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# Accounting for sizing parameters and boundary geometry in the bond graph representation of energetic elements

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## ABSTRACT

In this paper we present how to take into account parameters and variables that were not initially present in the bond graph representation of some components. These bond graph considerations are justified mainly by design arguments but from the automatics point of view this is closely related to the state representation concept. We present this idea through the examples of a mechanical spring and a simple thermodynamic system. In each case one solution is to substitute a multiport energy storage element to the initial one port element. It emphasizes the close relationship between the order of this component and the output variables that we want to observe in this component.

## INTRODUCTION

A major preoccupation of engineers today is the design of more and more complicated systems. These systems are present in various areas like robotics, manufacturing machines, or earth and air transports to name a few. Depending on the given objective designers may create new concepts or adapt existing ones. In both cases this necessitates an integrated design answering to a broad range of constraints resulting from functional specifications, geometrical specifications, technological specifications, dynamical and energetic specifications, performance specifications, economical specifications [Chedmail 1997].

In the context of system design several approaches exist to answer a given objective. They aim at determining a set of sizing and topological parameters in terms of which function objective are formulated. For instance, in the context of

mechanism synthesis, these parameters are the number and the nature of joints, the length of links, etc. Due to the different nature of problems in the phases of a system design the answer is generally not trivial.

With the increasing power of computers the design is now achieved generally in a computer-aided environment. However this necessitates efficient methodologies and tools to be built for taking maximum benefits from this computer-aided environment. The bond graph language (e.g. [Karnopp et al. 1990]) revealed to be a good candidate for implementing these methodologies such that sensibility study, optimization study or inverse formulations. By displaying the energetic phenomena and the topology of the energy transfers between these phenomena a bond graph representation enable a broad range of design problems to be treated or potentially addressed.

However a bond graph representation must also show to the designer the pertinent parameters in a system i.e. the technological parameters which once determined will permit the system to be manufactured. This feature is not always fulfilled for certain technological components. For example the bond graph representation of a linear mechanical spring is generally a one port C element connected to a junction structure. In the context of the system design the need to account for its rest length necessitates its state representation to be changed or alternatively its bond graph representation.

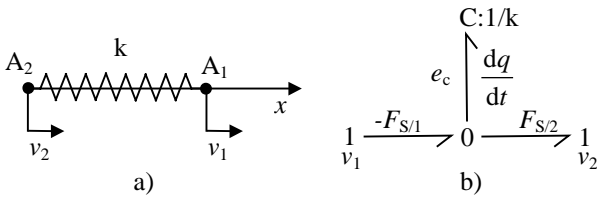
From a certain point of view, the definition of a system requires to define the system boundary. This boundary is composed of parts with different physical properties. A particular and interesting boundary definition case corresponds to deformable and moving parts. For example a simple thermodynamic system is assumed independent of any spatial structure (e.g. the pressure has the

same value for any point inside the system). Then the energetic description of the system in the potential domain requires only the volume to be known. This means that it is sufficient to know the initial value of the volume and its variation. This variation can be obtained by integrating its time rate of change and implies to know only the velocities of each deformable part and each moving part independently of their position. Then in the bond graph model, we need a storage element with a unique power port in the potential domain. However in a process design, the geometrical characteristics are required, and in this case it is necessary to know the position and consequently to introduce a number of power ports in the potential domain equal to the number of deformable and moving parts.

## EXAMPLE OF THE SPRING

### The rest length parameter

Let's consider the figure 1-a linear spring component of stiffness  $k$ . Independently on the subsystems it is connected to its partial bond graph representation is given on figure 1-b [Marquis-Favre et al. 2000].



**Figure 1.** Spring component and its bond graph representation

On this representation the C element (characterized by its stiffness  $k$ ) corresponds to the storage phenomena in the spring component.  $e_c$  and  $\dot{q}$  are the power variables on this element port.  $F_{S/1}$  and  $F_{S/2}$  represent the spring forces respectively on the subsystem attached at point  $A_1$  and on the subsystem attached at the point  $A_2$ . The 0-junction expresses that the spring works at a common effort.

The state representation of the C element is given by the equations 1. This enables the relations 2 for the forces and 3 for the stored energy to be written.

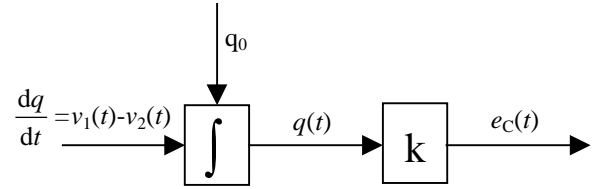
$$\begin{aligned} \frac{dq(t)}{dt} &= v_1(t) - v_2(t) \\ e_c(t) &= kq(t) \end{aligned} \quad [1]$$

$$\begin{aligned} -F_{S/1}(t) &= F_{S/2}(t) \\ &= e_c \\ &= kq(t) \\ &= k \int_0^t [v_1(\tau) - v_2(\tau)] d\tau + kq_0 \end{aligned} \quad [2]$$

$$E_c(t) = \frac{1}{2} kq(t)^2 + E_{c0} \quad [3]$$

where  $q$  is the generalized displacement corresponding to the spring elongation,  $q_0$  the associated initial condition, and  $E_{c0}$  is a constant corresponding to the initial (at  $t=0$ ) stored energy in the spring.

The equations 1 can be represented by the figure 2 calculus scheme.



**Figure 2.** Calculus scheme associated to the relation 2

Now from the mechanical point of view the spring forces may be expressed by:

$$F_{S/1}(t) = -k[l(t) - l_r] = -F_{S/2}(t) \quad [4]$$

where  $[l(t) - l_r] = q(t)$  is the spring elongation. It is calculated by:

$$[l(t) - l_r] = \int_0^t [v_1(\tau) - v_2(\tau)] d\tau + (l_0 - l_r) \quad [5]$$

with  $l$  the current spring length from point  $A_2$  to point  $A_1$ ,  $l_0$  the initial spring length and  $l_r$  the corresponding rest length.

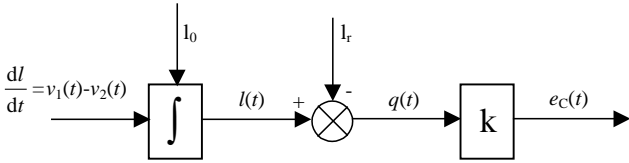
The figure 2 scheme shows that the only information required for the spring bond graph representation is the stiffness  $k$  and the initial condition on the elongation  $q_0$ . Nothing enables  $l$  and  $l_r$  to be here distinguished in the spring elongation while the rest length  $l_r$  of the spring may be crucial for a sizing problem. From the physics point of view the energy state of the C element is entirely determined by one state variable the generalized displacement corresponding to the spring elongation. To remedy this lack it is necessary to reconsider this state variable. Two solutions are proposed here to introduce the information on the rest length in the bond graph representation of a linear spring: (i) changing to another state variable; (ii) transforming the 1-port C element to a 2-port C element.

### First solution: changing to another state variable

From the figure 1-b bond graph and the equation 5, it is easy to see the direct relation between the derivatives of  $l$  and  $q$  (equation 6).

$$\frac{dl(t)}{dt} = v_1(t) - v_2(t) = \frac{dq(t)}{dt} \quad [6]$$

Noting that  $l_0 - l_r = q_0$ , the spring elongation is replaced by the spring length for the state variable of the figure 1-b C element. This suggests the figure 3 calculus scheme.

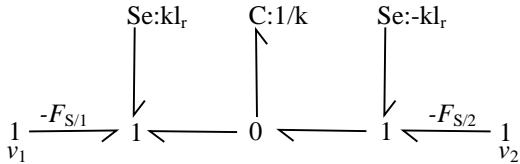


**Figure 3.** Calculus scheme associated to the relation 2

The bond graph representation is the same as the figure 1-b representation but the flow variable on the C element port is now the time derivative of length  $l$ . The state representation for the C element is now given by the equations 7 and the associated initial condition is  $l_0$  instead of  $q_0$ . Two parameters characterize the C element:  $k$  the stiffness and  $l_r$  the rest length.

$$\begin{aligned} \frac{dl(t)}{dt} &= v_1(t) - v_2(t) \\ e_c(t) &= k[l(t) - l_r] \end{aligned} \quad [7]$$

An alternative to the figure 1-b representation with  $l$  as the C element state variable is given figure 4. It is based on the fact that the constant term may be taken out from the output of equations 7. The advantage is that the C element is characterized now again with only one parameter (the stiffness  $k$ ) remembering that the initial condition corresponds to the initial spring length. The disadvantage is that two effort sources characterized by two parameters (the stiffness  $k$  and the rest length  $l_r$ ) inflate the representation.



**Figure 4.** Alternative representation with the spring length as the state variable for the C element

### Second solution: 2-port C element

From the equation 5 the spring length may be expressed by:

$$l(t) = \int_0^t v_1(\tau) d\tau + x_{10} - \int_0^t v_2(\tau) d\tau - x_{20} \quad [8]$$

where  $x_1$  and  $x_2$  are respectively the positions of connecting points  $A_1$  and  $A_2$ ,  $x_{10}$  and  $x_{20}$  the corresponding initial conditions such that  $x_{10} - x_{20} = l_0$ .

Equation 8 shows that it is necessary to compute  $x_1$  and  $x_2$  independently. However this is not done for the spring bond graph representation of figure 1. Even if  $x_1$  and  $x_2$  were calculated in subsystem attached respectively at point  $A_1$  and  $A_2$ , they would not be available for the spring since only power variables (effort and flow) are transported on power bonds. It is proposed here to substitute the figure 5 representation of the spring to the figure 1 representation.

$$1 \xrightarrow{-F_{S/1}} C \xleftarrow{-F_{S/2}} 1$$

**Figure 5.** Spring bond graph representation substitution

This representation is based on the idea of using the power ports of the component directly for the storage phenomena. Thus this 2- port C bond graph representation enables to integrate  $v_1$  and  $v_2$  separately to obtain  $x_1$  and  $x_2$ . In order to compute the spring elongation it is now necessary to give the rest length as a parameter for the spring. The characteristic laws are expressed from the storage energy in the spring:

$$\begin{aligned} E_c &= \frac{1}{2} k(l - l_r)^2 + E_{c0} \\ &= \frac{1}{2} k(x_1 - x_2 - l_r)^2 + E_{c0} \end{aligned} \quad [9]$$

Thus the efforts are:

$$\begin{aligned} e_1 &= -F_{S/1} = \frac{\partial E_c}{\partial x_1} = k(x_1 - x_2 - l_r) \\ e_2 &= -F_{S/2} = \frac{\partial E_c}{\partial x_2} = -k(x_1 - x_2 - l_r) = -e_1 \end{aligned} \quad [10]$$

Finally the state representation of the figure 5 2-port C element is given by:

$$\begin{aligned} \frac{dx_1(t)}{dt} &= v_1(t) \\ \frac{dx_2(t)}{dt} &= v_2(t) \\ e_1(t) &= k[x_1(t) - x_2(t) - l_r] \\ e_2(t) &= -k[x_1(t) - x_2(t) - l_r] \end{aligned} \quad [11]$$

The output relations of equations 11 show that they are not independent. This indicates that not all causality assignments are possible for the figure 5 C element. In particular derivative causality on both ports is impossible due to the outputs dependency. This is in agreement with the figure 1-b representation for the spring component. However it appears as a structural graphical property for the figure 1-b representation while it is only a structural mathematical property for the figure 5 representation.

For the 2-port C element one power bond has been changed in orientation to respect the inwards convention. So the sign of forces on the C ports are in agreement with the fact that the stored energy in the spring during a time interval is the work developed by the external forces on the spring (first principle for a spring).

With this second solution the bond graph representation the spring component is characterized by two parameters:  $k$  its stiffness and  $l_r$  its rest length. It can be noticed that the Maxwell reciprocity [Crandall et al. 1968] for the spring component is verified (equation 12).

$$\frac{\partial e_1}{\partial x_2} = -k = \frac{\partial e_2}{\partial x_1} \quad [12]$$

Karnopp explained that the order of a model depends on which physical variables are required as output variables [Karnopp 1979]. Here the order of the spring component model is increased of one for sizing parameter consideration, parameter that was not considered at first. However positions  $x_1$  and  $x_2$  are not really needed separately and just their difference is sufficient for characterizing the energetic state of the spring. This corresponds to the minimal state representation [Karnopp 1979].

Finally the figure 5 2-port C element may appear inconvenient for certain. It can be shown that the decomposition of this 2-port C element [Breedveld 1981] gives the figure 4 representation.

## A SIMPLE THERMODYNAMIC SYSTEM EXAMPLE

### Control volume boundaries

As previously recalled the potential energy stored in a spring depends on one energy variable the spring elongation. So for calculating the potential energy stored at time  $t$ , we only need the initial value of the elongation and the elongation change between  $t_0$  the initial time and time  $t$ , i.e. the integral of the relative velocity of the spring ends independently of the positions of the spring ends. Then, from an energetic point of view, a one port C element as shown on figure 1 can model the phenomenon of potential energy storage in a spring. If the analyst want to know the geometrical configuration of the spring he needs positions of the spring

moving ends which constitute the boundaries of the subsystem spring. From a bond graph point of view these positions are two displacement variables then the subsystem spring has to be modeled by the 2-port C element shown in figure 5.

Another examples can be found in the context of thermodynamics or fluid mechanics because the modeling of open systems is usually done with virtual control volumes or Eulerian frames fixed or moving. The control volume boundaries are in general composed of real parts that coincide with real walls and imaginary parts related to the control volume openings. The properties of the boundary real parts obviously depends on the properties of the walls whose the interior surface coincides with these boundaries. For example the boundaries can include deformable parts and/or moving parts if the walls have deformable and/or moving parts. On the assumption that the compressible fluid, contained inside the control volume, has a negligible kinetic energy and that its properties are independent of any spatial structure, it is well known that the internal energy stored in this fluid is a function of its mass  $m(t)$ , its volume  $V(t)$  and its entropy  $S(t)$ . Thus the phenomenon of the internal energy storage is modeled in bond graph by a 3-port C element, having one port in the material potential domain, one port in the elastic potential domain, one port in the thermal domain. The flow variables associated with these ports are respectively the time derivatives of the mass  $m(t)$ , of the volume  $V(t)$  and of the entropy  $S(t)$  of the fluid contained inside the control volume. This 3-port C element is shown in figure 6. Keeping constant one of these three energy variables implies to connect the corresponding port of the phenomenon bond graph model to a null flow source or to suppress this port.

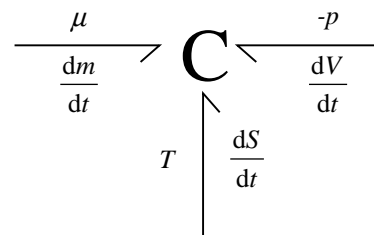
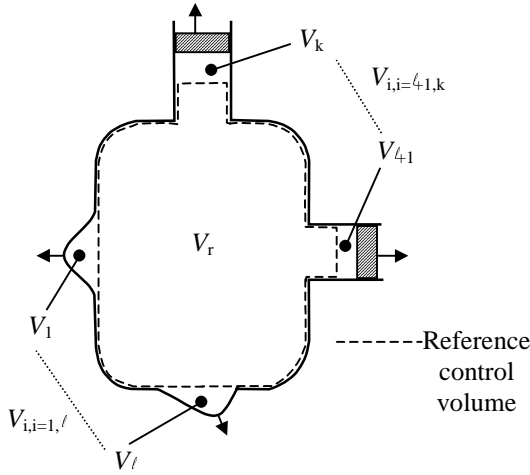


Figure 6. Internal energy storage bond graph model.

The fluid contained inside the control volume and the virtual boundaries of the control volume constitute a subsystem. The bond graph model of this subsystem takes into account the energetic phenomenology inside the fluid but also the energy exchanges at the boundaries. These energy exchanges obviously depends on the properties of the boundaries, i.e. the properties of the coinciding walls. For example, if the coinciding walls are assumed closed and adiabatic, the mass and the entropy are kept constant, then the material port and the thermal port of the 3-port C

element modeling the storage of internal energy can be suppressed. Furthermore, suppose that there are  $l$  deformable parts and  $k-l$  moving parts in the adiabatic boundaries of the closed and not moving control volume (figure 7).

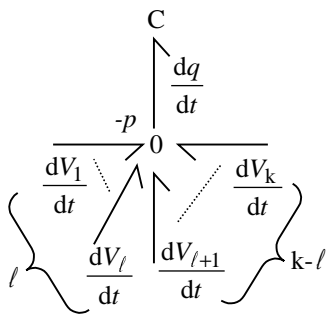


**Figure 7.** Control volume.

The bond graph model of the corresponding subsystem is composed of:

- a one port C element, which models the phenomenon of the internal energy storage, assumed here reversible, at constant mass and constant entropy and;
- a 0 junction connected to this one port C element and having  $k$  power bonds in the elastic potential domain.

This 0 junction takes into account the power balance. Since the fluid pressure is common for all the ports, the rate of the control volume change is equal to the sum of the  $k$  rates of the volume changes respectively due to the deformation of the  $l$  deformable parts and to the motion of the  $k-l$  moving and indeformable parts of the boundaries. The corresponding bond graph is shown in figure 8.



**Figure 8.** Bond graph model of the subsystem control volume.

This bond graph model corresponds to a state model with one state variable the volume change  $q(t)$  calculated from the volume  $V_r$  chosen as the reference control volume. Consequently there is one initial condition  $q(0)$  which globally takes into account, the initial deformation of each deformable part ( $V_{0i,i=1,l}$ ) and the initial volume corresponding to the initial position of each moving part ( $V_{0i,i=4,1,k}$ ). The state equation of the subsystem control volume is:

$$\frac{dq}{dt} = -\sum_{i=1}^k \frac{dV_{ie}}{dt} \quad [13]$$

with  $\frac{dV_{ie}}{dt}$  volume change rate corresponding to the deformation of a deformable part or due to the motion of a moving part imposed by the surrounding.

As the process is assumed reversible and adiabatic the output equation can be written:

$$-p = -\frac{p_r V_r^\gamma}{(V_r + q)^\gamma} \quad [14]$$

In this output equation there are two sizing parameters: the volume  $V_r$  and the pressure  $p_r$  corresponding to the reference state.

### Solution: k-port C element

It is also possible to choose the total volume  $V(t)$  as state variable. Then the state and the output equation has to be written:

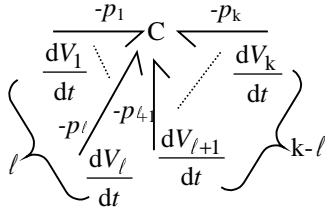
$$\frac{dV}{dt} = -\sum_{i=1}^k \frac{dV_{ie}}{dt} \quad [15]$$

$$-p = -\frac{p_r V_r^\gamma}{V^\gamma} \quad [16]$$

It can be noticed that if the analyst want to make appear both the volume changes due to the deformation of each deformable part ( $V_{i,i=1,l}$ ) and due to the motion of each moving part ( $V_{i,i=4,1,k}$ ) of the boundaries, he has to replace the previous bond graph (figure 8) by a  $k$ -port C element (figure 9).

Then the state model of the subsystem will have  $k$  state variables: i.e. the  $k$  volume changes due to the deformation of the  $l$  deformable parts ( $V_{i,i=1,l}$ ) and due to the motion of the  $k-l$  moving parts of the control volume boundaries ( $V_{i,i=4,1,k}$ ). At these  $k$  state variables correspond  $k$  initial conditions, i.e. the  $l$  volumes corresponding to the initial deformation of the  $l$  deformable parts ( $V_{0i,i=1,l}$ ) and the  $k-l$  volumes

corresponding to the initial position of the  $k-l$  moving and indeformable parts ( $V_{0i,i=41,k}$ ). The state equations of the subsystem control volume are:



**Figure 9.**  $k$ -port  $C$  element representation for the control volume.

$$\frac{dV_i}{dt} = -\frac{dV_{ie}}{dt} \quad i \in [1, \dots, k] \quad [17]$$

It can be noticed that the hypothesis of independence of the fluid properties of any spatial structure implies the equality of all the pressures  $p_i$ . Then the output equations, i.e. the constitutive equations of the  $k$ -port  $C$  element, are identical and have for expression:

$$-p_j = -\frac{p_r (V_r)^\gamma}{\left( V_r + \sum_{i=1}^k V_i \right)^\gamma} \quad j \in [1, \dots, k] \quad [18]$$

So the complete derivative causal form of the  $k$ -port  $C$  element is not possible,  $k-1$  power bonds remain in integral causality.

Furthermore it can be noticed that this expression makes appear the same sizing parameters  $V_r$  and  $p_r$ .

## CONCLUSION

Concerning the spring example the problem of the rest length has already been evoked in the context of rigid mechanical objects [Breedveld 1997]. However the proposed approach was an answer for making explicitly available the position of each rigid object in a bond graph but does not explicitly introduce the rest length parameter for a spring. Here the first solution for taking into account the rest length of a spring showed an alternative for choosing a  $C$  element state variable. The second solution was to substitute a 2-port  $C$  element to the original bond graph representation of the spring.

Concerning the simple thermodynamic example accounting for the boundary geometry has necessitated the initial one port  $C$  element representation into a  $k$  port  $C$  element representation to be transformed. This alternative is analogous to the one proposed for the spring second solution representation. For the spring this enables a sizing parameter to be explicit in the bond graph representation while for the

thermodynamic system this enables the boundary geometry of the system to be explicit.

Finally, Karnopp [Karnopp 1979] has underlined that the number of state variables of a given system depends on the number of independent energy storage elements and of the number of integrators necessary to obtain the chosen outputs. It is shown here that beside the energy state of a system the spatial configuration also affect the way of modeling (e.g. positions of the spring attachment points or boundary geometry of the thermodynamic control volume). This emphasizes once more the specificity of displacement variables in mechanics [Karnopp 1979], [Breedveld 1997].

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