

# Semantic difference in $\mathcal{ALN}$

A. Léger<sup>1</sup>   **C. Rey**<sup>2</sup>   F. Toumani<sup>2</sup>

<sup>1</sup>Orange Labs, Rennes, France  
alain.leger@francetelecom.com

<sup>2</sup>Université Blaise Pascal, Clermont-Ferrand, France  
christophe.rey@univ-bpclermont.fr  
ftoumani@isima.fr

International Workshop on Description Logics, 2007

# Presentation outline

- 1 Motivation
- 2  $\mathcal{ALN}$ -materials
- 3 Computation of the semantic difference in  $\mathcal{ALN}$
- 4 Computation of the syntactic difference in  $\mathcal{ALN}$
- 5 Semantic VS syntactic ?
- 6 Conclusion

# Generalities

- Non standard reasonings (generate description(s))
- Informal definitions, with  $A$  and  $B$  two descriptions:
  - ▶ "subtraction operator"  
 $B - A$  : what remains of  $B$  once  $A$  has been removed
  - ▶ "difference operation"  
 $B - A$  : what has to be added to  $A$  to get  $B$

# Two difference operators

- Semantic difference [Teege, 94]
  - ▶ Principle:  $B - A$  is what has to be added by conjunction with  $A$  in order to get an description (semantically) equivalent to  $B$
  - ▶ Application: concept decomposition in tutorial systems, best covers using a terminology [Hacid et al, 02]
- Syntactic difference [Brandt et al, 02]
  - ▶ Principle: (syntactically) remove from  $B$  all subdescriptions that are redundant wrt  $A$
  - ▶ Applications: evaluating the accuracy of an approximated concept

# Contributions of this work

- (Sound and complete) algorithm to compute the semantic difference in  $\mathcal{ALN}$
- (Sound and complete) algorithm to compute the syntactic difference in  $\mathcal{ALN}$
- Comparison of both to justify which one to use in best covers

# $\mathcal{ALN}$ -clausal form

- $\mathcal{ALN} = \{\sqcap, \forall R.D, \neg A, \leq nR, \geq nR\}$
- Set of all conjuncts of a normalized description
- A clause: one conjunct in this set
- Example:  

$$C \equiv \forall T.(P \sqcap \neg P) \sqcap (\forall Q.A) \sqcap (\forall Q.(\forall R.(\forall S.(B \sqcap \neg B))))$$

$\mathcal{ALN}$ -normal form:

$$\widehat{C} = (\leq 0 T) \sqcap (\forall Q.(A \sqcap \forall R.(\leq 0 S)))$$

$\mathcal{ALN}$ -clausal form:

$$\widehat{\mathcal{G}}_C^\# = \{\leq 0 T, \forall Q.A, \forall QR. \leq 0 S\}$$

# ALN-reasonings

- Subsumption
- Lcs
- Weak approximation:

$$\widehat{\mathcal{G}}_{Approx_{\uparrow}}^{\#}(\neg \forall R_1 R_2 \dots R_n . P) = \left\{ \begin{array}{l} \forall R_1 R_2 \dots R_n . (\neg P), \\ \forall R_1 R_2 \dots R_{n-1} . (\geq 1 R_n), \\ \forall R_1 R_2 \dots R_{n-2} . (\geq 1 R_{n-1}) \\ \forall R_1 . (\geq 1 R_2) \\ (\geq 1 R_1) \end{array} \right\}$$

# Semantic difference definition

## Definition ([Teege, 94])

Let  $\mathcal{L}$  a description logic,  $B$  and  $A$  two  $\mathcal{L}$ -concept descriptions such that  $B \sqsubseteq A$ . The semantic difference between  $B$  and  $A$ , noted  $B - A$ , is defined by :

$$B - A := \text{Max}_{\sqsubseteq} \{ C \mid C \sqcap A \equiv B \}$$

If the lcs always exists between two  $\mathcal{L}$ -descriptions, then this definition is extended to couples of descriptions  $B$  and  $A$  with  $B \not\sqsubseteq A$  by :

$$B - A := B - \text{lcs}(B, A)$$



# Main result

## Lemma

Let  $B$  and  $A$  two  $\mathcal{ALN}$ -descriptions such that  $B \sqsubseteq A$ . Let  $C$  be an  $\mathcal{ALN}$ -description in  $B - A$ .  $\widehat{\mathcal{G}}_C^\#$  can be built as follows:

First, if  $\widehat{\mathcal{G}}_A^\# = \widehat{\mathcal{G}}_B^\#$ , then  $\widehat{\mathcal{G}}_C^\#$  must be  $\{\top\}$ .

Else,  $\widehat{\mathcal{G}}_C^\#$  is initialized at  $\emptyset$ , and then, for all  $c_B$  in  $\widehat{\mathcal{G}}_B^\#$  :

- **Inconsistency case:**

if  $c_B = \forall R_1 R_2 \dots R_{n-1}. (\leq 0 R_n)$ , with  $n \geq 0$  (if  $n = 0$  then  $c_B = \perp$ )  
and  $\exists c_A \in \widehat{\mathcal{G}}_A^\# \mid c_A = \forall R_1 R_2 \dots R_n R_{n+1} \dots R_{n+m}. P$ ,  $m \geq 0$

then we add to  $\widehat{\mathcal{G}}_C^\#$  all clauses  $c$  verifying:

$\left( c = \forall R_1 R_2 \dots R_n. c', c' \in \widehat{\mathcal{G}}_{\text{Approx}_1(\neg \forall R_{n+1} \dots R_{n+m}. P)}^\# \right)$  and  $(A \not\sqsubseteq c)$

- **General case:**

else if  $c_B \notin \widehat{\mathcal{G}}_A^\#$ , then we add  $c_B$  to  $\widehat{\mathcal{G}}_C^\#$ .

Example  $B - A = \{C_1, C_2, C_3\}$

$\widehat{\mathcal{G}}_B^\#$	$\widehat{\mathcal{G}}_A^\#$	$\widehat{\mathcal{G}}_{C_1}^\#$	$\widehat{\mathcal{G}}_{C_2}^\#$	$\widehat{\mathcal{G}}_{C_3}^\#$
------------------------------	------------------------------	----------------------------------	----------------------------------	----------------------------------

Example  $B - A = \{C_1, C_2, C_3\}$

$\widehat{G}_B^\#$	$\widehat{G}_A^\#$	$\widehat{G}_{C_1}^\#$	$\widehat{G}_{C_2}^\#$	$\widehat{G}_{C_3}^\#$
$P_1$	$P_1$			

Example  $B - A = \{C_1, C_2, C_3\}$

$\widehat{G}_B^\#$	$\widehat{G}_A^\#$	$\widehat{G}_{C_1}^\#$	$\widehat{G}_{C_2}^\#$	$\widehat{G}_{C_3}^\#$
$P_1$ $\leq 1S$	$P_1$ $\leq 2S$	$\leq 1S$	$\leq 1S$	$\leq 1S$

Example  $B - A = \{C_1, C_2, C_3\}$

$\widehat{G}_B^\#$	$\widehat{G}_A^\#$	$\widehat{G}_{C_1}^\#$	$\widehat{G}_{C_2}^\#$	$\widehat{G}_{C_3}^\#$
$P_1$ $\leq 1S$ $\forall T.P_1$	$P_1$ $\leq 2S$ $\forall T.P_1$	$\leq 1S$	$\leq 1S$	$\leq 1S$

# Example $B - A = \{C_1, C_2, C_3\}$

$\widehat{G}_B^\#$	$\widehat{G}_A^\#$	$\widehat{G}_{C_1}^\#$	$\widehat{G}_{C_2}^\#$	$\widehat{G}_{C_3}^\#$
$P_1$ $\leq 1S$ $\forall T.P_1$ $\forall TV.P_3$	$P_1$ $\leq 2S$ $\forall T.P_1$	$\leq 1S$ $\forall TV.P_3$	$\leq 1S$ $\forall TV.P_3$	$\leq 1S$ $\forall TV.P_3$

# Example $B - A = \{C_1, C_2, C_3\}$

$\widehat{G}_B^\#$	$\widehat{G}_A^\#$	$\widehat{G}_{C_1}^\#$	$\widehat{G}_{C_2}^\#$	$\widehat{G}_{C_3}^\#$
$P_1$ $\leq 1S$ $\forall T.P_1$ $\forall TV.P_3$ $\forall S.(\geq 6R)$	$P_1$ $\leq 2S$ $\forall T.P_1$ $\forall S.(\geq 3R)$	$\leq 1S$ $\forall TV.P_3$ $\forall S.(\geq 6R)$	$\leq 1S$ $\forall TV.P_3$ $\forall S.(\geq 6R)$	$\leq 1S$ $\forall TV.P_3$ $\forall S.(\geq 6R)$

Example  $B - A = \{C_1, C_2, C_3\}$ 

$\hat{G}_B^\#$	$\hat{G}_A^\#$	$\hat{G}_{C_1}^\#$	$\hat{G}_{C_2}^\#$	$\hat{G}_{C_3}^\#$
$P_1$ $\leq 1S$ $\forall T.P_1$ $\forall TV.P_3$ $\forall S.( \geq 6R)$ $\forall S.( \leq 0T)$	$P_1$ $\leq 2S$ $\forall T.P_1$  $\forall S.( \geq 3R)$ $\forall S.( \leq 6T)$ $\forall ST.P_2$ $\forall STV.( \geq 4W)$  $\forall STVWX.( \leq 3R)$	$\leq 1S$  $\forall TV.P_3$ $\forall S.( \geq 6R)$	$\leq 1S$  $\forall TV.P_3$ $\forall S.( \geq 6R)$	$\leq 1S$  $\forall TV.P_3$ $\forall S.( \geq 6R)$



# Example $B - A = \{C_1, C_2, C_3\}$

$\hat{G}_B^\#$	$\hat{G}_A^\#$	$\hat{G}_{C_1}^\#$	$\hat{G}_{C_2}^\#$	$\hat{G}_{C_3}^\#$
$P_1$ $\leq 1S$ $\forall T.P_1$ $\forall TV.P_3$ $\forall S.(\geq 6R)$ $\forall S.(\leq 0T)$	$P_1$ $\leq 2S$ $\forall T.P_1$  $\forall S.(\geq 3R)$ $\forall S.(\leq 6T)$ $\forall ST.P_2$ $\forall STV.(\geq 4W)$  $\forall STVWX.(\leq 3R)$	$\leq 1S$  $\forall TV.P_3$ $\forall S.(\geq 6R)$  $\forall ST.(\neg P_2)$	$\leq 1S$  $\forall TV.P_3$ $\forall S.(\geq 6R)$	$\leq 1S$  $\forall TV.P_3$ $\forall S.(\geq 6R)$

Example  $B - A = \{C_1, C_2, C_3\}$ 

$\hat{G}_B^\#$	$\hat{G}_A^\#$	$\hat{G}_{C_1}^\#$	$\hat{G}_{C_2}^\#$	$\hat{G}_{C_3}^\#$
$P_1$ $\leq 1S$ $\forall T.P_1$ $\forall TV.P_3$ $\forall S.(\geq 6R)$ $\forall S.(\leq 0T)$	$P_1$ $\leq 2S$ $\forall T.P_1$  $\forall S.(\geq 3R)$ $\forall S.(\leq 6T)$ $\forall ST.P_2$ $\forall STV.(\geq 4W)$  $\forall STVWX.(\leq 3R)$	$\leq 1S$  $\forall TV.P_3$ $\forall S.(\geq 6R)$  $\forall ST.(\neg P_2)$	$\leq 1S$  $\forall TV.P_3$ $\forall S.(\geq 6R)$  $\forall STV.(\leq 3W)$ $\forall ST.(\geq 1V)$	$\leq 1S$  $\forall TV.P_3$ $\forall S.(\geq 6R)$

Example  $B - A = \{C_1, C_2, C_3\}$ 

$\hat{G}_B^\#$	$\hat{G}_A^\#$	$\hat{G}_{C_1}^\#$	$\hat{G}_{C_2}^\#$	$\hat{G}_{C_3}^\#$
$P_1$ $\leq 1S$ $\forall T.P_1$ $\forall TV.P_3$ $\forall S.( \geq 6R)$ $\forall S.( \leq 0T)$	$P_1$ $\leq 2S$ $\forall T.P_1$  $\forall S.( \geq 3R)$ $\forall S.( \leq 6T)$ $\forall ST.P_2$ $\forall STV.( \geq 4W)$  $\forall STVWX.( \leq 3R)$	$\leq 1S$  $\forall TV.P_3$ $\forall S.( \geq 6R)$  $\forall ST.( \neg P_2)$	$\leq 1S$  $\forall TV.P_3$ $\forall S.( \geq 6R)$  $\forall STV.( \leq 3W)$ $\forall ST.( \geq 1V)$	$\leq 1S$  $\forall TV.P_3$ $\forall S.( \geq 6R)$   $\forall STVWX.( \geq 4R)$ $\forall STVW.( \geq 1X)$ $\forall ST.( \geq 1V)$

# Full algorithm:

- Principle: extends previous lemma by combining all inconsistency cases that generate many clauses
- compute  $\mathcal{ALN}$  SemDiff
  - ▶ sound and complete
  - ▶ PTIME wrt the sizes of  $B$  and  $A$  (i.e. the numbers of clauses in their clausal forms and the maximal number of roles in any of their clauses)

# Syntactic difference definition

- based on the notion of subdescription
- Associated ordering :  $D \preceq_d E$  iff  $D$  is a subdescription of  $E$

## Definition (Syntactic difference [Kuesters, 01, Brandt et al, 02])

Let  $A$  and  $B$  be two  $\mathcal{AL}\mathcal{E}$  descriptions. The syntactic difference  $B - A$  of  $B$  and  $A$  is defined as the  $\mathcal{AL}\mathcal{E}$  description  $C$  which is minimal w.r.t.  $\preceq_d$  such that  $C \sqcap A \equiv A \sqcap B$ .

- Extension to  $\mathcal{ALN}$

# The compute $\mathcal{ALN}$ SyntDiff algorithm

## Theorem

Let  $A$  and  $B$  be two  $\mathcal{ALN}$ -descriptions given in their clausal form. The syntactic difference  $B - A$  is a unique

$\mathcal{ALN}$ -description  $C$  such that

if  $A \sqsubseteq B$ , then  $C = \top$ ,

else  $\widehat{\mathcal{G}}_C^\# = \widehat{\mathcal{G}}_{A \sqcap B}^\# \setminus \widehat{\mathcal{G}}_A^\#$ .

- Direct algorithm compute $\mathcal{ALN}$ SyntDiff
- sound and complete
- PTIME wrt the numbers of clauses in their clausal forms

# Comparison of both operators

- basically, amounts to the same computation
- except when processing inconsistencies

$B - A$	$B \equiv \forall R. \perp$ $A \equiv \forall R. (\neg P \sqcap P')$	$B \equiv \forall R. P$ $A \equiv \forall R. \neg P$
Semantic diff.	$\{\forall R. P, \forall R. \neg P'\}$	$\{\forall R. P\}$
Syntactic diff.	$\forall R. \perp$	$\forall R. \perp$
Conclusion	Sem. diff. more accurate, less convenient Synt. diff. less accurate, more intuitive	Sem. diff. more intuitive (?)

# Conclusion

- Semantic difference
  - ▶ more accurate  $\Rightarrow$  automatic knowledge processing (query rewriting, services discovery ,...)
- Syntactic difference
  - ▶ more intuitive  $\Rightarrow$  knowledge hand-manipulating : knowledge engineer, user interface,...
- No definitive conclusion: application-dependent



# Thank you

# Bibliography



F. Baader, R. Küsters, and R. Molitor.

Computing Least Common Subsumer in Description Logics with Existential Restrictions.  
In *Proceedings of IJCAI'99*.



F. Baader, R. Küsters, and R. Molitor.

Rewriting concepts using terminologies.  
In *Proc. of KR2000*.



S. Brandt, R. Küsters, and A.-Y. Turhan.

Approximation and Difference in Description Logics.  
In *Proc. of KR2002*.



F. Goasdoué and M.-C. Rousset.

Compilation and approximation of conjunctive queries by concept descriptions.  
In *Proc. of DL2002, 2002*.



M.S. Hacid, A. Léger, C. Rey, and F. Toumani.

Computing concept covers: a preliminary report.  
In *Proc. of DL02 Toulouse, France, April 2002*.



R. Küsters.

*Non-Standard Inferences in Description Logics*, volume 2100 of *Lecture Notes in Artificial Intelligence*.  
Springer-Verlag, 2001.  
Ph.D. thesis.



R. Molitor.

Structural subsumption for  $\mathcal{ALN}$ .  
Technical Report LTCS-98-03, LuFG Theoretical Computer Science, RWTH Aachen, March 1998.



G. Teege.

Making the difference: A subtraction operation for description logics.