



# Perspectives of seismic imaging using FWI with reciprocity misfit functional

Florian Faucher

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# Perspectives of seismic imaging using FWI with reciprocity misfit functional

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Giovanni Alessandrini<sup>2</sup>, H      Barucq<sup>1</sup>, Maarten V. de Hoop<sup>3</sup>,  
Romina Gaburro<sup>4</sup> and Eva Sincich<sup>2</sup>.

**Journ      Ondes Sud-Ouest, CEA Cesta, France**

Marth 12<sup>th</sup>–14<sup>th</sup>, 2019



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<sup>4</sup>Department of Mathematics and Statistics, Health Research Institute (HRI), University of Limerick, Ireland.

# Overview

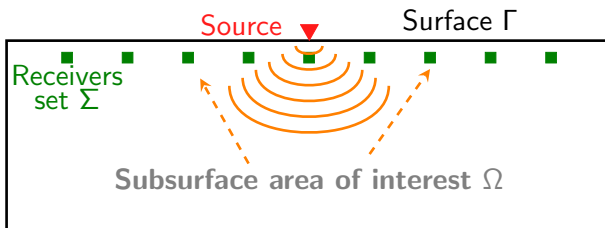
- ① Introduction
- ② Time-Harmonic Inverse Problem, FWI
- ③ Reconstruction procedure using dual-sensors data
- ④ Numerical experiments
  - Comparison of misfit functions
  - Changing the numerical acquisition with  $\mathcal{I}_{\mathcal{G}}$
- ⑤ Conclusion

# Plan

## 1 Introduction

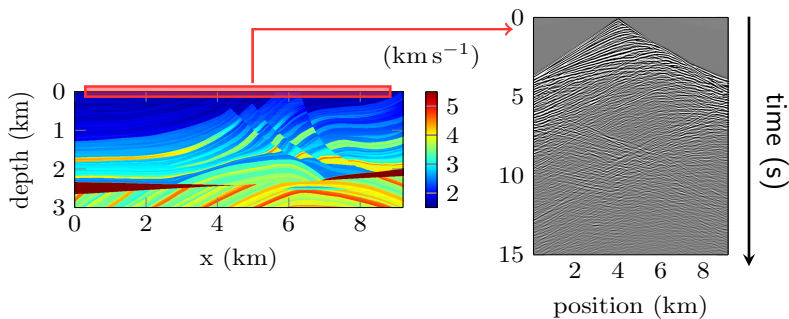
# Seismic inverse problem

Reconstruction of subsurface Earth properties from seismic campaign: collection of **wave** propagation data at the surface.



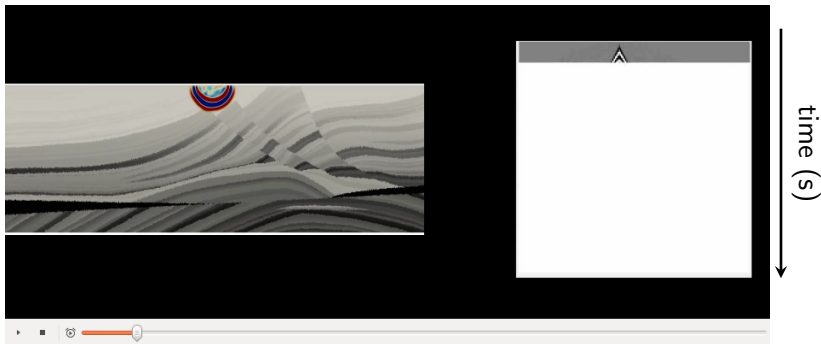
# Seismic data

We work with **back-scattered partial data** from **one-side** illumination on large domain.



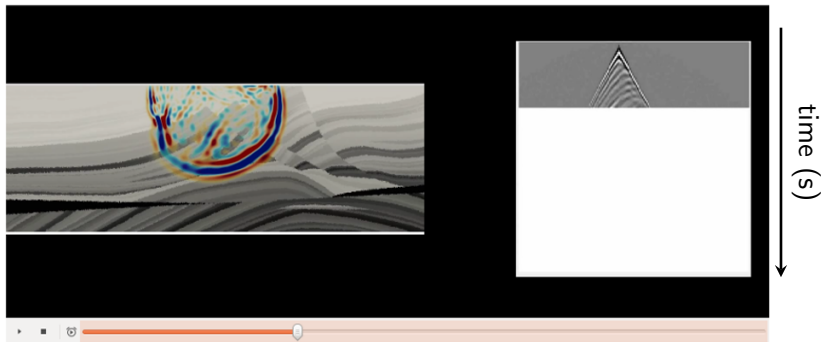
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# Seismic data

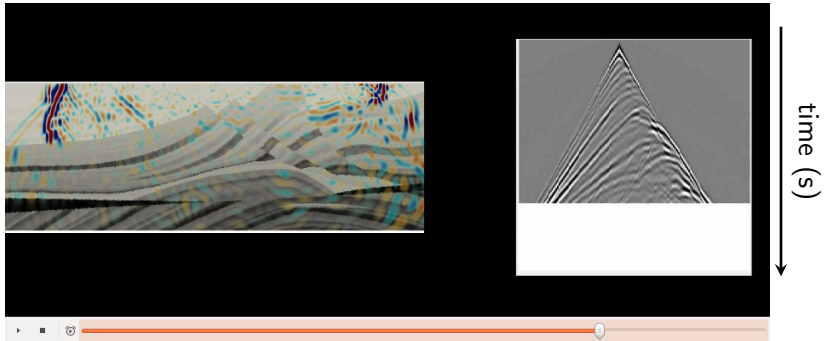
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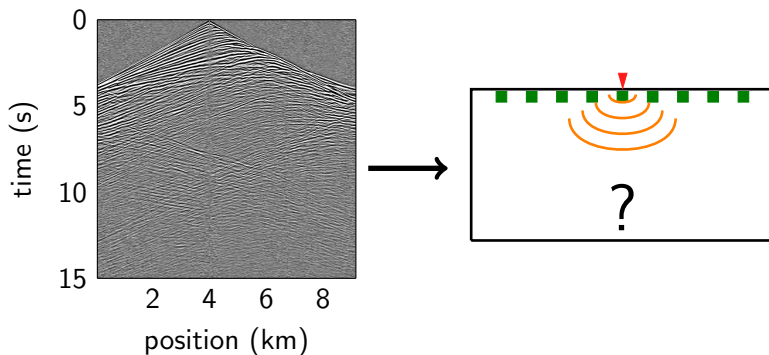
# Seismic data

We work with **back-scattered partial data** from **one-side** illumination on large domain.



# Seismic data

**Inverse problem:** from seismic traces to subsurface?



**nonlinear, ill-posed inverse problem.**

# Plan

## 2 Time-Harmonic Inverse Problem, FWI

# Time-harmonic wave equation

We consider propagation in acoustic media, given by the Euler's equations, **heterogeneous** medium parameters  $\kappa$  and  $\rho$ :

$$\begin{cases} -i\omega\rho(\mathbf{x})\mathbf{v}(\mathbf{x}) = -\nabla p(\mathbf{x}), \\ -i\omega p(\mathbf{x}) = -\kappa(\mathbf{x})\nabla \cdot \mathbf{v}(\mathbf{x}) + f(\mathbf{x}). \end{cases}$$

$p$ : scalar pressure field,  
 $\mathbf{v}$ : vectorial velocity field,  
 $f$ : source term,

$\kappa$ : bulk modulus,  
 $\rho$ : density,  
 $\omega$ : angular frequency.

# Time-harmonic wave equation



We consider propagation in acoustic media, given by the Euler's equations, **heterogeneous** medium parameters  $\kappa$  and  $\rho$ :

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$p$ : scalar pressure field,

$\kappa$ : bulk modulus,

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$\omega$ : angular frequency.

The system reduces to the Helmholtz equation when  $\rho$  is constant,

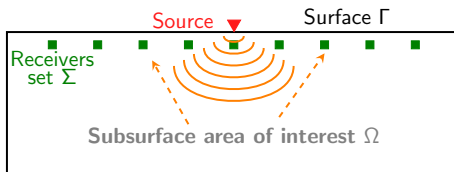
$$(-\omega^2 c(\mathbf{x})^{-2} - \Delta)p(\mathbf{x}) = 0,$$

$$\text{with } c(\mathbf{x}) = \sqrt{\kappa(\mathbf{x})\rho(\mathbf{x})^{-1}}.$$

# Dual-sensors devices

The inverse problem aims the recovery of the subsurface medium parameters from surface measurements of pressure **and** normal (vertical) velocity:

$$\mathcal{F} : m = (\kappa, \rho) \rightarrow \{\mathcal{F}_p ; \mathcal{F}_v\} = \{p(\mathbf{x}_1), p(\mathbf{x}_2), \dots, p(\mathbf{x}_{n_{rcv}}); v_n(\mathbf{x}_1), v_n(\mathbf{x}_2), \dots, v_n(\mathbf{x}_{n_{rcv}})\}.$$



D. Carlson, N. D. Whitmore *et al.*

Increased resolution of seismic data from a dual-sensor streamer cable – Imaging of primaries and multiples using a dual-sensor towed streamer

SEG, 2007 – 2010



CGG & Lundun Norway (2017–2018)

TopSeis acquisition ([www.cgg.com/en/What-We-Do/Offshore/Products-and-Solutions/TopSeis](http://www.cgg.com/en/What-We-Do/Offshore/Products-and-Solutions/TopSeis))

# Full Waveform Inversion (FWI)

FWI provides a **quantitative reconstruction** of the subsurface parameters by solving a minimization problem,

$$\min_{m \in \mathcal{M}} \mathcal{J}(m) = \frac{1}{2} \|\mathcal{F}(m) - d\|^2.$$

- ▶  $d$  are the observed data,
- ▶  $\mathcal{F}(m)$  represents the simulation using an initial model  $m$ :



[P. Lailly](#)

The seismic inverse problem as a sequence of before stack migrations  
[Conference on Inverse Scattering: Theory and Application, SIAM, 1983](#)



[A. Tarantola](#)

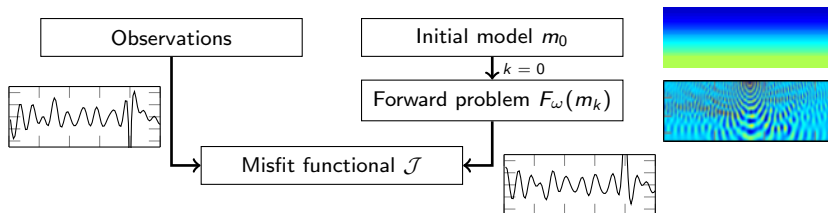
Inversion of seismic reflection data in the acoustic approximation  
[Geophysics, 1984](#)



[A. Tarantola](#)

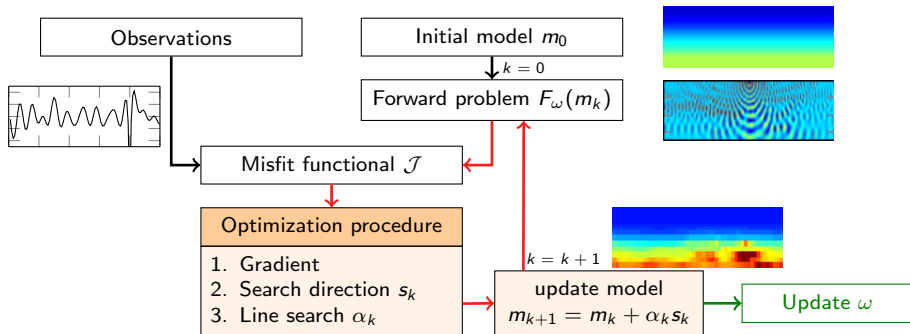
Inversion of travel times and seismic waveforms  
[Seismic tomography, 1987](#)

# FWI, iterative minimization

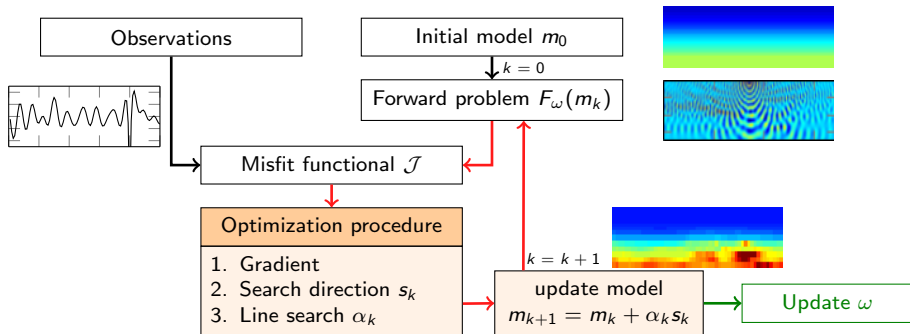




# FWI, iterative minimization



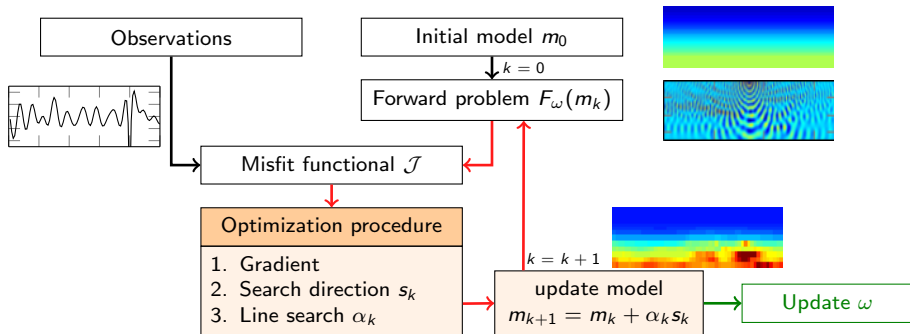
# FWI, iterative minimization



## Numerical methods

- ▶ Adjoint-method for the gradient computation, L-BFGS method,
- ▶ forward problem resolution with Discontinuous Galerkin methods,
- ▶ parallel computation, HPC, large-scale optimization,
- ▶ Rk: the code also works for **elastic** anisotropy and **viscous** media.

# FWI, iterative minimization



- ▶  $> 10^5$ : unknowns per physical parameter,
- ▶  $> 10^6$ : matrix size for discretization,
- ▶ we also study stability and convergence of the algorithm ...

# Plan

## 3 Reconstruction procedure using dual-sensors data

# Minimization of the cost function

The appropriate misfit functional to minimize with pressure and vertical velocity measurements.

- Compare the pressure and velocity fields separately:

$$\mathcal{J}_{L2} = \sum_{source} \frac{1}{2} \|\mathcal{F}_p^{(s)} - d_p^{(s)}\|^2 + \frac{1}{2} \|\mathcal{F}_v^{(s)} - d_v^{(s)}\|^2.$$

# Minimization of the cost function

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- ▶ Compare the fields multiplication for all combinations:

$$\mathcal{J}_g = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$



G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich

Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization

[preprint](#)

# Minimization of the cost function

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

From Euler's equation,  $v_n(\mathbf{x}_i) = -i(\omega\rho)^{-1}\partial_n p(\mathbf{x}_i)$ .

# Minimization of the cost function

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

From Euler's equation,  $v_n(\mathbf{x}_i) = -i(\omega\rho)^{-1}\partial_n p(\mathbf{x}_i)$ .

- **Cauchy data:** the cost function follows **Green's identity**.



# Minimization of the cost function

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

From Euler's equation,  $v_n(\mathbf{x}_i) = -i(\omega\rho)^{-1}\partial_n p(\mathbf{x}_i)$ .

- ▶ **Cauchy data:** the cost function follows **Green's identity**.
- ▶ **Reciprocity gap functional** in inverse scattering.



D. Colton and H. Haddar

An application of the reciprocity gap functional to inverse scattering theory  
[Inverse Problems 21 \(1\) \(2005\), 383398.](#)



G. Alessandrini, M.V. de Hoop, R. Gaburro and E. Sincich

Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data  
[arXiv:1702.04222, 2017](#)



G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich

Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization  
[preprint](#)

# Stability results

Lipschitz type stability is obtained for the Helmholtz equation with partial Cauchy data.

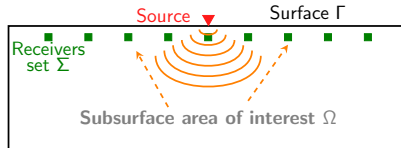
$$\|c_1 - c_2\| \leq \mathcal{C}(\mathcal{J}_{\mathcal{G}}(c_1, c_2))^{1/2}$$

# Stability results

**Lipschitz type stability** is obtained for the Helmholtz equation with **partial Cauchy data**.

$$\|c_1 - c_2\| \leq \mathcal{C}(\mathcal{I}_{\mathcal{G}}(c_1, c_2))^{1/2}$$

- Using back-scattered data from one side in a domain with free surface and absorbing conditions,



- for piecewise linear parameters.



G. Alessandrini, M.V. de Hoop, R. Gaburro and E. Sincich

Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data  
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# Additional possibilities

It allows the non-collocation of numerical and observational sources:

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

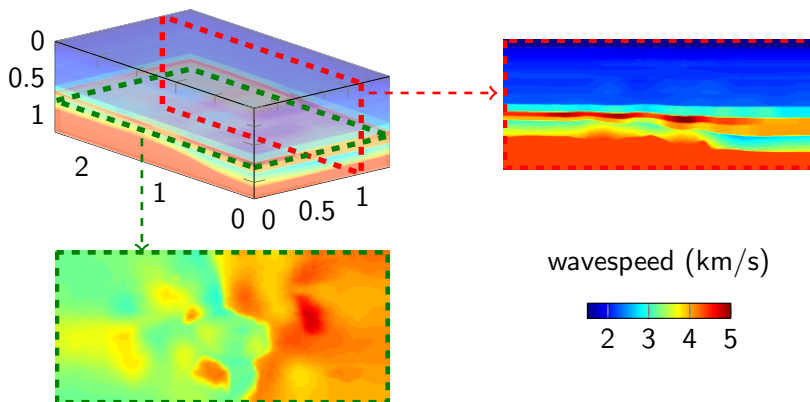
- ▶  $s_1$  is fixed by the observational setup,
- ▶  $s_2$  is chosen for the numerical comparisons.

## 4 Numerical experiments

- Comparison of misfit functions
- Changing the numerical acquisition with  $\mathcal{I}_g$

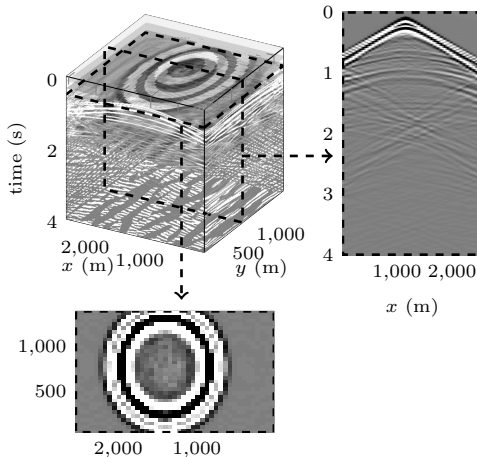
# Experiment setup

3D velocity model  $2.5 \times 1.5 \times 1.2\text{km}$  using dual-sensors data.



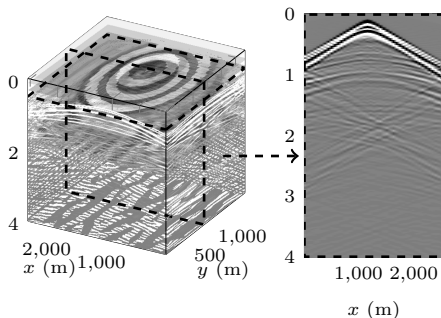
# Experiment setup

We work with time-domain data acquisition.



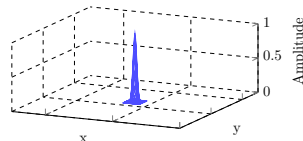
# Experiment setup

We work with time-domain data (pressure and velocity).



Acquisition for the measures

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,



For the reconstruction, we apply a Fourier transform of the time data.



# Comparison of misfit functional

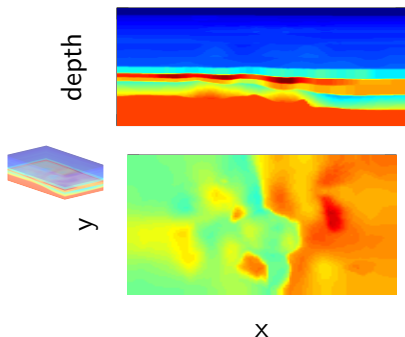
We respect the observational acquisition setup and perform the minimization of  $\mathcal{J}_{L2}$  or  $\mathcal{J}_{\mathcal{G}}$ , frequency from 3 to 15Hz.

$$\mathcal{J}_{L2} = \sum_{source} \frac{1}{2} \|\mathcal{F}_p^{(s)} - d_p^{(s)}\|^2 + \frac{1}{2} \|\mathcal{F}_v^{(s)} - d_v^{(s)}\|^2.$$

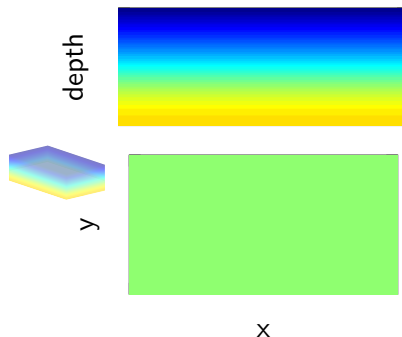
$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{source} \sum_{source} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

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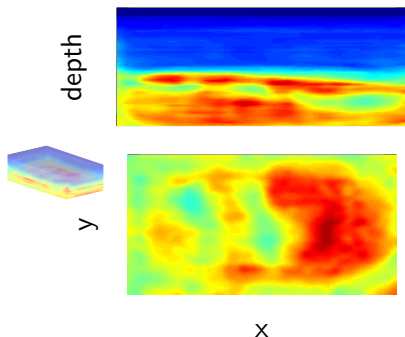
(a) True velocity



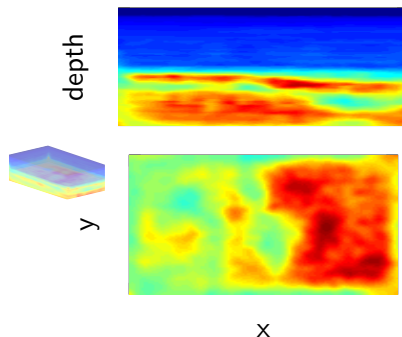
(b) Starting velocity

# Comparison of misfit functional

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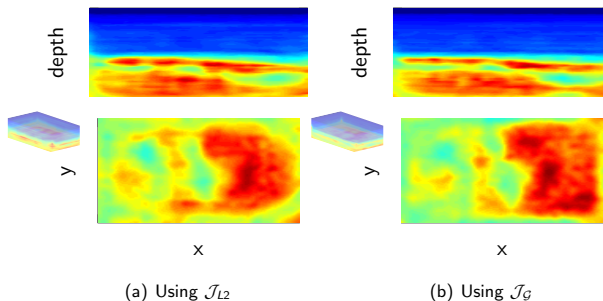
(a) Using  $\mathcal{J}_{L2}$



(b) Using  $\mathcal{J}_{\mathcal{G}}$

# Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of  $\mathcal{J}_{L2}$  or  $\mathcal{J}_G$ , frequency from 3 to 15Hz.



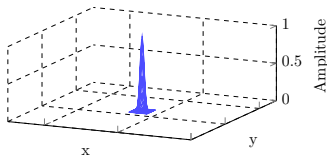
**But the major advantage of  $\mathcal{J}_G$  is the possibility to consider alternative acquisition setup.**

# Experiment with different obs. and sim. acquisition *inria* informatics mathematics

$$\min \mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

Acquisition for the measures  $s_1$

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,

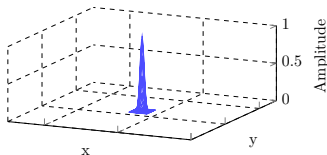


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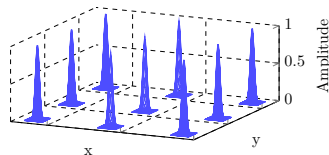
Acquisition for the measures  $s_1$

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,



Arbitrary numerical acquisition  $s_2$

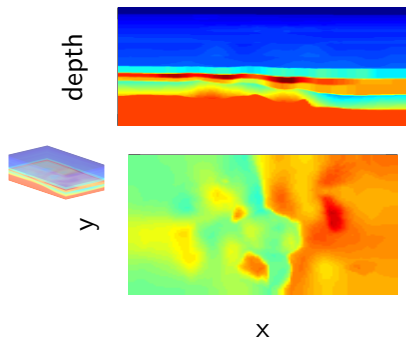
- ▶ **5 sources,**
- ▶ **80m depth,**
- ▶ **multi-point sources,**



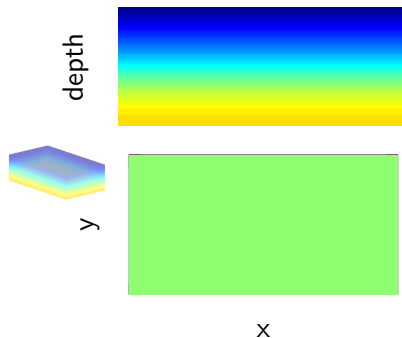
- ▶ No need to know observational source wavelet.
- ▶ Differentiation impossible with least squares types misfit.

# Experiment with different obs. and sim. acquisition

Data from frequency between 3 to 15 Hz, domain size  $2.5 \times 1.5 \times 1.2$  km, **Simulation using 5 sources only.**



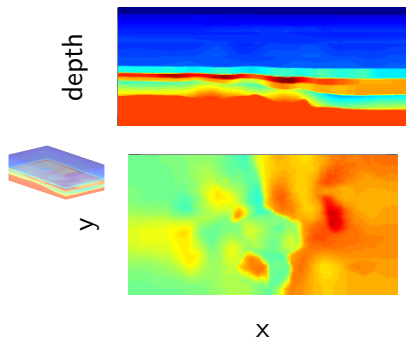
(a) True velocity



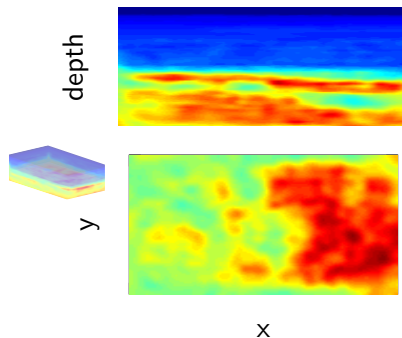
(b) Starting velocity

# Experiment with different obs. and sim. acquisition *inria* informatics mathematics

Frequency from 3 to 15 Hz,  $2.5 \times 1.5 \times 1.2$  km,  
Simulation using 5 sources only. -33% computational time.



(a) True velocity



(b) 15 Hz reconstruction



# Plan

## 5 Conclusion

# Conclusion



Seismic inverse problem using pressure and vertical velocity data:

- ▶ appropriate cost function to minimize,
- ▶ allow minimal information on the acquisition setup,
- ▶ other applications,
- ▶ **perspective:** design the most efficient numerical setup,
- ▶ Rk: possible for elastic media with measures of traction.

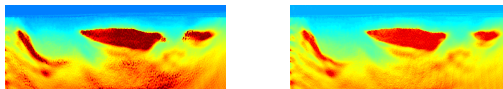
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Quantitative reconstruction algorithm toolbox for time-harmonic wave,

- ▶ **Discontinuous Galerkin** discretization in HPC framework,
- ▶ large scale optimization scheme using back-scattered data,
- ▶ **acoustic, elastic, anisotropy, 2D, 3D, attenuation**.



P- and S-wavespeed reconstructions

# Conclusion

Seismic inverse problem using pressure and vertical velocity data:

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- ▶ other applications,
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Quantitative reconstruction algorithm toolbox for time-harmonic wave,

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THANK YOU
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# APPENDIX

$$\|c_1^{-2} - c_2^{-2}\| \leq C(\|F(c_1^{-2}) - F(c_2^{-2})\|)$$



[G. Alessandrini](#)

Stable determination of conductivity by boundary measurement  
[Applicable Analysis 1988](#)



[N. Mandache](#)

Exponential instability in an inverse problem for Schrödinger equation  
[Inverse Problems 2001](#)

$$\|c_1^{-2} - c_2^{-2}\| \leq C(\overbrace{\|F(c_1^{-2}) - F(c_2^{-2})\|}^{\delta})$$

target      initial model      observation      simulation

- ▶ Stability associate data and model correspondence
- ▶ Reconstruction is based on the iterative minimization of the difference between observation and simulation using an initial model.



G. Alessandrini

Stable determination of conductivity by boundary measurement  
*Applicable Analysis* 1988



N. Mandache

Exponential instability in an inverse problem for Schrödinger equation  
*Inverse Problems* 2001

$$\|c_1^{-2} - c_2^{-2}\| \leq C(\|F(c_1^{-2}) - F(c_2^{-2})\|)$$

Diagram illustrating the stability of the Helmholtz inverse problem. The equation shows the difference in the squared inverse of the coefficients,  $\|c_1^{-2} - c_2^{-2}\|$ , bounded by a constant  $C$  times the difference in the forward problem,  $\|F(c_1^{-2}) - F(c_2^{-2})\|$ . The forward problem difference is labeled with a bracket as  $\delta$ . Arrows indicate the mapping from labels to terms: 'target' points to  $c_1^{-2}$ , 'initial model' points to  $c_2^{-2}$ , 'observation' points to  $F(c_1^{-2})$ , and 'simulation' points to  $F(c_2^{-2})$ .

- ▶ Stability associate data and model correspondence
- ▶  $C(\delta) \leq C(\log(1 + \delta^{-1}))^{-\alpha}$



G. Alessandrini

Stable determination of conductivity by boundary measurement  
*Applicable Analysis* 1988



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Exponential instability in an inverse problem for Schrödinger equation  
*Inverse Problems* 2001



- ▶  $c(x)$  is bounded  $B_1 \leq c^{-2}(x) \leq B_2$  in  $\Omega$
- ▶  $c(x)$  has a **piecewise constant** representation of size  $N$

$$c(x)^{-2} = \sum_{k=1}^N c_k \chi_k(x)$$

- ▶  $\Omega$  has Lipschitz boundary

$$\|c_1^{-2} - c_2^{-2}\|_{L^2(\Omega)} \leq \mathcal{C} \|F(c_1^{-2}) - F(c_2^{-2})\| \quad (1)$$



G. Alessandrini and S. Vessella

Lipschitz stability for the inverse conductivity problem  
[Advances in Applied Mathematics 2005](#)



E. Beretta, M. V. de Hoop, F. and O. Scherzer

Inverse boundary value problem for the Helmholtz equation: quantitative conditional Lipschitz stability estimates.

[SIAM Journal of Mathematical Analysis 2016](#)

The stability constant is bounded

$$\frac{1}{4\omega^2} e^{K_1 N^{1/5}} \leq \mathcal{C} \leq \frac{1}{\omega^2} e^{(K(1+\omega^2 B_2) N^{4/7})} \quad (2)$$

- depends on the partitioning  $N$  and the frequency  $\omega$



E. Beretta, M. V. de Hoop, F. and O. Scherzer

Inverse boundary value problem for the Helmholtz equation: quantitative conditional Lipschitz stability estimates [2016](#)

In the case of partial Cauchy data ( $p$  and  $\partial_\nu p$ ), we have that, we can obtain a Lipschitz type stability:

$$\|c_1^{-2} - c_2^{-2}\| \leq \mathcal{C}(\mathcal{J}_g(c_1^{-2}, c_2^{-2}))^{1/2}$$

Where  $c_1^{-2}$  and  $c_2^{-2}$  are piecewise linear.