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A PROCEDURE FOR GENERATING THE LAGRANGE EQUATIONS FROM THE BOND GRAPH REPRESENTATION USING THE λ -MULTIPLIER METHOD

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ABSTRACT

In the context of mechanical system modelling using bond graph language, this paper proposes a procedure consisting of a systematic bond graph causality assignment and the analytical exploitation of the causal bond graph obtained. The equations issuing from this analytical exploitation are the Lagrange equations using the λ -multiplier method. The set of DAE (Differential-Algebraic Equations) implicit Lagrange and kinematical equations are obtained by expressing an effort balance at the "generalized 1-junctions" and a flow balance at the "constraint 0-junctions".

The specialist can work with his own preferred form of equations (namely the Lagrange equations) and the non-specialist can manipulate high domain dependant analyses with a relative simple graphic concept (namely the bond graph). Moreover, due to the pluridisciplinary feature of bond graph language, the Lagrange equations, using the λ -multiplier method, can be used for different physical domains.

The paper is organized as follows : an introduction traces an overview of existing procedures for obtaining the Lagrange equations from a bond graph representation (Karnopp 1977; van Dijk 1994). The first part presents the procedure proposed in this paper. The second part details the application of this procedure to the example of a crank-slider mechanism. The third part explains the important issues of this procedure. Finally the conclusion draws some perspectives.

INTRODUCTION

Lagrangian formalism has certainly had retrospectively the greatest impact in the modern engineering world. It combines highly analytical mathematics and the pure foundations of physics so that abstract physics can be undertaken in a rigorously and systematically manner. It is now so widely used, however, that the fundamentals of Lagrangian formalism are often forgotten. The systematic aspect of its use is even so firmly fixed in the engineer's mind that its real issues are overlooked. Moreover we probably do not completely measure the universal feature

of Lagrangian formalism on physical domains. One might find some valuable clues in (Crandall *et al.* 1968) and (Bonvalet 1993) for example.

For a uniform approach, no graphic language can probably bring a sensitive understanding to physics more than the bond graph language does. Bond graph displays, with letter elements and half arrows, both energy phenomena and energy exchanges between these phenomena that are present in a multidisciplinary physical model (we stress upon the use of the word 'model' terminology to indicate that in the end, a bond graph representation will be always an approximation of reality). Apart from the level of a physical description, a bond graph representation contains potentially analytical information concerning the model. Among this analytical information we can generate the mathematical model of a system. The classical exploitation of the bond graph, however, is often restrictive (at least in the software based upon bond graph) in the form of the dynamic equations that can be obtained from it. In particular the state variables are systematically chosen through the set of energy variables of the bond graph i. e. the set generalized displacements of C-type elements and generalized momenta of I-type elements. This choice is restrictive in the sense that a specialist often uses another type of variable in his own domain to express the dynamic equations. In the hydraulic field pressures in the cylinder chambers are chosen as part of the state variables. They are the co-energy variables of C-type elements (Scavarda *et al.* 1997). In the electromagnetic field current is chosen as a state variable. It is a co-energy variable of an I-type element (Scavarda and Sesmat 1997). In the mechanical field, generalized coordinates are chosen as part of the state variables. They are generally the integrated variable of the co-energy variables of the I-type elements (Favre 1997).

Nevertheless, beside the restrictive aspect of the classical exploitation of the bond graph, some work (Karnopp 1977) and (van Dijk 1994) has dealt with obtaining the Lagrange equations of a system. Lagrange equations are second order differential equations, in terms of the generalized coordinates, describing the configuration of a system in the context of mechanics.

In his paper (Karnopp 1977) KARNOPP proposed a procedure for choosing the generalized coordinates and velocities.

It is important to note here that he works in a pluridisciplinary context. After the choice of the generalized coordinates and velocities, the energy associated with C elements and the co-energy associated with I elements, IC elements and gyrators can be constructed. From these expressions together with the efforts from effort sources, Lagrange equations can be derived. Firstly in this procedure the choice of the generalized coordinates and velocities is systemized. Secondly and for us the greatest drawback is that the equations are not directly derived from the bond graph representation but necessitate their calculation *a posteriori* from the expressions of energies and co-energies. Thirdly in some complex cases Karnopp's procedure does not prevent causal conflict.

In his thesis (van Dijk 1994) VAN DIJK generalized the concept of generalized coordinates in the physical domains. He brings the systematization of the procedure a step further by deriving Lagrange equations directly from bond graph without writing energy and co-energy functions (procedure LCAP for Lagrangian Causality Assignment Procedure). His approach results from considering the more general definition of the Lagrangian as a Legendre transform of the (total) energy of the system with respect to both some generalized displacements and generalized momenta. However the problem of causal conflicts and class 5 zero-order causal path is not addressed. Nevertheless this problem can be resolved by considering the global causality constraints (Favre 1997).

One important concept linked to the Lagrange equations is that of Lagrange-multiplier (or λ -multiplier). It is used in the case where the generalized coordinates are not independent and so constraint equations exist between them. In the context of bond graph applied in the mechanical field this has been first addressed by Bos in his thesis (Bos 1986). He has proposed a partial bond graph representation relaxing derivative causality of the dependant I elements and so accentuating the presence of the λ -multipliers. This presentation of the λ -multipliers through the bond graph language is also carried out by VAN DIJK in his thesis (van Dijk 1994) in a more general context with respect to the physical domains. They are installed where dependant storage elements appear during the application of MSCAP (Modified Sequential Causality Assignment Procedure). This allows the dynamic equations using Lagrange-multipliers to be derived. However we find this procedure restrictive in the sense that it installs λ -multipliers in places that the modeller perhaps does not want. Furthermore these may not be the best places for a convenient derivation of Lagrange equations using λ -multipliers. The aim of our paper is to raise this restriction in the exploitation of a causal bond graph.

In the following section we present the details of our procedure for deriving Lagrange equations using Lagrange-multipliers. The third section is devoted to the application of this procedure to the example to a crank-slider mechanism. The fourth section draws important issues from the application of this procedure. Finally the conclusion gives some perspectives.

λ LCAP PROCEDURE

The modeller may want to derive Lagrange equations using Lagrange-multipliers. For this reason, which can find its justification in the modelling process, this form of dynamic equation must be available from the exploitation of a causal bond graph. Also the modeller must have the choice of the generalized coordinates in terms of which he expresses his equations and the choice of the form of the possible appending constraint equations. This is the motivation for the procedure of causality assignment that we present in this paper. For the moment we restrain ourselves to the mechanical domain.

Our objective in the λ LCAP procedure (for λ -multiplier Lagrangian Causality Assignment Procedure) is to adapt the causality propagation to the choice of generalized coordinates by the modeller. The causality assignment will then adapt the places where the λ -multipliers will be installed. This is achieved by first giving the priority to the Artificial Flow Sources (ASF) corresponding to the generalized velocities. Then the causality is not systematically propagated which results in the acceptance of possible causal conflicts on some 0-junctions. Then Artificial Effort Sources (ASE) are installed where the causal conflicts appear showing λ -multipliers. Before presenting the steps of our procedure we give three definitions which will help in the following presentation.

Definition 1 [*Strong and weak causal determination*] (Ort and Martens 1973; Perelson 1975)

A bond gives a strong (or weak) causal determination to a connected junction if the causal assignment on the bond is such that the common variable on the junction is (or not) determined by this bond.

Definition 2 [*Generalized I-junction*]

A generalized I-junction is a 1-junction to which a generalized velocity is attached and where a Lagrange equation will be expressed by an effort balance.

Definition 3 [*Constraint 0-junction*]

A constraint 0-junction is a 0-junction to which a Lagrange multiplier is attached and where a constraint equation will be expressed in its kinematic form by a flow balance.

Procedure 1 (λ LCAP)

(Lagrangian Causality Assignment Procedure using λ -multipliers)

1. For every source (S_e and S_f) and every non-inversible element, assign the causality to the elementport with fixed causality.
2. Choose the generalized I-junctions (see definition 2) and connect an Artificial Flow Source (ASF) to each of them. The flow imposed by this source is a generalised velocity and the corresponding effort is equal to zero.
3. Propagate causality from the ASF and through (M)GY, (M)TF, the I-junctions and only the 0-junctions receiving a strong causality (see definition 3) on one of their connected bonds (introduce 0-junctions if necessary). Causal conflicts may appear at this step.

4. If the bonds are not all causally determined, assign a strong causality at a junction not already causally determined and propagate the causality in the same way as for the step 3.
5. Repeat the previous step until the bond graph is entirely causally determined.
6. Connect Artificial Effort Source (AES) to each of the 0-junctions where a causal conflict appears, there are then called constraint 0-junctions (see definition 3), (the causality violation is of type I (Joseph and Martens 1974; van Dijk 1994)). The effort imposed by this source is a λ -multiplier and the corresponding flow is equal to zero.

The Lagrange equations using λ -multipliers are obtained by writing effort balances at the generalized 1-junctions (the set of second order differential equations). They are expressed in an implicit form in terms of the generalized accelerations, velocities and coordinates, the λ -multipliers, and the input of the system. The constraint equations are obtained by writing the flow balances at the constraint 0-junctions (the set of kinematic constraints). The difference between the number of generalized 1-junctions and the number of constraint 0-junctions shows the degree of freedom of the system.

APPLICATION TO THE EXAMPLE OF A CRANK-SLIDER MECHANISM

The sketch of the example of a crank-slider mechanism is given figure 1.

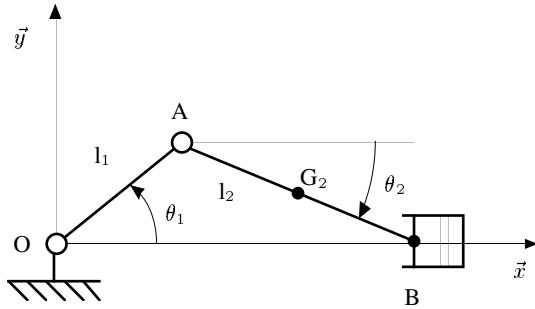


Figure 1: Sketch of a crank-slider mechanism

This planar model consists of three bodies. The crank rotates around the point O. Its angular position with respect to the fixed axis (O, \vec{x}) is θ_1 , its inertia at the point O is J_1 and the length OA is l_1 . The slider has a mass m and is constrained to move (point B) along the axis (O, \vec{x}) . Finally the connecting rod of mass center G_2 and length l_2 is connected to the crank at point A and to the slider at point B. Its mass is m_2 and its inertia is J_2 . We neglect the gravity action and we suppose the presence of viscous friction on the slider. Finally a torque C_m is applied to the crank.

The bond graph representation of this crank-slider mechanism has been undertaken with the Karnopp and Rosenberg procedure (Karnopp *et al.* 1990). The following key vectors have been identified: $[\dot{q}_k] = [\dot{\theta}_1 \quad \dot{\theta}_2]^T$ for the generalized velocities

which are not independent; $[v_1] = [\omega_1 \quad \dot{x}_{G_2} \quad \dot{y}_{G_2} \quad \omega_2]^T$ for the key vector associated to the kinetic phenomena where x_{G_2} and y_{G_2} are the inertial coordinates of the mass center G_2 . The bond graph representation is finally given in figure 2 where the junction structure reflects the kinematics of the mechanism, four I-type elements correspond to the kinetic phenomena, the R-type element corresponds to the phenomenon of dissipation (x_B is the position of point B on the axis (O, \vec{x})) and the effort source corresponds to the torque applied to the crank. We can note that the bond path between the figure 2 central 1-junctions reveals the constraint equation between the generalized coordinate in the kinematic form.

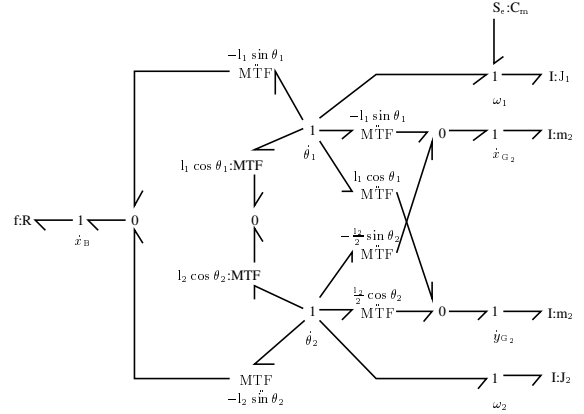


Figure 2: Bond graph representation of a crank-slider mechanism

We choose θ_1 and θ_2 as the generalized coordinates of this system. The corresponding generalized 1-junctions are the central 1-junctions of the figure 2 bond graph. We attach an ASF to each of them. The figure 3 bond graph shows the application of the three first steps of the λ LCAP procedure. We can note on this bond graph how the weak causalities are stopped on 0-junctions and that a causal conflict appears on the central 0-junction.

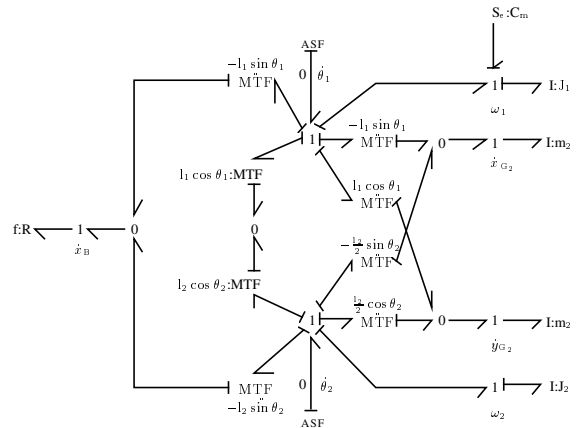


Figure 3: Partial causal bond graph after the three first steps of the procedure λ LCAP

Now steps four and five of the λ LCAP procedure are applied. This produces the figure 4 causal bond graph where a strong causality has been applied to the 0-junctions on the sides of the elements $I:m_2$ and $R:f$. At this stage the bond graph is entirely causally determined but it remains a causal conflict on the central 0-junction. This 0-junction is then a constraint 0-junction and the last step of λ LCAP procedure is applied. The result is given in figure 5.

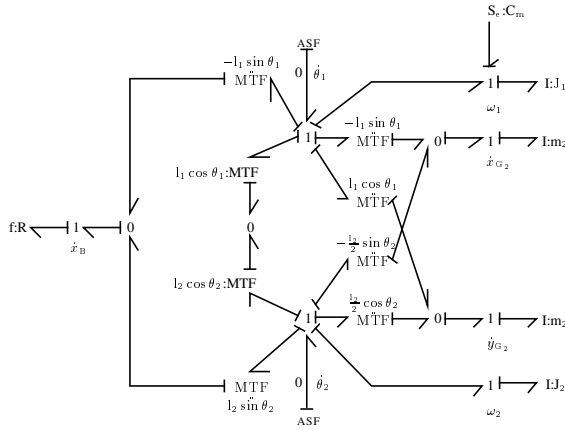


Figure 4: Causal bond graph after steps four and five of the λ LCAP procedure

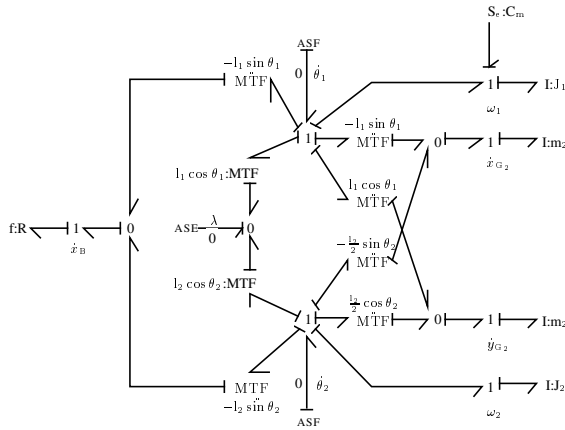


Figure 5: Causal bond graph augmented with the λ LCAP procedure

Now we can write the dynamic equations by first expressing the effort balances at the generalized 1-junctions. This gives:

$$0 = -(-C_m) - J_1 \ddot{\theta}_1 - (-l_1 \sin \theta_1) m_2 \frac{d}{dt} [(-l_1 \sin \theta_1) \dot{\theta}_1 + (-\frac{l_2}{2} \sin \theta_2) \dot{\theta}_2] - l_1 \cos \theta_1 m_2 \frac{d}{dt} [l_1 \cos \theta_1 \dot{\theta}_1 + \frac{l_2}{2} \cos \theta_2 \dot{\theta}_2] - (-l_1 \sin \theta_1) f (-l_1 \sin \theta_1 \dot{\theta}_1 - l_2 \sin \theta_2 \dot{\theta}_2) - l_1 \cos \theta_1 \lambda \quad (1)$$

$$0 = -J_2 \ddot{\theta}_2 - (-\frac{l_2}{2} \sin \theta_2) m_2 \frac{d}{dt} [(-l_1 \sin \theta_1) \dot{\theta}_1 + (-\frac{l_2}{2} \sin \theta_2) \dot{\theta}_2] - \frac{l_2}{2} \cos \theta_2 m_2 \frac{d}{dt} [l_1 \cos \theta_1 \dot{\theta}_1 + \frac{l_2}{2} \cos \theta_2 \dot{\theta}_2] - (-l_2 \sin \theta_2) f (-l_1 \sin \theta_1 \dot{\theta}_1 - l_2 \sin \theta_2 \dot{\theta}_2) - l_2 \cos \theta_2 \lambda \quad (2)$$

These equations are finally rearranged to give:

$$(J_1 + m_2 l_1^2) \ddot{\theta}_1 + m_2 l_1 \frac{l_2}{2} \cos(\theta_1 - \theta_2) \ddot{\theta}_2 + m_2 l_1 \frac{l_2}{2} \sin(\theta_1 - \theta_2) \dot{\theta}_2^2 + f l_1^2 \sin^2 \theta_1 \dot{\theta}_1 + f l_1 l_2 \sin \theta_1 \sin \theta_2 \dot{\theta}_2 + l_1 \cos \theta_1 \lambda = C_m \quad (3)$$

$$(J_2 + m_2 \frac{l_2^2}{4}) \ddot{\theta}_2 + m_2 l_1 \frac{l_2}{2} \cos(\theta_1 - \theta_2) \ddot{\theta}_1 - m_2 l_1 \frac{l_2}{2} \sin(\theta_1 - \theta_2) \dot{\theta}_1^2 + f l_2^2 \sin^2 \theta_2 \dot{\theta}_2 + f l_1 l_2 \sin \theta_1 \sin \theta_2 \dot{\theta}_1 + l_2 \cos \theta_2 \lambda = 0 \quad (4)$$

These equations are the Lagrange equations expressed in terms of the generalized coordinates θ_1 , θ_2 and the Lagrange-multiplier λ . The set of dynamic equations is completed by the constraint equation corresponding to the λ -multiplier. This equation is obtained by expressing a flow balance on the constraint 0-junction. This gives:

$$0 = l_1 \cos \theta_1 \dot{\theta}_1 + l_2 \cos \theta_2 \dot{\theta}_2 \quad (5)$$

The set of Differential-Algebraic Equations 3, 4 and 5 are finally the equations which govern the motion of the crank-slider mechanism.

IMPORTANT ISSUES

The procedure which has been proposed in this paper is more general than the one presented by KARNOPP and ROSENBERG, and the one presented by VAN DIJK. It directly derives equations from bond graph without writing energy and co-energy functions. It enables generalized coordinates to be chosen and it also shows λ -multipliers not necessarily on dependant storage elements. The way the procedure installs these λ -multipliers offers flexibility. In fact step 4 of the λ LCAP procedure gives the initiative to the modeller for achieving the causality assignment. In some respect this allows the modeller to choose the places where the λ -multipliers will be installed. This

feature is useful when the junction structure plays an important role the bond graph representation. This has been shown on the example of a cam-follower mechanism (Favre 1997) where some generalized velocities have been chosen through artificial parameters (Nikravesh 1988) and the corresponding λ -multipliers have naturally resulted on 0-junctions inside the junction structure.

The way the generalized velocities and the λ -multipliers are revealed through the bond graph representation during the application of λ LCAP emphasizes the dual aspect of power variables. Both generalized velocities and λ -multipliers are shown by artificial sources, respectively flow sources and effort sources. These sources are qualified artificial because the corresponding variables are naturally present in the bond graph representation without adding any element. These artificial sources are just used as a guide for the analytical exploitation of the causal bond graph in the context of Lagrange equations using λ -multipliers. Moreover the dual aspect is also clearly underlined with the generalized 1-junctions and constraints 0-junctions. For the first type of junction, an effort balance is accomplished in order to write the dynamic equations, for the second type of junction, a flow balance enables the kinematical constraint equations to be obtained.

In the classical procedure of causality assignment (SCAP for Sequential Causality Affectation Procedure), VAN DIJK emphasized that global causality constraints had to be taken into account. This has resulted in a modified procedure (MSCAP for Modified SCAP) which is to be used in case where zero-order causal paths of class four and five are susceptible to appear during the causality affectation (van Dijk 1994). In the case of the Lagrange equations VAN DIJK does not address this problem though it does exist as well i. e. that global constraints in causality affectation must be taken care of (Favre 1997) with the classical procedure allowing the Lagrange equations to be obtained. When using the method of Lagrange-multipliers the problem is different. The acceptance of causal conflict appearance at step 3 of λ LCAP and the flexibility in the causality propagation at step 4 frees oneself from global causality constraints during the causality assignment.

Finally the procedure proposed in this paper allows two types of analysis on the resulting causal bond graph. Firstly the generalized 1-junctions and the constraint 0-junctions show a partial junction structure from which we can identify the type of constraint equation namely if they are holonomic or non-holonomic (Favre 1997). Secondly the constraint 0-junctions, where artificial effort sources are connected, indicate where partial bond graph structure can be added in view of using certain numerical techniques by bond graph representation. These techniques allows in a sense the constraints to be relaxed and so it is natural to install the corresponding partial bond graphs at the places where the λ -multipliers are located. Among these techniques are the method of Baumgarte stabilization (Baumgarte 1984; Favre 1997; Favre and Scavarda 1998), the penalty formulation method (García de Jalón and Bayo 1994; Bayo *et al.* 1994; Favre 1997), and the

technique using the Singular Perturbation Formulation (Zeid and Overholt 1995a, 1995b, 1995c; Favre 1997).

CONCLUSION

In this paper a procedure of causality assignment has been proposed in order to obtain the Lagrange equations using the method of Lagrange-multipliers directly from bond graph representation. It has been proposed principally in the context of mechanics but it is not restricted to this domain. In fact by referring to the dual aspect of power variables, one can envisage a general application to the different physical domains like VAN DIJK did with his procedure LCAP (van Dijk 1994). This would consist for example of considering on one hand generalized 0-junctions where artificial effort sources would be connected and flow balances undertaken and on the other hand constraint 1-junctions where artificial flow sources would be connected and effort balances undertaken.

Furthermore we would like to emphasize the following point. We do not wish to substitute our procedure for the existing ones but rather to offer an alternative for modelling through bond graph representation. This is important for two reasons. On one hand alternatives increase the possibility of solving a problem. On the other hand they allow the specialist to work with the modelling techniques with which he is confident. Our philosophy on the subject is to raise the restrictive situation of a single methodology like the exploitation of bond graph by the MSCAP procedure only. For this we use as arguments in (García de Jalón and Bayo 1994) page 156: "There is not a consensus among the experts as to which method is the best for all cases. A method can be advantageous over another under certain conditions and vice versa"; or in (Huston 1991) page 109: "In spite of many notable advances in multibody dynamics analyses there are still disagreements among analysts about which methods are the best for specific applications and for computational efficiency. There are disagreements about which are the best methods for obtaining governing dynamical equations".

Finally in the context of a software implementation it is interesting to have the possibility of accessing at different procedures of causality assignment (for the same reasons invoked previously). In the λ LCAP procedure choices may be taken at two levels: (i) when the generalized velocities are chosen, (ii) at step 4 of the procedure when the bond graph is not completely causally augmented. These *a priori* arbitrary choices may be an obstacle for the modeller in particular if he is not a specialist of the domain concerned. At this moment the modeller must also have the choice to let the software to take a decision on his behalf (like any classical and systematical procedure does when taking the first storage elements for assigning an integral causality). From our point of view an efficient simulation may be undertaken if the modeller can choose between different modelling strategies. Bond graph language has the potential ability to conduct different strategies, tools, however, must be built for this.

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