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► **To cite this version:**

Wilfrid Marquis-Favre, Xiaoguang Xia, Serge Scavarda. Bicausality-based Procedures for Transfer and Transmission Matrix Determination of Single Source Single Load Linear Systems. ICBGM'03, International Conference on Bond Graph Modeling and Simulation, Jan 2003, Orlando, United States. pp.128-133. hal-02084646

**HAL Id: hal-02084646**

**<https://hal.science/hal-02084646v1>**

Submitted on 29 Mar 2019

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# Bicausality-based Procedures for Transfer and Transmission Matrix Determination of Single Source Single Load Linear Systems

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**Keywords:** Bicausal bond graph, transfer matrix, transmission matrix, inversion, loop rule

## Abstract

In linear SISO systems the energy flows from the source to the load through each actuating power line. Bond graph representation is well suited for studying the energy transfer in these systems because, in most of the systems, we can consider that energy flows through a power line embedded in two-port subsystems. Moreover, with bond graph language, we benefit from the ease of analytical exploitation introduced by the calculus causality and bicausality concepts.

In the bond graph representation of a system the power in some port is associated to a power bond to which two power variables are attached. The determination of the transmission matrix linking two power variables is an efficient tool for studying the energy transfer. Furthermore it is often useful to study the bi-directional energy transfer i.e. from the source to the load and vice versa. The latter case (namely from the load to the source) is quite useful for the energy sizing of a system from its dynamic specifications on an output variable.

This paper proposes different procedures for determining direct and inverse transmission matrices and the transfer matrix between an internal bond and an environmental one for Single Source Single Load Linear systems represented by bond graphs. The procedures are based on the bicausality concept and the loop rule.

## INTRODUCTION

In the context of linear model analysis, as well as control design, the transfer function concept is a well-dedicated tool for inspecting the dynamic behavior of linear systems. For MIMO linear systems the transfer matrix is the extrapolation of the transfer function. It relates the inputs to the outputs by a number of transmittance series. A specific dynamic point of view is the power transmission between two power ports of a system [Karnopp and Rosenberg 1968]. In that case the inputs and outputs are pairs of conjugate power variables i.e. efforts and flows. For actuating power line systems whose the function is to drive the loads by converting the energy furnished by the sources, the transmission matrix enables the energy transfer to pass from the source to the load (direct transmission) or from the load to the source (inverse transmission) to be investigated [Xia 2000]. The transmission matrix is also a means for calculating the wave matrix and then the scattering matrix [Kurokawa 1969, River and Sardos 1982, Scavarda et al. 1991, Amara and Scavarda 1992].

In this context the bond graph is an appropriate language for both modelling pluridisciplinary mechatronic systems and also conducting analyses for dynamic behavior investigation [Karnopp et al., 1990, Dauphin-Tanguy 2000]. Bond graph language proposes powerful tools for analysing and simulating the dynamic behavior of systems. In particular causality combined with the loop rule [Brown 1972] helps determine a transfer matrix between inputs and outputs in a linear model. More recently bicausality has opened new fields of analysis such as system inversion, state estimation, parameter estimation, input reconstruction, dynamic sizing, control synthesis [Gawthrop 1995, Ngwompo et al., 1996, Ngwompo and Scavarda 1999, Gawthrop 2000, Junco 2001, Ngwompo et al., 2001].

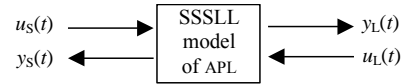
This paper proposes three procedures based on bicausality and the use of the loop rule for calculating both the direct and inverse

transmission matrices, also for calculating a transfer matrix from an internal bond to an environmental bond for Single Source Single Load Linear (SSSSL) systems. The next section presents the concepts and the tools used in the procedures. Then the procedures are proposed with illustrating examples. Finally the conclusion examines some benefit of these procedures and proposes some perspectives.

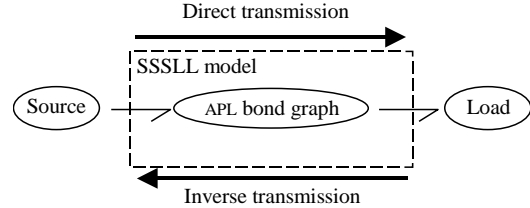
## CONCEPTS AND TOOLS

### Transfer and Transmission Matrices

Consider an Actuating Power Line (APL) represented by a Single Source Single Load Linear (SSSSL) model with its generic block diagram (Fig. 1) and its generic bond graph representation (Fig. 2). The sources and the loads constitute the model environment and define the inputs and outputs between which the transfer, and the transmission matrices may be calculated. The source is considered here as a non-ideal source and, without loss of generality, the inputs and the outputs are restricted here in the bond graph representation to physical power variables.



**Figure 1.** Generic block diagram representation of an SSSSL model



**Figure 2.** Generic bond graph representation of an SSSSL model

The transfer matrix is the physical input/output representation in the Laplace domain that enables the outputs to be expressed in terms of the inputs by a series of transfer functions (Equ. 1) [Borne et al., 1992, Dauphin-Tanguy 2000].

$$\underline{Y}(s) = \mathbf{M}(s) \cdot \underline{U}(s) \quad [1]$$

where  $\underline{Y}(s) = [Y_s(s) \ Y_L(s)]$  is the output Laplace transform vector,

$\underline{U}(s) = [U_s(s) \ U_L(s)]$  is the input Laplace transform vector,

$\mathbf{M}(s)$  is the transfer matrix composed of the individual input/output pair of transmittances (Equ. 2),

and  $s$  is the Laplace operator.

$$\mathbf{M}(s) = [m_{ij}(s)] \quad i=1,2; \ j=1,2 \quad [2]$$

with  $m_{ij}(s)$  the transfer function or transmittance between the  $j^{\text{th}}$  input and the  $i^{\text{th}}$  output.

Depending on the causality imposed by the source and the load, inputs and outputs are either efforts or flows.  $u_s(t)$  and  $y_s(t)$  are the source bond power variables and  $u_L(t)$  and  $y_L(t)$ , the load bond ones. According to the definitions given in [Karnopp and Rosenberg 1968], the transfer matrix of Fig. 2 APL is called: (i) admittance, if both the source and the load impose efforts; (ii) adpedance, if the source imposes an effort and the load a flow; (iii) immitance, if the source imposes a flow and the load an effort; and (iv) impedance, if both the source and the load impose flows.

This transfer matrix calculation corresponds to the physical cause and effect relationship in a model. If input/output role exchange is envisaged, the transfer matrix then refers to an inverse model and the associated mathematical model does not correspond to the natural way the physical system behaves. A particular case is such that either the source or the load imposes both the effort and the flow to the APL. This introduces the concept of transmission matrix [Karnopp and Rosenberg 1968]. If the source (resp. load) imposes both the effort and the flow the concept of direct (resp. inverse) transmission is defined [Xia 2000].

It is also worthwhile noting that direct and inverse notions for a transmission matrix correspond to the energy flow direction between the source and the load (Fig. 2). The direct transmission corresponds to the natural function of the APL, which is to drive the load by converting the energy furnished by the source. However the inverse notion also refers to an input/output mathematical reorganization of the equations. The concept of bicausality, which is used for calculating the transmission matrix, is now recalled.

### Bicausality and Inversion

The determination of the transfer matrix from the bond graph representation is straightforward by assigning the preferential integral causality by SCAP (Sequential Causality Assignment Procedure) [Karnopp et al., 1990, Van Dijk 1994] or by assigning the preferential derivative causality and then applying the loop rule on each pair of input/outputs [Brown 1972]. The different transfer functions thus obtained between the inputs and the outputs are the transmittances of the transfer matrix (Equ. 2).

In the Fig. 2 bond graph representation the transfer matrix is derived by the means of a causality assignment. The transfer matrix of an inverse model and the transmission matrix are possible to be derived straightforwardly by means of the bicausality concept. The essence of bicausality can be easily introduced by considering that a causal stroke ultimately consists of two half strokes each attached respectively to the effort and flow of a power bond (Fig. 3).

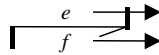


Figure 3. Bicausality concept: causality assignment decoupling

Assignment coupling characterizes the causality assignment on a bond and bicausality authorizes breaking this assignment coupling. In this way bicausality assignment corresponds exactly to deriving an inverse model from a bond graph representation. For the Fig. 2 APL causal bond graph, this means that at ports, causal strokes split into two according to the transmission matrix calculation. A double source which imposes both effort and flow, and a double detector which receives both effort and flow enable bicausality initialization and termination to be carried out.

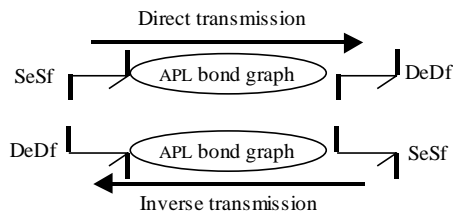


Figure 4. Bicausality assignment for the transmission matrix calculation

In the context of bond graph language, inversion has been largely treated in literature [Gawthrop 1995, Ngwompo et al., 1996, Fotsu Ngwompo 1997, Ngwompo and Scavarda 1999, Gawthrop 2000, Junco 2000, Junco 2001, and Ngwompo et al., 2001]. Bond graph language enables the structural invertibility properties, directly and at a graphical level, to be investigated. It is based on the existence of a set of disjoint input/output causal paths. If this set does not exist between the inputs and outputs involved in the inverse transfer or transmission matrix to be calculated, then the corresponding inverse model cannot be determined. In the context of SSSL models the invertibility condition is reduced to the existence of a power line between the source and the load. This condition is assumed to be realized according to the APL function between the source and the load.

Thus, the procedures presented in this paper for calculating an inverse transfer matrix or a transmission matrix are based largely on SCAPi (Sequential Causality Assignment Procedure for Inversion

[Ngwompo et al., 2001]) but without the condition of the minimal order. The second important calculus tool used in these procedures is the loop rule that is recalled in the next section.

### Loop Rule

The transfer matrix consists of transmittances between the inputs and outputs. One method for calculating these transmittances is based on the loop rule applied to bond graph [Brown 1972]. The loop rule has been adapted from Mason's rule presented in the context of signal flow graphs [Mason 1956]. Since bicausal bond graph corresponds to a certain organization of the equations, it is postulated that it can be associated to a signal flow graph and that the loop rule still works. Thus the loop rule is also used for determining the transfer matrix of an inverse model and the transmission matrix.

The loop rule enables, for a linear system represented by a causal or a bicausal bond graph, the transfer function between an input and an output to be calculated by:

$$G(s) = \frac{\sum_i P_i(s) A_{pi}(s)}{A(s)} \quad [3]$$

with  $A(s)$  as the bond graph determinant determined by the expression:

$$A(s) = 1 - \sum_i A_i^{(1)}(s) + \sum_i A_i^{(2)}(s) - \sum_i A_i^{(3)}(s) + \Lambda \quad [4]$$

where  $A_i^{(j)}(s)$  is the  $i^{\text{th}}$  product of the loop gains for  $j$  causal loops of the causal or bicausal bond graph which do not touch one another. The summations are over all possible combinations,  $P_i(s)$  is the  $i^{\text{th}}$  input/output path gain,  $A_{pi}(s)$  is the  $i^{\text{th}}$  reduced determinant of the causal loops which do not touch the  $i^{\text{th}}$  causal path. This reduced determinant is calculated in the same manner as for the determinant expressed by Equation 4.

By the principle of superposition the transfer or the transmission matrix is simply obtained by appending the different input/output transfer functions in rows and columns into a matrix [Borne et al., 1992].

### PROCEDURES

Three bicausality based procedures are presented. The first procedure enables the transmission matrices (direct and inverse) to be derived for a given SSSL model represented by its bond graph. The two other procedures enables a transfer matrix to be calculated, this gives the power variables of an environmental bond in terms of those of an internal bond power variables. This transfer matrix is that of an inverse model. Here it is an inverse model in the sense that one power variable of the involved environmental bond changes its input/output role. For the conciseness of the proposed procedures the concept of a power line is defined. A power line between two components is a series of bonds and junction structure elements connecting two components [Wu and Youcef-Toumi 1995, Ngwompo et al., 2001]. An input/output power line is associated to an input/output causal path if the causal path goes through one variable of each bond of that power line. Contrary to the causal path, the power line is an acausal concept.

### Transmission Matrix

Given a SSSL linear model and its bond graph representation the procedure is applied as follows:

#### Procedure 1:

- If the direct (resp. inverse) transmission matrix is required, replace the source by a double source (resp. detector) and the load by a double detector (source).
- Assign bicausality from the double source to the double detector along an input/output power line.
- Complete the causality assignment to the remaining acausal bond graph according to the classical causality constraints of the junction structure.
- Apply the loop rule to calculate the transmittances of the transmission matrix researched.

Procedure 1 is now illustrated by an example. Consider the Fig. 5 bond graph representation of an APL system (it could correspond to an electrical circuit for instance).

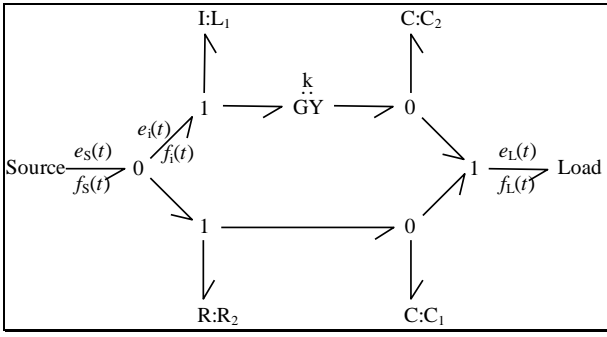


Figure 5: Example of an APL bond graph representation

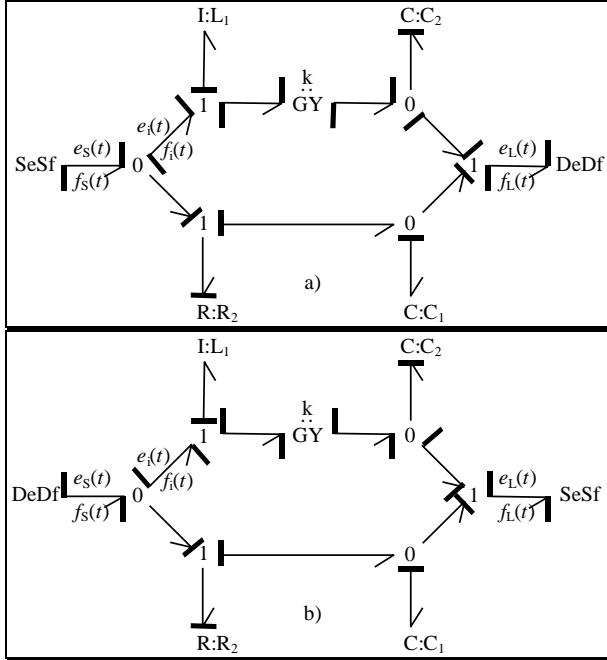


Figure 6. Example: the bicausal representations a) for the direct transmission matrix; b) for the inverse transmission matrix calculation

Fig. 6a (resp. b) illustrates procedure 1 steps a, b, and c for calculating the direct (resp. inverse) transmission matrix. The source and the load have been respectively replaced by double source and double detector elements. Then bicausality has been propagated from the double source to the double detector and finally causality has been completed.

**Example: Direct Transmission (Fig. 6a)**

The bicausal bond graph representation shows three causal loops with the following respective gains:

$$\Delta_1(s) = \frac{L_1}{kR_2C_1} ; \Delta_2(s) = \frac{kC_2}{R_2C_1} ; \Delta_3(s) = -\frac{1}{R_2C_1s} \quad [5]$$

These loops touch one another so the bicausal bond graph determinant is:

$$\Delta(s) = 1 - \frac{L_1}{kR_2C_1} - \frac{kC_2}{R_2C_1} + \frac{1}{R_2C_1s} \quad [6]$$

Table 1 shows, for each combination of input/output pairs, the input/output causal path gains and the corresponding transmittance numerator that composes the direct transmission matrix. This results from the application of the last step of the procedure.

**Example: Inverse Transmission (Fig. 6b)**

The bicausal bond graph representation shows three causal loops with the following respective gains:

$$\Delta_1(s) = -\frac{L_1}{kR_2C_1} ; \Delta_2(s) = -\frac{kC_2}{R_2C_1} ; \Delta_3(s) = -\frac{1}{R_2C_1s} \quad [7]$$

These loops touch one another so the bicausal bond graph determinant is:

$$\Delta(s) = 1 + \frac{L_1}{kR_2C_1} + \frac{kC_2}{R_2C_1} + \frac{1}{R_2C_1s} \quad [8]$$

Table 2 shows, for each combination of input/output pairs, the input/output causal path gains and the corresponding transmittance numerator that composes the inverse transmission matrix. This results from the application of the last step of the procedure.

It can be easily verified that both transmission matrices obtained are the inverse, one of the other.

Table 1. Results of the loop rule application for the direct transmission matrix calculation

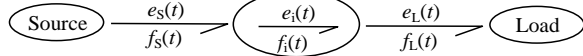
I/O pair	I/O causal path gains and corresponding reduced determinant		Transmittance numerator
$e_s(t)/e_l(t)$	$p_1(s) = \frac{1}{R_2C_1s} ; \Delta_{p1}(s) = 1$	$p_2(s) = -\frac{k}{R_2} ; \Delta_{p2}(s) = 1$	$-\left(\frac{k}{R_2} + \frac{1}{kC_1s} + \frac{kC_2}{R_2C_1} + \frac{L_1}{kR_2C_1}\right)$
	$p_3(s) = -\frac{1}{kC_1s} ; \Delta_{p3}(s) = 1$	$p_4(s) = -\frac{kC_2}{R_2C_1} ; \Delta_{p4}(s) = 1$	
	$p_5(s) = -\frac{L_1}{kR_2C_1} ; \Delta_{p5}(s) = 1$	$p_6(s) = -\frac{1}{R_2C_1s} ; \Delta_{p6}(s) = 1$	
$e_s(t)/f_l(t)$	$p_1(s) = \frac{L_1s}{kR_2} ; \Delta_{p1}(s) = 1$	$p_2(s) = \frac{kC_2s}{R_2} ; \Delta_{p2}(s) = 1$	$\frac{1}{k} \left(1 + \frac{1}{R_2C_1s}\right) + \frac{kC_2s}{R_2} + \frac{L_1s}{kR_2}$
	$p_3(s) = \frac{1}{k} ; \Delta_{p3}(s) = 1 + \frac{1}{R_2C_1s}$		
$f_s(t)/e_l(t)$	$p_1(s) = \frac{L_1}{kC_1} ; \Delta_{p1}(s) = 1$	$p_2(s) = \frac{kC_2}{C_1} ; \Delta_{p2}(s) = 1$	$\frac{L_1}{kC_1} + \frac{kC_2}{C_1} + k \left(1 + \frac{1}{R_2C_1s}\right)$
	$p_3(s) = k ; \Delta_{p3}(s) = 1 + \frac{1}{R_2C_1s}$		
$f_s(t)/f_l(t)$	$p_1(s) = -\frac{L_1s}{k} ; \Delta_{p1}(s) = 1 + \frac{1}{R_2C_1s}$		$-\left(\frac{L_1s}{k} + kC_2s\right) \left(1 + \frac{1}{R_2C_1s}\right)$
	$p_2(s) = -kC_2s ; \Delta_{p2}(s) = 1 + \frac{1}{R_2C_1s}$		

**Table 2:** Results of the loop rule application for the inverse transmission matrix calculation

I/O pair	I/O causal path gains and corresponding reduced determinant		Transmittance numerator
$e_1(t)/e_s(t)$	$p_1(s) = \frac{L_1 s}{k}; \Delta_{p1}(s) = 1 + \frac{1}{R_2 C_1 s}$		$\left( \frac{L_1 s}{k} + k C_2 s \right) \left( 1 + \frac{1}{R_2 C_1 s} \right)$
	$p_2(s) = k C_2 s; \Delta_{p2}(s) = 1 + \frac{1}{R_2 C_1 s}$		
$e_1(t)/f_s(t)$	$p_1(s) = \frac{L_1 s}{k R_2}; \Delta_{p1}(s) = 1$	$p_2(s) = \frac{k C_2 s}{R_2}; \Delta_{p2}(s) = 1$	$\frac{1}{k} \left( 1 + \frac{1}{R_2 C_1 s} \right) + \frac{k C_2 s}{R_2} + \frac{L_1 s}{k R_2}$
	$p_3(s) = \frac{1}{k}; \Delta_{p3}(s) = 1 + \frac{1}{R_2 C_1 s}$		
$f_i(t)/e_s(t)$	$p_1(s) = \frac{L_1}{k C_1}; \Delta_{p1}(s) = 1$	$p_2(s) = \frac{k C_2}{C_1}; \Delta_{p2}(s) = 1$	$\frac{L_1}{k C_1} + \frac{k C_2}{C_1} + k \left( 1 + \frac{1}{R_2 C_1 s} \right)$
	$p_3(s) = k; \Delta_{p3}(s) = 1 + \frac{1}{R_2 C_1 s}$		
$f_i(t)/f_s(t)$	$p_1(s) = \frac{1}{R_2 C_1 s}; \Delta_{p1}(s) = 1$	$p_2(s) = \frac{k}{R_2}; \Delta_{p2}(s) = 1$	$\frac{k}{R_2} + \frac{1}{k C_1 s} + \frac{k C_2}{R_2 C_1} + \frac{L_1}{k R_2 C_1}$
	$p_3(s) = \frac{1}{k C_1 s}; \Delta_{p3}(s) = 1$	$p_4(s) = \frac{k C_2}{R_2 C_1}; \Delta_{p4}(s) = 1$	
	$p_5(s) = \frac{L_1}{k R_2 C_1}; \Delta_{p5}(s) = 1$	$p_6(s) = -\frac{1}{R_2 C_1 s}; \Delta_{p6}(s) = 1$	

### From an Internal Bond to an Environmental Bond

This procedure now considers a bond within the bond graph representation of the APL (Fig. 7). Only the transfer from this bond to an external bond is envisaged. One motivation for calculating such a transfer is to look at the energy repartition in the Laplace domain. Also this transfer can be considered as an intermediate transfer between environmental ports. Two methods are proposed here: an indirect method and a direct one.



**Figure 7:** Bond graph generic representation of the SSSLL model with an internal bond *i*

#### Indirect Method

The indirect method necessitates calculating two matrices. In fact the transfer matrix that relates the environmental bond power variables to the internal bond variables cannot be obtained by the classical use of the loop rule. Here, in the envisaged transfer, the internal bond power variables are not inputs for the model. Thus, due to its nature, the loop rule cannot be directly applied. Instead other transfer matrices are determined and it is shown that the transfer matrix researched can be expressed in terms of these other matrices. Given an SSSLL model and its bond graph representation, the procedure is applied as follows:

*Procedure 2:*

- Choose the internal bond and the environmental bond (source or load bond) between which the transfer matrix is required.
- Replace the chosen (resp. not chosen) environmental element (source or load) by a double detector (resp. double source).
- Assign bicausality from the double source to the double detector along a power line between the source and the load.
- Complete the causality assignment to the remaining acausal bond graph according to the classical causality constraints of the junction structure.
- Apply the loop rule to calculate the transmittances of the transfer matrix between the power variables of the double detector bond (noted dd) and those of the double source element bond (noted ss). Denote the obtained matrix  $M^{ss \rightarrow dd}(s)$ . This matrix is a transmission matrix (either direct or inverse).
- Apply the loop rule to calculate the transmittances of the transfer matrix between the power variables of the internal bond (noted i) and those of the double source element bond (noted ss). Denote the

obtained matrix  $M^{ss \rightarrow i}(s)$ . In this step the internal bond power variables can be seen as outputs without changing the global model formulation.

- The researched transfer matrix between the double detector element bond and the internal bond power variables is calculated from the matrix product:  $M^{ss \rightarrow dd}(s) \cdot [M^{ss \rightarrow i}(s)]^{-1}$ . It is conjectured here

that the matrix  $M^{ss \rightarrow i}(s)$  is invertible due to the assumption of the existence of a power line between the source and the load.

#### Proof

Consider the Fig. 7 bond graph representation of an SSSLL model. The internal bond is denoted bond *i* and its power variables may be detected respectively by a flow detector element for  $f_i(t)$  and by an effort detector element for  $e_i(t)$ .

Whatever the source or the load chosen for the environmental bond involved in the transfer matrix, denote respectively  $[e_{ss}(t) f_{ss}(t)]$  and  $[e_{dd}(t) f_{dd}(t)]$  the power variables of the double source and of the double detector for the inverse model. After bicausality assignment and by the use of the loop rule, the transfer between the environmental bonds on one hand (which here is a transmission) and the transfer between the internal bond and the double source bond on the other hand are expressed by:

$$\begin{bmatrix} E_{dd}(s) \\ F_{dd}(s) \end{bmatrix} = M^{ss \rightarrow dd}(s) \begin{bmatrix} E_{ss}(s) \\ F_{ss}(s) \end{bmatrix} \quad [9]$$

$$\begin{bmatrix} E_i(s) \\ F_i(s) \end{bmatrix} = M^{ss \rightarrow i}(s) \begin{bmatrix} E_{ss}(s) \\ F_{ss}(s) \end{bmatrix}$$

By manipulating equations 9 and with the conjecture that the first transfer matrix of these equations is invertible the results are obtained straightforwardly:

$$\begin{bmatrix} E_i(s) \\ F_i(s) \end{bmatrix} = M^{ss \rightarrow i}(s) \cdot [M^{ss \rightarrow dd}(s)]^{-1} \begin{bmatrix} E_{dd}(s) \\ F_{dd}(s) \end{bmatrix} \quad [10]$$

#### End of proof

It must be emphasized here that at no instant the power variables of the internal bond became an input of the inverse model. An illustrative example is given at the end of the following section.

#### Direct method

A drawback of the indirect method for calculating a transfer matrix between an environmental bond and an internal one is the double application of the loop rule for determining two transfer matrices. Moreover one of these two matrices must be inverted and a matrix product

has to be calculated. The direct method proposes calculating only one matrix without a product. Furthermore the different terms of this matrix can be directly deduced from the application of the loop rule.

Given an SSSL model and its bond graph representation the procedure is applied as follows:

*Procedure 3:*

- Choose the internal bond and the environmental bond (source or load bond) between which the transfer matrix is required.
- Replace the chosen (resp. not chosen) environmental element (source or load) by a double source (resp. double detector).
- Assign bicausality from the double source to the double detector along a power line between the source and the load.
- Complete the causality assignment to the remaining acausal bond graph according to the classical causality constraints of the junction structure.
- Apply the loop rule to calculate the transmittances of the transfer matrix between the power variables of the internal bond (noted *i*) and those of the double source element bond (noted *ss*). Denote the matrix obtained  $M^{ss \rightarrow i}(s) = 1/\Delta(s) \cdot [m_{jk}^{ss \rightarrow i}]$  with  $\Delta(s)$  as the bond graph determinant. In this step the internal bond power variables can be seen as outputs without changing the global model formulation.

- The researched transfer matrix between the chosen environmental bond (here the double source element bond) and the internal bond power variables is expressed by:  $\Delta(s)/\delta(s) \cdot [\text{cofactor}(m_{jk}^{ss \rightarrow i})]$  with  $\delta(s) = m_{11}^{ss \rightarrow i} m_{22}^{ss \rightarrow i} - m_{21}^{ss \rightarrow i} m_{12}^{ss \rightarrow i}$ . It is conjectured here that the matrix  $M^{ss \rightarrow i}(s)$  is invertible due to the assumed existence of a power line between the source and the load.

**Proof**

Consider again the Fig. 7 bond graph representation of an SSSL model. From the step b of the procedure the environmental element attached to the chosen environmental bond is replaced by a double source even if, in this purpose of transfer calculation, the power variables of the chosen environmental bond are outputs. Then the transfer matrix between the internal bond power variables and the double source bond power variables is expressed by:

$$\begin{bmatrix} E_i(s) \\ F_i(s) \end{bmatrix} = M^{ss \rightarrow i}(s) \begin{bmatrix} E_{ss}(s) \\ F_{ss}(s) \end{bmatrix} = \frac{1}{\Delta(s)} \begin{bmatrix} m_{11}^{ss \rightarrow i}(s) & m_{12}^{ss \rightarrow i}(s) \\ m_{21}^{ss \rightarrow i}(s) & m_{22}^{ss \rightarrow i}(s) \end{bmatrix} \begin{bmatrix} E_{ss}(s) \\ F_{ss}(s) \end{bmatrix} \quad [11]$$

Then the researched transfer matrix is deduced from the inversion of equation 11:

$$\begin{bmatrix} E_{ss}(s) \\ F_{ss}(s) \end{bmatrix} = [M^{ss \rightarrow i}(s)]^{-1} \begin{bmatrix} E_i(s) \\ F_i(s) \end{bmatrix} = \frac{\Delta(s)}{m_{11}^{ss \rightarrow i}(s)m_{22}^{ss \rightarrow i}(s) - m_{21}^{ss \rightarrow i}(s)m_{12}^{ss \rightarrow i}(s)} \begin{bmatrix} m_{22}^{ss \rightarrow i}(s) & -m_{12}^{ss \rightarrow i}(s) \\ -m_{21}^{ss \rightarrow i}(s) & m_{11}^{ss \rightarrow i}(s) \end{bmatrix} \begin{bmatrix} E_i(s) \\ F_i(s) \end{bmatrix} \quad [12]$$

**End of proof**

Compared to the indirect method the direct one is far advantageous since the matrix terms researched are obtained directly by the application of the loop rule and more simply, in particular by manipulating the bond graph determinant and the transmittance numerators. An illustrative example is now presented to compare both the indirect and direct methods.

The two previous procedures are illustrated with the Fig. 5 example. The transfer matrix between the internal power variables  $e_i(t)$  and  $f_i(t)$  and the source bond power variables  $e_s(t)$  and  $f_s(t)$  is researched.

**Example: Indirect method (Fig. 6b)**

Fig. 6b illustrates the application of the procedure 2 steps a, b, c, and d. The source and the load have been respectively replaced by double detector and double source elements. Then bicausality has been propagated from the double source to the double detector and finally causality has been completed. Step e of the procedure leads to determining a transmission matrix. Here this is the inverse transmission matrix that has been already calculated in section 'Transmission Matrix' (cf. Table 2). Table 3 shows, for each pair combination of double source bond variables/internal bond variables, the input/output causal path gains and the corresponding transmittance numerators that compose the matrix calculated at step f of the procedure. The bond graph determinant is given by equation 8.

Finally the application of the last step of the procedure requires inverting the previous matrix (Equ. 13) and deriving the product of this inverse matrix by the transmission matrix, obtained one step before the previous one. This gives the transfer matrix researched between the internal bond power variables and the source bond ones (Equ. 14).

$$[M^{ss \rightarrow i}(s)]^{-1} = \frac{1}{1 + \frac{1}{R_2 C_1 s}} \begin{bmatrix} \frac{1}{k C_1 s} & \frac{1}{R_2 C_1 s} & -\frac{L_1}{k C_1} - \frac{k C_2}{C_1} - k \left( 1 + \frac{1}{R_2 C_1 s} \right) \\ -\frac{1}{k} \left( 1 + \frac{1}{R_2 C_1 s} \right) & \left( \frac{L_1 s}{k} + k C_2 s \right) & \left( 1 + \frac{1}{R_2 C_1 s} \right) \end{bmatrix} \quad [13]$$

$$M^{ss \rightarrow dd}(s) \cdot [M^{ss \rightarrow i}(s)]^{-1} = \frac{1}{1 + \frac{1}{R_2 C_1 s}} \begin{bmatrix} 1 + \frac{1}{R_2 C_1 s} & 0 \\ \frac{1}{k R_2 C_1 s} + \frac{1}{R_2} & 1 + \frac{1}{R_2 C_1 s} - \frac{L_1}{k R_2 C_1} - \frac{k C_2}{R_2 C_1} \end{bmatrix} \quad [14]$$

**Example: Direct method (Fig. 6a)**

Fig. 6a illustrates the application of procedure 3 steps a, b, c, and d. The source and the load have been respectively replaced by double source and double detector elements. Then bicausality has been propagated from the double source to the double detector and finally causality has been completed. Table 4 shows, for each pair combination of double source bond variables/internal bond variables, the input/output causal path gains and the corresponding transmittance numerators that compose the matrix calculated at step e of the procedure. The bond graph determinant is given by equation 6.

Finally the application of the last step of the procedure leads easily to the Equ. 14.

**Table 3:** Results of the loop rule application for the indirect method

I/O pair	I/O causal path gains and corresponding reduced determinant	Transmittance numerator
$e_i(t)/e_i(t)$	$p_1(s) = \frac{L_1 s}{k} ; \Delta_{p1}(s) = 1 + \frac{1}{R_2 C_1 s}$	$\left( \frac{L_1 s}{k} + k C_2 s \right) \left( 1 + \frac{1}{R_2 C_1 s} \right)$
	$p_2(s) = k C_2 s ; \Delta_{p2}(s) = 1 + \frac{1}{R_2 C_1 s}$	
$e_i(t)/f_i(t)$	$p_1(s) = \frac{1}{k} ; \Delta_{p1}(s) = 1 + \frac{1}{R_2 C_1 s}$	$\frac{1}{k} \left( 1 + \frac{1}{R_2 C_1 s} \right)$
$f_i(t)/e_i(t)$	$p_1(s) = \frac{L_1}{k C_1} ; \Delta_{p1}(s) = 1$	$\frac{L_1}{k C_1} + \frac{k C_2}{C_1} + k \left( 1 + \frac{1}{R_2 C_1 s} \right)$
	$p_2(s) = \frac{k C_2}{C_1} ; \Delta_{p2}(s) = 1$	
	$p_3(s) = k ; \Delta_{p3}(s) = 1 + \frac{1}{R_2 C_1 s}$	
$f_i(t)/f_i(t)$	$p_1(s) = \frac{1}{k C_1 s} ; \Delta_{p1}(s) = 1$	$\frac{1}{k C_1 s} - \frac{1}{R_2 C_1 s}$
	$p_2(s) = -\frac{1}{R_2 C_1 s} ; \Delta_{p2}(s) = 1$	

**Table 4: Results of the loop rule application for the direct method**

I/O pair	I/O causal path gains and corresponding reduced determinant		Transmittance numerator
$e_s(t)/e_i(t)$	$p_1(s) = 1; \Delta_{p1}(s) = 1 - \frac{L_1}{kR_2C_1} - \frac{kC_2}{R_2C_1} + \frac{1}{R_2C_1s}$		$1 - \frac{L_1}{kR_2C_1} - \frac{kC_2}{R_2C_1} + \frac{1}{R_2C_1s}$
$e_s(t)/f_i(t)$	$p_1(s) = -\frac{1}{kR_2C_1s}; \Delta_{p1}(s) = 1$	$p_2(s) = -\frac{1}{R_2}; \Delta_{p2}(s) = 1$	$-\left(\frac{1}{kR_2C_1s} + \frac{1}{R_2}\right)$
$f_s(t)/e_i(t)$	No path		No path
$f_s(t)/f_i(t)$	$p_1(s) = 1; \Delta_{p1}(s) = 1 + \frac{1}{R_2C_1s}$		$1 + \frac{1}{R_2C_1s}$

**CONCLUSION**

This paper proposes a series of three procedures, on one hand for calculating transmission matrices and on the other hand for calculating a transfer matrix between internal bond power variables and environmental bond power variables, from the bond graph representation of Single Source Single Load Linear (SSLL) systems. The procedures are based on a bicausality assignment coupled to the use of the loop rule.

The concepts of transmission matrix, internal bond transfer matrix and bicausality are associated to inverse models. In that context the SCAP procedure (Sequential Causality Assignment Procedure for Inversion) had been presented for sizing mechatronic systems on dynamic and energy criteria [Ngwompo et al., 2001]. SCAP enables a minimal order inverse model to be calculated from its bond graph representation. This corresponds to expressing the input in terms of the output with a minimal number of differentiations of this output. Thus, in the bicausal bond graph representation, a minimal number of energy storage elements are in derivative causality. On the contrary, for the purpose of transmission matrix or internal bond transfer matrix calculation, the minimal order condition is not so crucial. All the more it might even be interesting to have the maximal number of energy storage elements in derivative causality. In certain cases this is researched when applying the loop rule on a bond graph representation with the preferential derivative causality [Brown 1972]. It must be also emphasized here that due to the fact that the calculation is made in the Laplace domain, it is not necessary to find a minimal inverse model.

Calculation of the transmission matrix has been already proposed using classical causality [Scavarda et al. 1991]. However this calculation required assigning successively four different causality schemas in order to pick up, for each one, a coefficient in the respective admittance, adpedance, imittance and impedance matrices obtained. The use of bicausality gives a great improvement since it only needs one assignment to obtain the transmission matrix straightforwardly. Furthermore it has been shown how it was possible to derive a transfer matrix involving an internal bond. Using this transfer may be interesting for investigating the energy repartition in the junction structure.

A perspective is now to generalize these procedures to Multi Sources Multi Loads Linear (MSMLL) systems. In this case careful inspection of model invertibility properties will be required. Also it has been shown that the condition on the model order was not crucial. This leaves several possibilities for propagating bicausality depending on the number of existing input/output power lines. Another perspective could be to investigate the consequence of the power line choice for bicausality propagation from the point of view of the calculation.

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