



HAL
open science

Dynamics of propagating front into sand ripples under regular waves

Julie Lebunetel-Levaslot, Armelle Jarno-Druaux, Alexander Ezersky, François Marin

► **To cite this version:**

Julie Lebunetel-Levaslot, Armelle Jarno-Druaux, Alexander Ezersky, François Marin. Dynamics of propagating front into sand ripples under regular waves. *Physical Review E: Statistical, Nonlinear, and Soft Matter Physics*, 2010, 82 (3), 10.1103/PhysRevE.82.032301 . hal-02095110

HAL Id: hal-02095110

<https://hal.science/hal-02095110>

Submitted on 10 Apr 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Dynamics of propagating front into sand ripples under regular waves

J. Lebunetel-Levaslot,¹ A. Jarno-Druaux,¹ A. B. Ezersky,² and F. Marin^{1,*}

¹Laboratoire Ondes et Milieux Complexes, FRE CNRS 3102, Université du Havre, 25 rue Philippe Lebon, BP 540, 76058 Le Havre Cedex, France

²Laboratoire Morphodynamique Continentale et Côtière, UMR CNRS 6143, Université de Caen, 2-4 rue des Tilleuls, 14000 Caen, France

(Received 9 April 2010; revised manuscript received 20 July 2010)

The results of an experimental study of pattern formation on sandy bottom under the action of regular harmonic surface waves are reported. It is found that two modes of pattern formation occur: sand ripples form uniformly on the whole bottom or from localized nucleation sites. In the second regime, the ripples appear in isolated regions (patches) increasing in size, and front propagation speed is measured. A simple dynamical model based on the Ginzburg-Landau equation is proposed to explain the characteristics of patches.

DOI: XXXX

PACS number(s): 45.70.Qj

I. INTRODUCTION

Pattern formation on a bottom under the action of surface waves has been investigated theoretically and experimentally for many years. The morphological characteristics of sand ripple patterns observed in the near shore region are important for the prediction of the dissipation of waves energy, and for the sediment transport. Ripples also influence the biological processes occurring on the bottom and the dispersion of pollutants. Ayrton [1] and Bagnold [2] carried out the pioneering works on these structures. Detailed investigations of the onset of instability caused by oscillating water over sand were performed in [3,4]. The formation of vortices at the lee side of the ripple crest and their ejection upward at flow reversal were considered in [5,6]. These vortices control the mass transfer between neighboring ripples during their formation [6,7], and the wavelength of fully developed ripples is proportional to the amplitude of the oscillatory flow [6,8]. The stability of bottom patterns in relation to changes of amplitude and frequency of water oscillations was studied in [9].

In this paper, we focus on the investigation of the amplification of initial perturbations of small amplitude leading to the formation of sand ripples. The front propagation plays a key role in the involved processes, and we present in this brief report a detailed investigation of the characteristics of this front. The study of front propagation has been considered in different unstable systems [10], and in particular in numerous hydrodynamic systems [11–13]. The phenomenon of front propagation for sand ripples under waves was mentioned in [2]. However, this propagation has not been accurately investigated to our knowledge. The aim of the present paper is to study the features of front propagation in sand ripple patterns. The main difference between sand ripples and the systems considered in [11–13] is the following. The action of regular surface waves (propagating usually in one direction) results in anisotropy of the sand ripples instability, and consequently of the front propagation velocity, whereas the systems studied in [11–13] may be considered as isotropic. The characteristics of fronts propagating in the same

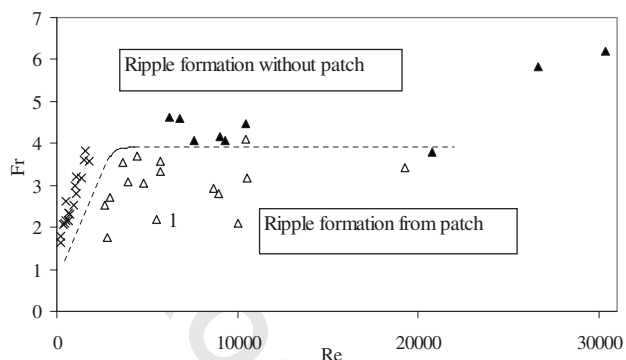
direction as surface waves or in the opposite direction are investigated in detail in the present study.

II. EXPERIMENTAL SETUP AND RESULTS

The experiments were performed in a 10 m long, 0.5 m high, and 0.49 m wide wave flume at Le Havre University. Regular surface waves were produced by an oscillating paddle at one end of the flume. At the other end a porous beach was installed to minimize wave reflection. The temporal evolution of the free surface was measured with two fixed resistive probes and analyzed with Goda's method [14]. The reflection coefficient was less than 5% for all of the tests. The mean water depth at rest was $d_* = 27$ cm. Experiments were carried out in a large range of wave and sediment parameters: $0.95 \text{ s} \leq T \leq 2.2 \text{ s}$, $0.045 \text{ m} \leq H \leq 0.099 \text{ m}$, $111 \mu\text{m} \leq d_{50} \leq 375 \mu\text{m}$, where T and H are the wave period and height, respectively, and d_{50} the median grain size. For each test, the bed was initially flat and covered by a 25 mm sand layer. In spite of the care taken to flatten the bottom, some defects of flatness are observed. The maximum amplitude of perturbations is approximately 2 mm. The bed deformation was measured from the first excitation cycles with an optical method, detailed in [15]. The spatial resolution in the horizontal and vertical directions was 0.5 mm/pixel. The dimensions of the processed field were 5.46 m long and 0.325 m width. The ripples wavelengths at the equilibrium state were in the range $28.4 \text{ mm} \leq \lambda \leq 77 \text{ mm}$ for present tests. Characteristics of ripples were obtained using the one-dimensional (1D) -Hilbert transform. A great advantage of this technique is that in each patch the amplitude and phase of ripples may be determined.

Two distinct modes of ripple patterns formation are observed. In the first mode, any perturbation on the bottom is enough to trigger ripple formation and ripples form uniformly on the whole bed. In the second mode, patterns form from isolated rippled zones (described as patches by Faraci and Foti [16]). For present tests, patches appeared in zones where the characteristic amplitude of disturbances was greater or equal to the critical value of 2 mm. Two nondimensional parameters were used to characterize the regime of pattern formation: the Reynolds number Re and the Froude number Fr . These parameters are defined as follows:

*Corresponding author. francois.marin@univ-lehavre.fr



× Jarno-Druaux et al., 2004 △ Present tests, patch ▲ Present tests, without patch

FIG. 1. Delineation of the two observed modes of ripple formation in the (Re,Fr) plane, indicating the boundary (dashed line) between the modes with and without patch (the test for which Re = 5512 and Fr = 2.2 is identified with the label 1).

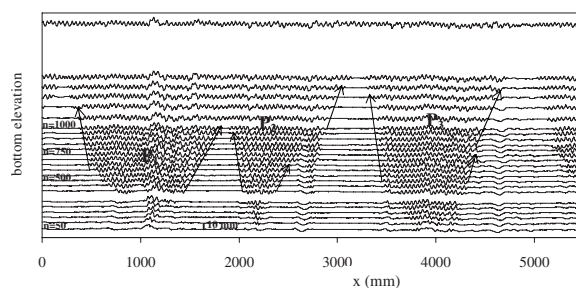


FIG. 3. Bottom elevation as a function of the x -longitudinal position and number of excitation cycles (Re=5512, Fr=2.2). The arrows show the ripple front positions detected for the three patches P1, P2, and P3.

95 $Re = U_\infty b / \nu$, $Fr = U_\infty / \sqrt{(s-1)gd_{50}}$, where b and U_∞ are the
 96 fluid particle semicursion and the fluid velocity amplitude
 97 at the edge of the bed boundary layer, respectively, s is the
 98 relative density of sediment, g the acceleration due to grav-
 99 ity, and ν the water kinematic viscosity. The delineation of
 100 the two observed modes of ripple formation in the (Re,Fr)
 101 plane is presented in Fig. 1. The data of Jarno-Druaux *et al.*
 102 [17] obtained in the same wave flume with lightweight grains
 103 of relative density 1.35 and median grain diameter $170 \mu\text{m}$
 104 are also shown in this figure. Present tests show that for fixed
 105 values of Re, there is a critical value Fr_{cr} of the Froude
 106 number below which ripples form from localized sites; for
 107 $Fr > Fr_{cr}$, no “patch” is observed. The critical Froude number
 108 becomes independent of the Reynolds number for Re
 109 > 5000 . This suggests that for $Fr_{cr} = Fr_{cr,max} \approx 3.9$, the sedi-
 110 ments move all over the bottom with a very low resistance to
 111 motion for every hydrodynamic conditions and ripples can
 112 form everywhere on the bottom. When $Fr < Fr_{cr,max}$, the in-
 113 ertial effects become more important for increasing values of
 114 Re (keeping constant the value of Fr), and the bed local
 115 perturbations lead to patches formation. The critical value
 116 Fr_{cr} grows with Re when $Fr < Fr_{cr,max}$. This may result from
 117 a decrease of the gravity effects in comparison with the in-
 118 ertial effects acting on the grains for increasing values of Fr
 119 for a given value of Re (< 5000), leading to a higher mobil-
 120 ity of the grains and preventing patch formation.

121 In order to study the front propagation, we focus on a test
 122 for which a slow dynamics of pattern formation from ampli-
 123 tude defects is observed. We have Re=5512 and Fr=2.2 for
 124 this test, and the mean ripple wavelength is $\lambda = 42 \text{ mm}$ at the
 125 equilibrium state. An example of bed image in gray levels is
 126 given in Fig. 2 for $n = 800$ excitation cycles where three main
 127 patches are clearly identified (P1 to P3). The temporal evo-
 128 lution of the bottom elevation is plotted as a function of the

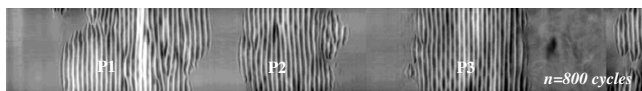


FIG. 2. Example of bed image in gray levels for $n = 800$ cycles (Re=5512, Fr=2.2). P1 to P3 refer to the three processed patches.

x -longitudinal position in Fig. 3, for $y = 0.20 \text{ m}$ where the y 129
 axis refers to the cross-section direction. The origin of the x 130
 axis is situated at 2.3 m from the wave paddle, increasing 131
 values of x corresponding to shorter distances from the ab- 132
 sorbing beach. The time step is equal to 50 cycles for the first 133
 1000 cycles, and afterward to 100 cycles. Isolated systems of 134
 propagating ripples can be observed during more than 1000 135
 cycles before the invasion of the whole bottom. The estima- 136
 tion of front velocities can then be performed on a long time 137
 for the three observed patches. The bottom elevation signal 138
 $\eta(x,t)$ of each patch is cut into two parts in order to process 139
 the two fronts separately. The Fourier spectra of signals are 140
 then calculated and harmonics are filtered. After this filtering 141
 process, we get $\eta(x,t) = \eta_m(x,t)\cos[kx + \varphi(x,t)]$, where 142
 $\eta_m(x,t)$ is for the slow varying amplitude, $\varphi(x,t)$ is the slow 143
 varying phase of the bottom profile, and k is the bottom wave 144
 number. 145

Using the Hilbert transform, 146

$$\hat{\eta}(x,t) = \frac{1}{\pi} PV \left[\int_{-\infty}^{+\infty} \frac{\eta(x,t)}{x-\chi} d\chi \right] = \eta_m(x,t) \sin[kx + \varphi(x,t)] \quad (1) \quad 147$$

where PV denotes principal value, it is possible to determine 148
 the phase and amplitude of sand ripples on the flume bottom 149
 and compare with the theoretical predictions. We can consid- 150
 er the bottom profile as the real part of a complex function 151
 $\eta(x,t) = \text{Re}\{A(x,t)\exp[i(kx)]\}$ with $A(x,t) = |A(x,t)|e^{i\varphi(x,t)}$ and 152
 where 153

$$|A(x,t)| = a = \sqrt{\hat{\eta}^2(x,t) + \hat{\eta}^2(x,t)}, \quad \varphi(x,t) = \arctan\left(\frac{\hat{\eta}}{\eta}\right) - kx. \quad (2) \quad 154$$

We extract the module of the complex amplitude a and the 155
 phase $\varphi(x,t)$ from the signals. An example of the spatial 156
 dependence for $a(x)$ and $\varphi(x)$ is shown in Fig. 4. It is impor- 157
 tant to emphasize that large changes of phase occur at wave 158
 front. The wave front is localized in the region where a tran- 159
 sition from a low amplitude to a high (nearly constant) value 160
 is detected. We have chosen the following criterion to deter- 161
 mine the front position: the front is situated in the region 162
 where the value of the amplitude is equal to 15% of the 163
 maximum value for the patch. The ripple fronts are presented 164
 in Fig. 3. They propagate linearly with time, and a good 165
 coefficient of regression (in the range 0.70–0.98) is obtained. 166
 The up-flow (v_{p-} ; propagation in the direction opposite to the 167

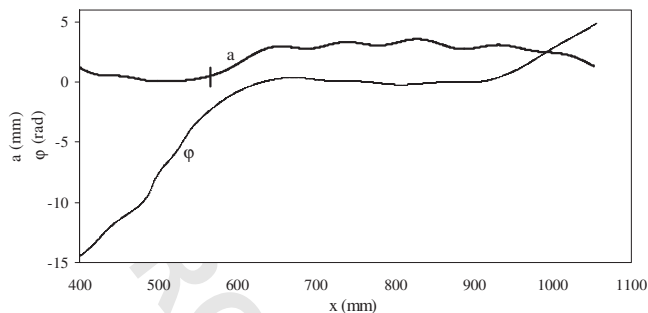


FIG. 4. Example of spatial distribution for the amplitude (bold line) and phase (thin line) of sand ripples for a front propagating upward (Patch P1, $t=550$ wave excitation cycles). The segment crossing the bold line delineates the border between the flat bottom and the ripple patch according to the criterion of threshold amplitude.

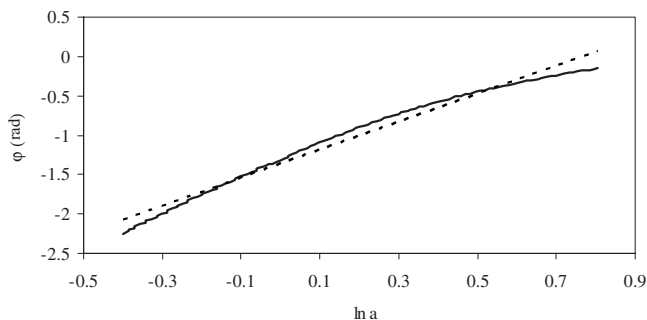


FIG. 5. Dependence of the wave phase φ on the logarithm of wave amplitude $\ln a$ for $462 \text{ mm} < x < 520 \text{ mm}$ (bold curve) with its best linear fit approximation ($\varphi=1.82 \ln a-1.52$; square of the correlation coefficient $R^2=0.96$; dashed line); $t=700$ wave excitation cycles.

168 surface waves propagation) and down-flow (v_{p+} ; propagation
169 in the same direction as the surface waves) front velocities of
170 the patches are given in Table I. The results show that the
171 fronts propagating in the direction of surface wave propaga-
172 tion have greater velocities than the fronts propagating in the
173 opposite direction ($|v_{p+}| > |v_{p-}|$).

174 III. DISCUSSION OF RESULTS

175 According to the present experimental results, the front
176 propagation may be considered as an envelope wave, and we
177 have found the amplitude and phase of this envelope. Let us
178 compare the experimental results with the solution of an
179 equation describing the envelope amplitude, the complex
180 Ginzburg-Landau equation (GLE) which is widely used to
181 investigate pattern dynamics [18]. Present experiments show
182 us that there is a threshold value of the initial bed perturba-
183 tions in the front propagation regime: perturbations with an
184 amplitude less than a critical value decay with time, whereas
185 perturbations with an amplitude greater than the critical
186 value grow. To take this effect into account, it is necessary to
187 keep the nonlinear terms proportional to the third and fifth
188 degrees of amplitude in the GLE (quintic version of GLE)
189 [10]. The cubic version of the GLE is able to describe the
190 linear instability of infinitely small perturbations and the
191 nonlinear amplitude saturation. The simplest model to de-
192 scribe the front propagation in our system is the following:

193
$$\frac{\partial A}{\partial t} = (1 + ic_1) \frac{\partial^2 A}{\partial x^2} + \varepsilon A + (1 + ic_3)|A|^2 A - (1 - ic_5)|A|^4 A, \quad (3)$$

194 where A is the complex amplitude of sand ripples, ε the
195 super criticality ($\varepsilon < 0$ in our case), and c_1, c_3, c_5 are real

TABLE I. Up-flow and down-flow patch velocities for the three patches P1, P2, and P3 ($\text{Re}=5512$, $\text{Fr}=2.2$).

Patch	Up-flow patch velocity v_{p-} (mm s ⁻¹)	Down-flow patch velocity v_{p+} (mm s ⁻¹)
1	-0.23	0.62
2	-0.19	0.64(400 < n < 650); 0.43(1000 < n < 1400)
3	-0.16	0.37(500 < n < 800); 0.52(900 < n < 1400)

coefficients for dispersion (c_1), cubic nonlinearity (c_3), and
quintic nonlinearity (c_5). The analytical solution of Eq. (3)
has the following form [19]: $A = e^{-i\Omega t} a(\xi) e^{i\phi(\xi)}$,
where V is the front velocity and Ω the frequency of sand
ripples. The amplitude and phase obey the following differ-
ential equations (ansatz): $\partial a / \partial \xi = K_{L\mp} a(1 - a^2(\xi)/a_N^2)$,
 $\partial \phi / \partial \xi = q_L + (q_N - q_L) a^2(\xi)/a_N^2$. For propagating fronts, the
solution has the following form:

204
$$a = a_N e^{K_{L\mp} \xi / \sqrt{1 + e^{2K_{L\mp} \xi}}}, \quad (4)$$

The amplitude grows exponentially from an infinitely
small value to a constant value a_N . The six constants $K_{L\mp}$, q_L ,
 q_N , Ω , V , a_N are determined by inserting the ansatz into
Eq. (3) [10]. The sign “+” corresponds to a front which
propagates in the positive direction (direction of surface
waves propagation), $K_{L+} < 0$, $a(x=-\infty, t=0) = a_N$, $a(x$
 $=+\infty, t=0) = 0$, and the sign “-” corresponds to a front
propagating in the opposite direction: $K_{L-} > 0$, $a(x=-\infty, t$
 $=0) = 0$, $a(x=+\infty, t=0) = a_N$. It should be noted that for re-
gions where $a = a_N$, we have $\phi = q_N(x \mp Vt)$, and for $a \ll a_N$,
 $\phi \approx q_L(x \mp Vt)$. The profile of the sandy bottom may be
written as $\eta(x, t) = \text{Re}[A(x, t) \exp(ikx)] = a(x \mp Vt) \cos(\Omega t$
 $- q_{N,L}(x \mp Vt) - kx)$. This means that q_L and q_N may be con-
sidered as infinitesimal and finite amplitude additional terms
for the wave number of sand ripples, respectively. Using the
 a_N^2 expression from the first equation of ansatz, we find a
correlation between the phase and amplitude derivatives:
 $\partial \phi / \partial \xi = q_N - [(q_N - q_L) \partial a / \partial \xi] / a K_{L\pm}$, and after integration we
get

224
$$\phi = q_N \xi - [(q_N - q_L) / K_{L\mp}] \ln a. \quad (5)$$

Excluding a linear growth of the phase with space for a given
instant, we are able to present a local correlation between the
wave amplitude $a(x)$ and the wave phase $\varphi(x)$: $\varphi(x) = (q_L$
 $- q_N) \ln a / K_{L\pm}$. Such correlation really occurs for the wave
front in sand ripples. Figure 5 shows an example of the
variation of the phase φ with the amplitude a . Using the best
linear fit approximation (dashed line in Fig. 5), we deter-
mined the coefficient $\kappa_{\pm} = (q_L - q_N) / K_{L\pm}$. The results are dif-
ferent for the fronts propagating in the direction of surface
waves and in the opposite direction. The values of the coef-
ficient κ_{\pm} are estimated for both fronts of Patch 1, for dif-
ferent numbers of excitation cycles; these values are given in

237 Table II. It can be noted that the linear dependence between
 238 the phase φ and $\ln a$ is obtained with lower values of the
 239 regression coefficient for fronts propagating in the direction
 240 of surface waves (except when $t=950$ excitation cycles). In
 241 this case, for some instants, no estimation of κ_+ could be
 242 proposed. The fronts propagating in the direction of surface
 243 waves are then not as regular as the fronts propagating in the
 244 opposite direction. We were able to estimate the coefficient
 245 $K_{L\pm}$: the solution (4) shows us that this exponent may ap-
 246 proximate the amplitude growth on the wave front. Using an
 247 exponential approximation of experimental data, we have
 248 found different coefficients for the fronts: the averaged value
 249 for K_{L+} was $K_{L+} = -0.047 \text{ mm}^{-1}$, and for the front propagat-
 250 ing in the opposite direction, $K_{L-} = 0.03 \text{ mm}^{-1}$. The front
 251 propagating in the direction of surface waves is “steeper”
 252 than the front propagating in the opposite direction. We have
 253 estimated the changes in wave number due to the finite am-
 254 plitude of sand ripples: $q_L - q_N \approx 0.039 \text{ mm}^{-1}$ for waves co-
 255 directed with the surface waves, and $q_L - q_N \approx 0.025 \text{ mm}^{-1}$
 256 for waves propagating in the opposite direction. In both
 257 cases, the finite amplitude leads to a decrease in wave num-
 258 ber $k + q_N$ in comparison with the linear wave number $k + q_L$,
 259 but this effect is larger for the front propagating in the direc-
 260 tion of surface waves.

261 The differences between the characteristics of the fronts
 262 propagating in the same direction as the surface waves and in
 263 the opposite direction may be due to the drift induced by
 264 surface waves. It is well known [20] that in the bed boundary
 265 layer above a flat bed, induced flows lead to mass transport
 266 in the direction of waves propagation. Above sand ripples,
 267 the momentum transfer and suspended sediment dynamics
 268 are dominated by the formation and shedding at flow reversal
 269 of lee wake vortices [21]. Present data involve weakly asym-
 270 metrical waves ($B < 0.1$ where $B = 3bk_{sw}/4 \sinh^2(k_{sw}d_*)$ and
 271 k_{sw} is the surface wave number). Using a one-dimensional
 272 vertical (1DV) two-layer model where vortex shedding is
 273 represented in the lower layer by a time-varying eddy vis-
 274 cosity, Davies and Thorne [21] have shown that the near-bed
 275 sand transport is in the direction of surface waves propaga-
 276 tion for weakly asymmetrical waves. Such transport of sand
 277 increases the front velocity v_{p+} and decreases the velocity

278 v_{p-} .
 303
 304
 305

306 [1] H. Ayrton, *Proc. R. Soc. London, Ser. A* **84**, 285 (1910).
 307 [2] R. A. Bagnold, *Proc. R. Soc. London, Ser. A* **187**, 1 (1946).
 308 [3] P. Blondeaux, *J. Fluid Mech.* **218**, 1 (1990).
 309 [4] E. Foti and P. Blondeaux, *Coastal Eng.* **25**, 227 (1995).
 310 [5] V. Marieu *et al.*, *J. Geophys. Res.* **113**, C09007 (2008).
 311 [6] K. H. Andersen *et al.*, *Phys. Rev. E* **63**, 066308 (2001).
 312 [7] E. K. O. Hellén and J. Krug, *Phys. Rev. E* **66**, 011304 (2002).
 313 [8] T. Schnipper *et al.*, *Phys. Rev. E* **78**, 047301 (2008).
 314 [9] J. L. Hansen *et al.*, *Nature (London)* **410**, 324 (2001).
 315 [10] W. van Saarloos, *Phys. Rep.* **386**, 29 (2003).
 316 [11] J. Fineberg and V. Steinberg, *Phys. Rev. Lett.* **58**, 1332 (1987).
 317 [12] J. Fineberg *et al.*, *Phys. Rev. A* **41**, 5743 (1990).
 318 [13] M. Fermigier *et al.*, *J. Fluid Mech.* **236**, 349 (1992).

TABLE II. Values of the inclination coefficient for the upstream and downstream fronts and different numbers of excitation cycles (Patch P1). The linear regression coefficients are given in parentheses.

Number of excitation cycles	$\kappa_- = \frac{q_L - q_N}{K_{L-}}$	$\kappa_+ = \frac{q_L - q_N}{K_{L+}}$
550	+1.30(R ² =0.99)	
600	+0.33(R ² =0.99)	
650	+0.45(R ² =0.99)	-1.20(R ² =0.95)
700	+1.80(R ² =0.97)	
750	+0.66(R ² =0.99)	-0.12(R ² =0.75)
800	+1.80(R ² =0.99)	-0.49(R ² =0.81)
850	+0.60(R ² =0.99)	-0.51(R ² =0.85)
900	+0.31(R ² =0.94)	-2.30(R ² =0.93)
950	+0.28(R ² =0.87)	-0.33(R ² =0.98)

IV. CONCLUSIONS

We have shown that depending on the values of the control parameters (Froude and Reynolds numbers), sand ripples on the bottom may arise as a result of two types of bifurcation: spatially homogeneous growth of small perturbations, and appearance of patches. In the last case, wave front propagation occurs. Using the Hilbert transform, we measured the amplitude and phase of ripple waves, and we have found coincidences between the experimental characteristics of propagating fronts and the analytical solution of van Saarloos and Hohenberg [19]. Such coincidences allowed us to find a correlation between the sand ripples amplitude and wave number, and conclude that there exists an effect of wave number decrease due to the finite amplitude of sand ripples. Our measurements agree with the measurements of other researchers (see, for example, [16]): the spatial period of sand ripples increases with increasing ripples amplitude. We have found that the propagating front characteristics depend on the direction of surface waves which generate ripples. If the front propagates in the direction of surface waves, it has a larger celerity, is steeper and more irregular than the front which propagates in the opposite direction. In our opinion such differences are caused by the mean flow induced by surface waves near the bottom.

[14] Y. Goda, *Random Seas and Design of Marine Structures* (World Scientific, Singapore, 2000).
 [15] F. Marin and A. B. Ezersky, *Eur. J. Mech. B/Fluids* **27**, 251 (2008).
 [16] C. Faraci and E. Foti, *Phys. Fluids* **13**, 1624 (2001).
 [17] A. Jarno-Druaux *et al.*, *Eur. J. Mech. B/Fluids* **23**, 695 (2004).
 [18] I. S. Aranson and L. Kramer, *Rev. Mod. Phys.* **74**, 99 (2002).
 [19] W. van Saarloos and P. C. Hohenberg, *Phys. Rev. Lett.* **64**, 749 (1990).
 [20] M. S. Longuet-Higgins, *Philos. Trans. R. Soc. London, Ser. A* **245**, 535 (1953).
 [21] A. G. Davies and P. D. Thorne, *J. Geophys. Res.* **110**, C05017 (2005).