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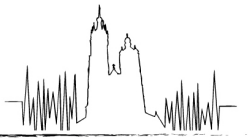
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Synthesis of a Mach cone using a speaker array

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Abstract

The interest of the authors concerns sniper detection using time-reversal techniques on the Mach cone in a reverberant urban environment. In order to setup a safe and reproducible experimental framework at a reduced scale, it is possible to synthesize a N-wave with a conical geometry by means of loudspeakers disposed along a hypothetical firing axis. The supersonic nature of the simulated displacement leads to a set of constraints, both on spatial and temporal samplings, correlated to the structure of the medium and to the digital sampling of the N-shaped signal. Those constraints are theoretically studied to ensure reconstruction of the conical wavefront. A first experiment has been realized, that allowed the synthesis of a Mach wave using 15 speakers spaced by 4.36 cm. Taking into account the directivity of each speaker and the diffraction effects due to the line array, the symmetry of revolution of the cone is studied. Since the loudspeakers are in their linear regime, nonlinear behaviors of the wave are no longer present. However, inverse filtering methods are possible for improving the quality of the signal. We show that it is possible to visualize the spatio-temporal evolution of the pressure field in planes containing the firing axis using a linear microphone array mounted on a translation robot. Comparisons between experiments and simulations show encouraging results for the following.

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1. Introduction

The general context of the present study is the application of time-reversal to the shock wave generated by a supersonic projectile, typically a sniper bullet, in a reverberant urban environment. The authors aim at synthesizing a Mach cone without shooting any real bullet for obvious safety reasons, but also to be able to lead reproducible experiments at a reduced scale, both in anechoic and reverberant environments.

This paper introduces a method for generating an acoustic wave reproducing some of the properties of the Mach cone in an anechoic room. In the following, we propose to use a linear speaker array disposed along the firing axis and to simulate the passing of the bullet at a speaker position by the emission of a N-shaped signal. To the knowledge of the authors, there is no preexisting bibliographical reference concerning the experimental synthesis of a Mach cone using the

proposed method. However, Wunderli et al described a similar simulation framework using only sinusoidal excitation of discrete monopolar sources [1].

The proposed method is based on *a priori* knowledge concerning the Mach wave, which is a localized wave of conical shape with half-angle α , verifying $\sin \alpha = c/v$, where c is the speed of sound in the air and v the speed of the projectile. The temporal signal associated to a Mach wave is a N-shaped signal called *N-wave* [2]. The spatial decreasing of the wave amplitude Δp according to distance r from the firing axis (*Distance at Closest Point of Approach*, DCPA) is proportional to $r^{-3/4}$, according to [4]

$$\frac{\Delta p(r)}{P_0} = \frac{0.53(M^2 - 1)^{\frac{1}{8}}}{r^{\frac{3}{4}}} \frac{d}{l^{\frac{1}{4}}}, \quad (1)$$

for a bullet of length l and diameter d . $M = v/c$ stands for the Mach number and P_0 for the unperturbed pressure.

Although the Mach wave is a nonlinear propagating phenomenon, it is necessary to work in the linear

domain of the speakers to properly emit the signals. The pressure should decrease far from the axis like a radiating cylinder, proportionally to $r^{-1/2}$ [5], instead of $r^{-3/4}$.

At high frequencies, speakers are not omnidirectional. It is necessary to check the influence of the speakers directivities in order to evaluate their effect on the array axisymmetry.

The small number of speakers only allows a restrained portion of a firing axis. This implies that the conical geometry will only be reproduced in a localized portion of the space-time domain. The authors would like to emphasize that they do not intend to perfectly reproduce a Mach wave, but only some of the properties that might be useful to test time-reversal techniques. We look for a reproducible wave whose geometry is conical in a reasonable portion of space and whose temporal dependance is a N wave, which approaches the characteristics of a theoretical shock wave.

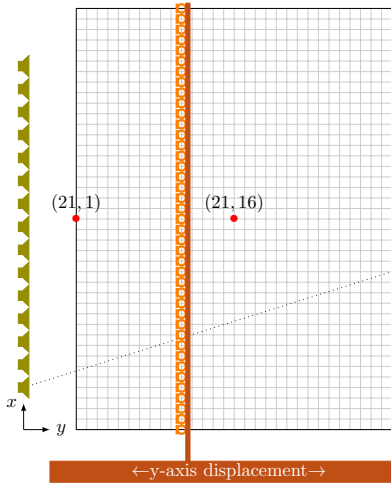


Figure 1. Disposition of speaker (green) and microphone (orange) arrays. Two points on the measurement grid are highlighted (red dots).

2. Material and methods

2.1. Material

All the presented experiments take place in an anechoic room, so that reverberation can be neglected.

2.1.1. Speaker array

The speaker array contains 15 Audax TW010I1 speakers (see figure 2) vertically disposed along the x -axis and driven by a 16 channel sound device. The interspeaker distance $\Delta x = 4.36$ cm, that has to be as small as possible for good spatial definition, is limited by the size of the speakers. The chosen loudspeaker model is a good compromise for a small speaker being as omnidirectional as possible in the frequency domain of the N signal, whose principal component lies between 3000 and 4000 Hz.

2.1.2. Microphone array

The measurement system consists of a line array of 41 microphones spaced by 2 cm and vertically disposed along the x -axis in front of the antenna. The distance y between both arrays is controlled by a motorized translation robot, allowing displacement with a precision inferior to 1 mm. Assuming reproducibility of the synthesis, it is possible to measure the cone in a plane of space containing the firing axis, as shown in figure 1. The relative sizes and shapes of speaker and microphone arrays can be seen on figure 2.



Figure 2. Photo showing details of microphone (left) and speaker (right) arrays

2.2. Emission of the conical wave

Using theoretical aspects concerning the displacement of a supersonic source given in [3], the source term can be modeled as a linear mass distribution:

$$m(\mathbf{r}, t) = q(t)\delta(\mathbf{r} - \mathbf{r}_S) \quad (2)$$

where \mathbf{r}_S is the axis of displacement of the projectile. In this model, the mass of the source term is allowed to vary with respect to time for each point of the trajectory.

Consider the linear array as a discrete distribution of point-like sources. We make the assumption that the i^{th} speaker emits a signal $s_i(t)$. Under the assumption of a linear behavior of the loudspeakers, the global source term associated to the antenna leads to a discretization of the continuous source-term (2) as follows:

$$\sum_i s_i(t)\delta(\mathbf{r} - \mathbf{v}t_i) \approx q(t)\delta(\mathbf{r} - \mathbf{r}_S) \quad (3)$$

It may be convenient to define constant steps of spatial sampling and temporal delay, respectively noted Δx and Δt , such that $x_i = i\Delta x$ and $t_i = i\Delta t$ (see further eq. (6)).

The conical wave is synthesized by emitting a N-wave signal in each speaker, with a time delay Δt between each emission that depends on the space Δx between two speakers according to $v = \Delta x / \Delta t$. Thus, a conical wave of half-angle $\alpha = \arcsin(c/v)$ is created.

In this study, we use for bullet parameters those corresponding to an Elephant gun calibre : $d = 14.5$ mm,

$l = 64.4$ mm, $v = 1029$ m·s⁻¹. This leads to $M \approx 3$ and $T = 211$ μ s at 1 m from the axis. The corresponding N-wave has its principal frequency component centered around 3500 Hz (see figure 5).

2.3. Sampling constraints

2.3.1. Spatial sampling

Considering that the signal $s(t)$ is sampled at frequency $F_s = 48$ kHz, the highest frequency f_{max} contained in $s(t)$ has to respect the Shannon criterion $f_{max} < \frac{F_s}{2}$.

The speaker array is sampled by a step $\Delta x = 4.36$ cm (see fig. 2) so that the maximum wave number respects $k_{max} < \pi/\Delta x$.

Signals propagating into the air at speed c , their frequency content thus verifies $f = kc/2\pi$, so one gets $f_{max} < \frac{c}{2\Delta x}$.

Constrained by the size of the speakers, we take $\Delta x = 4.36$ cm. Then one has $f_{max} < 3922$ Hz. This is an acceptable result bearing in mind that the central frequency of the desired N-wave is 3500 Hz.

2.3.2. Temporal sampling

The time interval $\Delta t = \frac{\Delta x}{v} = 42.4$ μ s between the emission of two signals is clearly not a time sampling interval. It must verify $f_{max}\Delta t < \frac{1}{2M} \approx 0.167$.

As working on sampled signals it is necessary that all durations are expressed as multiples of the inverse of the sampling frequency: $F_s\Delta t \in \mathbb{N}$, $F_sT \in \mathbb{N}$.

Those integers are the durations given in number of samples: $\lfloor F_s\Delta t \rfloor = 2$, $\lfloor F_sT \rfloor = 10$, that is why we took the approximations $\Delta t = 2/F_s \approx 41.7$ μ s (compared to 42.4 μ s) and $T = 10/F_s \approx 208$ μ s (compared to the theoretical value 211 μ s).

2.3.3. High sampling frequency

The constraints on Δx and Δt may be ensured if F_s is so high that every frequency contained in the signal $s(t)$ is small compared to F_s by at least several times the Mach number, say if there exists $n \in \mathbb{N}$ such that $2n\lfloor M+1 \rfloor f_{max} < F_s$. Then comes $\frac{n}{F_s} < \frac{1}{2\lfloor M+1 \rfloor f_{max}} \Rightarrow \frac{nv}{F_s} < \frac{c}{2f_{max}}$. Taking $\Delta x = n\frac{v}{F_s}$, one both has $\Delta x < \frac{c}{2f_{max}}$ and $F_s\Delta t = n \in \mathbb{N}$.

In other words, a proper spatiotemporal sampling is ensured when the energy of the N signal vanishes at a limit frequency that is determined both by the sampling frequency and the Mach number. The ideal N-wave spectrum has been studied in [7] for aircraft sonic booms. The high-frequency components are spectral sinc lobes due to the temporal Heaviside function, that become negligible for a sufficiently high sampling frequency. $F_s = 48$ kHz is considered as a suitable value and is the upper limit of the multichannel sound device used for emission. However, the propagating wavefront being an analogic pressure field, it is possible to measure it with a different sampling frequency. The measurements are achieved at 192 kHz for better temporal resolution.

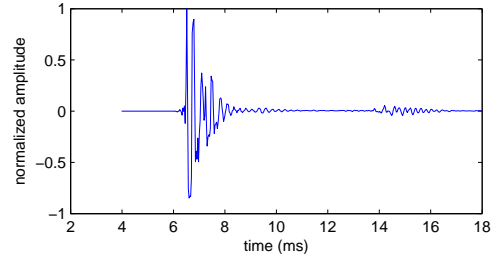


Figure 3. Pressure measurement at position (21, 1) – corresponding to $(x = 42$ cm, $y = 8.5$ cm) – of the emission of a numerical N-shaped signal

3. Inverse filtering for N-wave synthesis

In the previous sections, for the sake of simplicity, we assumed that a perfectly N-shaped signal $q(t)$ is emitted by the speakers. In other words, we assumed the loudspeakers have a perfectly frequency response, which is of course untrue. As a consequence, it is of critical importance to determine by inverse filtering which numerical signal should be given to the computer sound device so that the measured pressure field has the desired shape. It is clearly observed on figure 3 that the emission of a numerical N-shaped signal does not return a N-shaped pressure field. The impulse response of the loudspeakers and the propagation through space, implying attenuation and diffraction, deforms the N signal. Under the assumption of linear behavior of the speakers, it is possible to proceed an inverse filtering method for correcting this deformation.

Name s the numerical signal that is to be injected to a speaker so that the measured signal w is a perfect N wave. Under the assumption of linearity, there exists a function h such that $w = h * s$.

Using a simplified version of the theory developed by Kirkeby and Nelson [6], the inversion of the system is based on the fact that it is possible to write the convolution as a simple matrix product $w = Hs$, where H is a Toeplitz matrix containing h in its first column. It can be pseudo-inverted by the formula $s = (H^*H)^{-1}H^*w$, the star denoting conjugate transpose.

The main task consists in determining the matrix H or, equivalently, the vector h . The estimation of h is performed using a Welch procedure [8] for a single microphone located in the middle of the antenna (#21 for a total of 41 mics.) and disposed at a distance of $a = 38.5$ cm from the middle speaker.

Thanks to the disposal of absorbing materials on the walls, floor and ceiling of the anechoic room, reverberation effects are enough reduced to approach a free-field propagation by monopolar sources. Nevertheless, inverse filtering is optimal only at the measurement point. We assume that in the free-field approximation, the inverse filter is almost invariant un-

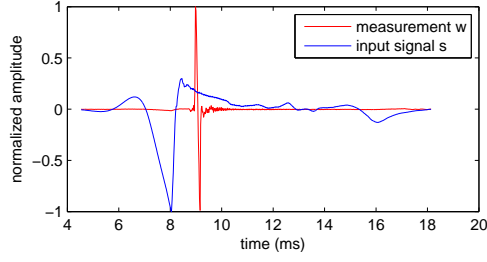


Figure 4. Inverse filtering for one speaker at position (21, 16): comparison between the inverse signal (blue) and the measured signal (red)

der translation. The experimental results prove that this approximation is valid on the grid shown in figure 1, although some deterioration is noticed on the boundaries, probably because of diffraction effects. Proceeding one inverse filtering per microphone position would have led to a heavy, inconvenient experimental protocol and to the absence of physical existence of the synthesized wave, each optimized measure being valid only in a single point of space.

There has been one inverse filtering per speaker, so that we obtain one transfer function h_i per loudspeaker and its associated Toeplitz matrix H_i , each of these leading to an inverse signal

$$s_i = (H_i^* H_i)^{-1} H_i^* w \quad (4)$$

Each signal s_i has to be temporally shifted by two time intervals.

- The first delay allows to correct the fact that h_i is calculated to concentrate all the N-shaped signals at the same time and at the same place, according to $H_i s_i = w$ for all i :

$$\tau_i = \frac{\sqrt{a^2 + (i - k)^2 \Delta x^2} - a}{c} \quad (5)$$

k being the index of the speaker in front of the microphone used for the learning procedure. The introduction of τ_i leads to the synthesis of a cylindrical wave.

- The second delay can be seen as the correction from a cylindrical to a conical wave:

$$t_i = \frac{i \Delta x}{v} \quad (6)$$

The relative amplitudes of the different signals have to be corrected of a factor ξ_i taking into account free-field propagation verifying

$$\xi_i = \frac{a}{\sqrt{a^2 + (i - k)^2 \Delta x^2}}$$

In conclusion, the signal $s'_i(t)$ that is to be sent to the i^{th} channel is

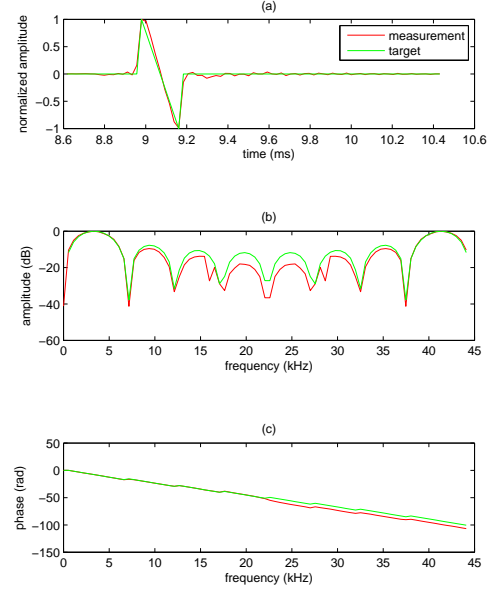


Figure 5. Inverse filtering for one speaker at position (21, 16) – corresponding to $(x = 42 \text{ cm}, y = 38.5 \text{ cm})$: comparison between a numerical N-shaped signal and the measurement after inverse filtering (a) in time domain, (b) for spectral amplitudes, (c) for spectral phases.

$$s'_i(t) = \xi_i s_i(t - t_i - \tau_i)$$

On figure 4, one can see that an inverse signal is very different from an N-wave. However, the transfer function is efficiently inverted because the resulting acoustic signal w has the expected good N shape. Figure 5 presents the comparison between a numerical N-wave and the resulting measured N-wave, both in time and frequency domains. One can see that the spectrum is correctly reproduced around 3500 Hz that is the zone of maximal component.

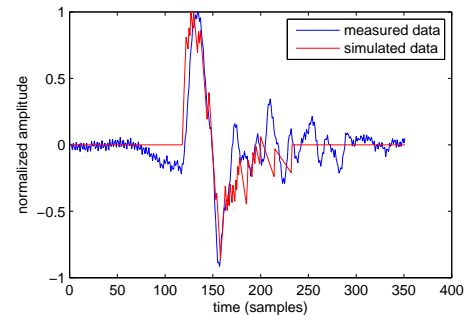


Figure 6. Inverse filtering for the whole speaker array. Comparison between the measure and the numerical simulation of the superposition of N-waves at position (21, 16).

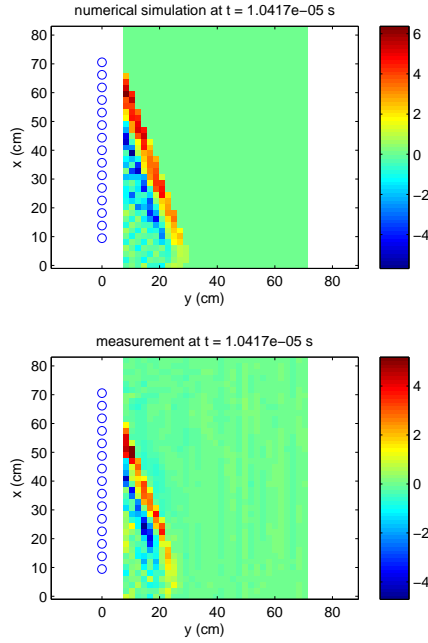


Figure 7. Early reconstruction of the wave front. Up: numerical simulation. Down: experimental result (Pa).

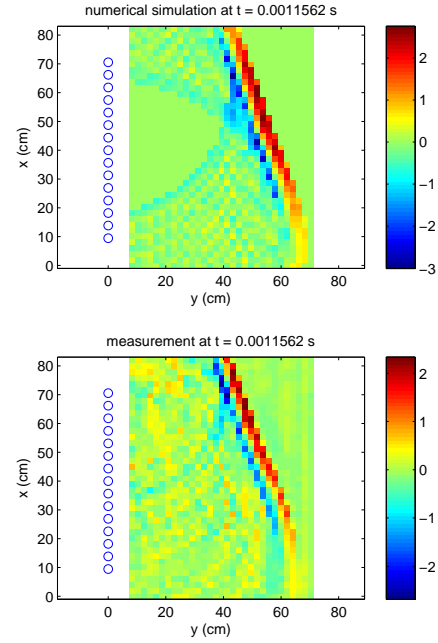


Figure 8. Late reconstruction of the wave front. Up: numerical simulation. Down: experimental result (Pa).

4. Numerical simulation

A numerical simulation of the discrete source has been performed using Matlab in order to predict the behavior of the wave. This simulation consists in analytically solving the D'Alembert equation, considering each speaker of the array as a monopolar source emitting a perfect N-shaped signal. The obtained results emphasize that some artifacts are inherent to the reconstruction, due to both spatial and temporal samplings. They are unavoidable even with perfect numerical N-waves propagating in ideal free field.

Despite those artifacts, the signal remains of small temporal support and of recognizable shape, as shown in figure 6. Numerical simulation and measurement agree, validating the free-field approximation.

5. Main results of the experiment

5.1. Spatio-temporal reconstruction of the wavefront

Our purpose is to reproduce the geometry of the wave. Numerical simulations give satisfactory results for wavefront reconstruction in the case of ideal point sources. The experiments are achieved bearing in mind the numerical simulations, so both are presented for comparison purpose.

Figures 7 and 8 show the spatial reconstruction of the conical wave fronts in a portion of the plane facing the speaker array. Figure 7 shows an early instant of the reconstruction while figure 8 shows the wave a few samples before it begins to leave the microphone grid.

Experiments match numerical simulations for the shape of the wavefront. Up to the directivity of the speaker array (studied later in this article), the conical geometry is reproduced in the region of space constrained by the finiteness of the array and estimated by geometric considerations. Under the dotted line shown in figure 1, the reconstruction is not accurate, due to the finite size of the speaker. As a consequence, a mask is used in further data processing and analysis, in order to neglect the measurements in this zone.

On figure 8, one can see that the region of space after the passing of the wave is more noisy in the experiment than in the simulation. This noise probably comes from random fluctuations of the air that avoid a perfect opposition between the top and the bottom of the N.

It is to remark that a less intense noise is also present in the numerical simulation: it has a purely mathematical origin. It comes from the discretization of the source term. Residual spherical wavefronts that theoretically oppose one to each other to vanish can be seen both in the simulation and the measurement. Once again, they are due to the finiteness of the source region.

5.2. Symmetry of revolution

We also propose to study the symmetry of revolution by rotating the speaker array in order to measure spatiotemporal planes in various directions of emission, namely $\{0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ\}$. Obtaining a 4-D spatiotemporal pressure field, it is then possible to plot a directivity diagram taking the RMS in time for chosen x and DCPA values, x being inside the accurate reconstruction zone — above the dotted line

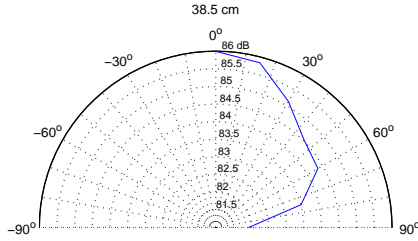


Figure 9. Directivity diagram for position (21, 16).

of figure 1. On figure 9, the directivity diagram measured in position (21, 16), corresponding to a DCPA of 38.5 cm, is shown. One can see that there is not a perfect symmetry of revolution: the array loses approximately 4 dB between 0° and 90°. The directivity might slightly vary with respect to x and/or DCPA, depending on the quality of the reconstruction of the N and on the number of microphones involved in the computation.

5.3. Decrease of amplitude with respect to DCPA

The decrease of the amplitude of the N wave is also studied by estimating the exponent β such that $\Delta p(r) \sim \frac{1}{r^\beta}$. Whitham theory predicts $\beta = 0.75$, linear behavior of a cylindrical source would give $\beta = 0.5$. Taking the maximum over t , that corresponds to the value of synthesized Δp , and average over all x positions: $p_1(r, \theta) = \langle \max_t p(x, r, \theta, t) \rangle_x$. β is estimated for each θ by finding the coefficient of the linear fit of $\log |p_1(\log |r|, \theta)|$.

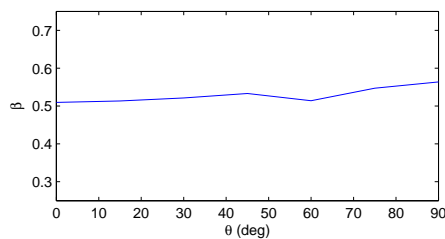


Figure 10. Variations of coefficient β with respect to the angle θ .

Figure 10 shows that β poorly depends on θ and only takes values between 0.50 and 0.55, which is coherent with a freefield propagation of a cylindrical source.

6. Conclusion and perspectives

In this paper, we proposed a method to synthesize a supersonic conical wave by means of a speaker array without involving any projectile. We show that inverse filtering allows one to synthesize accurately the N-shaped signal resulting from a supersonic source. The impulse response of each speaker is corrected,

and diffraction effects due to the periodicity of arrays are suppressed. Though the inversion is mathematically exact only in a single point of space, it is still acceptable in a neighborhood of reasonable size of 1 m² around this point. Our experimental results fit numerical simulation with good precision, conforing us that the method is accurate for a safe and reproducible synthesis of a Mach cone.

For speakers spaced by 4.36 cm, the reconstruction of the wave front is made possible at a sampling frequency of 48 kHz, taking into account that the desired N signal has a spectral component centered around 3500 Hz. The N signal being of very short duration – around 10 samples at 48 kHz –, it is necessary to work at a very high sampling frequency both for the emission and the measurement of signals. The word clocks of all sound devices have to be very precisely synchronized for better emission and reception of the signals, because an error of 1 sample may change the geometry of the cone. In future works, we intend to work with smaller speakers closer to each other at a higher sampling frequency for better precision both in space and time domains.

The bullet speed decrease is not taken into account in this study. This may be considered in a future work, by taking a non-constant law between Δx and Δt , leading to dispose the speakers unequally spaced and/or to send N-signals at different time intervals.

All the experiments take place in an anechoic environment, avoiding any reverberation effect and guaranteeing free-field-like propagation. Keeping the anechoic inverse filtering, it is possible to put reverberant material to test the reflexion of the cone on reduced-scale city walls. This will be done in further experiments.

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