

## CORRIGENDUM

YVES CAPDEBOSCQ

ABSTRACT. ©Capdeboscq, 2014. The definitive, peer reviewed and edited version of this article is published in *Asymptotic Analysis*, IOS Press, 88 (3), pp.185–186 2014. DOI: 10.3233/ASY-141240.

Correction to : On the scattered filed generated by a ball inhomogeneity of constant index

The statement of Lemma A.3 is incorrect and its proof is incomplete. Below is the corrected statement, and its proof.

**Lemma.** For any  $x > 0$  the function  $x \rightarrow \ln \left| H_0^{(1)}(x) \right|$  is convex. For any  $y > 1$  the function

$$x \rightarrow \frac{\left| H_0^{(1)}(xy) \right|}{\left| H_0^{(1)}(x) \right|}$$

is decreasing on  $(0, \infty)$  and

$$1 \geq \frac{\left| H_0^{(1)}(xy) \right|}{\left| H_0^{(1)}(x) \right|} \geq \frac{1}{\sqrt{y}}.$$

*Proof.* Define

$$f := x \rightarrow \ln \left| H_0^{(1)}(x) \right|.$$

The proof (which unfortunately contains obvious typos) given in the paper shows the convexity (not the concavity) of  $f$ . Several inequalities of this type are shown in [1]. An asymptotic expansion shows that  $\lim_{x \rightarrow \infty} x^4 f''(x) + x^3 f'(x) = -\frac{1}{4}$ , and by inspection we see that

$$x^4 f''(x) + x^3 f'(x) < 0 \text{ for all } x > 0.$$

Furthermore,  $x \rightarrow x^2 f''(x)$  is increasing. Therefore, for every  $0 < \lambda \leq 1$ , and for every  $x > 0$  there holds

$$x^2 f''(x\lambda) + \frac{x}{\lambda^2} f'(x) \leq \frac{1}{\lambda^2} (x^2 f''(x) + x f'(x)) < 0,$$

thus, for any  $x > 0$ ,  $y \rightarrow x f'(x\lambda) - \frac{x}{\lambda} f'(x)$  is decreasing, and therefore cancels only at  $\lambda = 1$ . We have obtained that  $f'(x\lambda) - \lambda^{-1} f'(x) > 0$  for all  $\lambda < 1$  and all  $x > 0$ . Now set  $t = x\lambda$ , and  $y = \lambda^{-1}$ , to derive that

$$y f'(ty) < f'(t)$$

for all  $y > 1$ , and all  $t > 0$ . In particular,  $x \rightarrow \exp(f(xy) - f(x))$  is decreasing on  $(0, \infty)$ , which is our thesis. The bounds follow from the limits the origin and infinity.  $\square$

There is a typo in the statement of Theorem 2.1, Theorem 3.1 and one before last line of the proof of Theorem 2.1. Instead of  $u_s^\epsilon (|x| = (\epsilon\omega)^{-1})$ , one should read  $u_i (|x| = \epsilon)$ . There is also a typo in the first line of the proof of Theorem 5.1 . Instead of  $x \left| \frac{H_n^{(1)}(\sqrt{q_0\omega}R)}{H_n^{(1)}(\lambda\omega_\epsilon)} \right|^2$ , one should read  $x \left| H_n^{(1)}(x) \right|^2$ .

## REFERENCES

- [1] Á Baricz, S. Ponnusamy, and M. Vuorinen. Functional inequalities for modified Bessel functions. *Expo. Math.*, 29(4):349–414, 2011.