# A Simple ADMM Solution to Sparse-Modeling-Based Detectors for Massive MIMO Systems

Yacine Meslem[1], Abeldjalil Aïssa-El-Bey[2], and Mustapha Djeddou[1]

[1] Laboratoire Télécommunications, UER- Signal et Communications
École Militaire Polytechnique, B.P 17, Bordj el Bahri (16111),
Algiers, Algeria
yacinemeslem@hotmail.fr
[2] IMT Atlantique, UMR CNRS 6285 Lab-STICC, UBL,
Brest F-29238, France

**Abstract.** We give a simple yet efficient Alternating Direction Method of Multipliers algorithm for solving sparse-modeling-based detectors [7, 9] for massive MIMO systems. Our solution relies on a special reformulation of the associated optimization problem by describing the constraints as a Cartesian power of the probability simplex. Simulation results show that the proposed algorithm is as accurate as the best known solvers (interior point methods), while its complexity remains linear with respect to the size of the system.

**Keywords:** Alternating Direction Method of Multipliers(ADMM), Detection, Massive MIMO, Optimization, Sparse-modeling.

## 1 Introduction

Large-scale Multiple-Input Multiple-Output, more commonly known as massive MIMO are attractive wireless communication systems that utilizes tens to hundreds antennas for transmission and reception. This powerful technology has emerged as an effective solution to the growing tremendous demand on high quality wireless communication in a limited radio spectrum. This is because it can provide high data transmission rates, when applied in a spatial multiplexing mode (V-BLAST), without needing additional bandwidth allocation. For that, massive MIMO systems are one of the key enablers for deploying the next generation cellular systems [11]. However, as a result of requiring a large number of antennas, the detection task becomes very challenging. Thought the maximum likelihood (ML) is the optimal detector in the sense that it leads to the smallest probability of error, it is equivalent to solve an integer least square minimization, and thus it is extremely expensive from a complexity perspective. Sphere decoding can effectively resolve this problem for small size MIMO systems but not for large scale ones [10]. As an alternative to the ML optimal approach, many suboptimal detectors have been proposed and studied in the literature. Those

techniques range from the simple linear detectors such as zero forcing (ZF) and minimum mean square error (MMSE) to the more sophisticated semidefinite relaxation (SDR) based detectors. All these detectors have their floors. The decoders which rely on SDR still not so tolerable in terms of complexity while the performances (in terms of BER) of linear decoders are so poor.
On another front, an emerging detection strategy relying on sparse and redundant representations theory has recently shown promising results [7, 9] and [6]. Sparse and redundant representations theory is a powerful and emerging paradigm that is at the core of compressive sensing and it has been successfully applied to a wide range of signal and image processing problems [5]. As we will briefly show in section 3, the references above have all adopted the sparse decomposition approach introduced in [1]. Subsequently, their proposed detectors are the same from an optimization point of view. On the side of numerical solutions, the first two papers have suggested interior point (IP) methods while the last one have employed ADMM after converting the optimization model to Least Absolute Shrinkage and Selection Operator (LASSO) problem. The ADMM technique [2] is a first order optimization method. It has been extensively applied in a variety of fields thanks to its ability to break down a hard minimization to easier sub-problems, and separately solves them. Let us return to [6]. It is well known that the penalized LASSO form requires an additional regularization parameter, which is, in general, very difficult to determine in advance.
In this work, we propose simple way to solve all those detectors using ADMM. In order to do that, we describe the constraints set in terms of the standard (probability) simplex. Based on this new formulation of the detection problem, our algorithm is derived. The rest of this paper proceeds as follows. In section 2, we give the system model. We shortly review sparse-modeling-based detectors in section 3. Our proposed ADMM-based solution is presented in section 4. Section 5 is devoted to simulation results. Finally, we conclude this paper in section 6

*Definitions and Notation.* The proximity operator of a scaled function as $\lambda f(\cdot)$ is defined as $\mathbf{prox}_{\lambda f}(\mathbf{v}) = \underset{\mathbf{x}\in\mathcal{D}}{\operatorname{argmin}} \left(f(\mathbf{x}) + \frac{1}{2\lambda}\|\mathbf{x}-\mathbf{v}\|_2^2\right)$. The orthogonal projection of a vector $\mathbf{v}$ onto a convex closed set $\mathcal{D}$ is $\mathit{\Pi}_{\mathcal{D}}(\mathbf{v}) = \underset{\mathbf{x}\in\mathcal{D}}{\operatorname{argmin}} \left(\frac{1}{2}\|\mathbf{x}-\mathbf{v}\|_2^2\right)$. As a widely adopted convention, we use uppercase boldface letters for matrices and lowercase boldface letters for vectors. The $i^{\text{th}}$ standard unit vector of an Euclidean space $\mathbb{R}^n$ is denoted as $\mathbf{e}_i$. We denote the $d \times d$ identiy matrix by $\mathbf{I}_d$ and the all ones vector of length $d$ by $\mathbf{1}_d$. The symbols $(\cdot)^T$ and $(\cdot)^H$ stand for the transpose and the Hermitian of a matrix respectively. As usual, we denote the set of reals by $\mathbb{R}$ and the set of complexes by $\mathbb{C}$. $\Re(\cdot)$ and $\Im(\cdot)$ are used for the real part and the imaginary part of a complex number (resp. vector). By the symbol $\otimes$, we denote the Kronecker product. For an integer $n$, $[\![1, n]\!]$ denotes the set $\{1, 2, \ldots, n\}$. We write, $(\cdot) \succeq (\cdot)$ to emphasize that the first object is greater or equal to the second one component-wise. The indicator function of closed convex set $\mathcal{D}$ is defined as

$$\mathcal{I}_{\mathcal{D}}(\mathbf{z}) = \begin{cases} 0 & \mathbf{z} \in \mathcal{D} \\ +\infty & \mathbf{z} \notin \mathcal{D} \end{cases}$$

## 2 System Model

We assume that our large-scale MIMO system is constituted of $N_t$ transmit and $N_r$ receive antennas (uplink scenario). The transmission of the data symbols $\mathbf{x} \in \mathcal{M}^{N_t}$ , where $\mathcal{M}$ is the signaling set, is made through a flat fading channel modeled as a matrix $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$. The entries of $\mathbf{H}$ are assumed to be i.i.d CSGS random variables of zero mean and unit variance. The received vector $\mathbf{y} \in \mathbb{C}^{N_r}$ is then given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\nu}, \tag{1}$$

where, $\mathbf{y} \in \mathbb{C}^{N_r}$ is the received vector and $\boldsymbol{\nu}$ represent the AGWN noise at the receiver, which is their entries are also i.i.d CSGS random variable of zero mean and a variance equal to $\sigma^2$. Furthermore, we assume that the receiver knows perfectly the CSI (Channel Information state) but the emitter does not. The ML detector requires the solution of the following least square problem under discrete constraints,

$$\begin{array}{l} \text{minimize } \ \frac{1}{2}\|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2 \\ \text{subject to } \mathbf{x} \in \mathcal{M}^{N_t}. \end{array} \tag{2}$$

Some detection approaches work on the real equivalent model to (1) which is

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\boldsymbol{\nu}}, \tag{3}$$

where, $\tilde{\mathbf{y}} = [\Re(\mathbf{y})\ \Im(\mathbf{y})]^T$, $\tilde{\mathbf{x}} = [\Re(\mathbf{x})\ \Im(\mathbf{x})]^T$ and

$$\tilde{\mathbf{H}} = \begin{bmatrix} \Re(\mathbf{H}) & -\Im(\mathbf{H}) \\ \Im(\mathbf{H}) & \Re(\mathbf{H}) \end{bmatrix}.$$

Note that $\tilde{\mathbf{x}} \in \mathcal{P}^{2N_t}$, where, $\mathcal{P}$ is the equivalent PAM to the original (generally) QAM constellation.

## 3 Review on sparse-modeling-based massive MIMO detection

For the sake of completeness, we present in this section a brief review on the sparse-modeling-based massive MIMO detection strategy. The sparse decomposition in [1, 7] exploits the finteness propriety of the modulation set in the way that we shortly explain hereafter. Let $\mathbf{m} = [m_1, \ldots, m_L]^T$, then any symbol $m_i \in \mathcal{M}$ can be written as

$$m_i = \mathbf{m}^T \mathbf{e}_i,$$

Clearly, $\mathbf{e}_i$ is sparse since it has only one nonzero entry. Extending this to the transmitted signal $\mathbf{x}$ gives

$$\mathbf{x} = (\underbrace{\mathbf{I}_{N_t} \otimes \mathbf{m}^T}_{\mathbf{D}})\mathbf{s}, \tag{4}$$

where, $\mathbf{s} \in \mathbb{R}^{LN_t}$ and $\mathbf{D} \in \mathbb{C}^{N_t \times LN_t}$ is the dictionary.

One drawback of this transform is its ultimate dependency on the modulation being used and thus, its dimension may be burden for higher order alphabets [9]. A solution to this issue has been given in [9], where it has been proposed to apply the previous transform after converting rectangular $L$-QAM to BPSK constellation using the technique of [4]. Leveraging these representations for (2) and relaxing the hard constraint give the following minimization problem [7, 9]

$$\begin{array}{l} \text{minimize} \;\; \frac{1}{2}\|\mathbf{y}-\mathbf{\Phi s}\|_2^2 \\ \text{subject to } \mathbf{Bs}=\mathbf{1} \\ \qquad\qquad \mathbf{s}\succeq\mathbf{0}. \end{array} \tag{5}$$

The minimization (5) represents what we refer to as sparse-modeling-based detectors. Here, $\mathbf{\Phi}=\mathbf{DH}$ or $\mathbf{\Phi}=\mathbf{WD'H}$ depending of the transform being used. The matrix $\mathbf{W}$ mapping $L$-QAM symbols to BPSK ones [4], where $\mathbf{D'}=\mathbf{I}\otimes[-1,+1]$. The matrix $\mathbf{B}=\mathbf{I}\otimes\mathbf{1}^T$, where, again, the dimensions are determined with respect to the chosen decomposition. Therefore, to keep our derivations general, we assume, in the rest of this paper, that our sparse representation vector $\mathbf{s}\in\mathbb{R}^{pN}$, where $N$ is the size of the equivalent data vector to be sparsely represented and $p$ is the dimension of the associated standard unit vector. For the sparse decomposition used in [6], it is exactly that of [1, 7] applied to the PAM equivalent constellation (to QAM) when the real model (3) is used, and its LASSO formulation has been demonstrated to be equivalent to (5) [7, 9]. At the end of this section, we want to mention that the minimization in (5) is convex and straightforward separable (in objective and constraints), and hence many ADMM solutions are possible and they are guaranteed to converge.

## 4 Proposed ADMM solution

We denote by $\mathcal{A}_+$ the feasible set (5), *i.e.*,

$$\mathcal{A}_+=\left\{\mathbf{s}\in\mathbb{R}^{pN}\mid\mathbf{Bs}=\mathbf{1}_N,\mathbf{s}\succeq\mathbf{0}_{pN}\right\}.$$

Since the vector $\mathbf{s}$ is a concatenation of $N$ unit vectors $_i$ of length $p$, we can regard the set $\mathcal{A}_+$as the $N$-Cartesian power of the unit simplex $\Delta_p$ of dimension $p$. So, we can write

$$\mathcal{A}_+=\Delta_p^N, \tag{6}$$

where, $\Delta_p=\left\{\mathbf{s}\in\mathbb{R}^p\mid\mathbf{1}^T\mathbf{s}=1,\mathbf{s}\succeq\mathbf{0}_p\right\}$. For the simplicity of notation, we omit hereafter the subscript $p$. Using (6), we can rewrite (5) as follows

$$\begin{array}{l} \text{minimize} \;\; \frac{1}{2}\|\mathbf{y}-\mathbf{\Phi s}\|_2^2 \\ \text{subject to } \mathbf{s}_i\in\Delta,\forall i\in[\![1,N]\!]. \end{array} \tag{7}$$

In order to apply ADMM for solving (7), we need first to write it in a split form as follows

$$\begin{array}{l} \text{minimize} \;\; \frac{1}{2}\|\mathbf{y}-\mathbf{\Phi s}\|_2^2+\sum\limits_{i=1}^{N_t}\mathcal{I}_\Delta(\mathbf{z}_i) \\ \text{subject to } \mathbf{s}=\mathbf{z}, \end{array} \tag{8}$$

where, $\mathbf{z} = \left[\mathbf{z}_1^T, \ldots, \mathbf{z}_N^T\right]^T \in \mathbb{R}^{pN}$ is a slack variable. The ADMM method solves a convex optimization problem such as (8) in three steps. The first two steps are an alternated minimization of the augmented Lagrangian associated with the problem of interest with respect to the primal variables $\mathbf{s}$ then $\mathbf{z}$ (respectively $\mathbf{z}$ then $\mathbf{s}$ ). The last step is an updating of the dual variable $\mathbf{w}$. For our problem (8), its augmented Lagrangian reads

$$\mathcal{L}_\rho\left(\mathbf{s}, \mathbf{z}, \mathbf{w}\right) = \frac{1}{2}\|\mathbf{y} - \mathbf{\Phi}\mathbf{s}\|_2^2 + \sum_{i=1}^{N_t} \mathcal{I}_\Delta(\mathbf{z}_i) + \mathbf{w}^T\left(\mathbf{s} - \mathbf{z}\right) + \frac{\rho}{2}\|\mathbf{s} - \mathbf{z}\|_2^2 \tag{9}$$

By simple algebraic manipulations, we can put (9) in the so-called scaled form as follows

$$\mathcal{L}_\rho\left(\mathbf{s}, \mathbf{z}, \mathbf{u}\right) = \frac{1}{2}\|\mathbf{y} - \mathbf{\Phi}\mathbf{s}\|_2^2 + \sum_{i=1}^{N_t} \mathcal{I}_\Delta(\mathbf{z}_i) + \frac{\rho}{2}\|\mathbf{s} - \mathbf{z} + \mathbf{u}\|_2^2 - \frac{\rho}{2}\|\mathbf{u}\|_2^2, \tag{10}$$

where, $\mathbf{u} = \frac{\mathbf{w}}{\rho}$. Note that the last term in (10) does not affect the optimization with respect to both $\mathbf{s}$ and $\mathbf{z}$ and thus it will be omitted.

*s-update.* Updating $\mathbf{s}$ requires the optimization of the following problem

$$\mathbf{s}^{k+1} = \underset{\mathbf{s}}{\operatorname{argmin}} \frac{1}{2}\|\mathbf{y} - \mathbf{\Phi}\mathbf{s}\|_2^2 + \frac{\rho}{2}\|\mathbf{s} - \mathbf{z}^k + \mathbf{u}^k\|_2^2. \tag{11}$$

This is a smooth quadratic convex problem and it solution can be easily obtained by seeking the value of $\mathbf{s}$ for which the gradient vanishes. Subsequently, the solution is given by

$$\mathbf{s}^{k+1} = \left(\frac{1}{2}\left[\left(\mathbf{\Phi}^H\mathbf{\Phi}\right) + \left(\mathbf{\Phi}^H\mathbf{\Phi}\right)^T\right] + \rho\mathbf{I}\right)^{-1}\left(\Re\left(\mathbf{\Phi}^H\mathbf{y}\right) + \rho\left(\mathbf{z}^k - \mathbf{u}^k\right)\right). \tag{12}$$

It worth mentioning that, for the derivation of the gradient, we have used the fact that our optimization variable $\mathbf{s}$ is real. That is to say, that for a complex-valued function $h$ of a real variable $\alpha$, we simply define its derivative as

$$\frac{dh(\alpha)}{d\alpha} = \frac{d\Re(h(\alpha))}{d\alpha} + \jmath\frac{d\Im(h(\alpha))}{d\alpha}.$$

Note that the matrix to be inverted is constant in throughout all the process. So, its inverse is needed to be determined only once.

*z-update.* The problem to solve in this step is given by

$$\mathbf{z}^{k+1} = \underset{\mathbf{z}_1,\ldots,\mathbf{z}_N}{\operatorname{argmin}} \sum_{i=1}^{N} \mathcal{I}_\Delta(\mathbf{z}_i) + \frac{\rho}{2}\|\mathbf{s}^k + \mathbf{u}^k - \mathbf{z}\|_2^2 \tag{13}$$

**Algorithm 1** Proposed ADMM based algorithm

**Input:** $\mathbf{s}_0 \in \mathbb{R}^N, \mathbf{z}_0 \in \mathbb{R}^N, \mathbf{u}_0 \in \mathbb{R}^N, \rho > 0$

**preprocessig:**

$\quad \mathbf{\Psi} \leftarrow \left(\frac{1}{2}\left[\left(\mathbf{\Phi}^H\mathbf{\Phi}\right) + \left(\mathbf{\Phi}^H\mathbf{\Phi}\right)^T\right] + \rho\mathbf{I}\right)^{-1}$ %% save matrix inverse

$\quad \tilde{\mathbf{y}} \leftarrow \Re\left\{\mathbf{\Phi}^H\mathbf{y}\right\}$ %% save matrix multiplication

$k \leftarrow 0$

**repeat**

$\quad \mathbf{s}^{k+1} \leftarrow \mathbf{\Psi}\left[\tilde{\mathbf{y}} + \rho\left(\mathbf{z}^k - \mathbf{u}^k\right)\right]$

$\quad$ **for** $i \leftarrow 1, N$ **do**

$\quad\quad \mathbf{z}_i^{k+1} \leftarrow \mathit{\Pi}_{\Delta}\left(\mathbf{u}_i^k + \mathbf{s}_i^{k+1}\right)$

$\quad$ **end for**

$\quad \mathbf{u}^{k+1} = \mathbf{u}^k + \mathbf{s}^{k+1} - \mathbf{z}^{k+1}$

$\quad k \leftarrow k + 1$

**until** a halting criterion is met

$\tilde{\mathbf{x}} \leftarrow \mathbf{D}\mathbf{s}^K$ %% back to the $\mathbf{x}$ domain

$\hat{\mathbf{x}} \leftarrow \mathit{\Omega}(\tilde{\mathbf{x}})$ %% hard slicing

**Output:** $\hat{\mathbf{x}}$

Since the minimization (13) defines the proximity operator of a separable function, we can use the separability property of the proximity operator [12, p. 129] to write

$$\mathbf{z}^{k+1} = \begin{bmatrix} \mathbf{prox}_{\mathcal{I}_{\Delta}}(\mathbf{s}_1 + \mathbf{u}_1) \\ \vdots \\ \mathbf{prox}_{\mathcal{I}_{\Delta}}(\mathbf{s}_N + \mathbf{u}_N) \end{bmatrix}. \tag{14}$$

Furthermore, the proximity of an indicator function of any set is nothing but the orthogonal projection on the same set. Therefore, (14) becomes

$$\mathbf{z}^{k+1} = \begin{bmatrix} \mathit{\Pi}_{\Delta}(\mathbf{s}_1 + \mathbf{u}_1) \\ \vdots \\ \mathit{\Pi}_{\Delta}(\mathbf{s}_N + \mathbf{u}_N) \end{bmatrix}. \tag{15}$$

All we need now, is to determine how to perform the orthogonal projection onto the standard simplex. Clearly, the Euclidean projection onto a simplex is a convex optimization problem as any orthogonal projection onto any convex set. Although, there is no closed-form solution to this problem, one can still get it in different simple ways. One well known iterative method is presented in [12, p. 182]. A simpler, reasonably fast and easy-to-implement algorithm is derived in [3]. This algorithm is our choice to perform the $z$-update step.

*u-update* For the completeness of the algorithm, we recall that the scaled dual variable $\mathbf{u}$ updates as follows

$$\mathbf{u}^{k+1} = \mathbf{u}^k + \mathbf{s}^{k+1} - \mathbf{z}^{k+1}. \tag{16}$$

We summarize our proposed solution in algorithm 1. The derived algorithm is simple and it alternates between an updating of a regularized least square

solution ($s$-update) and a series of projections onto the standard probability simplex ($z$-update).

## 5 Computer Experiments

In this section, we assess the performances of our algorithm, in terms of i) the quality of solution that it provides, ii) the BER, specially for the under-determined scenario when the number of receive antennas is less than the number transmitters and iii) the runtime needed for the algorithm to execute. We Fellow the system model described in (2). The simulations have been conducted using a workstation with Xeon(R) $E5-2630$ v3 CPU and 8 GB of RAM. All the algorithms are implemented using Matlab. We should mention that due to space limitation, we only present the results corresponding to the flat fading channel mode. The extension to the selective channel environment is not difficult [7].

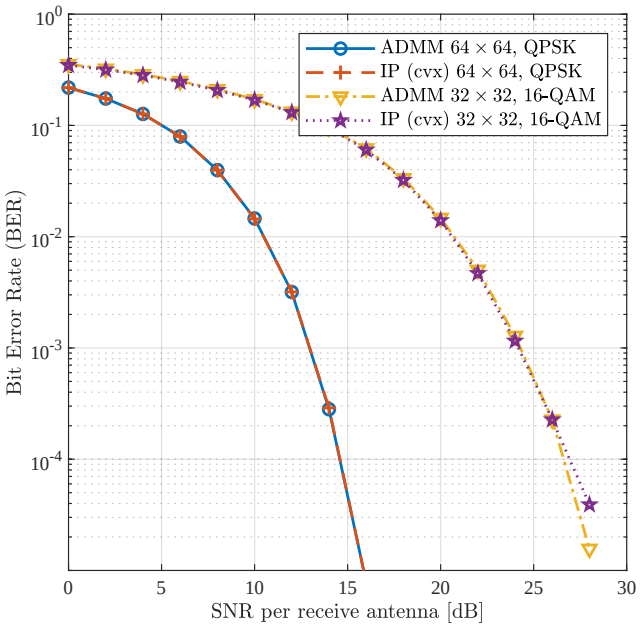



**Fig. 1.** BER performance comparison between the proposed ADMM and interior point solution for two different massive MIMO transmissions scenarios

First, we evaluate the proposed algorithm in terms of the reconstruction quality. To do that, we compare the solution of our algorithm with that of the highly precise interior point (IP) method, for two instances of (5), *i.e.*, for a QPSK $64 \times 64$ MIMO system and a $16-$QAM $32 \times 32$ one. We employed cvx [8], a Matlab toolbox for modeling and solving convex optimization problems relying

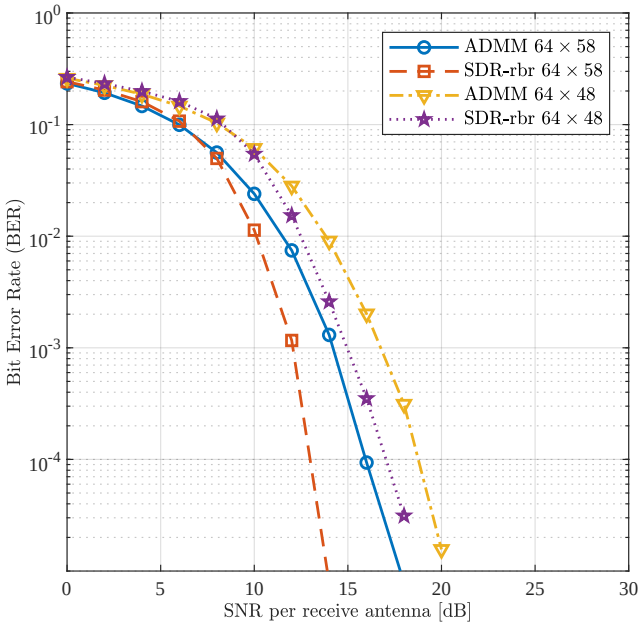



**Fig. 2.** BER performance comparison of the proposed ADMM solution versus MMSE and SDR-rbr for two overloaded mssive MIMO systems using QPSK signaling

on IP methods. The same hard slicing techique is used to round the solutions of both algorithms to the signaling set. The obtained results are depicted in Fig. 1, where we can see that our algorithm produces exactly the same quality of solutions as the IP method for the two MIMO settings that we have considered and this remains true for the whole SNR range.

In order to get some insight about the performances of the sparse-modeling-based detectors and in particular our proposed algorithm, we compare it to one of the high-performance SDR based detectors, (SDR-rbr), for QPSK and BPSK constellations [13]. We use the code provided by the authors, available at http://www.ee.cuhk.edu.hk/~wkma/mimo/rbr.html. The comparison is made for two under-determined (overloaded) massive MIMO systems scenarios using QPSK signaling. The results of this comparison are given in Fig. 2. It is clear that the SDR-rbr decoder outperforms our detector for both configurations $64 \times 58$ and $64 \times 48$. One important thing to notice here, is that the gap of performances between the two algorithms decreases ($64 \times 48$ case) as the system becomes more overloaded. This confirms that the sparse-modeling-based detection approach is more suitable for the under-determined case [7, 9]. Finally, we compare the run time of our algorithm with respect to one of the most low-complexity detectors, MMSE, and one of the cheapest SDR based detectors, SDR-br. THe results are shown in Fig. 3 We have used QPSK modulation for a variety of antennas number. The average runtime was obtained for 250 independent simulations

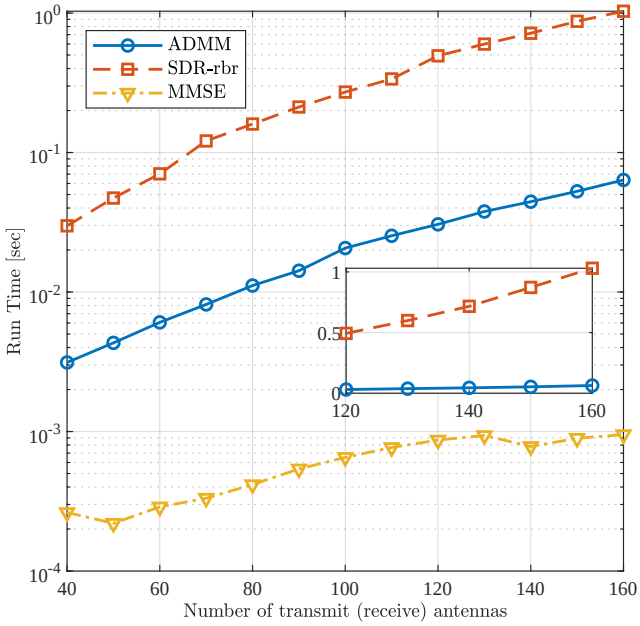



**Fig. 3.** Runtime of the proposed ADMM algorithm, SDR-rbr and MMSE for a variety of antennas number at SNR = 15 dB

at 15 dB. We notice that the complexity of our algorithm is in between the complexity of MMSE and that of the SDR-rbr. The ratio between our algorithm and the SDR-rbr one is roughly 10 in the favor of our method, and so it is when compared to the MMSE decoder in the favor of the latter. Furthermore, as an interesting remark, the proposed algorithm scales linearly with the dimension of the system. We should point out that, for all the results presented above, we have used a *standard* stopping criterion [2] for our algorithm without any acceleration technique. We believe, however, that a modification of the halting criterion of the proposed algorithm such as an early termination, *i.e.*, running the algorithm for a prefixed number of iterations and / or an incorporation of an acceleration scheme like using a variable step ($\rho$) [2], will help in further reducing the runtime of the proposed algorithm.

## 6 Conclusion

We have proposed a simple and fairly fast algorithm for solving the sparse-modeling-based detectors. We have shown, by simulations, that the proposed algorithm achieves the accuracy level as the IP methods with a lower complexity. Our solution is based on the ADMM framework after reformulating (in terms of the probability simplex) the optimization problem modeling the underlined detection strategy. Future work includes a further detailed study on the different

parameters of ADMM (step size, stopping criterion), and a comparison with other possible ADMM implementations. As another interesting directions, we will evaluate the robustness of our solution to multi-cell interference and the multiple access scenarios.

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