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# Manipulatives in special education: help or hindrance?

Catherine Houdement and Édith Petitfour

LDAR (EA 4434) Université de Rouen Normandie, UA UCP UPD UPEC;  
[catherine.houdement@univ-rouen.fr](mailto:catherine.houdement@univ-rouen.fr); [edith.petitfour@univ-rouen.fr](mailto:edith.petitfour@univ-rouen.fr)

*Our research focuses on semiotic issues in teaching and learning mathematics in special education. In this paper we examine the use and role played by manipulatives in a session that is intended to teach decimal numeration to students with mathematical learning difficulties. A semiotic analysis of the data (audio and video recordings) highlights discrepancies between the intention of the teacher providing manipulatives and the uses made by students, who are, however, very cooperative. We conclude with some suggestions for more appropriate use of the material.*

*Keywords: mathematical learning difficulties, special needs, manipulative materials, semiotic misunderstanding.*

## Introduction

French special education practice is strongly rooted in using everyday manipulative and concrete situations as a way to teach students with Mathematical Learning Difficulties (MLD) (Assude, 2018).

Our research focuses on the role of manipulatives in teaching and learning mathematics in special education: does it help students or does it create barriers? In this paper, we explore the issue by studying the use and effect of manipulatives on students with special needs, during a session that is intended to teach them about decimal numeration, specifically “*capturing the meaning of place value including decimal notation*” (Karagiannakis, Baccaglini-Frank, & Papadatos, 2014, p. 2).

## Theoretical stance and tools

### Observer posture

“*It is time to reveal what children with MLD can do, rather than they cannot*” (Van den Heuvel Panhuizen, quoted in Verschaffel, 2015, p. 623). Situations in special education often offer a greater degree of unpredictability than other settings (Giroux, 2013). We focus our observations on students’ own actions, and on the didactic and social interactions that are seen during the activity. We hypothesize that these actions and interactions are constitutive of their learning.

### Theoretical tools

To take into account the often-quirky use and interpretation of mathematical signs by students in special education, we adopt a semiotic approach (Peirce, 1931–1953). Several researchers working in the domain of teaching mathematics to MLD students (e.g. Giroux, 2013) have pointed out that the reduction of discrepancies between students’ and the teacher’s mathematical interpretations is an issue in teaching. These interpretations depend on tasks and interactions.

We name these interpretation discrepancies *semiotic misunderstanding* (Houdement & Petitfour, 2018). A semiotic misunderstanding can be a discrepancy between two interpretations of the same sign by a person, or a quirky personal interpretation of a mathematical sign. The term can apply to any type of signs, not only mathematical ones.

We adopt a semiotic approach to capture the multimodality of mathematics learning and teaching phenomena (e.g. Radford, Edwards, & Arzarello, 2009; Sabena, 2018) and use the concept of the *semiotic bundle*, defined as,

“a *system of signs*<sup>1</sup> (...) that is produced by one or more interacting subjects and that evolves in time. Typically, a semiotic bundle is made of the signs that are produced by a student or by a group of students while solving and/or discussing a mathematical question. Possibly the teacher too participates to this production and so the semiotic bundle may include also the signs produced by the teacher.” (Arzarello, Paola, Robutti, & Sabena, 2009, p. 100)

Following Sabena (2018), we construct a *semiotic table*, which is a multimodal transcript that reports the diachronic and synchronic dimensions of the semiotic bundle (Figures 2, 3, 4, 5): for each intervention, the first row records time markers; the second row records the intervention number; and the third row records the author. There are as many rows as necessary, given the data.

## Context – data


This paper is based on two filmed episodes from Lena’s class (Lena is the teacher), in a medical–educational institute. The class includes seven students, aged 12–13, with intellectual disabilities, who have difficulty understanding numeracy. In a preceding session, Lena had introduced a “groupable proportional” physical model (Van de Walle, 2007), consisting of sticks bundled with elastics into sets of ten or a hundred, in order to highlight place value in written numbers (Houdement & Tempier, 2018). She presents a situation that requires writing down the number of tens and units in the context of a placing an order for chalk (Figure 1). Students had already seen the sticks and were familiar with its use to illustrate quantities as this was the second time they had worked with the order form.

Here is the teachers order form

Customers	Number of chalks ordered	Chalk boxes delivered
Mrs C.	100	..... boxes of 10
Mrs F.	120	..... boxes of 10

Here is the school order form

School	Number of chalks ordered	Chalk boxes delivered
Grade 6	101	..... boxes of 10 and .....unit chalks



**Figure 1: Excerpt of an order form from the student’s file**

In this paper, we focus on Nick and Angele. Both students are mildly intellectually impaired. Nick has been diagnosed with dyspraxia and visual spatial disorders. Angele suffers from an emotional (or behavioral) disorder – Lena portrays Angele as ‘explosive’ as she has fits when she feels helpless. In the first extract, Lena responds to Angele’s request for help. In the second, Lena ends the session by

<sup>1</sup> “words (orally or in written forms); extra-linguistic modes of expression (gestures, glances...); different types of inscriptions (drawings, sketches, graphs...); various instruments (...)” (Arzarello et al., 2009, p. 97)







reviewing the order form with Angele and Nick. We were also interested to hear about the teacher's perceptions, and we therefore recorded exchanges with Lena about this session.

To recap, our aim was to describe and analyze interactions between students, using semiotic bundles, in order to capture uses of manipulatives and its influence on mathematical teaching and learning. The following sections introduce and discuss some of our findings.

### The worksheet, a source of misunderstanding

The task consisted of filling out two order forms (Figure 1). We consider the order form as material to be analyzed. In the first column, Lena presents schoolteachers as customers. In the interview, Lena informed us that she thought that this contextualization helped students to make sense of the task. She assumed that the situation presented in the order form helped students to understand the worksheet, the organization of the table, and how to read it. But reading this document does not produce the intended effects – neither for Angele, nor for Nick. Both students fail to understand what is expected of them. The tables are even a source of (sometimes amazing) semiotic misunderstandings.

Beginning with the first semiotic misunderstanding, Nick talks to Angele and Melanie (a third student) about the answer 'ten' in Mrs. C's row of the table, which, in theory, means ten boxes of ten chinks. Surprisingly, Nick interprets '10 boxes of 10' (Figure 2), as 'ten boxes to which ten are added'. Group work does not give Nick the opportunity to question this interpretation.

		1'38		1'41														
		82		83														
		Nick		Melanie														
WORDS	Nick	It's ten	plus ten	it's twenty														
	Melanie				Huh?													
BODY	Nick	<table><tr><td>Customers</td><td>Number of chalks ordered</td><td>Chalk boxes delivered</td></tr><tr><td>Mrs C.</td><td>100</td><td> boxes of 10</td></tr></table>		Customers	Number of chalks ordered	Chalk boxes delivered	Mrs C.	100	 boxes of 10	<table><tr><td>Customers</td><td>Number of chalks ordered</td><td>Chalk boxes delivered</td></tr><tr><td>Mrs C.</td><td>100</td><td> boxes of 10</td></tr></table>		Customers	Number of chalks ordered	Chalk boxes delivered	Mrs C.	100	 boxes of 10	Nick looks at Melanie
	Customers	Number of chalks ordered	Chalk boxes delivered															
Mrs C.	100	 boxes of 10																
Customers	Number of chalks ordered	Chalk boxes delivered																
Mrs C.	100	 boxes of 10																
	Melanie	Melanie counts ten by ten up to a hundred with her fingers			Melanie writes '10'													







		1'42																
		84																
		Nick																
WORDS		It's ten boxes	and then you take ten															
	BODY	Nick	<table><tr><td>Customers</td><td>Number of chalks ordered</td><td>Chalk boxes delivered</td></tr><tr><td>Mrs C.</td><td>100</td><td> boxes of 10</td></tr></table>		Customers	Number of chalks ordered	Chalk boxes delivered	Mrs C.	100	 boxes of 10	<table><tr><td>Customers</td><td>Number of chalks ordered</td><td>Chalk boxes delivered</td></tr><tr><td>Mrs C.</td><td>100</td><td> boxes of 10</td></tr></table>		Customers	Number of chalks ordered	Chalk boxes delivered	Mrs C.	100	 boxes of 10
Customers			Number of chalks ordered	Chalk boxes delivered														
Mrs C.	100	 boxes of 10																
Customers	Number of chalks ordered	Chalk boxes delivered																
Mrs C.	100	 boxes of 10																

Figure 2: Excerpt from the semiotic table from 1'38 to 1'43

We continue with the second semiotic misunderstanding, which concerns the first row of the second table (101 chinks, Figure 1) and the number of single units, Nick writes **1** to the right of his correct answer in the first row of the first table (100 chinks, Figure 1): resulting in "10**1** boxes of 10". We interpret this as a 'mis-seen' semiotic misunderstanding, which can probably be attributed to Nick's trouble in seeing the whole sheet of paper or discerning important details because of his visual spatial learning disabilities. He seems unable to provide an answer alone. 'Assembling' two numbers into one persists when correcting with manipulatives. Nick makes six attempts; each time he counts the number of bundles out loud as groups of ten (ten, twenty, thirty, forty, etc.), but nevertheless concludes by adding the numbers concerned to make one hundred and ten. For his first attempt, he

announces “It’s a hundred and ten”; the second and third times he proposes one hundred and twenty, then one hundred and thirty as it appears that he thinks that he has made a mistake. The following times, he affirms and reaffirms his answer “It’s one hundred and ten!” Lena then becomes aware of the misunderstanding.

Nick: It’s one hundred and ten, Miss!

Lena: You, you’ve done one hundred plus ten, Nick.

Nick: Well it’s true. I can’t, it makes me nervous!

Nick’s wish to add two numbers could stem from his interpretation of the order form. He thus evokes the problem to be solved in the previous session that is similar: “you had to order chalks for the class and for the teachers and count how many there are altogether.” In our opinion, there appears to be a lack of flexibility regarding the interpretation of an already-met sign (here, the order form), when applied to another problem. The stiffening of the signification of a sign is a common cognitive phenomenon in MLD students (Bloch, 2009).

## **Manipulative materials**

### **The teacher’s perspective**

Lena provides manipulatives to help her students: wooden sticks represent sticks of chalk. Sticks are put in a tray, either as single sticks, or grouped into bundles of ten or one hundred (ten bundles of ten) with elastic. She explains to her students that a stick represents a ‘single stick of chalk’, and that a bundle of ten sticks represents a box of ten sticks of chalk.

Students do not spontaneously use the sticks to complete the table. Lena has to encourage them, first by helping Angele to complete the table (246 sticks of chalk have to be ordered), then by helping Angele and Nick to correct the first table. In both cases, she assigns the same function to the material: to directly solve the specific problem without any prior explanation.

During the interview Lena confirmed that the procedure she had anticipated for Angele was to accumulate bundles of ten, while counting by ten, up to 240: we call this procedure based on the accumulation of bundles of ten, procedure L. We observed what happened in practice: she stopped any other procedure, particularly Angele’s repeated attempts to use pre-prepared bundles of a hundred (lines 117–121).

Angele: No, I prefer to do it with the hundreds, Miss!

Lena: Yes, but you can’t, just have a look Angele // *she shows the photo of the box of chalk*

Angele: Well, it’s a hundred, here it’s two hundred // *she takes two bundles of a hundred out of the tray.*

Lena: Yes, but you haven’t got a box of a hundred chalks, you’ve only got boxes of ten // *she shows the photo of the box of chalk on the worksheet.*

Angele: Oh, it’s complicated, huh // *she puts the bundle of a hundred back into the tray.*

And while correcting Angele and Nick's work, Lena specifies (at 18'22):

Lena: Anyway, the bundles of a hundred, we're not interested in them right now, you don't have to take them, you've only got bundles of ten // *she shows the picture of the chalk box on Angele and Nick's worksheet.*

After the session, Lena said that she wanted to help Angele by using the material as it would be used in the proposed situation: the seller does not have a box of a hundred sticks of chalk. She had not realized that Angele had started with another procedure (that we call procedure A), which could have succeeded: first construct the collection of 246 sticks by using 2 pre-prepared bundles of a hundred sticks, 4 bundles of ten sticks, and 6 individual sticks and then enumerate the bundles of ten in the collection.

### Angele's perspective

Angele agrees to use the material that is provided to order of 246 sticks of chalk. She takes two, pre-prepared bundles of a hundred out of the tray, and another 4 single sticks. Then she calls the teacher for help. For Angele, two bundles of a hundred sticks represent the '2' in 246 (you can hear it when you say the number 246), but how can you represent the '4' (as you do not hear 'four ten')? This behavior reflects a semiotic misunderstanding about the relation between sticks and numeration units. It seems that Angele focuses on the digit 4, without taking into account its value (40) in the number: indeed, she says "it's four tens" but puts four single sticks on the table. It is likely that she stops because she is surprised to find that she has no more material to represent the 6 in 246. The role of manipulatives is correct: to illustrate the meaning of place value. But Angele is unable to interpret this surprise alone. The teacher does not see her problem, interrupts procedure A (Figure 3) and initiates procedure L (only using bundles of ten and single sticks).



		4'59	5'00			5'04
		<b>119</b>	<b>120</b>			<b>121</b>
		Angele starts the procedure A again	Teacher blocks Angele's procedure A			Angele surrenders
WORDS	Angele	<i>Self-confident</i> Well it is, it's a hundred, that's two...	... hundred			Oh, it's too complicated, huh?
	Teacher		Yes, but you don't have..	... box of a hundred, you only have..	.. boxes of ten	
BODY	Angele	Angele puts the two packs of 100 in front of her			Angele quickly puts a 100-pack back in the bin..	... then the other one.
	Teacher			Teacher points to Unit chalk Box of 10 chalk	then to the chalk box drawn on the paper	

Figure 3: Excerpt from the semiotic table from 4'59 to 5'05

Angele finds it difficult to work with procedure L and encounters many problems in singing the skip counting-by-ten song from one hundred (110, 120, 130...); Lena does not encourage her to stop at one hundred and start again, instead she firmly guides her to continue the song beginning at one hundred. Angele skips some numbers and hesitates. Lena tries to help her to adjust using the 1 to 100



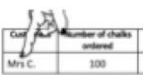
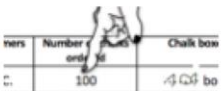
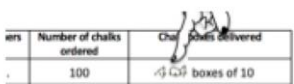

number chart. In the end, Angele just about manages to count up to 200 in tens with the help of Lena, whereas she had already reached this quantity when working alone (see line 119).

The procedure L requires knowledge that Angele had not yet acquired, namely counting by ten from one hundred to two hundred. At the same time, Angele had already begun another procedure (procedure A) that was within reach and required less memory, which could have resulted in success. Following procedure A would have meant that she did not have to attempt the unattainable, counting by ten procedure imposed by Lena. But Lena had planned for her to use procedure L, which seems to suggest that Lena believes there is only one way to use the material.

### Correcting Nick's work

Working alone, Nick filled in the table by drawing upon verbal exchanges with his peers: in particular, he wrote 101 in the first row of the first table (which required 100 sticks of chalk to be ordered, Figure 1). Lena focuses on this incorrect response, putting the tray of sticks in front of both students. First, Lena asks Angele and Nick how to get a hundred with bundles of ten: Lena encourages them to count, but Angele immediately gives the right answer (“well, ten”), confirming what she demonstrated when working individually, namely that she does not need the material to find the answer. Then, she retrieves bundles of ten from a bundle of a hundred and aligns them while reciting the counting-by-ten song. Nick joins her when she reaches sixty. Angele concludes “one hundred, and it’s ten”. She does not re-count the bundles; she recalls “ten” from memory.

When Lena asks again: “How many bundles of ten do we need to have a hundred?”, this time Angele counts them aloud by pointing to the bundles, one by one, up to ten. Angele and Nick answer together that it is ten. When Lena points her finger to the worksheet, Nick understands the link between Mrs. C’s order and the material (Figure 4).

		19'57		19'59		20'01		20'04	
		Lena		Nick		Lena		Nick	
WORDS	Nick			hundred					
	Teacher Lena	Mrs C.	she asked ... ?			Hundred chalks	How many boxes of ten you are going to send her?	... ?	ten
BODY	Nick			Nick reads the « 100 » pointed				Nick looks at the teacher	
	Teacher Lena		Teacher points « 100 » 						Teacher puts and moves one hand on the bundles

**Figure 4: Excerpt from the semiotic table from 19'57 to 20'05**

When Nick is asked again to “count again, ten-by-ten”, he incorrectly enumerates the bundles by reciting the skip counting-by-ten song. Although he points to the bundles from left to right with his index finger, his gesture is not always coordinated with his words: he says, “fifty sixty” while pointing to the fifth bundle, and the two words “seventy eighty” while pointing to the sixth bundle. Finally, he says “one hundred” without pointing to the last two bundles. The semiotic table illustrates Nick’s problems with using the material (Figure 5).

These enumeration mistakes (two number words for one object, objects that are forgotten) that may be due to Nick's dyspraxia, are not perceived by Lena. It should be noted that Nick does not find the answer "one hundred", or "it's ten" by using the material, but rather by repeating Angele's answer. The material is not helpful for Nick.









		20'14						20'19	20'21
		Nick							
WORDS	Nick	Ten twenty	thirty	forty	fifty	sixty	seventy eighty	ninety	hundred
	Teacher								It is ten
BODY	Nick								
	Teacher								

Figure 5: Excerpt from the semiotic table from 20'14 to 20'22

## Conclusion

In principle, manipulatives used in special education are intended to help students' mathematical learning. But a significant risk exists that this approach does not consider the variety of ways it can be used, and how to coordinate this with the student's reasoning, understood as "his/her intention to act" (Petitfour, 2017, p. 252). We have offered several illustrations of the issue in this text.

The teacher introduces manipulatives that are understood to be suited to numeracy work (Van de Walle, 2007). But she assigns a single function to it: accumulating bundles of ten while counting by ten up to the target quantity. This use of the material does not allow either Angele or Nick to solve the problem independently, given their respective difficulties (enumeration and counting words).

Nevertheless, both students find ways to use the material constructively: Angele initiates a different procedure (procedure A) that could have resulted in reaching the correct answer (the 246 example). A little later, Angele and Nick make a spontaneous association in order to fulfill the order for 200 chinks. When the teacher introduces a box to put the objects in, they both carry out an effective manipulation: each, in turn, places a bundle of ten in the box while counting by ten. When working together, Nick, guided by Angele, can enumerate correctly. This cooperation also enables Nick to correctly recite the counting-by-ten song, as he is sensitive to Angele's mistakes.

The teacher does not use the material to accomplish any other functions, for example to check proposed answers, to validate them (or not), or to highlight errors. For instance, if Nick had tried to check his answer by taking 101 bundles of ten, he might have realized his mistake.

In principle, suitable manipulatives have many uses, as it can give students ideas for potential actions that could become effective procedures, and it can provide a practical way to check students' answers. But, in special education as elsewhere, the teacher should allow students a degree of autonomy in problem solving, facilitate cooperation (e.g., working in pairs), and illustrate the organization of space (e.g., the introduction of a box, which was probably useful for dyspraxic Nick). On the other hand, even suitable material can become an obstacle if the teacher or student rigidifies its use or/and associated procedure.



## References

- Arzarello, F., Paola, D., Robutti, O., & Sabena, C. (2009). Gestures as semiotic resources in the mathematics classroom. *Educational Studies in Mathematics*, 70(2), 97–109.
- Assude, T. (2018). Relations entre systèmes sémiotiques, milieux et techniques mathématiques : malentendus, hybridité, inventivité. *Actes du 44<sup>e</sup> colloque de Formateurs de Professeurs des Écoles en mathématiques* (pp. 13–28). Épinal 2017.
- Bloch I. (2009). Enseignement des mathématiques à des élèves "en difficulté": situations, signes mathématiques, phénomènes de contrat. *Actes du 35<sup>e</sup> colloque de Formateurs de Professeurs des Écoles en Mathématiques* (pp. 63–79). Retrieved from <http://www.arpeme.fr/documents/7F525E0A267B2350EFA8.pdf>
- Giroux, J. (2013). Étude des rapports enseignement/apprentissage des mathématiques dans le contexte de l'adaptation scolaire. Problématique et repères didactiques. *Éducation et Didactique*, 7(1), 59–85.
- Houdement, C., & Petitfour, E. (2018). L'analyse sémiotique de l'activité mathématique, une nécessité didactique dans le contexte de l'adaptation scolaire. *Annales de didactique et de sciences cognitives*, 23, 9–40.
- Houdement, C., & Tempier, F. (2018). Understanding place value with numeration units. *ZDM Mathematics Education*, 2018, DOI 10.1007/s11858-018-0985-6.
- Karagiannakis, G., Baccaglini-Frank, A., & Papadatos, Y. (2014). Mathematical learning difficulties subtypes classification. *Frontiers in Human Neuroscience*, 8(57), 1–5. DOI 10.3389/fnhum.2014.00057.
- Petitfour, E. (2017). Outils théoriques d'analyse de l'action instrumentée, difficultés d'élèves dyspraxiques en géométrie. *Recherches en didactique des mathématiques*, 37(2-3), 247–288.
- Peirce C. S. (1931 à 1953). *Écrits sur le signe (rassemblés, traduits et commentés par Gérard Deledalle)*. Paris: Éditions du Seuil 1978.
- Radford, L., Edwards, L., & Arzarello, F. (2009). Beyond words. *Educational Studies in Mathematics*, 70(3), 91–95.
- Sabena, C. (2018). Exploring the Contribution of Gestures to Mathematical Argumentation Processes from a Semiotic Perspective. In G. Kaiser, H. Forgasz, M. Graven, A. Kuzniak, E. Simmt, & B. Xu (Eds.), *Invited Lectures from the 13th International Congress on Mathematical Education* (pp. 541–559). Cham: Springer.
- Van de Walle, J. A. (2007). *Elementary and middle school mathematics: Teaching developmentally* (Sixth edition). Boston, MA: Allyn & Bacon.
- Verschaffel, L. (2015). Panel on Special Needs. In X. H. Sun, B. Kaur & J. Novotna (Eds.), *Proceedings of the Twenty-third ICMI Study: Primary mathematics study on whole numbers*. (pp. 619–624). Macau, China. Retrieved from <http://www.umac.mo/fed/ICMI23/proceedings.html>.