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Supervised classification of multidimensional and irregularly sampled signals.

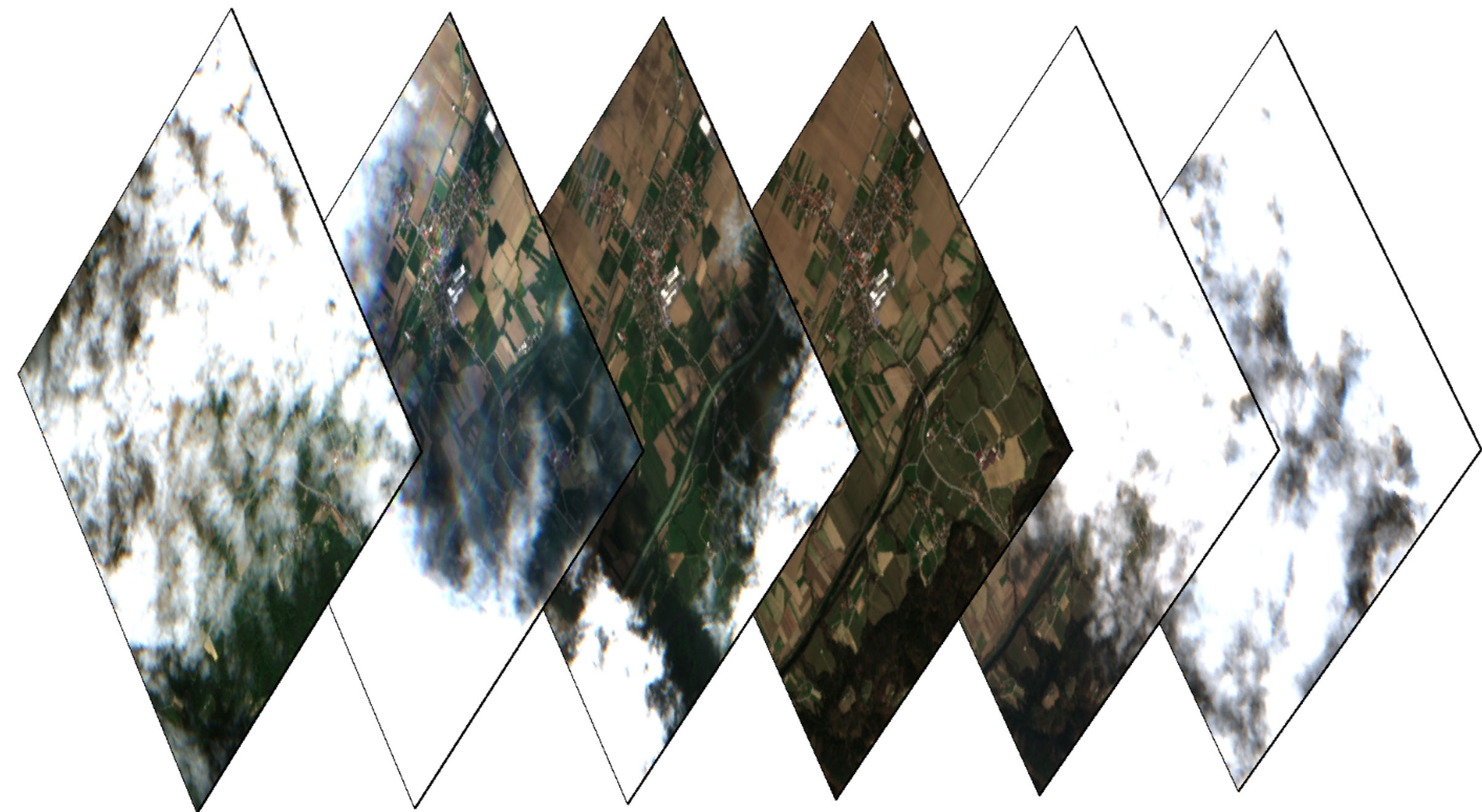
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Introduction



Background:

Recent space missions, such as Copernicus Sentinel-2^a, provide high resolution Satellite Image Time Series (SITS) to study continental surfaces, with a very short revisit period (5 days for sentinel-2). In order to process such data, statistical models are regularly used [1, 2], which usually require a regular temporal sampling. However, for SITS, clouds and shadows (eg. figure from [3]), as well as the satellite orbite, an irregular temporal sampling is common.

Contribution:

A new statistical approach using Gaussian processes is proposed to classify irregularly sampled signals without temporal rescaling. Moreover, the model offers a theoretical framework to impute missing values such as cloudy pixels.

^ahttps://www.esa.int/Our_Activities/Observing_the_Earth/Copernicus/Sentinel-2

Model

Gaussian Processes (\mathcal{GP}) model:

Let $\mathcal{S} = \{(\mathbf{y}_i, z_i)\}_{i=1}^n$ a set of multidimensional and irregularly sampled signals. A signal Y is modeled as a vector of p independent random processes $\mathcal{T} \rightarrow \mathbb{R}^p$, with $\mathcal{T} = [0, T]$. The associated label is modeled by a discrete random variable Z taking its values in $\{1, \dots, \mathcal{C}\}$. The model introduced here is based on two assumptions: 1) The coordinate processes Y_b , $b \in \{1, \dots, p\}$ of Y are independent, 2) Each process Y_b is, conditionally to $Z = c$, a Gaussian process. Then

$$Y_b(t)|Z = c \sim \mathcal{GP}(m_{b,c}(t), K_{b,c}(t, s)),$$

where $m_{b,c} : \mathcal{T} \rightarrow \mathbb{R}^p$ is a mean function, and $K_{b,c}$ a covariance kernel with hyperparameters $\theta_{b,c}$. For example $\theta_{b,c} = \{\gamma_{b,c}^2, h_{b,c}, \sigma_{b,c}^2\}$ with

$$K_{b,c}(t, s) = \gamma_{b,c}^2 k(t, s|h_{b,c}) + \sigma_{b,c}^2 \delta_{t,s}$$

An irregularly sampled noisy signal \mathbf{y}_i is observed on T_i time stamps $\{t_1^i, \dots, t_{T_i}^i\} \in \mathcal{T}$ and its b th coordinate is represented by a vector in \mathbb{R}^{T_i} . We write $\mathbf{y}_{i,b} = [Y_b^i(t_1^i), \dots, Y_b^i(t_{T_i}^i)]^T$, with

$$\mathbf{y}_{i,b}|Z_i = c \sim \mathcal{N}_{T_i}(\boldsymbol{\mu}_{i,b,c}, \boldsymbol{\Sigma}_{b,c}^i).$$

There $\boldsymbol{\mu}_{i,b,c} = \mathbf{B}_b^i \boldsymbol{\alpha}_{b,c}$ is the sampled mean projected on a finite-dimensional space (\mathbf{B}_b^i is the fixed design matrix, $\boldsymbol{\alpha}_{b,c}$ is the unknown vector of coordinates). $\boldsymbol{\Sigma}_{b,c}^i$ is the matrix kernel $K_{b,c}$ evaluations at $\{t_1^i, \dots, t_{T_i}^i\}$.

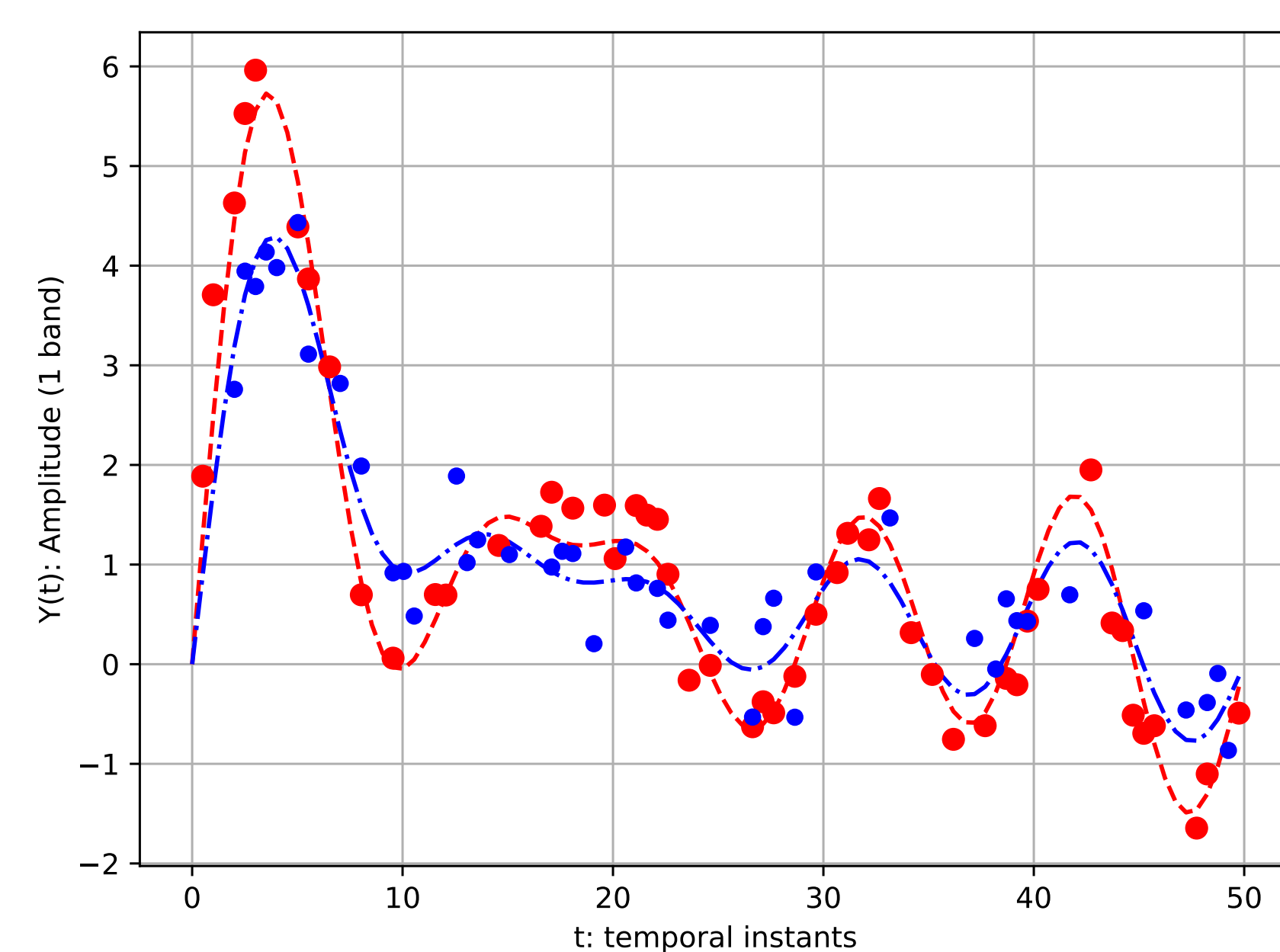
Estimation:

- $\boldsymbol{\alpha}_{b,c}$ and $\theta_{b,c}$ are estimated by maximizing the log-likelihood,

$$-\frac{1}{2} \sum_{i|Z_i=c} \log |\boldsymbol{\Sigma}^i(\theta_{b,c})| + (\mathbf{y}_{i,b} - \mathbf{B}_b^i \boldsymbol{\alpha}_{b,c})^\top \boldsymbol{\Sigma}^i(\theta_{b,c})^{-1} (\mathbf{y}_{i,b} - \mathbf{B}_b^i \boldsymbol{\alpha}_{b,c}).$$

- $\boldsymbol{\alpha}_{b,c}$ is given by an explicit formula, while $\theta_{b,c}$ is computed thanks to a gradient technique.

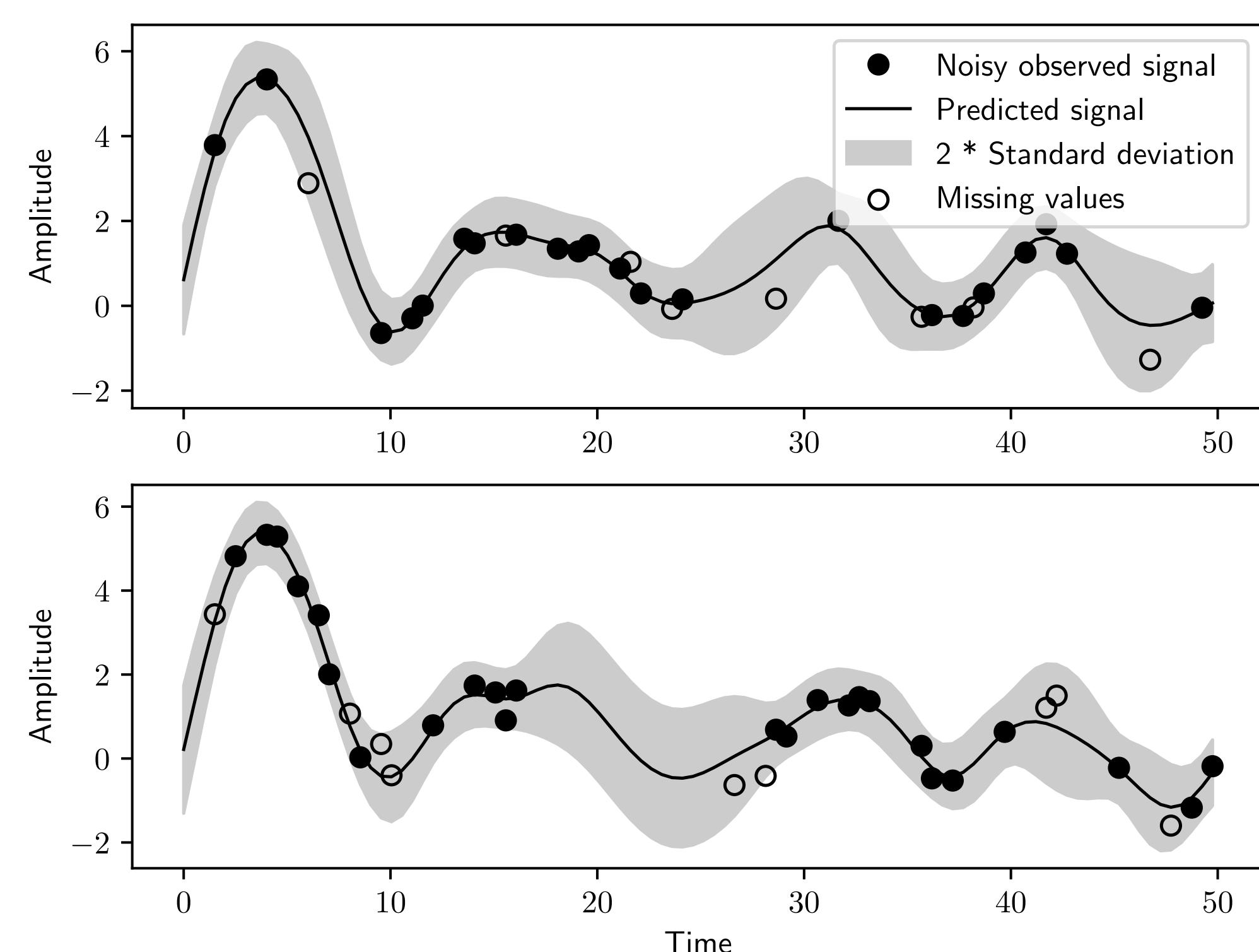
Validation (Synthetic data)



Example of two signals (dots) that belongs to two different classes

Classification rate based on average time samples

| n_t | 5 | 10 | 25 | 50 | 75 |
|--------------------------------|------|------|------|------|------|
| $\text{Acc}_{\text{exp}} (\%)$ | 52.8 | 52.9 | 74.3 | 93.9 | 94.2 |
| $\text{Acc}_{\text{sin}} (\%)$ | 64.3 | 85.3 | 100 | 100 | 100 |



Imputation on two signals belonging to the same class.

Classification and Imputation of missing values

The assigned class is given by the MAP rule from the posterior probability

$$P(Z = c|\mathbf{y}_j) = \frac{\hat{\pi}_c \prod_{b=1}^p f_{T_j}(\mathbf{y}_j, \mathbf{B}_b^j \hat{\boldsymbol{\alpha}}_{b,c}, \boldsymbol{\Sigma}^j(\hat{\boldsymbol{\theta}}_{b,c}))}{\sum_{\ell=1}^K \hat{\pi}_\ell \prod_{b=1}^p f_{T_j}(\mathbf{y}_j, \mathbf{B}_b^j \hat{\boldsymbol{\alpha}}_{b,\ell}, \boldsymbol{\Sigma}^j(\hat{\boldsymbol{\theta}}_{b,\ell}))}.$$

When the class is known to be c , the missing value at t^* is estimated through the computation of conditional expectation.

$$\begin{cases} \hat{Y}_{b,c}^i(t^*) = B_b^i(t^*) \hat{\alpha}_{b,c} + K_{b,c}(t^*, t_{1:T_i})^\top \boldsymbol{\Sigma}^i(\hat{\boldsymbol{\theta}}_{b,c})^{-1} (\mathbf{y}_{i,b} - B_b^i \hat{\alpha}_{b,c}) \\ \text{var}(\hat{Y}_{b,c}^i(t^*)) = K_{b,c}(t^*, t^*) - K_{b,c}(t^*, t_{1:T_i}) \boldsymbol{\Sigma}^i(\hat{\boldsymbol{\theta}}_{b,c})^{-1} K_{b,c}(t_{1:T_i}, t^*) \end{cases}$$

We also generalized this imputation when the class is unknown.

Future work

We are now implementing the model for massive real data (Sentinel-2). We are also working on a new model when the bands are correlated.

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- [1] P. J. Brockwell and R. A. Davis. *Time Series: Theory and Methods*. Springer-Verlag, Berlin, Heidelberg, 1986.
- [2] C. K. Williams and C. E. Rasmussen. *Gaussian processes for machine learning*. the MIT Press, 2006.
- [3] Sentinel hub blog. <https://medium.com/sentinel-hub>. Accessed: 2019-03-21.