Identification of heterogeneous elasto-plastic behaviors using the Constitutive Equation Gap Method

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Abstract — The constitutive equation gap method (CEGM) is applied to identify the spatial distribution of mechanical fields (material parameters and stress) in a heterogeneous elasto-plastic material. In order to apply the method in a controlled situation, a test producing a known state of residual stresses was designed. For such a material, the heterogeneous residual stress field can be interpreted as a distribution of material parameters (yield stress and hardening law). The ability of the CEGM to identify heterogeneous material properties is verified on the experimental results.
Key Words — Full field measurement, Material identification, Inverse method, Elastoplasticity.

Introduction

We focus here on the CEGM to identify local constitutive laws and mechanical parameters in heterogeneous materials. Our approach uses the secant elastoplastic tensor $\underline{\underline{B_n^s}}$ and a measured strain field $\underline{\varepsilon}\left(\overrightarrow{u_n^m}\right)$ deduced from the displacement field $\overrightarrow{u_n^m}$ measured at a load step n by a full-field measurement. The secant tensor depends on the elastic and plastic material properties, on the spatial position and on the load step n. This tensor is equal to the standard elastic tensor $\underline{\underline{B^e}}$ for an elastic step. The identification method proposed in this work enhances the one proposed by Latourte et al. [1] since it allows both to identify simultaneously all the plastic parameters and to deal with complex geometries.

General presentation of the CEGM

The concept of the CEG is based on the minimization of a cost function equal to the sum of the potential and complementary energies and on the deviation between the measured and computed strain fields. The CEGM uses a statically admissible stress field $\underline{\sigma_n^c}$ involving a secant stiffness tensor $\underline{\underline{B_n^s}}$ determined at each load step n:

$$\underline{\sigma_n^c} = \underline{\underline{B_n^s}} : \underline{\varepsilon_n^c} \tag{1}$$

with $\underline{\varepsilon_n^c} = \underline{\varepsilon}\left(\overrightarrow{u_n^c}\right)$ representing the strain tensor and $\overrightarrow{u_n^c}$ the associated displacement field.

We consider a quasi-static problem over a domain Ω with free surfaces $\partial\Omega_i$, boundaries $\partial\Omega_j$ loaded with overall known forces $\vec{R}_j$ and boundaries with imposed displacements $\partial\Omega_u$ satisfying $\partial\Omega_j \cup \partial\Omega_i \cup \partial\Omega_u = \partial\Omega$, $\partial\Omega_j \cap \partial\Omega_i = \emptyset$, $\partial\Omega_i \cap \partial\Omega_u = \emptyset$ and $\partial\Omega_j \cap \partial\Omega_u = \emptyset$:

$$\begin{cases} div\ \underline{\sigma_n^c} = \vec{0} \quad in\ \Omega \\ \vec{R}_j = \int_{\partial\Omega_j} \underline{\sigma_n^c}\vec{n}\ ds \quad on\ \partial\Omega_j \\ \underline{\sigma_n^c}\vec{n} = \vec{0} \quad on\ \partial\Omega_i \\ \overrightarrow{u_n^c} = \overrightarrow{u_n^m} \quad on\ \partial\Omega_u \end{cases} \tag{2}$$

where $\vec{n}$ is the outer unit vector. For a sequence of N successive load steps, the CEG functional depends on two sets of parameters, *i.e.* the stress fields $\underline{\sigma_n^c}$ and the mechanical material parameters on which $\underline{\underline{B_n^s}}$ depends, and reads:

$$F_{CEGM}\left(\overrightarrow{u_n^c}, \underline{\underline{B_n^s}}\right) = \sum_{n=1}^{N} \int_{\Omega} \left[\underline{\varepsilon}\left(\overrightarrow{u_n^c}\right) - \underline{\varepsilon}\left(\overrightarrow{u_n^m}\right)\right] : \underline{\underline{B_n^s}} : \left[\underline{\varepsilon}\left(\overrightarrow{u_n^c}\right) - \underline{\varepsilon}\left(\overrightarrow{u_n^m}\right)\right] d\Omega \tag{3}$$

According to Simo & Hughes [2], denoting $\Delta\gamma_n$ the plastic multiplier increment and k the hardening modulus, the secant tensor reads $\underline{\underline{B_n^s}} = \left[\underline{\underline{B^e}}^{-1} + \frac{\Delta\gamma_n}{1+\frac{2}{3}k\Delta\gamma_n}\underline{\underline{P}}\right]^{-1}$, where $\underline{\underline{P}} = \frac{1}{3}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 6 \end{bmatrix}$.

The convexity of the CEG functional allows the use of a relaxation method for minimizing the functional with respect to its first argument (the displacement field $\overrightarrow{u_n^c}$ associated with the stress field $\underline{\sigma_n^c}$) and then to its second argument (the material properties introduced in the expression of the secant tensor $\underline{\underline{B_n^s}}$) in order to identify mechanical parameters and stress fields.

First we identify the elastic properties, second we determine the time step at which plasticity is activated at least on one material domain and third we identify the plastic properties. This approach is suitable for multilinear kinematic hardening since the expression of the secant elastoplastic tensor remains valid for any kinematic hardening.

Experimental application

In this section some results obtained with the CEGM presented above are given. The identification method was applied to a heterogeneous material subjected to a tensile test. Due to its trapezoidal shape, the specimen presented in Figure 1 allows to obtain heterogeneous residual strain fields after a tensile loading. The specimen was machined after this pre-hardening in order to obtain a standard dog-bone specimen. The residual plastic strain field (between the reference trapezoidal specimen and the machined dog-bone specimen) was measured by DIC. The elasto-plastic response of the material was previously identified using a standard tensile test. It allows to determine the value of the yield stress locally reached during this first process. A tensile test is the performed on the obtained dog-bone specimen. The strain fields are measured by DIC and we perform a multilinear elastoplastic identification. The mesh used for elastic identification is composed of a single quadrangular element, since homogeneous elastic properties are expected. The mesh used for plastic identification contains 20 elements and the mesh used for the DIC has 160 quadrangular elements. The identification is carried out on 20 material domains.

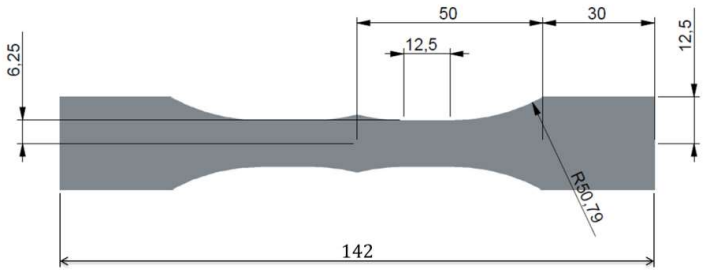

Figure 1: “Trapezoidal” specimen: dimensions in mm

The elastic identification gives a Young's modulus of 184 GPa and a Poisson's ratio of 0.27 with a maximum relative error of 5%. These values are in agreement with the reference values identified on the standard tensile test. The distribution of the identified yield stress is shown in Figure 2. It should be noted that the identified values are similar to the reference values with a maximum relative error of 10% (see Figure 2.a and 2.b). Figure 2.c shows that once the initial state has been taken into account, the evolutions of the hardening curves of the 20 material domains are in agreement with the reference plastic behavior (in black).

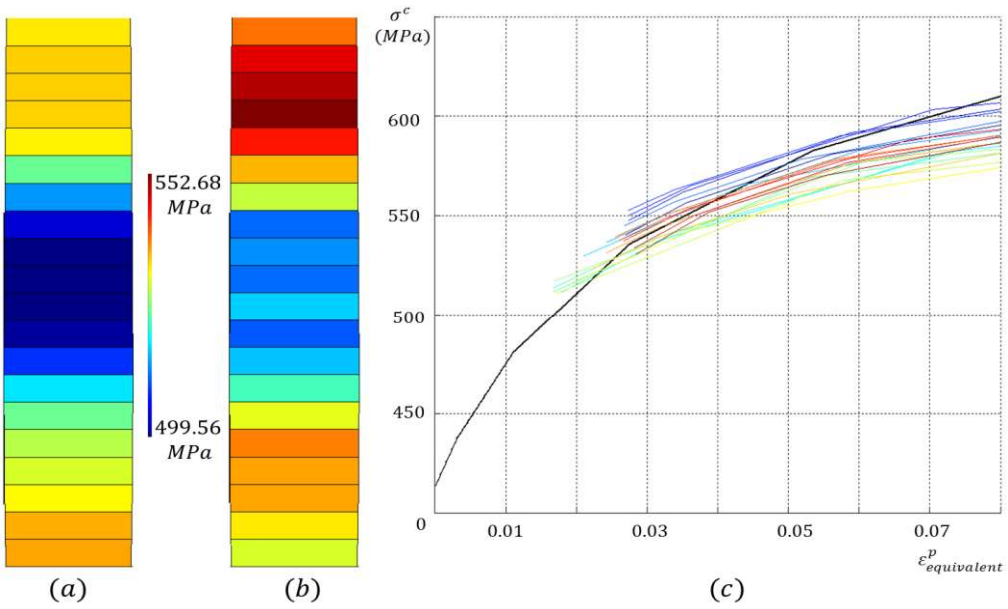

Figure 2: Reference yield stress (a), identified yield stress (b) and illustration of the stress as a function of the equivalent plastic deformation of the reference model (in black) and those of the different material domains identified (in color) (c).

Conclusion

To conclude, the identification of heterogeneous mechanical fields has been experimentally validated for elasto-plasticity with kinematic multi-linear hardening. Measuring the residual plastic strain and determining the yield stress locally reached during the first loading allows to estimate the heterogeneous initial state of the material. The identification results are very encouraging and must, from the same perspective, be validated on even more heterogeneous situations while still being controlled as for example polycrystalline samples.

References

[1] Latourte F., Chrysochoos A., Pagano S. and Wattrisse B., Elastoplastic behavior identification for heterogeneous loadings and materials. *Experimental Mechanics* 48 435-449, 2008.
[2] Simo J. and Hughes T., *Computational Inelasticity*. Springer-Verlag, 1998.