A bond graph representation of an optimal control problem: the example of the DC motor

Wilfrid Marquis-Favre, Bogdan Chereji, Daniel Thomasset, Serge Scavarda

HAL Id: hal-02084745
https://hal.science/hal-02084745

Submitted on 29 Mar 2019

A bond graph representation of an optimal control problem: the example of the DC motor

Wilfrid Marquis-Favre   Bogdan Chereji   Daniel Thomasset
Serge Scavarda
Laboratoire d'Automatique Industrielle,
Institut National des sciences Appliquées de Lyon,
Bât. St Exupéry,
25, avenue Jean Capelle,
F-69621 Villeurbanne Cedex,
e-mail: firstname.lastname@insa-lyon.fr

Abstract

Optimization has become of great importance in present day Systems Engineering. Also, bond graph language proves to be a very efficient tool for modelling, analysing and designing mechatronic systems from an energy and dynamic point of view. The idea presented in this paper is to combine an optimization formulation in the optimal control context with bond graph language. The objective is to transpose the optimization problem into bond graph formalism so that its exploitation will solve this optimization problem. This paper, being a primary investigation, restricts itself to an optimal control problem formulation applied to the example of the classical DC motor. Developments are based on the use of Pontryagin's classical Maximum Principle where the cost function is expressed as the integral of a quadratic form of the state space vector and the control input to be determined. This type of expression may correspond to a certain energy loss–minimization. It is shown that the formulation of this optimal control problem in bond graph makes logical use of the adjoint system concept. Later variable mapping enables an augmented bond graph representation of the whole problem to be set up. Finally the bicausality assignment to this augmented bond graph representation furnishes the solution to the optimal control problem under consideration.

Keywords: Pontryagin's Maximum Principle, adjoint system, bond graph, optimal control, bicausality.

1   Introduction

The motivation for introducing optimization formulation into bond graph language is in the perspective of coupling this to other methodologies which use bond graph. One such methodology has been the sizing of mechatronic systems on dynamics and energy criteria developed at the Laboratoire d'Automatique Industrielle [1, 2, 3, 4]. This coupling will enable, for example, multivariable control, optimal control, and sizing with energy minimization to be undertaken.

As a first step we focuse our work on an optimal control formulation. The developments used for the bond graph implementation are based on Pontryagin's Maximum Principle (e. g. [5, 6, 7]). This is explained by the fact that we are dealing with dynamic systems represented by bond graph. Thus optimization is viewed as a dynamics optimization and the modelling is carried out from an energy point of view.

The developments presented in this paper are in an initial stage of a work that requires far more investigation, so the optimization formulation is restricted to a simple category of optimal control problems. The objective is to show the potentiality for a systematization of bond graph construction representing the optimal control formulation and the model for which the control is designed. As a result of this approach, a bond graph representation furnishing the optimal control law is the objective and this will enable the analytical developments to be *a priori* skipped.

As mentioned previously, the bond graph construction is established at this point for limited problem cases, i. e. optimal control for linear SISO systems having a dissipative cost function. To be more precise, having as the input the bond graph of the system subjected to a desired optimal control and the specified type cost function, the procedure will be able to synthesize a bond graph corresponding to the particular optimization problem. As the procedure is only in the initial stage, the method has been developed using the example of the DC motor and the analytical calculus is performed with a view to extrapolating the procedure under research.

Section 2 briefly recalls the key steps of Pontryagin's Maximum Principle application. In section 3, with the DC motor as a basis for the developments, the particular optimal control considered in this paper is formulated analytically and through bond graph language in parallel. Section 4 synthesizes a conjectural procedure and summarizes the conditions of how the procedure can be applied. Finally a conclusion discusses the key issues and the perspectives for future developments.

2 Recall on Pontryagin's Maximum Principle

Even though the fact that Pontryagin's Maximum Principle is a classical optimal control technique, we take the liberty of recalling the application of this principle. The basic formulation can be summarized as follows [5, 7]. Consider the state space model (1) and the integral performance index or cost function to minimize (2).

$$\dot{\boldsymbol{x}} = \boldsymbol{f}\left(\boldsymbol{x}, u, t\right) \tag{1}$$

$$J = \int_{t_0}^{t_f} \mathcal{L}\left(\boldsymbol{x}, u, t\right) dt \tag{2}$$

where $\boldsymbol{x}$ is the state space vector of dimension n, u, here supposed unbounded, is the control to be determined (here a scalar for a SISO system) and $\mathcal{L}$ is supposed to be a positive definite function of $\boldsymbol{x}$ and u. The integral performance index minimization is taken between the given initial state $\boldsymbol{x}_0 = \boldsymbol{x}\left(t_0\right)$ at time t_0 and final state $\boldsymbol{x}_f = \boldsymbol{x}\left(t_f\right)$ at time t_f.

The first step of Pontryagin's method is to form the Pontryagin H function given by equation (3).

$$H\left(\boldsymbol{x}, \boldsymbol{\lambda}, u, t\right) = \mathcal{L}\left(\boldsymbol{x}, u, t\right) + \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{f}\left(\boldsymbol{x}, u, t\right) \tag{3}$$

where $\boldsymbol{\lambda}$ is the vector of co-state variables (or covariant vector), usually called Lagrange multipliers of the associated constrained variational problem. The superscript 'T' denotes the symbol of a matrix or vector transpose.

Then the optimal control u^0 is obtained from the partial differentiation of H with respect to u (equation 4).

$$u^0\left(\boldsymbol{x}, \boldsymbol{\lambda}, t\right) \text{ solution of } \frac{\partial H\left(\boldsymbol{x}, \boldsymbol{\lambda}, u, t\right)}{\partial u} = 0 \tag{4}$$

where the superscript '0' denotes optimal values.

From the optimal control u^0 the optimal H^0 function can be computed (equation 5).

$$H^0\left(\boldsymbol{x}, \boldsymbol{\lambda}, t\right) = H\left(\boldsymbol{x}, \boldsymbol{\lambda}, u^0, t\right) \tag{5}$$

Finally a set of 2n first order differential equations (6) provide the optimal solution for $\boldsymbol{x}$ (n–state equations) and $\boldsymbol{\lambda}$ (n–costate equations) with the boundary conditions $\boldsymbol{x}_0$ and $\boldsymbol{x}_f$. This solution can then be substituted into the expression of u^0 to obtain the optimal control solution.

$$\begin{cases} \dot{\boldsymbol{x}} = \frac{\partial H^0(\boldsymbol{x},\boldsymbol{\lambda},t)}{\partial \boldsymbol{\lambda}} \\ \dot{\boldsymbol{\lambda}} = -\frac{\partial H^0(\boldsymbol{x},\boldsymbol{\lambda},t)}{\partial \boldsymbol{x}} \end{cases} \tag{6}$$

The above recalled method is now used in the next section as a basis for the bond graph language formulation of our given optimal control problem.
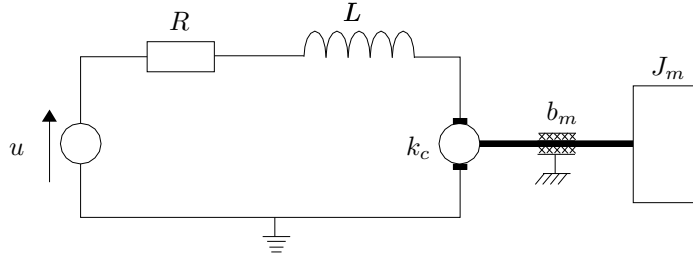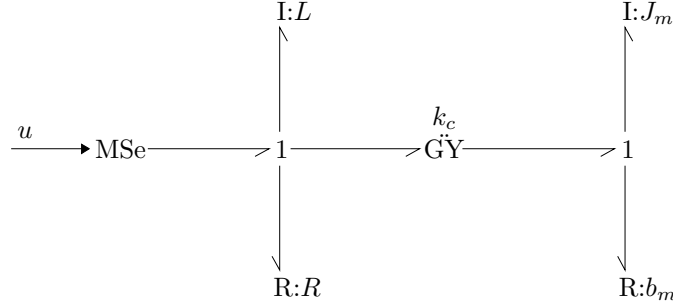
Figure 1: DC motor schema

Figure 2: DC motor bond graph representation

3   Bond graph optimal control formulation

The transposition of an optimal control problem into bond graph language is presented by the figure 1 DC motor example. Its bond graph representation is given in figure 2. This very simple example has been chosen to better understand the whole bond graph generation mechanism. It consists of the armature electrical circuit composed of a voltage source u, a resistance R and an inductance L. Electromechanical coupling is characterized by the torque constant k_c and, on the mechanical side, we only consider the rotor inertia J_m and the viscous friction on the rotor (parameter b_m). The model is linear and in optimal control context, we aim at determining u with the cost function (7) that corresponds to the input and some dissipative energy minimization (on the electrical side for this example).

$$J = \int_{t_0}^{t_f} \left(\frac{u^2}{R_x} + P_{d,elec} \right) dt \tag{7}$$

where $P_{d,elec}$ is the electrical power dissipation and R_x a certain input factor. At this point the electrical dissipation choice was strictly arbitrary and either mechanical or both electrical and mechanical could have been chosen.

The performance index 7 takes into account the effort to be furnished at the input of the system through a quadratic term weighted by the input factor R_x. We implement an extra R–element that does not change the DC motor model on its own but takes into account the corresponding performance index term in the figure 3 bond graph representation.

On the figure 3 bond graph representation basis we now develop the optimal control calculation by applying Pontryagin's method (recalled in section 2). First the state space model (8) is obtained by applying the classical integral causality assignment procedure [8, 9]. Since this step is not a key issue in the development of the present paper, we do not detail the way of obtaining the equation system and the reader is referred to the previous bibliography.

$$\begin{cases} \dot{p}_1 = -\frac{R}{L}p_1 - \frac{k_c}{J_m}p_2 + u \\ \dot{p}_2 = \frac{k_c}{L}p_1 - \frac{b_m}{J_m}p_2 \\ y = \frac{1}{L}p_1 + \frac{u}{R_x} \end{cases} \tag{8}$$
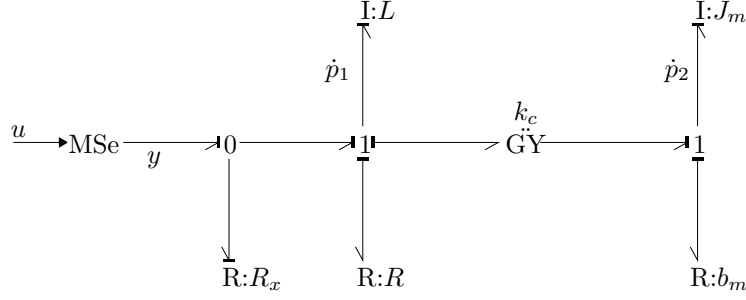
3

Figure 3: Bond graph representation in integral causality of the DC motor with power converter loss

where the energy storage variables p_1 and p_2 of the I–elements, respectively in electrical and mechanical domains, are the state variables, and the output variable y represents the power conjugate variable of u on the figure 3 bond graph MSe element.

From the cost function (7) and the dynamic system (8) the corresponding Pontryagin function (9) is constructed.

$$H = \lambda_1 \left(-\frac{R}{L}p_1 - \frac{k_c}{J_m}p_2 + u \right) + \lambda_2 \left(\frac{k_c}{L}p_1 - \frac{b_m}{J_m}p_2 \right) + \frac{u^2}{R_x} + \frac{R}{L^2}p_1^2 \tag{9}$$

where power of dissipation in the electrical circuit has been expressed in the state variable term p_1 from the figure 3 integral bond graph.

From the control equation (4) we obtain the optimal control u^0, then the H^0 function and finally from the equation system (6) we can express the optimal control system consisting of two state and two costate equations (10)–(12).

$$u^0 = -\frac{R_x}{2}\lambda_1 \tag{10}$$

$$H^0 = \lambda_1 \left(-\frac{R}{L}p_1 - \frac{k_c}{J_m}p_2 - \frac{R_x}{2}\lambda_1 \right) + \lambda_2 \left(\frac{k_c}{L}p_1 - \frac{b_m}{J_m}p_2 \right) + \frac{R_x}{4}\lambda_1^2 + \frac{R}{L^2}p_1^2 \tag{11}$$

$$\begin{cases} \dot{p_1} = \frac{\partial H^0}{\partial \lambda_1} = -\frac{R}{L}p_1 - \frac{k_c}{J_m}p_2 - \frac{R_x}{2}\lambda_1 \\ \dot{p_2} = \frac{\partial H^0}{\partial \lambda_2} = \frac{k_c}{L}p_1 - \frac{b_m}{J_m}p_2 \\ \dot{\lambda_1} = -\frac{\partial H^0}{\partial p_1} = -\frac{2R}{L^2}p_1 + \frac{R}{L}\lambda_1 - \frac{k_c}{L}\lambda_2 \\ \dot{\lambda_2} = -\frac{\partial H^0}{\partial p_2} = \frac{k_c}{J_m}\lambda_1 + \frac{b_m}{J_m}\lambda_2 \end{cases} \tag{12}$$

The equation (12) system together with the boundary conditions $p_{10} = p_1(t_0)$, $p_{20} = p_2(t_0)$, $p_{1f} = p_1(t_f)$, and $p_{2f} = p_2(t_f)$ provide the optimal state solution to the optimal control problem given by the example of the DC Motor. At this point it is convenient to put system (12) into the matrix form (13).

$$\begin{pmatrix} \dot{p_1} \\ \dot{p_2} \\ \dot{\lambda_1} \\ \dot{\lambda_2} \end{pmatrix} = \begin{pmatrix} -\frac{R}{L} & -\frac{k_c}{J_m} & -\frac{R_x}{2} & 0 \\ \frac{k_c}{L} & -\frac{b_m}{J_m} & 0 & 0 \\ -\frac{2R}{L^2} & 0 & \frac{R}{L} & -\frac{k_c}{L} \\ 0 & 0 & \frac{k_c}{J_m} & \frac{b_m}{J_m} \end{pmatrix} \cdot \begin{pmatrix} p_1 \\ p_2 \\ \lambda_1 \\ \lambda_2 \end{pmatrix} \tag{13}$$

Equation (13) clearly shows the part appearance of the adjoint of the DC motor model [10, 11] (second block 2×2 sub–matrix on the main diagonal). This is in fact classical in a dynamic optimization problem formulated with state space equation constraints. The above remark constitutes the first important rendezvous with the bond graph formulation of the optimal control problem presented. Based on recent works aiming at building the bond graph representation of a linear model adjoint system [12, 13], it is possible at this stage to associate a generic bond graph representation to the 2×2 costate block matrix of equation (13). Detailing the procedure
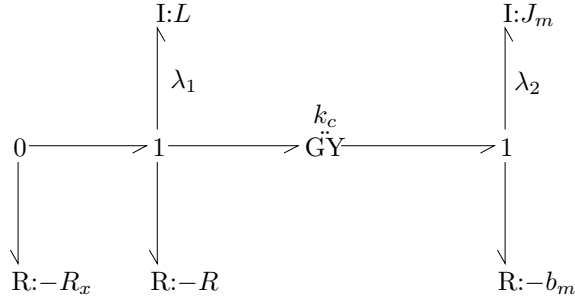
Figure 4: Bond graph representation of the DC motor adjoint system

is out of the scope of this paper and the reader is referred once again to the bibliography given previously. The most important steps to retain in our objective are (i) that the state variables of the adjoint system are coenergy variables; (ii) that the bond graph junction structure and the I–, C– and R–elements are conserved mirroring the original bond graph; and (iii) that each dissipative element changes its parameter sign. For the DC motor treated here, this provides the figure 4 bond graph representation.

Before going further into the bond graph implementation, we undertake variable mapping (14) since it is more traditional to read the bond graph in terms of energy variables. This does not change the figure 4 bond graph representation and the optimal state space representation is now given by the matrix equation (15). Since the figure 4 bond graph results from an optimization formulation, we propose to call it the optimizing bond graph from now on.

$$\begin{cases} p_{\lambda_1} = L\lambda_1 \\ p_{\lambda_2} = J_m\lambda_2 \end{cases} \tag{14}$$

$$\begin{pmatrix} \dot{p_1} \\ \dot{p_2} \\ \dot{p_{\lambda_1}} \\ \dot{p_{\lambda_2}} \end{pmatrix} = \begin{pmatrix} -\frac{R}{L} & -\frac{k_c}{J_m} & -\frac{R_x}{2L} & 0 \\ \frac{k_c}{L} & -\frac{b_m}{J_m} & 0 & 0 \\ -\frac{2R}{L} & 0 & \frac{R}{L} & -\frac{k_c}{J_m} \\ 0 & 0 & \frac{k_c}{L} & \frac{b_m}{J_m} \end{pmatrix} \cdot \begin{pmatrix} p_1 \\ p_2 \\ p_{\lambda_1} \\ p_{\lambda_2} \end{pmatrix} \tag{15}$$

The second key rendezvous with bond graph formulation is now imminent. From equation 15 there are two issues that deserve our attention. The first one is related to the bottom left 2×2 submatrix that clearly represents a coupling between the original and its adjoint systems. We propose to transpose this coupling into bond graph language by a 2–port R–element characterized by the matrix equation (16) and replacing the two 1–port R–elements respectively characterized by R and $-R$. In this way the 2–port R–element has one port connected to the original system bond graph and the second one to the optimizing bond graph. At this point, it is worthwhile noting that the dissipative phenomenon associated with the multiport R–element is the one involved in the cost function which was considered at the beginning. Taking account of the mechanical dissipation in the performance index would involve the two mechanical side 1–port R–elements. Moreover the changed sign for the coefficient $2R$ comes from the junction structure involved, namely from the 1-junction that re–introduces a negative sign agreeing with the matrix coefficient $-\frac{2R}{L}$ of equation (15).

$$\begin{pmatrix} e_{sys} \\ e_{opt} \end{pmatrix} = \begin{pmatrix} R & 0 \\ 2R & -R \end{pmatrix} \cdot \begin{pmatrix} f_{sys} \\ f_{opt} \end{pmatrix} \tag{16}$$

where the subscripts '$_{sys}$' and '$_{opt}$' denote respectively the original system and the optimizing bond graph representations.

The second issue of equation 15 concerns the top right 2×2 submatrix. In order to transpose this second coupling into bond graph language between the original and its adjoint systems, it is more convenient to come a step backward in the application of Pontryagin's principle. As presented in section 2 the optimal system (6) is expressed after having obtained the optimal control u^0 from (4) and re–introduced it into the Pontryagin

H function. Chronologically speaking the variational approach from which Pontryagin's Maximum Principle is issued shows that we should initially consider the set of state and co–state equations together with the Euler equation with respect to u (equations 17) [5, 7]. It can be shown that the presentation in section 2 is strictly equivalent but for a matter of convenience we now exploit form (17).

$$\begin{cases} \dot{\boldsymbol{x}} = \frac{\partial H(\boldsymbol{x}, \boldsymbol{\lambda}, u, t)}{\partial \boldsymbol{\lambda}} \\ \dot{\boldsymbol{\lambda}} = -\frac{\partial H(\boldsymbol{x}, \boldsymbol{\lambda}, u, t)}{\partial \boldsymbol{x}} \\ \frac{\partial H(\boldsymbol{x}, \boldsymbol{\lambda}, u, t)}{\partial u} = 0 \end{cases} \tag{17}$$

The application to the DC motor gives equations (18) and (19). While the input u role is already expressed in the DC motor bond graph (figure 3), these equations show that the bond graph transposition of the top right 2×2 matrix coupling in equation (15) is now transferred onto the bond graph formulation of the Euler equation with respect to u (equation 19).

$$\begin{pmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_{\lambda_1} \\ \dot{p}_{\lambda_2} \end{pmatrix} = \begin{pmatrix} -\frac{R}{L} & -\frac{k_c}{J_m} & 0 & 0 \\ \frac{k_c}{L} & -\frac{b_m}{J_m} & 0 & 0 \\ -\frac{2R}{L} & 0 & \frac{R}{L} & -\frac{k_c}{J_m} \\ 0 & 0 & \frac{k_c}{L} & \frac{b_m}{J_m} \end{pmatrix} \cdot \begin{pmatrix} p_1 \\ p_2 \\ p_{\lambda_1} \\ p_{\lambda_2} \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot u \tag{18}$$

$$\lambda_1 + \frac{2}{R_x} u = 0 \tag{19}$$

Here now is the third important rendezvous with the bond graph formulation of our optimal control problem. Concerning the electrical dissipative phenomenon, we propose to introduce again a 2–port R–element replacing the two 1–port R–elements respectively characterized by R_x and $-R_x$. This 2–port R–element has the same constitutive matrix structure (equations 20). It has one port connected to the original system bond graph and one port connected to the optimizing bond graph.

$$\begin{pmatrix} e_{sys} \\ e_{opt} \end{pmatrix} = \begin{pmatrix} R_x & 0 \\ 2R_x & -R_x \end{pmatrix} \cdot \begin{pmatrix} f_{sys} \\ f_{opt} \end{pmatrix} \tag{20}$$

With this 2–port R–element and on the optimizing bond graph side it is now essential, so as not to change the adjoint system state space, to impose a null effort (corresponding to e_{opt} in equation 20) onto the 0–junction of the figure 4 bond graph. It only remains to transpose the Euler condition (19) with respect to u (corresponding to e_{sys} in equation 20). It is not difficult to see that, combined to the previous imposed null effort, forcing the 2–flow balance onto the same 2–port 0–junction enables condition (19) to be verified since with equation (20), $u = e_{sys} = R_x f_{sys}$ and $0 = e_{opt} = 2R_x f_{sys} - R_x f_{opt} = 2R_x f_{sys} + R_x \lambda_1$. The bond graph element that enables both a null effort and a 2–flow balance to be imposed onto a 2–port 0–junction is a double source null effort and null flow. It is connected to the 0–junction of the figure 4 bond graph. This type of element initializes a bicausality [14, 15] propagation in the bond graph and thus requires the presence of a double detector [2, 3, 4]. In the mathematical formulation of the optimal control design problem, the role of the control u is changed into an output and the power conjugate variable y keeps its output role, thus the double detector replaces the original MSe element in the figure 3 bond graph.

Finally the compilation of the three bond graph formulation rendezvous enables the figure 5 bond graph representation to be implemented. Here the figure 4 bond graph has been horizontally mirrored for obvious graphical convenience reasons. The bicausal exploitation of the figure 5 bond graph enables the optimal system and the optimal control to be computed.

Now if we can develop a procedure that automatically derives this bond graph, all Pontryagin's method developments can be skipped. The next section proposes an attempt towards a systematic procedure for generating the bond graph related to this type of optimal control problem.
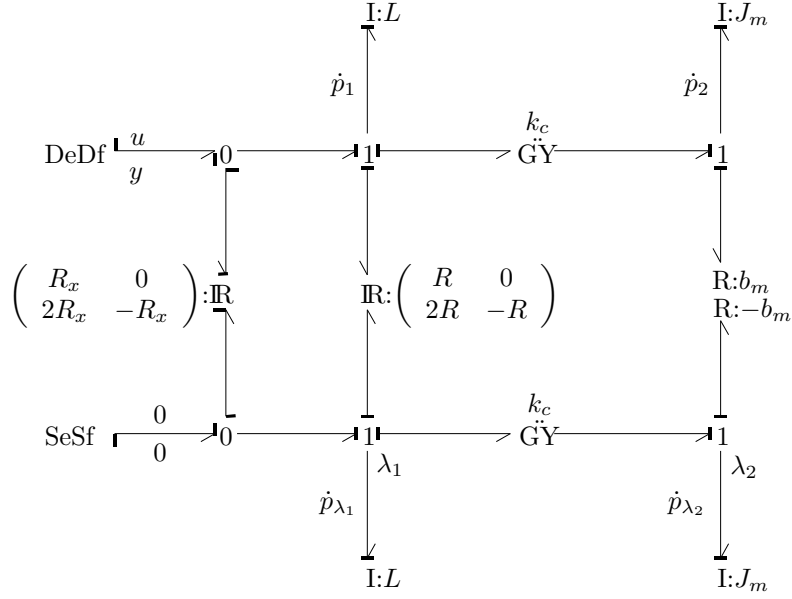
Figure 5: Bond graph representation of the DC motor optimal control problem

4 Towards a procedure

From the developments presented in the previous section, the three bond graph formulation rendezvous can specify certain steps towards a potential procedure. This procedure aims at automatically generating the bond graph representation of an optimal control problem given the bond graph representation of the original system for which the optimal control is designed. So this optimal control bond graph representation would be exploitable for directly determining the optimal control solution without developing the analytical steps of Pontryagin's method. However, due to the relative novelty of the approach presented, it will be essential to recall its application conditions. These different steps are now proposed.

Given:

- a linear time invariant model of a SISO system and its bond graph representation,
- the input control to be designed with respect to a dissipative– and control–based quadratic cost function which is the object of the minimization, and
- the fixed-boundary conditions for the time and state space,

the following steps enable the bond graph representation of the given optimal control problem to be obtained.

Steps: Optimal control bond graph construction

1. Establish the partial adjoint bond graph representation (hereafter called optimizing bond graph) by taking the junction structure, the I–, C–, and R–elements of the original bond graph representation and by reversing the R–element parameter signs. At this point, the energy source element associated to the control disappears from this bond graph representation.

2. For each dissipative phenomenon involved in the given cost function, replace both corresponding 1–port R–elements respectively in the original system and the optimizing bond graph by a coupling 2–port R–element with the constitutive matrix (21).

$$\left(\begin{array}{cc} R & 0 \\ 2R & -R \end{array} \right) \tag{21}$$

where R is the parameter characterizing the original dissipative phenomenon, the first row and column are associated to the power conjugate variables on the original system bond graph side port, and the second row and column are associated to the power conjugate variables on the optimizing bond graph side port.

At this point, it is important to have, at the beginning, taken into account the R–element corresponding to the input factor.

3. Replace the source element in the original system bond graph by a double detector and mirror it with a double source at the same place on the optimizing bond graph. The double source imposes both null effort and flow.

4. Assign bicausality to the bond graph obtained. The analytical exploitation of the bicausal bond graph representation obtained provides the system equation and the optimal control solutions to the initial problem.

End of steps

At this point, several remarks deserve our attention. It is worthwhile noting that the coupling R–element has a generic structure and is related to the cost function under consideration. This provides a certain systematization and flexibility of the step 2 multiport R–element implementation with respect to dissipative energy minimization. Also the bond graph representation obtained is a basis for other bicausal exploitations and thus several possible formulations of optimal control solutions. For instance, figure 5 bicausal bond graph shows the bicausal path going through the left–hand side multiport R–element. A second possibility can be investigated for this example with the bicausal path going through the right–hand side multiport R–element.

Concerning the application of Pontryagin's method, it is important to recall that formulation provides only a necessary optimality condition. In order to determine the nature of optimisation, i. e., whether it is a minimum or a maximum, the corresponding sufficient conditions must be investigated with the second variation of the integral performance index (2) [7]. Moreover the application conditions of the steps are relatively restrictive for the moment. Although the authors are convinced of the potentiality of the approach proposed, the application of the procedure is restricted to the following conditions, (i) the linear time invariant systems represented by bond graph with no dependent storage elements, (ii) the dissipative–based cost function, (iii) boundary conditions involving fixed–final time and fixed–final state.

5 Conclusion

This paper presents an approach for building a bond graph representation of a certain category of optimal control problem within a given hypothetical framework. Bicausal bond graph exploitation then provides the optimal system and the control solution. The appearance of the bicausality concept is not surprising since, in the end, an optimal control design problem may be viewed as an inverse problem [3, 4, 16].

We insist on the fact that the steps presented for the optimal control problem within bond graph language is conjectural and at this moment work is being carried out for proving its effectiveness completely. However it seemed interesting to the authors to present this first step towards an optimization problem formulation into bond graph language. This is the very first step but it has opened a number of doors for future developments and the integration of many concepts associated to dynamic optimization in bond graph language. Basically a constant in the optimisation formulation under consideration is the state space differential equation constraints that must be taken into account. Thus the term $\boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{f}(\boldsymbol{x}, u, t)$ will always be present in the Pontryagin function (3). This corresponds to an adjoint model coupled to the initial one. This feature is clearly systematic and the corresponding bond graph is generic at least for linear time invariant models. Further investigation must be carried out with time variant and nonlinear models to obtain a bond graph set up for an optimal control problem.

For the type of cost functions dealt with in this paper, namely those based on dissipation minimization, the coupling is reflected by a multiport R–element with a given structure. For a more general optimization objective (like for instance that with minimal time objective), this point will have to be worked out.

Also various boundary conditions will have to be investigated. They will generally add extra boundary conditions that have to be considered from the bond graph representation point view. The work presented in

this paper provides an analytical framework for optimization formulation using bond graph but one point that has not been discussed is the numerical simulation. Due to the boundary conditions (namely both the initial and final conditions), optimization problems make numerical methods dedicated to the initial value problem totally inefficient and this will lead to investigation of specific codes for adapting bond graph tools. This is another research perspective.

References

[1] Fotsu-Ngwompo, R.: *Contribution au Dimensionnement des Systèmes sur des Critères Dynamiques et Energétiques - Approche par Bond Graph*. Ph. D. Thesis: Institut National des Sciences Appliquées de Lyon, 1997.

[2] Fotsu-Ngwompo, R., Scavarda, S.: Dimensioning Problems in System Design Using Bicausal Bond Graphs. *Simulation Practice and theory*, vol. 7, 1999, pp. 577–587.

[3] Fotsu-Ngwompo, R., S. Scavarda, Thomasset, D.: Physical Model-Based Inversion in Control Systems Design Using Bond Graph Representation. Part 1: Theory. *Proceedings of the* IMECHE *Part I Journal of Systems and Control Engineering*, vol. 215, 2001, pp. 95–103.

[4] Fotsu-Ngwompo, R., S. Scavarda, Thomasset, D.: Physical Model-Based Inversion in Control Systems Design Using Bond Graph Representation. Part 2: Applications. *Proc. of the* IMECHE *Part I Journal of Systems and Control Engineering*, vol. 215, 2001, pp. 105–112.

[5] Schultz D. G., Melsa J.: *State Functions and Linear Control Systems*, ed. McGraw Hill Book Company, 1967.

[6] Takahashi Y., Rabins M., Auslander D.: *Control and Dynamic Systems*, ed. Addison–Wesley, 1972.

[7] Naidu D. S.: *Optimal Control Systems*, CRC Press, 2003.

[8] Karnopp, D.C., Margolis, D.L., Rosenberg, R.C.: *System Dynamics : Modeling and Simulation of Mechantronic Systems*. John Wiley & Sons, New York, 2000.

[9] Dauphin-Tanguy, G.: *Les Bond Graphs*. Herms Science Publications, Paris, 2000.

[10] Zadeh, L. A., Desoer, C. A.: *Linear System Theory*. McGraw Hill Book Company, New York, 1963.

[11] Kailath, T.: *Linear Systems*. Prentice Hall, Englewood Cliffs, NJ, 1980.

[12] Xia, X.: *Contribution à l'Analyse du Comportement Dynamique et Energétique d'un Système Linéaire et Stationnaire Modélisé par un Bond Graph Bicausal*. Ph. D. Thesis: Institut National des Sciences Appliquées de Lyon, 2000.

[13] Xia, X., Scavarda, S.: Adjoint System by Using the Representation of Bond Graph. In: *Proc. of the* ICBGM'01, *Int. Conf. on Bond Graph Modeling and Simulation, Phoenix,* USA, *January 07–11, 2001*, pp. 15–20.

[14] Gawthrop, P. J.: Bicausal Bond Graphs. In: *Proc. of the* ICBGM '95, *2nd Int. Conf. on Bond Graph Modeling and Simulation, Las Vegas,* USA, *January, 1995*, pp. 83–88.

[15] Gawthrop, P. J.: Physical Interpretation of Inverse Dynamics Using Bicausal Bond Graphs. *Journal of the Franklin Institute* 337 (2000), pp.743–769.

[16] Groetsch, C. W.: *Inverse Problems*. The Mathematical Association of America, USA, 1999.