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Perspectives of seismic imaging using FWI with reciprocity misfit functional

Florian Faucher

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Perspectives of seismic imaging using FWI with reciprocity misfit functional

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Romina Gaburro⁴ and Eva Sincich².

Journ ees Ondes Sud-Ouest, CEA Cesta, France

Marth 12th–14th, 2019



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⁴Department of Mathematics and Statistics, Health Research Institute (HRI), University of Limerick, Ireland.

Overview

- 1 Introduction
- 2 Time-Harmonic Inverse Problem, FWI
- 3 Reconstruction procedure using dual-sensors data
- 4 Numerical experiments
 - Comparison of misfit functions
 - Changing the numerical acquisition with $\mathcal{J}_{\mathcal{G}}$
- 5 Conclusion

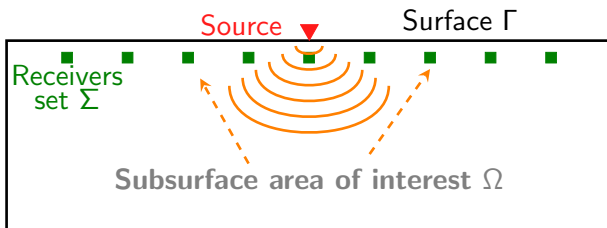
Plan



1 Introduction

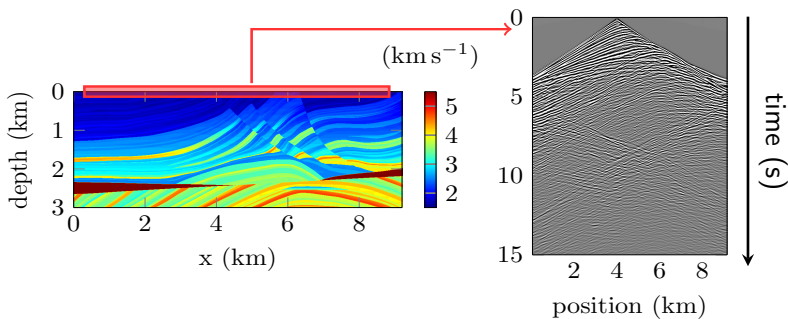
Seismic inverse problem

Reconstruction of subsurface Earth properties from seismic campaign: collection of **wave** propagation data at the surface.



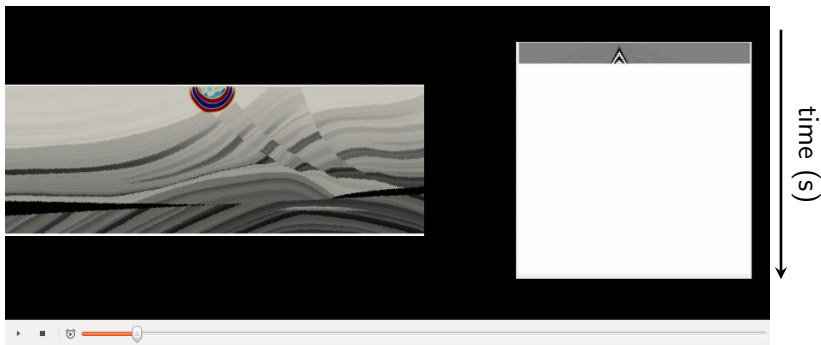
Seismic data

We work with **back-scattered partial data** from **one-side illumination** on large domain.



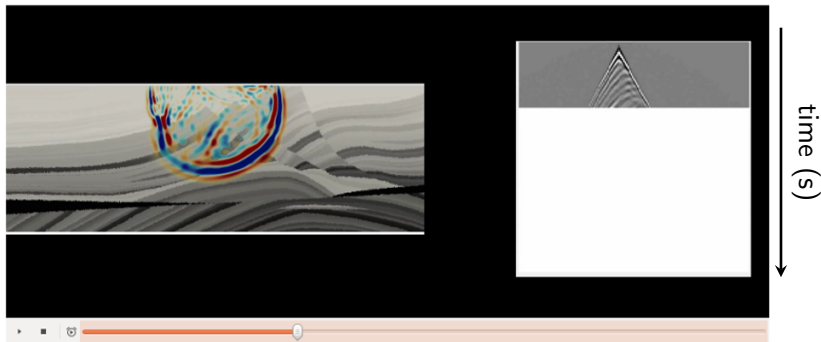
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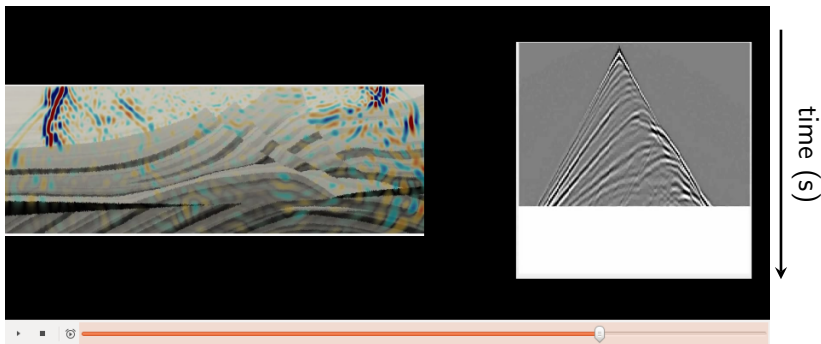
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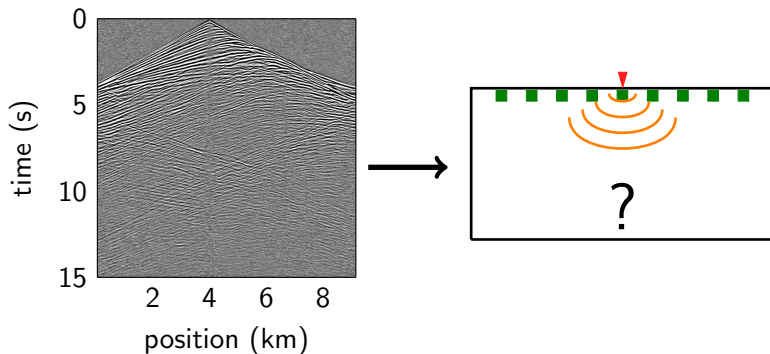
Seismic data

We work with **back-scattered partial data** from **one-side** illumination on large domain.



Seismic data

Inverse problem: from seismic traces to subsurface?



nonlinear, ill-posed inverse problem.

2 Time-Harmonic Inverse Problem, FWI

Time-harmonic wave equation

We consider propagation in acoustic media, given by the Euler's equations, **heterogeneous** medium parameters κ and ρ :

$$\begin{cases} -i\omega\rho(\mathbf{x})\mathbf{v}(\mathbf{x}) = -\nabla p(\mathbf{x}), \\ -i\omega p(\mathbf{x}) = -\kappa(\mathbf{x})\nabla \cdot \mathbf{v}(\mathbf{x}) + f(\mathbf{x}). \end{cases}$$

p : scalar pressure field,
 \mathbf{v} : vectorial velocity field,
 f : source term,

κ : bulk modulus,
 ρ : density,
 ω : angular frequency.

Time-harmonic wave equation

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p : scalar pressure field,

\mathbf{v} : vectorial velocity field,

f : source term,

κ : bulk modulus,

ρ : density,

ω : angular frequency.

The system reduces to the Helmholtz equation when ρ is constant,

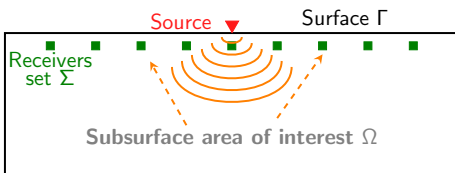
$$(-\omega^2 c(\mathbf{x})^{-2} - \Delta)p(\mathbf{x}) = 0,$$

$$\text{with } c(\mathbf{x}) = \sqrt{\kappa(\mathbf{x})\rho(\mathbf{x})^{-1}}.$$

Dual-sensors devices

The inverse problem aims the recovery of the subsurface medium parameters from surface measurements of pressure **and** normal (vertical) velocity:

$$\mathcal{F} : m = (\kappa, \rho) \rightarrow \{\mathcal{F}_p ; \mathcal{F}_v\} = \{p(\mathbf{x}_1), p(\mathbf{x}_2), \dots, p(\mathbf{x}_{n_{rcv}}); v_n(\mathbf{x}_1), v_n(\mathbf{x}_2), \dots, v_n(\mathbf{x}_{n_{rcv}})\}.$$



D. Carlson, N. D. Whitmore *et al.*

Increased resolution of seismic data from a dual-sensor streamer cable – Imaging of primaries and multiples using a dual-sensor towed streamer

SEG, 2007 – 2010



CGG & Lundun Norway (2017–2018)

TopSeis acquisition (www.cgg.com/en/What-We-Do/Offshore/Products-and-Solutions/TopSeis)

Full Waveform Inversion (FWI)

FWI provides a **quantitative reconstruction** of the subsurface parameters by solving a minimization problem,

$$\min_{m \in \mathcal{M}} \mathcal{J}(m) = \frac{1}{2} \|\mathcal{F}(m) - d\|^2.$$

- ▶ d are the observed data,
- ▶ $\mathcal{F}(m)$ represents the simulation using an initial model m :



P. Lailly

The seismic inverse problem as a sequence of before stack migrations
Conference on Inverse Scattering: Theory and Application, SIAM, 1983



A. Tarantola

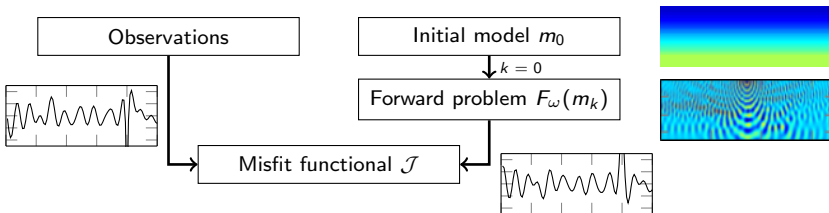
Inversion of seismic reflection data in the acoustic approximation
Geophysics, 1984



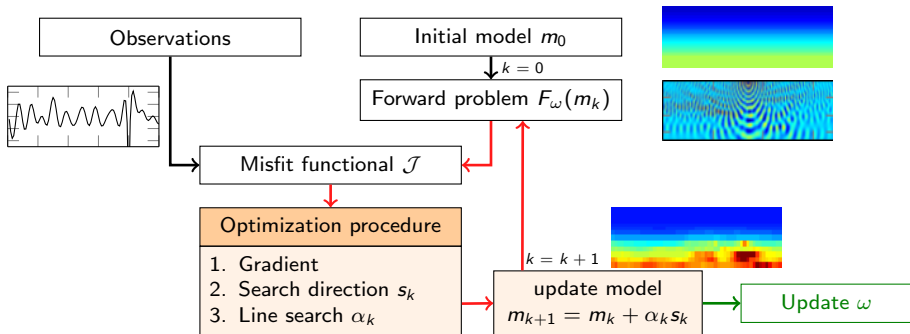
A. Tarantola

Inversion of travel times and seismic waveforms
Seismic tomography, 1987

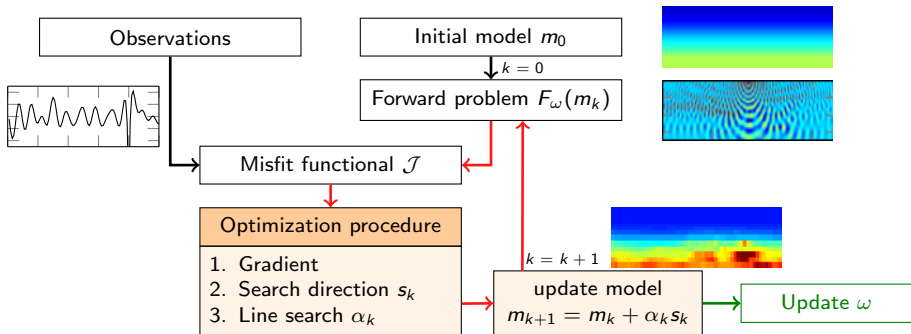
FWI, iterative minimization



FWI, iterative minimization



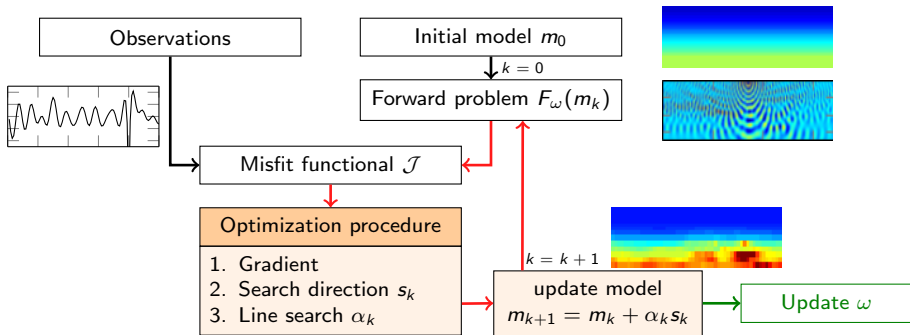
FWI, iterative minimization



Numerical methods

- ▶ Adjoint-method for the gradient computation, L-BFGS method,
- ▶ forward problem resolution with Discontinuous Galerkin methods,
- ▶ parallel computation, HPC, large-scale optimization,
- ▶ Rk: the code also works for **elastic** anisotropy and **viscous** media.

FWI, iterative minimization



- ▶ $> 10^5$: unknowns per physical parameter,
- ▶ $> 10^6$: matrix size for discretization,
- ▶ we also study stability and convergence of the algorithm ...

3 Reconstruction procedure using dual-sensors data

Minimization of the cost function

The appropriate misfit functional to minimize with pressure and vertical velocity measurements.

- ▶ Compare the pressure and velocity fields separately:

$$\mathcal{J}_{L2} = \sum_{source} \frac{1}{2} \|\mathcal{F}_p^{(s)} - d_p^{(s)}\|^2 + \frac{1}{2} \|\mathcal{F}_v^{(s)} - d_v^{(s)}\|^2.$$

Minimization of the cost function

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- ▶ Compare the fields multiplication for all combinations:

$$\mathcal{J}_G = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$



G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich

Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization
[preprint](#)

Minimization of the cost function

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

From Euler's equation, $v_n(\mathbf{x}_i) = -i(\omega\rho)^{-1} \partial_n p(\mathbf{x}_i)$.

Minimization of the cost function

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

From Euler's equation, $v_n(\mathbf{x}_i) = -i(\omega\rho)^{-1}\partial_n p(\mathbf{x}_i)$.

- ▶ **Cauchy data:** the cost function follows **Green's identity**.

Minimization of the cost function

$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

From Euler's equation, $v_n(\mathbf{x}_i) = -i(\omega\rho)^{-1} \partial_n p(\mathbf{x}_i)$.

- ▶ **Cauchy data:** the cost function follows **Green's identity**.
- ▶ **Reciprocity gap functional** in inverse scattering.



D. Colton and H. Haddar

An application of the reciprocity gap functional to inverse scattering theory
[Inverse Problems 21 \(1\) \(2005\), 383398.](#)



G. Alessandrini, M.V. de Hoop, R. Gaburro and E. Sincich

Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data
[arXiv:1702.04222, 2017](#)



G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich

Inverse problem for the Helmholtz equation with Cauchy data: reconstruction with conditional well-posedness driven iterative regularization
[preprint](#)

Stability results

Lipschitz type stability is obtained for the Helmholtz equation with **partial Cauchy data**.

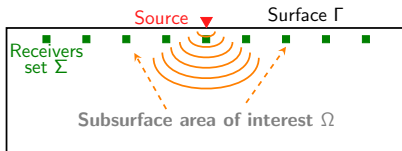
$$\|c_1 - c_2\| \leq \mathcal{C}(\mathcal{J}_{\mathcal{G}}(c_1, c_2))^{1/2}$$

Stability results

Lipschitz type stability is obtained for the Helmholtz equation with **partial Cauchy data**.

$$\|c_1 - c_2\| \leq \mathcal{C}(\mathcal{J}_{\mathcal{G}}(c_1, c_2))^{1/2}$$

- ▶ Using back-scattered data from one side in a domain with free surface and absorbing conditions,



- ▶ for piecewise linear parameters.



G. Alessandrini, M.V. de Hoop, R. Gaburro and E. Sincich

Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data
arXiv:1702.04222, 2017



G. Alessandrini, M.V. de Hoop, F. F., R. Gaburro and E. Sincich

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Additional possibilities

It allows the non-collocation of numerical and observational sources:

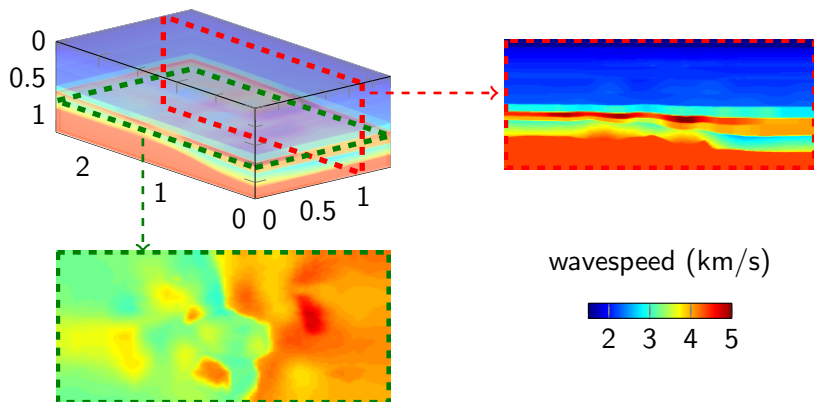
$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

- ▶ s_1 is fixed by the observational setup,
- ▶ s_2 is chosen for the numerical comparisons.

- 4 Numerical experiments
 - Comparison of misfit functions
 - Changing the numerical acquisition with \mathcal{J}_g

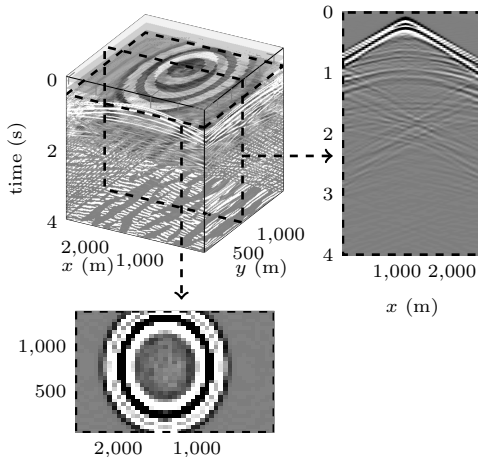
Experiment setup

3D velocity model $2.5 \times 1.5 \times 1.2\text{km}$ using dual-sensors data.



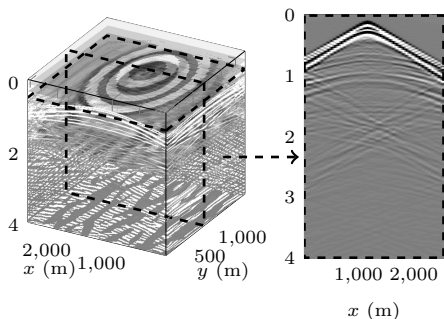
Experiment setup

We work with time-domain data acquisition.



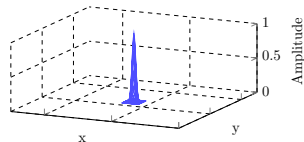
Experiment setup

We work with time-domain data (pressure and velocity).



Acquisition for the measures

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,



For the reconstruction, we apply a Fourier transform of the time data.

Comparison of misfit functional

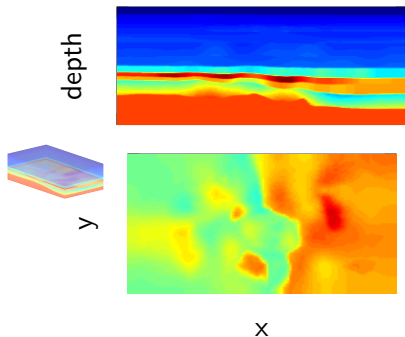
We respect the observational acquisition setup and perform the minimization of \mathcal{J}_{L2} or $\mathcal{J}_{\mathcal{G}}$, frequency from 3 to 15Hz.

$$\mathcal{J}_{L2} = \sum_{source} \frac{1}{2} \|\mathcal{F}_p^{(s)} - d_p^{(s)}\|^2 + \frac{1}{2} \|\mathcal{F}_v^{(s)} - d_v^{(s)}\|^2.$$

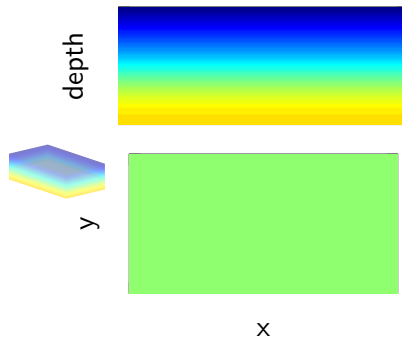
$$\mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{source} \sum_{source} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

Comparison of misfit functional

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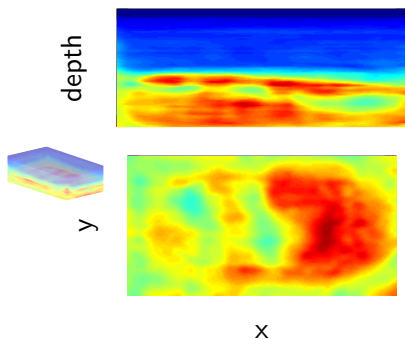
(a) True velocity



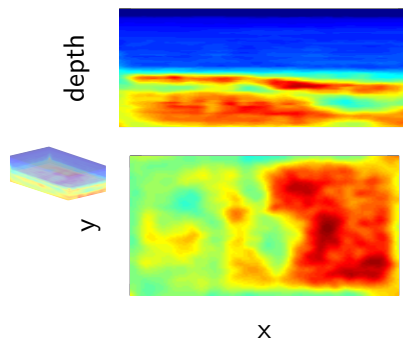
(b) Starting velocity

Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of \mathcal{J}_{L2} or $\mathcal{J}_{\mathcal{G}}$, frequency from 3 to 15Hz.



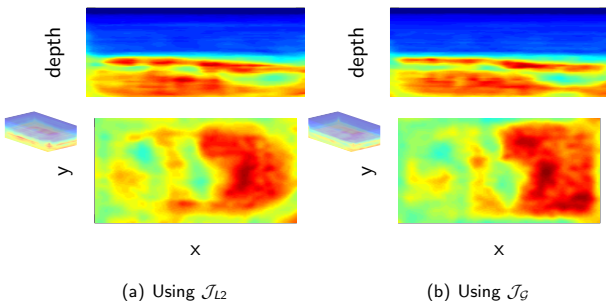
(a) Using \mathcal{J}_{L2}




(b) Using $\mathcal{J}_{\mathcal{G}}$

Comparison of misfit functional

We respect the observational acquisition setup and perform the minimization of \mathcal{J}_{L2} or $\mathcal{J}_{\mathcal{G}}$, frequency from 3 to 15Hz.



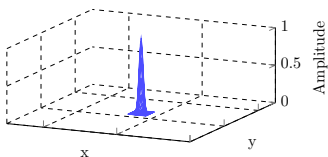
But the major advantage of $\mathcal{J}_{\mathcal{G}}$ is the possibility to consider alternative acquisition setup.


Experiment with different obs. and sim. acquisition 

$$\min \mathcal{J}_{\mathcal{G}} = \frac{1}{2} \sum_{s_1} \sum_{s_2} \|d_v^{(s_1)T} \mathcal{F}_p^{(s_2)} - d_p^{(s_1)T} \mathcal{F}_v^{(s_2)}\|^2.$$

Acquisition for the measures s_1

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,

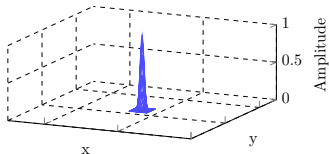


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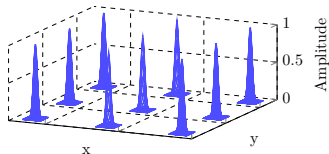
Acquisition for the measures s_1

- ▶ 160 sources,
- ▶ 100 m depth,
- ▶ point source,




Arbitrary numerical acquisition s_2

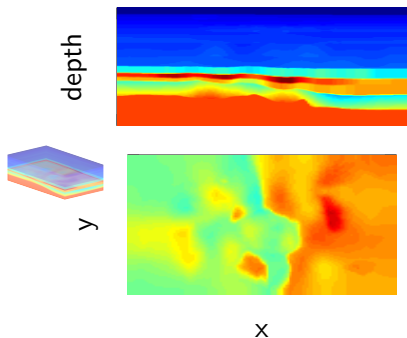
- ▶ **5 sources,**
- ▶ **80m depth,**
- ▶ **multi-point sources,**



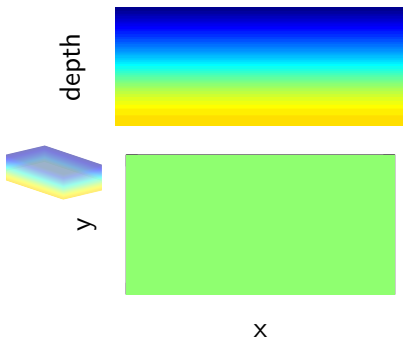
- ▶ No need to know observational source wavelet.
- ▶ Differentiation impossible with least squares types misfit.

Experiment with different obs. and sim. acquisition 


Data from frequency between 3 to 15 Hz, domain size $2.5 \times 1.5 \times 1.2$ km, Simulation using 5 sources only.



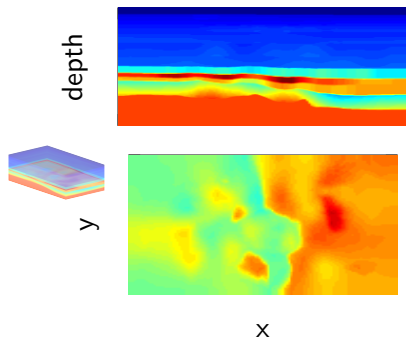
(a) True velocity



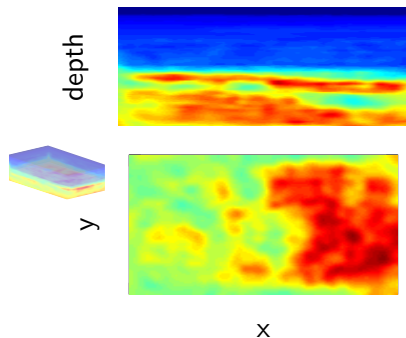
(b) Starting velocity

Experiment with different obs. and sim. acquisition 

Frequency from 3 to 15 Hz, $2.5 \times 1.5 \times 1.2$ km,
Simulation using 5 sources only. -33% computational time.



(a) True velocity



(b) 15 Hz reconstruction

Plan



5 Conclusion

Conclusion



Seismic inverse problem using pressure and vertical velocity data:

- ▶ appropriate cost function to minimize,
- ▶ allow minimal information on the acquisition setup,
- ▶ other applications,
- ▶ **perspective:** design the most efficient numerical setup,
- ▶ Rk: possible for elastic media with measures of traction.

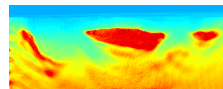
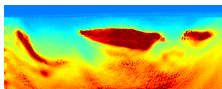
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Quantitative reconstruction algorithm toolbox for time-harmonic wave,

- ▶ **Discontinuous Galerkin** discretization in HPC framework,
- ▶ large scale optimization scheme using back-scattered data,
- ▶ **acoustic, elastic, anisotropy, 2D, 3D, attenuation.**



P- and S-wavespeed reconstructions

Conclusion

Seismic inverse problem using pressure and vertical velocity data:

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- ▶ other applications,
- ▶ **perspective:** design the most efficient numerical setup,
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- ▶ **acoustic, elastic, anisotropy, 2D, 3D, attenuation.**

THANK YOU

APPENDIX

$$\|c_1^{-2} - c_2^{-2}\| \leq C(\|F(c_1^{-2}) - F(c_2^{-2})\|)$$



G. Alessandrini

Stable determination of conductivity by boundary measurement
[Applicable Analysis 1988](#)



N. Mandache

Exponential instability in an inverse problem for Schrödinger equation
[Inverse Problems 2001](#)

$$\|c_1^{-2} - c_2^{-2}\| \leq C(\|F(c_1^{-2}) - F(c_2^{-2})\|)$$

The diagram illustrates the stability of the Helmholtz inverse problem. The equation shows the difference between two conductivity values, c_1^{-2} and c_2^{-2} , bounded by a constant C times the difference between their corresponding observations, $F(c_1^{-2})$ and $F(c_2^{-2})$. A bracket above the observation terms is labeled δ , representing the error in the data. Arrows point from the labels 'target' and 'initial model' to c_1^{-2} and c_2^{-2} respectively, and from 'observation' and 'simulation' to $F(c_1^{-2})$ and $F(c_2^{-2})$ respectively.

- ▶ Stability associate data and model correspondence
- ▶ Reconstruction is based on the iterative minimization of the difference between observation and simulation using an initial model.



G. Alessandrini

Stable determination of conductivity by boundary measurement
Applicable Analysis 1988



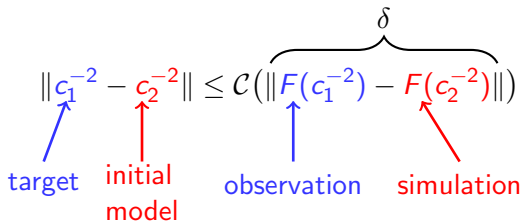
N. Mandache

Exponential instability in an inverse problem for Schrödinger equation
Inverse Problems 2001

$$\|c_1^{-2} - c_2^{-2}\| \leq C(\|F(c_1^{-2}) - F(c_2^{-2})\|)$$

target initial model observation simulation

δ



- ▶ Stability associate data and model correspondence
- ▶ $C(\delta) \leq C(\log(1 + \delta^{-1}))^{-\alpha}$



G. Alessandrini

Stable determination of conductivity by boundary measurement
Applicable Analysis 1988



N. Mandache

Exponential instability in an inverse problem for Schrödinger equation
Inverse Problems 2001

- ▶ $c(x)$ is bounded $B_1 \leq c^{-2}(x) \leq B_2$ in Ω
- ▶ $c(x)$ has a **piecewise constant** representation of size N

$$c(x)^{-2} = \sum_{k=1}^N c_k \chi_k(x)$$

- ▶ Ω has Lipschitz boundary

$$\|c_1^{-2} - c_2^{-2}\|_{L^2(\Omega)} \leq C \|F(c_1^{-2}) - F(c_2^{-2})\| \quad (1)$$



G. Alessandrini and S. Vessella

Lipschitz stability for the inverse conductivity problem
[Advances in Applied Mathematics 2005](#)



E. Beretta, M. V. de Hoop, F. and O. Scherzer

Inverse boundary value problem for the Helmholtz equation: quantitative conditional Lipschitz stability estimates.

[SIAM Journal of Mathematical Analysis 2016](#)

The stability constant is bounded

$$\frac{1}{4\omega^2} e^{K_1 N^{1/5}} \leq C \leq \frac{1}{\omega^2} e^{(K(1+\omega^2 B_2) N^{4/7})} \quad (2)$$

- ▶ depends on the partitioning N and the frequency ω



E. Beretta, M. V. de Hoop, F. and O. Scherzer

Inverse boundary value problem for the Helmholtz equation: quantitative conditional Lipschitz stability estimates 2016

In the case of partial Cauchy data (p and $\partial_\nu p$), we have that, we can obtain a Lipschitz type stability:

$$\|c_1^{-2} - c_2^{-2}\| \leq \mathcal{C}(\mathcal{J}_G(c_1^{-2}, c_2^{-2}))^{1/2}$$

Where c_1^{-2} and c_2^{-2} are piecewise linear.