



**HAL**  
open science

# Geographic Cellular Automata for Realistic Urban form Simulations: How Far Should the Constraint be Contained?

Jean-Philippe Antoni, Gilles Vuidel, Hichem Omrani, Olivier Klein

## ► To cite this version:

Jean-Philippe Antoni, Gilles Vuidel, Hichem Omrani, Olivier Klein. Geographic Cellular Automata for Realistic Urban form Simulations: How Far Should the Constraint be Contained?. Luca D'Acci. The Mathematics of Urban Morphology, Birkhäuser Basel; Springer Nature Switzerland AG, pp.147-162, 2019, Modeling and Simulation in Science, Engineering and Technology, 978-3-030-12381-9. 10.1007/978-3-030-12381-9\_7. hal-02087028

**HAL Id: hal-02087028**

**<https://hal.science/hal-02087028>**

Submitted on 1 Apr 2019

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Geographic cellular automata for realistic urban form simulations: how far should the constraint be contained?

Jean-Philippe Antoni <sup>1</sup> - [jean-philippe.antoni@u-bourgogne.fr](mailto:jean-philippe.antoni@u-bourgogne.fr)

Gilles Vuidel <sup>1</sup>

Hichem Omrani <sup>2</sup>

Olivier Klein <sup>2</sup>

<sup>1</sup>Laboratoire ThéMA, UMR 6049 French National Centre of Scientific Research (CNRS), University of Bourgogne-Franche-Comte

<sup>2</sup>Luxembourg Institute of Socio-Economic Research, Maison des Sciences Humaines, Luxembourg

This is a postprint version, the definitive version of this chapter has been published in D'Acci, L. (Éd.). (2019). The Mathematics of Urban Morphology.

<https://www.springer.com/gb/book/9783030123802>

DOI : 10.1007/978-3-030-12381-9\_7

---

# Geographic Cellular Automata for Realistic Urban form Simulations: How Far Should the Constraint be Contained?



Jean-Philippe Antoni, Gilles Vuidel, Hichem Omrani and Olivier Klein

**Abstract** Cellular automata (CA) are discrete models that are being ever more widely used to study urban forms and, more broadly, to understand, simulate, and forecast land use changes (LUC). But LUC models are not based on CA dynamics alone and so they are not fully consistent with mathematical definitions of CA. Accordingly, to study urbanization, authors often use “constraint CA” or “geographic CA” (GCA), i.e., CA which are coupled with other models in order to integrate geographical assumptions related to urban form and to provide more realistic results. These complementary models are usually calibrated according to expert knowledge and do not lead to reproducible deterministic results. Consequently, there is often a sizeable gap between the theory of CA as defined in mathematics and their practical use for LUC. In this chapter, cellular automata are constrained by a Markovian process helping to determine the number of cells that can change from one land use category to another. Second, a potential model is used to create a suitability map and define the probability of a cell changing from one category to another. Finally, all these additional constraints lead to a suite of models which is clearly more complex than classical CA as it can be considered mathematically. Nevertheless, as far as possible, it presents GCA as a mathematical adaptation of CA integrating the geographical assumptions necessary for studying urban forms in a realistic way.

AQ1

**Keywords** Cellular automata · Markov chain · Potential model · Urban form · Spatial modeling

---

J.-P. Antoni (✉) · G. Vuidel  
Laboratoire ThéMA, UMR 6049—CNRS et Université Bourgogne  
Franche-Comté, 6 Boulevard Gabriel, 21000 Dijon, France  
e-mail: [jean-philippe.antoni@u-bourgogne.fr](mailto:jean-philippe.antoni@u-bourgogne.fr)

H. Omrani · O. Klein  
Luxembourg Institute of Socio-Economic Research, Maison des  
Sciences Humaines, 11, Porte des Sciences, 4366 Esch-sur-Alzette,  
Belval, Luxembourg

© Springer Nature Switzerland AG 2019  
L. D'Acci (ed.), *The Mathematics of Urban Morphology*,  
Modeling and Simulation in Science, Engineering and Technology,  
[https://doi.org/10.1007/978-3-030-12381-9\\_7](https://doi.org/10.1007/978-3-030-12381-9_7)

1

# 1 Introduction

Urban form may be defined as the relationship between buildings and open spaces within agglomerations or different types of urban aggregates according to the specificities of local context. It refers more specifically to the outer envelope or contours of the city (Antoni 2008). This form, which is characterized by developments at different scales (from the entire agglomeration to a single building), is the result of human occupation of the territory. As a result of individual behaviors, it reflects urban lifestyles generated through several factors such as the urban fabric, the built environment, density/compactness, and the spatial distribution of activities and facilities.

Geographers tackling urban forms seek primarily to understand the mechanisms that lead to the current form of a given urban context, to provide procedures for designing optimal forms, and to simulate future developments. Such approaches rely on adapted modeling tools that integrate features based on explanatory and predictive models, which can be used as tools to support reflection and decision-making. Among the mathematical models, many computer-based solutions attempt to simulate the evolution of cities and specially to understand how urban forms change over time, past and/or future. Among them, cellular automata (CA) stand out as a form of mathematical computation models based on a discrete dynamic modeling system. They are structured into procedures based on the nesting of simple rules that reflect the complexity of real systems. This approach is attractive because it relies on a generic development principle that fits very well with the way systems in general, and urban systems in particular, evolve. In this framework, LucSim<sup>1</sup> was designed and developed by ThÉMA Laboratory (Antoni et al. 2017) as a cellular automata model specially designed for geographical analysis and spatial simulation for both researchers and advanced planning institutes. This user-friendly software is well adapted for analyzing and simulating land use changes and spatial dynamics at different scales for decision-making in urban and land planning.

After having recalled some definitions of CA as used in geography, the next section shows why they remain difficult to apply directly to concrete urban form planning or studies. Section 3 then presents two major constraints (temporal and spatial) that can improve CA simulation results by refining basic assumptions related to land use change. Section 4 presents the results in a new theoretical CA formalization, leading to more realistic urban form simulations illustrated by the example of Wrocław in Poland. These results are then discussed in Sect. 5 questioning the extent to which these constraints need to be contained.

---

<sup>1</sup> See <https://sourcesup.renater.fr/lucsim/> for more details.

## 2 CA-Based Discrete Modeling

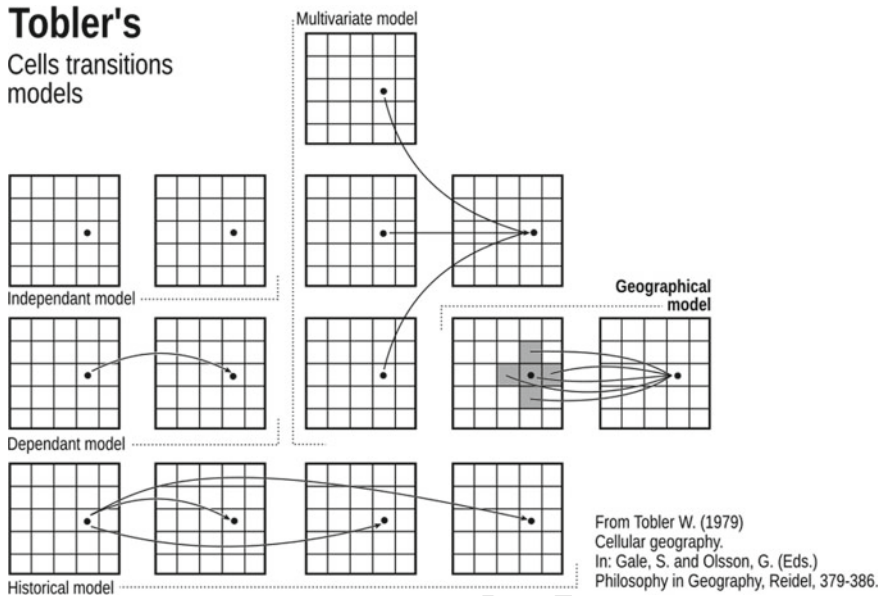
Starting from a formal definition of CA-based discrete modeling, this section focuses on the strengths and limitations of this type of approach when considering urban form simulations.

### 2.1 CA Formal Definition

CA are discrete computer models composed of a grid of regular cells assigned to one particular state (among a finite number of states) which may change into another state over time. They were invented in the 1940s through the works of S. Ulam and J. von Neumann (1963) and popularized in the 1970s by John Conway's *Game of Life* (Conway 1970). Initially, they were of interest only to a few theorists of mathematics or computer science, who used them to solve puzzles or to build mathematical games in scientific journals. In the 1980s, a number of papers, especially those of Wolfram (1983, 1985) made CA fashionable, or rather showed them in a new light with a multitude of possible applications for very different fields. Having been focused initially on problems in physics and chemistry, several innovative experiments opened up biology, medicine, and ecology to CA before they were introduced into spatial studies, particularly geography and urban planning. In geography, the use of CA indeed echoes the cellular conception of geographical space defended by Tobler (1979) and Couclelis (1985) and reveals the deeply geographical character of this kind of tool (Couclelis 1988). For these authors, this cellular conception is more advantageous than considering space through the irregular spatial polygons defined by political and administrative jurisdictions. It provides a notational simplification allowing a cell of an array to be indexed in the same way as in matrix algebra. In such a notation,  $g_{ij}^t$  is a cell characterized by a land use category (urban, forest, industry, etc.) at the location  $i, j$  at time  $t$ , and  $g_{ij}^{t+\Delta t}$  corresponds to the change in the land use category at the same location at time  $t + t$ .

From this basis, Tobler (1979) was probably the first geographer to envisage and describe all the formal possibilities of cell transitions according to different processes involving their neighborhood (Fig. 1):

1. An independent model where  $g_{ij}^{t+\Delta t}$  is not related to  $g_{ij}^t$  in any way.
2. A dependent model where the land use at location  $i, j$  at time  $t + t$  depends on the previous land use at that location, such that  $g_{ij}^{t+\Delta t} = f(g_{ij}^t)$ .
3. An historical model where the land use at position  $i, j$  in the future depends on the initial land uses at that location, such that  $g_{ij}^{t+\Delta t} = f(g_{ij}^t, g_{ij}^{t-\Delta t}, g_{ij}^{t-2\Delta t}, \dots, g_{ij}^{t-k\Delta t})$ .
4. A multivariate model where the land use at location  $i, j$  is dependent on several other variables at that location, such that  $g_{ij}^{t+\Delta t} = f(u_{ij}^t, v_{ij}^t, w_{ij}^t, \dots, z_{ij}^t)$ .



**Fig. 1** Tobler's cells transitions models

- 94 5. A geographical model where the land use at location  $i, j$  is dependent on the land  
95 use at other locations, such that  $g_{ij}^{t+\Delta t} = f(g_{i+p, j+q}^t)$ .

96 This fifth model clearly corresponds to the process implemented in most CA  
97 models. Nevertheless, in the field of spatial studies and geographical sciences, formal  
98 definitions remain rare except for Tobler's former theoretical formalization.  
99 Researchers using CA seldom take the time to describe the mathematical form of the  
100 model they are using and refer only to other fundamental papers (White and Engelen  
101 1993; Benenson and Torrens 2004), or describe CA as *if-then-else* algorithms (Batty  
102 1997). Torrens (2000) is one of the rare geographers to use a mathematical notation  
103 to define the principles of CA transition according to the geographical process (fifth  
104 model) defined by Tobler.

## 105 2.2 CA Limits

106 A Tobler-like geographical notation is clearly pleasant mathematically and correctly  
107 describes how a transition can operate from one cell state to another according to  
108 theoretical neighboring configurations. But despite this advantage, it does not model  
109 land use change in an operative way, nor does it reproduce or create realistic simu-  
110 lations. An illustrative example based on a case study of Wroclaw (Poland) helps to  
111 explain why.

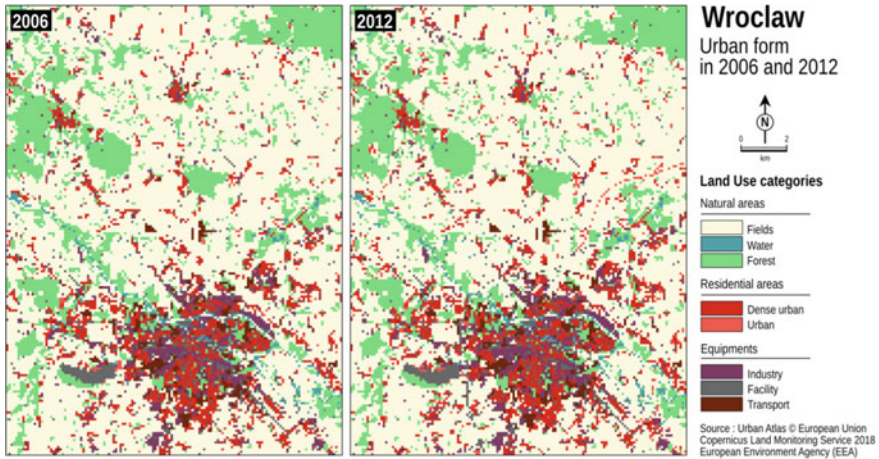


Fig. 2 The urban form of Wroclaw in 2006 and 2012

112 This example is built using data from the Urban Atlas (Copernicus Programme)<sup>2</sup>  
 113 which describes land use in 2006 and 2012. As shown in Fig. 2, land use is classified  
 114 into eight categories: water, fields, forest, dense urban, urban, industry, facilities,  
 115 and transport. To simulate the evolution of this territory in the future, we use four  
 116 transition rules implemented in the LucSim CA software (Antoni et al. 2017). These  
 117 rules are constructed from expert knowledge and are supposed to reproduce urban  
 118 expansion based on simple principles that have largely determined the evolution  
 119 of urban form in the past. Rules are here expressed in two ways: verbal language  
 120 and their conversion into a computer-based language specific to LucSim software  
 121 (dashes):

- 122 1. Fields will become urban if, within a neighborhood of two cells, the current  
 123 cell is surrounded by at least 10% of urban and dense urban cells, if, within a  
 124 neighborhood of fifteen cells, there is at least one cell of facilities, if there is no  
 125 direct connection to forest (within a neighborhood of one cell), and if within a  
 126 neighborhood of three cells less than 50% of the cells are urban and dense urban:
  - 127 – *Fields -> Urban:  $p_{CellCir(Urban,2)} + p_{CellCir(Urban\_dense,2)} + p_{CellCir(New,2)} \geq 10\%$  and  $nb_{CellCir(Facilities,15)} \geq 1$  and  $nb_{CellCir(Forest,1)} \leq 1$  and  $p_{CellCir(Urban,3)} + p_{CellCir(Urban\_dense,3)} \leq 50\%$ ;*
- 130 2. Urban cells will be densified if they are completely surrounded by urbanized  
 131 areas within a neighborhood of one cell:
  - 132 – *Urban -> Urban\_dense:  $p_{CellCir(Urban,1)} = 100\%$ ;*

<sup>2</sup>See <https://land.copernicus.eu/about> for more details.

- 133 3. Urban parks (forests) will be created from urban or dense urban if the density of  
 134 urbanized areas is more than 90% within a small radius (within a neighborhood  
 135 of 2 cells for an urban category and within a neighborhood of 1 cell for a dense  
 136 urban category):
- 137 – *Urban -> Forest:  $pCellCir(Urban,2) + pCellCir(Urban\_dense,2) \geq 90\%$ ;*
  - 138 – *Urban\_dense -> Forest:  $pCellCir(Urban,1) + pCellCir(Urban\_dense,1) \geq$*   
 139 *90%;*

140 However, the strict application of these rules within LucSim produces results that  
 141 have nothing to do with the current land use, nor with any logical development from  
 142 a town planning or land use planning perspective.

143 Figure 3 indeed shows resulting spatial configurations, which are supposed to  
 144 reproduce an urban sprawl process. It clearly shows that the urban sprawl process  
 145 simulated by the model leads to a credible expansion of the urban form during the  
 146 first iterations of the CA run, with an expansion of the city taking the form of an  
 147 oil slick. But very quickly, the number of newly urbanized cells produced by the



**Fig. 3** Simulation of the urban form of Wrocław from  $T_0$  to  $T + 6$ , (To make the images easier to read, newly urbanized areas appear in black before returning to their original color (red))



### Wroclaw

Land Use change simulation from T0 to T+15

Software : LucSim

Land Use categories

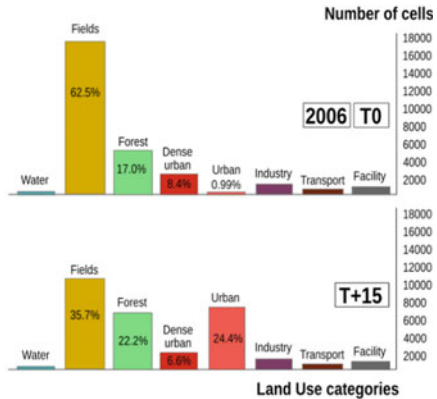
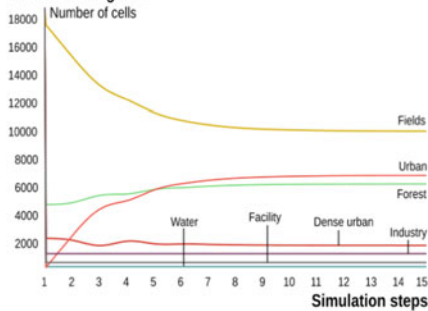


Fig. 4 Land use changes in Wroclaw from T0 to T + 15

148 model generates a snowball effect that far exceeds any realistic forecast of future  
 149 urbanization for a city like Wroclaw.

150 From this example, we can conclude that the simple use of CA with transition  
 151 rules is not sufficient for forecasting realistic and operational simulations of urban  
 152 forms. On the one hand, the recognition of spatial patterns leading to the application  
 153 of a transition rule corresponds only very partially to the reality of the urbanization  
 154 process. Indeed, the number of possible transitions appears to be much greater than  
 155 the real needs in terms of new housing and population growth. On the other hand, the  
 156 results give no information about the timing of this urbanization. Therefore, although  
 157 it seems clear that a time step does not correspond to a regular and stable duration,  
 158 we are unable to say whether the images produced from  $t + 1$  to  $t + 16$  lead us to  
 159 2020, 2050, 2200, or 3500 (Fig. 4).

AG3

160 It appears clearly, then, that a CA cannot be directly applied to simulate city  
 161 growth and more generally land use change. It must necessarily be constrained to  
 162 answer more precise questions about space and time, so that the results produced can  
 163 be integrated more easily, and realistically within a range of analysis and decision-  
 164 making for planning. Only these kinds of constraints, based on assumptions about  
 165 urban form, enable CA to be used in geography and allow for the fundamental  
 166 difference between classical mathematical cellular automata and geographic cellular  
 167 automata (GCA).

### 168 3 Suitable Constraints for Urban Modeling

169 There are many methods by which to constrain CA and the literature abounds with  
 170 examples using several methods and models. In this section, we shall focus on just two  
 171 kinds of fundamental constraints. The first is a time constraint to situate the results

172 produced by the automaton over time. As in many publications (Arsanjani et al. 2013),  
 173 it relies on a Markov chain process. The second constraint is a space constraint, which  
 174 aims to reduce the number of neighboring configurations for possible transitions and  
 175 focuses on the most realistic of them. It is based on a potential model.

### 176 3.1 Temporal Constraint

177 The first step uses a Markov chain to constrain the process of land use change  
 178 in quantitative terms. Comparison of two static land use images (2006, 2012) can  
 179 be used to determine what has happened between each image and so formulate a  
 180 transition process. By comparing the land use categories date by date and cell by  
 181 cell, it is possible to determine cellular changes between  $t$  and  $t + 1$  and to identify  
 182 the land use dynamics. Theoretically, each cell can either change from one land use  
 183 category to another or remain in its initial category. The dynamics of the model can  
 184 therefore be presented as a series of possible transitions from one land use category  
 185  $k$  at time  $t$  to another land use category  $l$  at  $t + 1$ . For a given cell  $N_i$ , a transition  $\Delta$   
 186 can be written as:

$$188 \quad \Delta N_{i,kl} = 1 \text{ if } N_{i,k}(t) = 1 \text{ and } N_{i,l}(t + 1) = 1$$

189 To simplify the complexity resulting from the large number of cells and possi-  
 190 ble transitions, changes can be aggregated by land use categories. The aggregate  
 191 transition for the complete system is then

$$192 \quad \Delta N_{kl} = \sum_{i=1}^n \Delta N_{i,kl}$$

194 This formulation allows us to build a contingency matrix indicating the number of  
 195 cell transitions from a category  $k$  to a category  $l$  between  $t$  and  $t + 1$  (i.e., between 2006  
 196 and 2012). This matrix can be easily converted into a transition matrix indicating the  
 197 probability of change between all land use categories (Table 1). When associated with  
 198 the previous vectors, this matrix provides all the elements needed for the construction  
 199 of a Markov chain (MC). In the literature, an MC is defined as a mathematical  
 200 process where transition probabilities are conditional on the past, and express the  
 201 state of a variable at a time  $t$  as a function of observations of this variable at  $t -$   
 202 1 (Feller 1968, Berchtold 1998). It relies on the connection of three items: (i) the  
 203 description of the relative values associated with an initial state (land uses visualized  
 204 as a vector for example); (ii) a transition matrix expressing the transition probabilities  
 205 of different groups of observations from one category to another; and (iii) a diachronic  
 206 transformation by an operator in the form of a matrix multiplication iteration.

207 If we follow this procedure, land use at time  $t + 1$  can be simulated by multiplying  
 208 the corresponding vector at time  $t$  by the corresponding contingency matrix, after the

**Table 1** The transition matrix for Wrocław between 2006 and 2012

	Water	Fields	Forest	Dense U	Urban	Industry	Facilities	Transp.
Water	<b>98.784</b>	0.608	0	0	0.304	0	0	0.304
Fields	0.08	<b>96.849</b>	0.017	0.028	2.178	0.364	0.034	0.049
Forest	0	0.063	<b>99.561</b>	0	0.104	0.125	0	0.146
Dense U	0	0	0	<b>99.831</b>	0.042	0.085	0.042	0
Urban	0	5.755	0.719	11.151	<b>69.784</b>	6.475	0.719	5.396
Industry	0	0.558	0	0	1.275	<b>98.088</b>	0	0.08
Facilities	0	0	0.159	0.638	1.115	0.159	<b>97.448</b>	0.478
Transp	0	0	0	0	0.662	0	0	<b>99.338</b>

209 transformation of the latter into transition probabilities from one land use category  
 210  $k$  to another  $l$ . To transform observed contingencies into transition probabilities, we  
 211 use the following:

$$212 \quad p_{kl}(t) = \frac{\Delta N_{kl}}{N_k(t)} \quad \text{and} \quad \sum_{k=1}^m p_{kl}(t) = 1$$

214 We then consider the MC as follows:

$$216 \quad N_i(t + 1) = \sum_{k=1}^m p_{ki} \cdot N_k(t)$$

$$218 \quad \text{where } p_{kl} = \frac{\Delta N_{kl}}{N_k(t)} = \frac{\Delta N_{kl}}{\sum_l \Delta N_{kl}} \quad \text{and} \quad \sum_l p_{kl} = 1$$

219 According to this formulation, the MC process gives us the chance to prospectively  
 220 calculate future states from known past states, based on observation of past trends and  
 221 probabilities. According to the method, this calculation is based on the assumption  
 222 that future changes will follow the trend of past changes, but as it is based on a  
 223 matrix calculation, this trend is not necessarily linear. Moreover, the values of the  
 224 transition matrix can also be modified by users of the model to integrate different  
 225 parameters for the quantification of future land use changes. In our case, LucSim  
 226 uses the original transition matrix to calculate the number of cells in each land use  
 227 category in 2018, 2024, 2030, etc., on the basis of 2006 and 2012 land uses (same  
 228 interval of 6 years between each date). This system gives us a more plausible picture  
 229 of urban dynamics by calculating land use vectors for each future date, as presented  
 230 in Table 2.

231 This table also indicates that the total number  $n_{l,t}$  of cells that should be urbanized  
 232 in 2030 must not exceed 984 “urban” cells and 2571 “dense urban” cells.

**Table 2** Expected future land use vectors

	Water	Fields	Forest	Dense U	Urban	Industry	Facilities	Transp.
<b>2018</b>	339	17058	4771	2401	613	1322	620	1006
<b>2024</b>	348	16568	4758	2474	836	1407	615	1121
<b>2030</b>	357	16107	4747	2571	984	1504	612	1245

### 3.2 Spatial Constraint

Among other methods, MCs are a way to quantify future land use changes when space is considered through cells. However, they say nothing about the location of those changes. The places where changes occur are strictly determined by the transition rules of the CA. To integrate information known elsewhere about the spaces most likely to be urbanized quickly (or on the contrary not to be if they are protected), it is therefore mandatory to add a second constraint capable of determining the most suitable locations. This second constraint is relatively conventional using GCA and is usually based on expert knowledge. It consists of constructing a *suitability map* based on *driving factors*, namely geographical features that are supposed to influence urbanization (Clarke 2008).

It seems obvious that working on locations requires a theoretical framework for geographical space. It is not surprising, then, that geographers have developed many models for this, often dedicated to residential location (Putman 1979). Diffusion models, for example, are spatial models that make it possible to locate certain elements on the assumption that they are generated through the diffusion of other elements. Fractal models are other models that can also simulate urban growth (Batty 2007). The city is then considered as a system that maximizes interactions between the elements it contains. Spatial interaction models are also another family of models derived from Newton's law of universal gravitation. They are based on identical principles and make it possible to locate changes where they are complementary to those around them by minimizing the distances between them. They have been used for calculating areas of traffic or influence (Helvig 1964) and for estimating residential or industrial locations (Abler et al. 1972).

Among spatial interaction models, potential models indicate that the probability of there being a relationship between places decreases with distance. Basically, they are used to measure "accessibility" aiming to evaluate the variation of the relative amount of relationship opportunity depending on the position of all places. Generally, the potential of a place is calculated from the analysis of the importance of all the other points of the system, an importance that is termed "mass" in reference to the Newtonian gravity model. The potential of a cell is usually the sum of all the potentials created at that location by the set of individual masses that make up the system (i.e., all the other cells). The calculation of the potential  $P$  of each point  $i$  therefore consists in applying to them a formula simultaneously taking into account the mass value  $m$  of all the points  $j$  located in a geographical area as a function of the

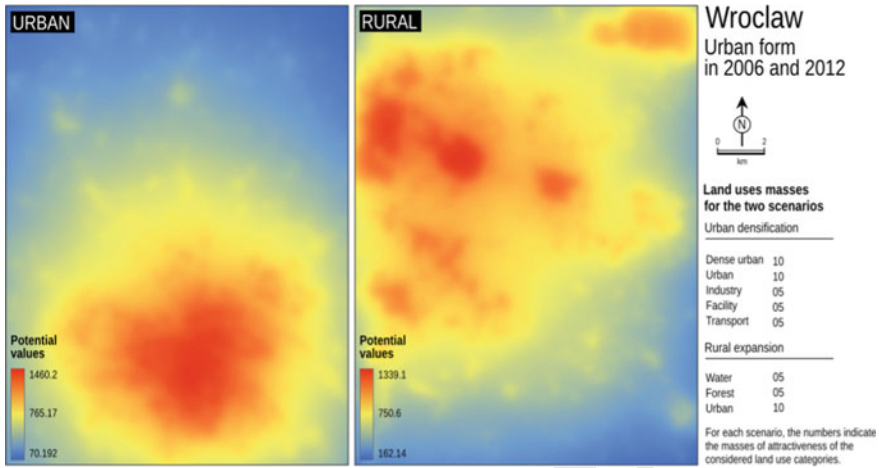


Fig. 5 Two contrasting realistic potential models

distance  $d_{ij}^\alpha$  separating those points from the one for which the calculation is made.  
The operative formula is defined as follows:

$$P_i = \sum_{j=1}^n \frac{m_j}{d_{ij}^\alpha}$$

Spatial constraints based on potential results were applied to the case study of Wroclaw by distinguishing two contrasting scenarios. The first scenario (S1) focuses on “Urban densification” and assigns significant masses to *Dense urban* and *Urban* ( $m = 10$ ) categories and medium masses to *Industry*, *Facility*, and *Transport* ( $m = 5$ ) categories. As a result, the potential map (Fig. 5, left) shows high potential for land use change around the heart of the city of Wroclaw. Areas with high potential for change—urbanization—are limited to very closed urban areas. The second scenario (S2) deals with “Rural expansion” and assigns significant masses to *Urban* ( $m = 10$ ) and medium masses to *Forest* and *Water* ( $m = 5$ ) categories. Therefore, in this case, the potential map (Fig. 5, right) highlights more sprawling areas with high potential for change, mainly in the northwestern part of the study area, relatively far from Wroclaw city center.

#### 4 Constraint Geographical CA

Based on the spatiotemporal constraints presented in Sect. 3, a new more integrated formalization of CA can be proposed. In this new formula, the state of a cell  $i$  at step  $t + 1$  still depends on the state of the cell at step  $t$  ( $c_{i,t}$ ) and the state of the cells in

the neighborhood ( $V_{i,t}^r$ ). This relation is clearly based on the classical CA definition defined par Tobler or Torrens (Sect. 2). But, it also integrates the MC results limiting land use development to within a number  $n_t$  of cells for each land use category. Moreover, the suitability of the simulations calculated by the CA is dependent on the cell's potential  $P_i$  based on land use attractiveness masses and distances. A synthetic expression of the model could be written

$$c_{i,t+1} = f(c_{i,t}, V_{i,t}^r, n_{l,t}, P_i)$$

where  $n_t$  = was defined in Sect. 3.1 and  $P_i$  was defined in Sect. 3.2.

This formula corresponds to the CA process integrated in the LucSim software and was applied to produce two contrasting and realistic scenarios in the case study of Wroclaw in 2006 and 2012. Scenario S1 seeks to densify the more urbanized areas in a pronounced manner. This urban development is thus concentrated around the previously densely built-up areas and seeks to fill the open space corresponding to fields or less dense urban categories. S2 is a peri-urban development scenario which takes place around villages relatively far from the most urbanized areas, where the fields have high potential for change into the less dense urban category.

Moreover, for each scenario, MCs make it possible to quantify future urbanization by 2030 by estimating the number of cells that change from a nonurban to an urban state with a distinction between two categories: urban and dense urban categories for the years 2018, 2024, and 2030 (Table 2). In a second step, two contrasting suitability maps (Fig. 5) constrain the spatial development according to the weighting of each land use category. The resulting suitability maps based on the potential model (Fig. 5) show two contrasting potentials for development: one that is more concentrated around the city core for S1, and the other that is more dispersed in the center and northern part of the case study for S2. Then in a third step, based on the results of steps 1 and 2, the AC could be run according to the three rules set out in Sect. 2.2.

As expected, S1 concentrates on urban development in the southern part of the study area around the core of Wroclaw. This concentration around the core is accompanied by a few outgrowths mainly in the northwestern and northeastern parts of the city. By contrast, S2 reveals a marked expansion in the urban category in the more rural northwestern part of the study area. Urbanization there is less dense and takes on more the form of urban sprawl. As can be seen from the example, LucSim makes it easy to simulate urban development scenarios and their consequences for urban forms, on the one hand reinforcing the compactness of the city and on the other fostering its expansion in rural areas. This kind of modeling process helps in inter-

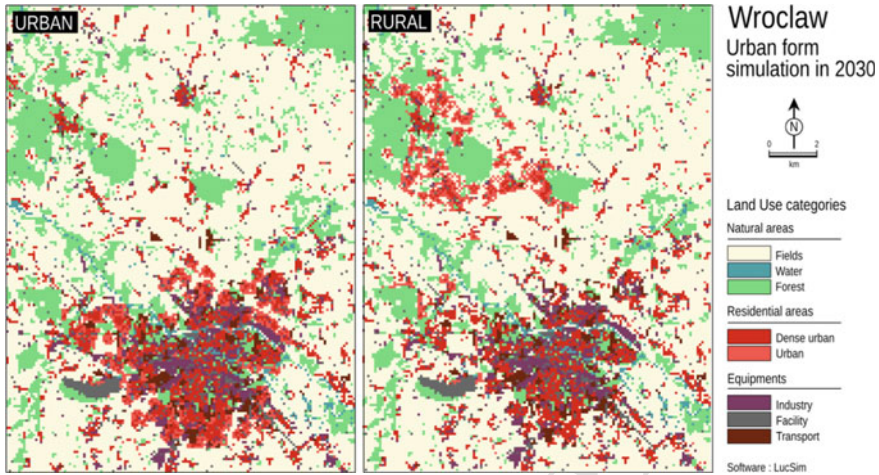


Fig. 6 Simulation of two contrasting scenarios by 2030

324 actively analyzing the direct consequences of spatial planning policies. Finally, it is  
 325 also important to emphasize that by integrating space and time constraints, the simu-  
 326 lation results are made much more realistic than in the total absence of constraints  
 327 (Fig. 6).

## 328 5 Discussion

329 The results presented in the previous section are certainly consistent, but they raise  
 330 a number of questions that call for discussion. First, the CA is driven by a dual  
 331 constraint system applied to the initial transition rules. However, although there is  
 332 no prior theoretical, conceptual, and formal incompatibility between the Markovian  
 333 model and the potential model, some inconsistencies may appear in its actual use.  
 334 For example, constraints derived from MCs, such as the transition rules themselves,  
 335 apply at the level of the land use categories. They are dependent on each transition  
 336 involving two-to-one states. On the other hand, the potential model produces a result  
 337 applicable for all of the categories. It is therefore independent of transitions and less  
 338 precise. One solution to overcome this problem would be not to calculate one single  
 339 potential model, but as many potential models as there are transitions by calibrating  
 340 each of them on masses of attractiveness corresponding actually and more precisely to  
 341 this transition. Such a solution might be attractive in theory, but in practice, it makes  
 342 modeling increasingly more complex by multiplying the problems of calibration,  
 343 which were already questionable in the example discussed here.

344 The calibration of models also raises a second set of questions. Among the mod-  
 345 els used here, only MCs can be considered “autonomous” since they automatically

346 produce results from two original images. The potential model and the definition of  
347 the transition rules require the use of expert knowledge. In the current state of our  
348 knowledge about the growth of urban forms, such recourse is a qualitative input that  
349 does not guarantee the reproducibility of the results produced by the modeling in  
350 any way. Moreover, insofar as these parameters are not directly derived from math-  
351 ematical calculation, they appear questionable. This fundamental point is obviously  
352 a limitation for the modeling exercise and for the application of mathematical tools  
353 for forecasting urban forms. To overcome this problem, many authors have proposed  
354 to use “machine learning” approaches so that transition rules are automatically gen-  
355 erated based on known past states. Indeed, recent work simulates transitions using  
356 decision trees (Samardžić-Petrović et al. 2015) or artificial neural networks (Li and  
357 Yeh 2002; Almeida et al. 2008; Tayyebi et al. 2011). Although still very exploratory,  
358 the results obtained so far seem promising and, through artificial intelligence pro-  
359 cesses, they offer an additional step to mathematical and geographical modeling.

360 But finally, these recent developments also raise the question of the importance  
361 of expert knowledge in forecasting urban growth. The results produced in this field  
362 are often considered as “images of the future” leading to a collective reflection on  
363 the future of territories, rather than final results. Given our difficulty in predicting the  
364 future, and the fact that it is very unlikely that this future will be a mere reproduction  
365 of the past, it is clear that these results will probably be wrong in the long run (Antoni  
366 2016). Consequently, using expert knowledge to involve local actors in defining a  
367 common future does not seem completely absurd. In essence, these reflections on  
368 CA calibration ask how far the constraint should be contained. Depending on their  
369 objectives, anyone can define the level of constraint they wish to apply to simulate  
370 land use change, from a calibration entirely defined by mathematical models or totally  
371 derived from expert knowledge. The best way might be a mixed approach combining  
372 both machine learning and expert knowledge.

## 373 6 Conclusion

374 After having shown the necessity for constraining CA in the study of urban forms,  
375 this paper has proposed a mathematical formalization for specific geographic cellu-  
376 lar automata (GCA) implemented in LucSim software. At this level of detail, such  
377 a formalization adapted to the social sciences is rare in the literature and therefore  
378 appears as one of the fundamental originalities of this chapter. In particular, it aims  
379 to link the cellular design of geographical space, the Markovian approach to tran-  
380 sition processes, the distance weighting included in gravity models (potential), and  
381 the phenomenon of emergence that defines artificial intelligence models in a single  
382 formal notation. That’s not so bad! In addition, the results produced using this set  
383 of methods and models appear quite realistic and are able to correctly reproduce a  
384 credible process of urban growth. But at the same time, this reproduction remains  
385 open to the intricacy of planning scenarios and allows us to consider a wider use




386 of CA in the framework of a more operational territorial forecast. And that's even  
387 better!

388 Finally, in a more general way, this chapter also shows that it is worth transferring  
389 methods developed in mathematics, physics, computer sciences, or mathematics to  
390 the social sciences. This transfer obviously requires a substantial effort of abstraction  
391 and what may be considerable investment for researchers or developers who are not  
392 immediately comfortable with mathematical tools. But since the quantitative revolu-  
393 tion started by geographers in the 1960s (Burton 1963), this approach is currently  
394 enabling us to work the latest advances in artificial intelligence for decision support  
395 into urban and land use planning.

## 396 References

- 397 Abler, R., Adams, J.S., Gould, P. (1972) *Spatial Organization. The Geographer's View of the World*,  
398 Prentice/Hall International, 587 p.
- 399 Almeida, C., Gleriani, J., Castejon, E., Soares-Filho, B. (2008) Using neural networks and cellular  
400 automata for modelling intra-urban land-use dynamics. *International Journal of Geographical*  
401 *Information Science*, 22(9), 943–963.
- 402 Antoni, J.P. (2008) *Lexique de la ville*, Ellipses, 184 p.
- 403 Antoni, J.P. (2016) *Concepts, méthodes et modèles pour l'aménagement et les mobilités: l'aide à*  
404 *la décision face à la transition eco-energetique*. Economica, 250 p.
- 405 Antoni, J.P., Judge, V., Vuidel, G., Klein, O. (2017) "Constraint Cellular Automata for Urban Devel-  
406 opment Simulation: An Application to the Strasbourg-Kehl Cross-Border Area." In: Camacho  
407 Olmedo M.T., Paegelow M., Mas J.F., Escobar F. (Eds.) *Geomatic Approaches for Modeling Land*  
408 *Change Scenarios*, Springer, pp. 293–306.
- 409 Arsanjani, J.J., Helbich, M., Kainz, W., Ali Darvishi, B. (2013) Integration of logistic regression,  
410 Markov chain and cellular automata models to simulate urban expansion. *International Journal*  
411 *of Applied Earth Observation and Geoinformation*, 21, 265–275.
- 412 Batty, M. (1997) Cellular automata and urban form: A primer. *Journal of the American Planning*  
413 *Association*, 63:2, 266–274.
- 414 Batty, M. (2007) *Cities and Complexity: Understanding Cities with Cellular Automata, Agent-Based*  
415 *Models, and Fractals*, MIT Press, 592 p.
- 416 Benenson, I., Torrens, P.M. (2004) *Geosimulation: Automata-Based Modelling of Urban Phenom-*  
417 *ena*. John Wiley, London, UK.
- 418 Berchtold, A. (1998) *Chaînes de Markov et modèles de transition. Applications aux sciences*  
419 *sociales*. Hermes Science Publications, Paris.
- 420 Burton, I. (1963) The quantitative revolution and theoretical geography. *The Canadian Geographer*,  
421 7, 151–162.
- 422 Clarke, K.C. (2008) *A Decade of Cellular Urban Modeling with SLEUTH: Unresolved Issues and*  
423 *Problems*, Ch. 3 in *Planning Support Systems for Cities and Regions* (Ed. Brail R. K., Lincoln  
424 Institute of Land Policy), Cambridge, MA, pp 47–60.
- 425 Conway, J. (1970) Mathematical games, *Scientific American*, October, 120–127.
- 426 Couclecis, H. (1985) Cellular world: A framework for modelling micro-macro dynamics. *Environ-*  
427 *ment and Planning A*, 17, 585–596.
- 428 Couclecis, H. (1988) Of mice and men: What rodent populations can teach us about complex spatial  
429 dynamics. *Environment and Planning A*, 20, 99–109.
- 430 Feller, W. (1968) *An Introduction to Probability Theory and Its Applications*. 3rd edition. Wiley.

- 431 Helvig, M. (1964) *Chicago's external Truck Movements: Spatial Interactions between the Chicago*  
 432 *Area and its Hinterland*, University of Chicago, Department of Geography, Research paper, 90,  
 433 132 p.
- 434 Li, X., Yeh, A.G.O. (2002) Neural-network-based cellular automata for simulating multiple land use  
 435 changes using GIS. *International Journal of Geographical Information Science*, 16(4), 323–343.
- 436 Neumann (von), J. (1963) The General and Logical Theory of Automata. *Collected Work*, no. 5,  
 437 Taub, A.H.
- 438 Samardžić-Petrović, M., Dragičević, S., Bajat, B., Kovačević, M. (2015) Exploring the decision  
 439 tree method for modelling urban land use change. *Geomatica*, 69(3), 313–325.
- 440 Tayyebi, A., Pijanowski, B.C., Tayyebi, A.H. (2011) An urban growth boundary model using neural  
 441 networks, GIS and radial parameterization: An application to Tehran, Iran. *Landscape and Urban*  
 442 *Planning*, 100(1), 35–44.
- 443 Tobler, W.R. (1979) "Cellular Geography." In: Gale S., Olsson G., *Philosophy in Geography*. Reidel  
 444 Pub., Dordrecht, Holland, pp. 379–386.
- 445 Torrens, P.M. (2000) How cellular models of urban systems work (1. Theory), *CASA Working Paper*  
 446 *Series*.
- 447 White, R., Engelen, G. (1993) Cellular automata and fractal urban form: A cellular modelling  
 448 approach to the evolution of urban land use patterns. *Environment and Planning A*, 25, 1175–1199.
- 449 Wolfram, S. (1983) Statistical mechanics of cellular automata. *Review of Modern Physics*, 55,  
 450 601–644.
- 451  Wolfram, S. (2002) *A New Kind of Science*. Champaign, 549 p.