tau_s$, where τ_s is the duration of the pulse and T is the period of the cnoidal waves. For the conditions of the experiment we have $n \approx 10$, which agrees well with the estimate of the elliptic parameter $m=0.9996$. The estimates presented in this work show that a sequence of solitons and a soliton riding on a harmonic wave have smaller velocities than one soliton without any background. The problem of the velocity of soliton propagation in a resonator needs a further, more detailed investigation. For example, in the interval between solitons, we observed an oscillating component that was not taken into account in the calculations. The velocity of soliton bound states is also an open problem. Although many theoretical and numerical studies on hydrodynamic solitons have been performed,³² there is still a need for their detailed experimental investigation.

V. CONCLUSION

In this work we have investigated the spatiotemporal properties of solitons excited by harmonic forcing on the surface of a shallow water in a hydrodynamic resonator. We have shown that solitons propagate synchronously with harmonic waves. It was revealed that the solitons are tied up with a definite phase of harmonic wave that depends on the amplitude and frequency of external forcing. We have isolated solitons generated on the background of a harmonic wave and have analyzed their dynamics using space-time

diagrams. We have evidenced that the duration of a soliton and the merging length of colliding solitons decrease with the amplitude. We found that there is a small change of velocities of colliding solitons, which may be attributed to a small dissipation of energy due to viscous damping and a turbulent boundary layer. An analytical model of soliton generation based on the amplitude and phase equations was developed and it is consistent with the experimental results. The numerical resolution of the Boussinesq equations with small dissipation terms has provided results that were in good agreement with our experimental data, in particular, for some values of excitation amplitude and frequencies; we have obtained bound states of solitons that require a detailed investigation in the future.

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APPENDIX

Let us derive an equation describing changes in the amplitude of a soliton propagating against the background of a harmonic wave. Toward this end, we consider a harmonic wave with frequency Ω and wavenumber χ propagating against the background of a variable flow, the characteristic length of which is much larger than the spatial period of the wave. We assume that $\chi H \ll 1$ and regard such a wave as a quasiparticle possessing energy E . Variation of the wave amplitude is caused by energy transfer from a large-scale flow. We estimate the changes in the wave energy E in the reference system moving with the velocity of the large-scale flow in the adiabatic approximation,

732
$$\frac{\partial}{\partial t} \left(\frac{E}{\Omega} \right) = 0. \tag{A1}$$

733 The frequency Ω depends on the velocity field u and the
734 surface displacement η in a large-scale flow according to

735
$$\Omega = u\chi + \sqrt{g(H + \eta)}\chi. \tag{A2}$$

736 If we assume that the large-scale flow is also a wave on
737 shallow water with frequency ω and wave number k , then we
738 have $\Omega \ll \omega$, $kh \ll \chi H \ll 1$. For the short-wave frequency, the
739 relation (A2) reduces to

740
$$\Omega \approx \eta \sqrt{\frac{g}{H}}\chi + \left(1 + \frac{1}{2} \frac{\eta}{H} \right) \sqrt{gH}\chi \approx \frac{3}{2} \eta \sqrt{\frac{g}{H}}\chi + \sqrt{gH}\chi. \tag{A3}$$

741 Then, from the expressions (A1) and (A2) and using the
742 shallow water condition ($\eta/H \ll 1$), we find the equation

743
$$\frac{1}{E} \frac{\partial E}{\partial t} = \frac{1}{\Omega} \frac{\partial \Omega}{\partial t} \approx \frac{3}{2H} \frac{\partial \eta}{\partial t}. \tag{A4}$$

744 Now assume that, instead of a short-wave harmonic
745 wave, we have a cnoidal wave consisting of a set of harmon-
746 ics. Since the right-hand side of the equation (A4) contains
747 neither the frequency nor the wavenumber of the short wave,
748 the equation (A4) is valid for each harmonic of the cnoidal
749 wave. Then, the cnoidal wave energy $E_{kn} = E_1 + E_2 + \dots$,
750 where E_n is the energy of harmonic components, obeys the
751 same equation (A4). The same equation is obtained for the
752 soliton, which is the limiting case of a cnoidal wave. The
753 necessary condition for the energy additivity is that the soli-
754 ton size should be much smaller than the wavelength of the
755 large-scale wave. Changes in the soliton energy in the refer-
756 ence system moving with velocity ω/k are described by

757
$$\frac{\partial E_s}{\partial t} = \frac{3}{2} \frac{\omega \eta_0}{H} E_s \cos \varphi_s. \tag{A5}$$

758 Here, E_s is the soliton energy and φ_s is the phase intro-
759 duced in Sec. III C. The relation (A5) gives the first term in
760 Eq. (4a). For this equation to describe soliton evolution, it
761 must be supplemented with a term describing energy dissipa-
AQ: #1 762 tion [the second term on the right-hand side of (4)]. A
763 similar equation was derived by Gorshkov *et al.*²³ for elec-
764 tromagnetic wave solitons propagating in LC lines.

765 ¹S. Leibovich and A. R. Seebass, in *Nonlinear Waves* (Cornell University
766 Press, Ithaca, NY, 1974).

767 ²M. Remoissenet, *Waves Called Solitons: Concepts and Experiments*.
768 (Springer-Verlag, Heidelberg, 1996). Recent books have been published
769 that address to some extent the hydrodynamic solitons. See, for example
770 E. Infeld and G. Rowlands, *Nonlinear Waves and Solitons and Chaos*,
771 Cambridge University Press, Cambridge, 2000). M. Peyrard and T. Dauxois,
772 *Physique des Solitons* (EDP Sciences, Paris, 2004).

773 ³T. Maxworthy, "Experiments on collision between solitary waves," *J.*
774 *Fluid Mech.* **76**, 177 (1976).

775 ⁴P. D. Weidman and T. Maxworthy, "Experiments on strong interaction
776 between solitary waves," *J. Fluid Mech.* **85**, 417 (1978).

⁵A. Bettini, T. A. Minelli, and D. Pascoli, "Solitons in undergraduate labo- **777**
ratory," *Am. J. Phys.* **51**, 977 (1983). **778**

⁶M. Olsen, H. Smith, and A. C. Scott, "Solitons in a tank," *Am. J. Phys.* **779**
54, 826 (1984). **780**

⁷Z. Feng, "Travelling solitary wave solutions to the generalized Boussinesq **781**
equation," *Wave Motion* **37**, 17 (2003). **782**

⁸D. P. Renouard, F. J. Seabra Santos, and A. M. Temperville, "Experimen- **783**
tal study of the generation, damping, and reflection of a solitary wave," **784**
Dyn. Atmos. Oceans **9**, 341 (1985). **785**

⁹M. J. Cooker, P. D. Weidman, and D. S. Bale, "Reflection of high ampli- **786**
tude wave at a vertical wall," *J. Fluid Mech.* **342**, 141 (1997). **787**

¹⁰M. W. Dingemans, *Water Wave Propagation over Uneven Bottoms. Part* **788**
2—Non-linear Wave Propagation (World Scientific, Singapore, 1997). **789**

¹¹A. Temperville, "Interaction of solitary waves in shallow water theory. **790**
Archives of Mechanics," *Arch. Mech.* **31**, 177 (1979). **791**

¹²M. Oikawa and N. Yajima, "Interactions of solitary waves — a perturba- **792 AQ:**
tion approach to nonlinear systems," *J. Phys. Soc. Jpn.* **34**, 1093 (1973). **793 #2**

¹³C. H. Su and R. M. Mirie, "On head-on collision between solitary waves," **794**
J. Fluid Mech. **98**, 509 (1980). **795**

¹⁴H. Power and A. T. Chwang, "On reflection of a planar solitary wave at a **796**
vertical wall," *Wave Motion* **6**, 183 (1984). **797**

¹⁵J. L. Hammack and H. Segur, "The Korteweg-de-Vries equation and water **798**
wave. Part 2. Comparison with experiments," *J. Fluid Mech.* **60**, 769 **799**
(1974). **800**

¹⁶W. Chester and J. A. Borns, "Resonant oscillations of water waves. II. **801**
Experiment," *Proc. R. Soc. London, Ser. A* **306**, 23 (1968). **802**

¹⁷W. Chester, "Resonant oscillations of water waves. I. Theory," *Proc. R. **803***
Soc. London, Ser. A **306**, 1 (1968). **804**

¹⁸E. Cox and M. P. Mortell, "The evolution of resonant water-wave oscilla- **805**
tions," *J. Fluid Mech.* **162**, 99 (1986). **806**

¹⁹H. Ockendon, J. R. Ockendon, and D. D. Waterhouse, "Multi-mode reso- **807**
nances in fluids," *J. Fluid Mech.* **315**, 317 (1996). **808**

²⁰H. Ockendon and J. R. Ockendon, "Nonlinearity in fluid resonances," **809**
Meccanica **36**, 297 (2001). **810**

²¹D. Amundsen, E. A. Cox, M. P. Mortell, and S. Reck, "Evolution of non- **811**
linear sloshing in a tank near half the fundamental resonance, *Stud.* **812**
Appl. Math. **107**, 103 (2001). **813**

²²J. Wu, R. Keolian, and I. Rudnik, "Observation of nonpropagating hydro- **814**
dynamic soliton," *Phys. Rev. Lett.* **52**, 1421 (1984); The theoretical ex- **815**
planation of this experiment is given by A. Larraza and S. Putterman, **816**
"Theory of non-propagating surface wave soliton," *J. Fluid Mech.* **148**, **817**
443 (1984). **818**

²³K. A. Gorshkov, L. A. Ostrovsky, and V. V. Papko, "Parametric amplifica- **819**
tion and generation of pulses in nonlinear distributed systems (in Rus- **820**
sian)," *Izv. Vyssh. Uchebn. Zaved., Radiofiz.* **16**, 1195 (1973); This re- **821**
search work was also described by L. A. Ostrovsky and A. I. Potapov, in **822**
the book *Modulated Waves. Theory and Applications* (The Johns Hopkins **823**
Univ. Press, Baltimore, 1999). **824**

²⁴A. I. Potapov and A. I. Vesnitsky, "Interaction of solitary waves under **825**
head-on collisions Experimental investigation," *Wave Motion* **19**, 29 **826**
(1994). **827**

²⁵L. D. Landau and E. M. Lifchitz, *Mécanique des Fluides* (Editions Mir, **828**
Moscou, 1990). **829**

²⁶R. Grimshaw, E. N. Pelinovsky, and T. G. Talipova, "Damping of large- **830**
amplitude solitary waves," *Wave Motion* **37**, 351 (2003). **831**

²⁷L. A. Ostrovsky and I. A. Soustova, "Upper mixed layer of the ocean as a **832**
source of energy sink for internal waves (in Russian)," *Okeanologiya* **833**
XIX, 973 (1979). **834 AQ:**

²⁸J. F. A. Sleath, *Sea Bed Mechanics* (Wiley, Chichester, 1984). **835 #3**

²⁹J. E. Zhang, C. Chen, and Y. Li, "On Boussinesq models of constant **836**
depth," *Phys. Fluids* **16**, 1287 (2004). **837**

³⁰H. Ockendon, J. R. Ockendon, and A. D. Johnson, "Resonant sloshing in **838**
shallow water," *J. Fluid Mech.* **167**, 465 (1986). **839**

³¹J. G. B. Byatt-Smyth, "The reflection of a solitary wave by a vertical **840**
wall," *J. Fluid Mech.* **197**, 503 (1988). **841**

³²G. Iooss and F. Dias, "Water-waves as a spatial dynamical system," in **842**
Handbook of Mathematical Fluid Dynamics, edited by S. Friedlander and **843**
D. Serre, Elsevier Science B.V., Amsterdam, 2003), Vol. 2, pp. 443-499. **844**

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