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# Parameter analysis of the compressible packing model for concrete application

Mokrane Bala<sup>1</sup>, Rachid Zentar<sup>1</sup> and Pascal Boustingorry<sup>2</sup>

<sup>1</sup> Institut Mines Telecom Lille-Douai,  
LGCgE-GCE, 764 bd Lahure, BP 10838, 59508 Douai, France

<sup>2</sup> CHRYSO France,  
7 rue de l'Europe - Z.I. - 45300 Sermaises du Loiret, France

## Abstract

Since its invention, concrete has been the most widely used building material in the world [1]. Among its constituents, the aggregates occupy up to 80% of the volume. The remaining volume is occupied by a cement paste. This generates colossal quantities of cement and a heavy carbon footprint. Packing density optimization for the granular mixtures allows not only minimizes the quantity of cement incorporated in the concrete but also enhances its performance and durability. Our work aims to determine the parameters of the compressible packing model (CPM) to predict the maximum compactness of a granular mixture from the characteristics of its constituents. The work will be undertaken on two typologies of grains: crushed and rolled aggregates.

## 1 Introduction

In the field of civil engineering, concrete is an indispensable raw material which often offering the best technical and economic choice for the construction of buildings and infrastructures. Aggregates in concrete come in different sizes and shapes. They reach up to 80% of the total volume and can arise from natural, artificial (manufactured, industrial by-product, ...) or recycled. After the invention of reinforced concrete in the 19th century, concrete formulators found themselves faced with new challenges regarding the composition of the most compact granular mixtures. In conventional concrete formulation methods, granular mixtures are determined empirically, often from particle size curves (ideal distribution curves), as for the works of Fuller & Thompson (1907) d'Andreasen & Andersen (1929) and Dreux (1970). These methods do not make it possible to predict the packing density and may require several series of experiments for the optimization of the granular mix.

With the emergence of modern concretes and special concretes in the 20th century, such as High Performance Concretes (BHP) and Self Compacting Concrete (SCC), concrete mixtures by "ideal" particle size curves proved to be difficult or unsuccessful, as we quickly arrived at five grain scales that are: ultrafine (mineral additions, silica fumes, etc ...), fines (quarry sands, fine limestones, ...), sands, small gravel and coarse gravel.

In order to appear to these difficulties, several models have been developed to predict the compactness of a granular mixture. The demonstration of the interaction effects between grains of different sizes, in particular by the work of Caquot (1937), which shows the major influence of the wall effect on granular mixtures, has prompted researchers to study granular classes in pairs. Inspired by the Mooney viscosity model (1950), Stovall [2] studied binary mixtures with and without interaction. He has developed the linear packing model for granular mixtures with multiple classes, taking into account both the wall effects and the loosening effect between the granular classes of different sizes. The linear model was refined to build the virtual packing model that predicted virtual packing density, including mixtures of grains of the same size but of different shapes. It involves the volume proportions of each of the granular classes ( $y_i$ ), their packing density when they are arranged separately ( $\beta_i$ ), the wall effect coefficient ( $b_{ij}$ ) and the loosening effect coefficient ( $a_{ij}$ ).

The virtual packing density for a mixture of "n" granular fractions where the class (i) is dominant, is given by the following formula [3]:

$$\gamma = \gamma_i = \frac{\beta_i}{1 - \sum_{j=1}^{i-1} y_j \left( 1 - \beta_i + \beta_i b_{ij} \left( 1 - \frac{1}{\beta_i} \right) \right) - \sum_{j=i+1}^n y_j \left( 1 - a_{ij} \frac{\beta_i}{\beta_j} \right)} \quad (1)$$

The granular interaction coefficients were calibrated on binary mixtures [1]. The coefficient of loosening effect  $a_{12}$ , exerted by the fine particles (2) in a dominant large grain population (1), and the wall effect  $b_{21}$  exerted by coarse grains on the preponderant fine grains population are given by [2]:

$$a_{12} = \beta_2 \left( \left| \frac{\partial e}{\partial y_2} \right|_{y_2=0} + \frac{1}{\beta_1} \right) \quad (2)$$

And:

$$b_{21} = \frac{\frac{1}{\beta_2} - 1 - \left| \frac{\partial e}{\partial y_2} \right|_{y_2=1}}{\frac{1}{\beta_1} - 1} \quad (3)$$

“e” is the void ratio of the granular mixture.

After calibration of the coefficients a and b on a series of experiments, de Larrard [2] proposed simplified formulas of the granular interaction coefficients in the following form:

$$a_{i,j} = \sqrt{1 - \left(1 - \frac{d_j}{d_i}\right)^{1,02}} \quad (4)$$

And:

$$b_{i,j} = 1 - \left(1 - \frac{d_i}{d_j}\right)^{1,5} \quad (5)$$

A first attempt to predict the packing density by the Solid Suspension Model was limited by two defects related to the notion of reference viscosity (considered as a description of the degree of compaction of the system) and the interaction functions that were not satisfactory in their mathematical form [5].

The Compressible Packing Model [3] represents the latest generation of models developed at “Laboratoire Central des Ponts et Chaussées” (LCPC) in France. This model makes it possible to predict the real packing density of a mixture of several granular classes from the knowledge of the compactness of each one-dimensional class and the energy of the setting up.

We find in the CPM the notion of virtual packing density and real packing density which are respectively the optimal packing density that can be achieved by placing the grains one by one to get a minimal void, and the real packing density of the mixture in which the granular arrangement is naturally disordered.

The real packing density depends on the compaction energy. This energy is introduced into the CPM by the concept of the compaction index “K”. The real packing density “C” of a mixture of aggregates is connected to “K” by the expression:

$$K = \sum_{i=1}^n K_i = \sum_{i=1}^n \frac{y_i / \beta_i}{\frac{1}{C} - \frac{1}{\gamma_i}} \quad (6)$$

Otherwise, a procedure called LCP No. 61 is associated with the CPM to measure the real packing density experimentally. Several methods existed to determine the real packing density which differs by the packing process: picketing, vibration, ... etc. The compaction table procedure according to the LPC method No. 61 [6] has the advantage that the compaction energy is constant and proportional to the number of shocks applied to the sample.

The experimental packing density can be calculated from the final height of the sample after compaction (h) and its dry density ( $\rho_{rd}$ ) through the following relation:

$$C = \frac{\rho_a}{\rho_{rd}} \quad (7)$$

$\rho_a = M/V$  : is the density of the compacted sample.

“M” is the weight of the compacted sample and “V” its volume.

The calibration of the CPM parameters (granular interaction coefficients and compaction index) on binary mixtures of aggregates of different types (crushed and rolled) has shown that the error does not exceed 0.77% for rolled aggregates and 1.71% for crushed aggregates [2], this taking a value equal to 9 for the compaction index “K”.

The analysis of the CPM parameters (interaction coefficients and compaction index) can be lead through the following flowchart (Fig. 1).

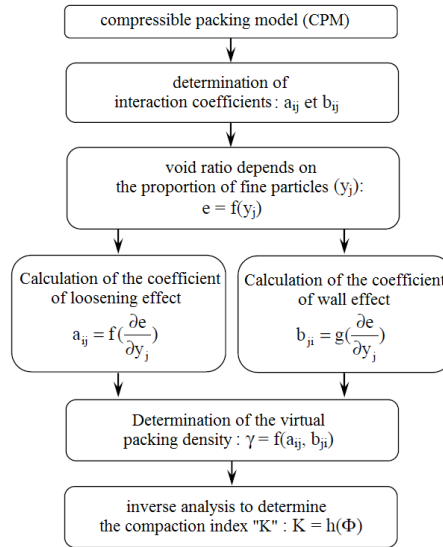


Fig. 1 Flowchart of the calibration of the CPM [3].

The CPM is a very important tool for optimizing and evaluating the compactness of granular mixtures for concrete. In this work, we are interested in determining the packing density of binary mixtures of aggregates in order to identify the different parameters that control this property as well as their degree of influence on the results of the CPM. This step is necessary before starting a study of concrete formulation by a "compactness" approach. Our study was conducted on aggregates of different sizes and shapes from the north and central region of France. First, we will apply the CPM as developed by de Larrard in order to qualify the prediction of the model and determine the most influential parameters. Subsequently, we will lead a phase of optimization of the parameters of the CPM on our granular mixtures.

## 2 Materials and methods

### 2.1 Materials

In the Table 1 are listed the aggregates of the present study. The size, shape and roughness of the grains are the three main parameters that affect the packing density that has influenced our choice. Rolled aggregates come from Chevrières quarry (Hauts-de-France region) and Decize quarry (Bourgogne-Franche-Comté region). The crushed aggregates come from the quarries of Boulonnais (Nord Pas-de-Calais). The granular fractions adopted for this study range from sand (0/4 mm) to gravel (10/20 mm).

Table 1 Origin and nature of the studied aggregates.

Type of aggregates	Origin	Granular class d/D	Density [t/m <sup>3</sup> ]
Crushed	Boulonnais	Sand 0/4	2.69
		Gravel 4/10	2.67
		Gravel 10/20	2.67
Rolled	Chevrières	Sand 0/4	2.55
		Gravel 4/10	2.43
	Decize	Gravel 10/20	2.54

The nature and state characteristics of the materials were identified according to standard tests.

## 2.2 Equipment

The compaction table for carrying out the compactness measurement test is shown in Fig. 2.

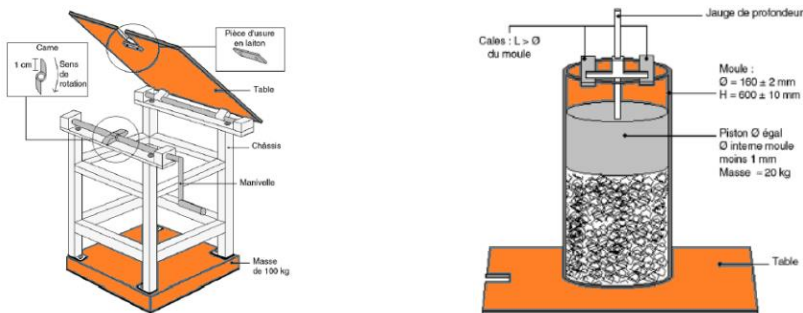


Fig. 2 Compaction table: Bottom part (left) and top part (right) [3].

A mass ( $M = 7$  kg) of the sample is placed in a cylindrical mould in three equivalent layers. Each layer is subjected to a series of shocks in the compaction table before putting the next layer. The sample will then undergo 40 additional shakes under a pressure of 10 kPa (an equivalent weight of 20 kg was used). The energy transmitted by shocks to the grains allows them to rearrange in a denser configuration, which improves the packing density of the sample. The process is characterized by a compaction index  $K = 9$  [4].

## 2.3 Experimental program

The capacities were measured on "Crushed-Crushed" and "Rolled-Rolled" mixtures to determine the optimal packing density that can be achieved with each type of aggregates on binary mixtures. A combination of sand and coarse gravel was analyzed. As shown in Table 2, considering the maximum nominal diameter of each class, the size ratio was fixed and the effect of grain shape on the packing density was experienced. Other combinations of binary mixtures between sand and gravel with an intermediate class (4/10 gravel) were tested. Trends in these mixtures will be given in conclusions. Otherwise, the variation of the proportions of a granular class in the binary mixture equal to 5% in the extreme parts of the curve of the packing density of the mixture and to 10% in the center of the curve. This is to measure the impact of the granular interaction effects we have seen previously (coefficient of wall effect and loosening effect).

Table 2 Experimental program of binary mixtures.

Type of aggregates	Granular class d/D	Binary mixtures	$D_1/D_2$ ratio
Crushed	Sand 0/4	SC 0/4 + GC 12/20	5
	Gravel 12/20		
Rolled	Sand 0/4	SR 0/4 + GR 11/22	5
	Gravel 11/22		

## 3 Experimental results

The main properties of the aggregates of the study are listed in Table 3. The absolute density was measured according to the European standard NF EN 1097-6 and the particle size analysis was carried out according to the European standard NF EN 933-1.

The particle size distribution of the different fractions are shown in Fig. 3.

Particle size analysis showed a high fines content of crushed sand (6% of grains smaller than 63 microns). Mixing curves are continuous and spread except for a small discontinuity for rolled aggregates between 10 and 12 mm sieves due to non-overlapping of granular classes 4/10 and 11/22.

Table 3 Characteristics of the aggregates of the study.

d/D	Crushed aggregates			Rolled aggregates		
	0/4 (Sand)	4/10 (Gravel)	12/20 (Gravel)	0/4 (Sand)	4/10 (Gravel)	11/22 (Gravel)
Fine content < 63 $\mu\text{m}$ [%]	6.2	1.0	0.8	1.3	1.7	0.1
$d_{50}$ [mm]	0.7	7.2	15.2	0.4	7.0	16.0
Finesse module (for sands)	3.2	-	-	3.4	-	-
Absolute density [ $\text{g}/\text{cm}^3$ ]	2.726	2.735	2.711	2.626	2.648	2.626
Water absorption [%]	0.43	0.59	0.49	1.18	3.06	1.79
Shape and roughness	Angular aggregates, flat and rough surface			Rounded shape, flat or elongated aggregates, smooth surface		

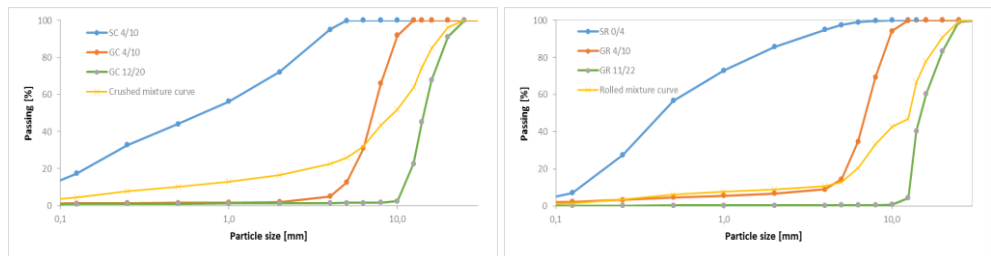


Fig. 3 Particle size distribution of crushed aggregates (left) and rolled aggregates (right).

Packing densities measurement tests were carried out by the compaction table according to LPC procedure No. 61 on binary mixtures of crushed and rolled aggregates. However, as the fines content is different from one granular fraction to another (especially crushed sand and rolled sand), it was decided to use the materials in their raw state (ie without sieving at 63 microns as recommended by LCP Procedure No. 61). Moreover, for each test, two different measurements of compactness were made. In each case, the average packing density and the possible differences are noted. It has been observed through the experimental program that the repeatability of the tests is ensured (maximum standard deviation of 0.015 for crushed aggregates and 0.004 for rolled aggregates).

The evolution curves of the packing density measured by the compaction table as a function of the introduction of small grains in a coarse grain population are shown in Figure 3. The small grains will be introduced into the intergranular voids of large grains if their size allows them to enter in those voids (otherwise, we will see a loosening effect). From the curves in Figure 3, it is observed a rapid increase of the packing density in the area of large grains dominant due to the ratio of large sizes ( $d_1 / d_2 = 5$ ) which minimizes the loosening effects. Subsequently, the packing density reaches an optimum before decreasing when entering the area where the small grains are dominant. Wall effects occur in this area.

The high fines content (high particle content <63 microns, minimum  $d_{50}$ ) of the crushed aggregates (Fig. 4. left) allows achieving high levels of packing density, particularly in the area of small grains dominant. In this case, shape effects were less visible for crushed aggregates. However, despite their low fines content, the rolled aggregates (Fig. 4. right) allowed us to reach the highest packing density. From these results, it is deduced that the maximum packing density that can be achieved on binary mixtures is influenced by the combination of the following parameters: the packing density of the elementary granular classes, the shape and roughness of the grains as well as the fines content.

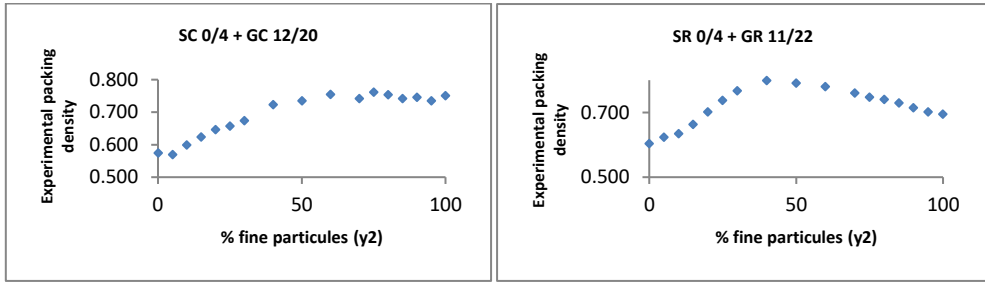


Fig. 4 Packing density measured by the compaction table of binary mixtures of crushed aggregates (left) and rolled aggregates (right).

After having made the evolution curves of the experimental packing density for our two binary mixtures (crushed and rolled), we proceed to the modeling phase of these packing densities by the CPM in which we will analyze the different parameters of the model which are: the coefficients of granular interactions (a and b) and the compaction index (K).

#### 4 Modeling packing density by CPM

In order to determine the packing densities by the CPM, the virtual packing densities are first calculated using Equation (1). For this, we need to determine the coefficient of loosening effect ( $a_{ij}$ ) and wall effect ( $b_{ij}$ ). These parameters will be calculated initially using the simplified formulas (Equations (4) and (5)). The real packing density of the mixtures is subsequently calculated through the compaction index which is taken as 9 because of this value is associated with the operating mode by the compaction table. Finally, we will proceed to a calibration of the coefficients a, b and K on our crushed and rolled aggregates.

##### 4.1 Coefficients a and b calculated by the simplified formulas (K = 9)

The simplified equations of the granular interaction coefficients were developed following series of experiments on aggregates of different natures (crushed and rolled). In order to verify the agreement of the experimental and theoretical results using these formulas, we have made a modelling of the experimental compactness by the CPM using the Equations (4) et (5) for the coefficients a and b. The results are shown in Figure 5.

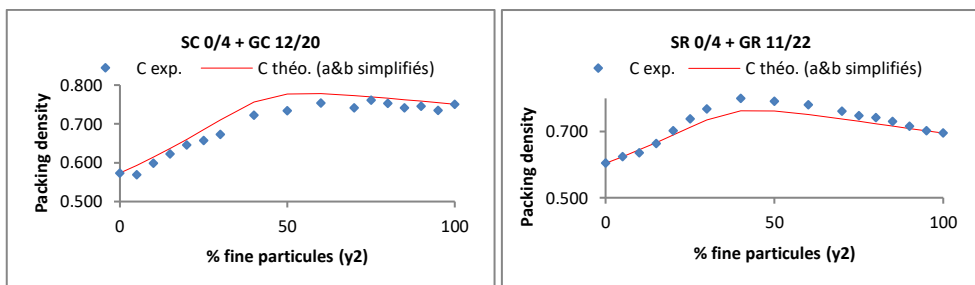


Fig. 5 Measured and modeled packing density of crushed (left) and rolled (right) binary Mixtures - Interaction Coefficients through the CPM Simplified Formulas.

Based on the simplified formulas of the interaction coefficients, the CPM allowed us to predict experimental packing densities with an average error of 2.8% for crushed aggregates and 2.1% for rolled aggregates. We note through the curves in Fig. 5 that the largest differences are found in the area of optimal packing density. In this zone, the packing density is overestimated by the MEC for crushed aggregate mixtures and underestimated in the case of rolled aggregate mixtures. This leads us to say

that the efficiency of the compaction mode (by the compaction table) is influenced by the parameters of particle size [8] and shape.

The parameters  $a$ ,  $b$  and  $K$  taken in this first modelling are given in Table 4.

However, the experimental and theoretical data analyzed by de Larrard (de Larrard 2000) revealed an average error of 1.71% for crushed aggregates and 0.77% for rolled aggregates. In order to optimize the results of the modelling, we decided to calibrate the granular interaction coefficients  $a$  and  $b$  on our binary granular mixtures using Formulas (2) and (3).

#### 4.2 Coefficients $a$ and $b$ calibrated on our aggregates ( $K = 9$ )

In the work of de Larrard, the coefficients  $a$  and  $b$  were determined at the limits of the curves of the void ratio as a function of the proportions of the small grains. Following this path and based on the equations (2) and (3) that we saw previously, we proceed to the calibration of coefficients  $a$  and  $b$  on our aggregates through Fig. 6.

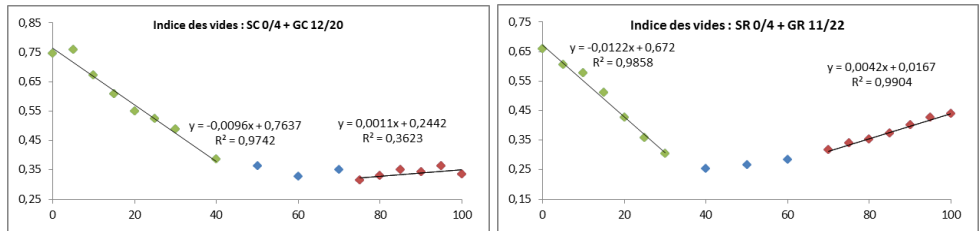


Fig. 6 Determination of the granular interaction coefficients  $a$  and  $b$  for crushed (left) and rolled (right) aggregates.

For some granular mixtures on void ratio curves, we observe dispersions and trends (positive or negative slope) are difficult to determine. This indicates the sensitivity of this method compared to the experimental tests. In fact, carrying out compactness tests between two adjacent points (5% fine-grained interval) can give similar results. The values of the coefficients  $a$  and  $b$  that could be calculated are given in Table 4 and the results of the modelling in Fig. 7.

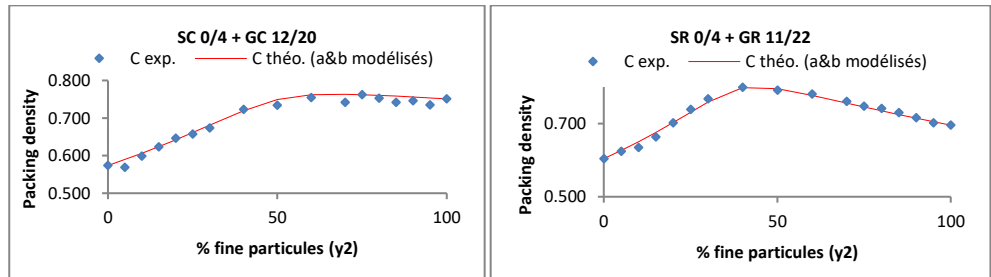


Fig. 7 Measured and modeled packing density of crushed (a) and rolled (b) binary mixtures - Calibrated interaction coefficients on our mixtures.

Table 4 Parameters of the modeling by simplified and calibrated formulas of  $a$  and  $b$  ( $K = 9$ ).

Parameters	Crushed aggregates	Rolled aggregates
$a$ (simplified formula)	0.451	0.430
$a$ (calibrated on our mixtures)	0.633	0.333
$b$ (simplified formula)	0.284	0.260
$b$ (calibrated on our mixtures)	0.344	0.039



After determining the interaction coefficients, we can minimize the difference between the model and the experiment at 1.2% for crushed aggregates and 0.6% for rolled aggregates.

## 5 Conclusions

In this paper, we studied the packing density of binary mixtures of crushed and rolled aggregates. The results of the modeling by the CPM have shown that the computation of the granular interaction coefficients by the simplified formulas is insufficient to have a good prediction. At the end of this study, we can draw the following conclusions:

- The simplified formulas of  $a$  and  $b$  take into account only the size parameters (maximum diameter of the granular classes "D") without the shape parameters (one law for the two types of aggregates).
- After calibration of the interaction coefficients, we can minimize the difference between the model and the experiment at: 1.60% for crushed and 0.90% for rolled aggregates.
- The accuracy of the model is influenced by the sensitivity of the coefficients  $a$ ,  $b$  and  $K$ :
  - ✓ The pre-established formulas for the granular interaction coefficients ( $a$  and  $b$ ) in CPM do not coincide in all cases with the values calculated on the experimental curves of the binary mixtures.
  - ✓ The compaction index ( $K$ ) may vary from one mixture to another (especially for crushed aggregates).
- For packing density measurements at the compaction table made with the fine fraction (<63 microns), the repeatability of the tests is ensured (maximum standard deviation of 0.015 between two tests). The segregation effects of fine particles can be neglected.
- Optimum packing density is influenced by the combination of the following three parameters:
  - ✓ The ratio of size of particles;
  - ✓ The shape and roughness of the grains;
  - ✓ The fines content.
- The model has a good accuracy in determining the optimum of binary granular mixtures.

The perspectives of the present study are the optimization of the packing density of the ternary granular mixtures for the concrete as well as the study of the parameters of form and fines content of the aggregates. The use of predictive algorithms of artificial intelligence may allow re-establishing new formulas for calculating model parameters, especially the granular interaction coefficients.

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## References

- [1] Planetoscope - Statistiques : Production Mondiale de Béton N.d. Site web: <https://www.planetoscope.com/matieres-premieres/1374-production-mondiale-de-beton.html>, accessed February 21, 2018.
- [2] Stovall, T., F. De Larrard, and M. Buil. 1986. "Linear Packing Density Model of Grain Mixtures." *Powder Technology* 48(1):1–12.
- [3] De Larrard, F. 2000. "Structures Granulaires et Formulation Des Bétons." *Etudes et Recherches Des Laboratoires Des Ponts et Chaussées OA* 34:414.
- [4] De Larrard, F. 1999. "Concrete Mixture Proportioning: A Scientific Approach." CRC Press.
- [5] Sedran, T., and F. De Larrard. 1999. "Une Nouvelle Approche de La Formulation Des Betons". *Annales Du Batiment*: 1–26.
- [6] Ledee, V., F. De Larrard, T. Sedran, and F. Brochu. 2004. "Essai de Compacité Des Fractions Granulaires à La Table à Secousses: Mode Opérateur. Techniques et Méthodes Des Laboratoires Des Ponts et Chaussées". *Méthode*.
- [7] Tran Ngoc, T. 2009. "Valorisation de Sédiments Marins et Fluviaux En Technique Routière". Thesis, Université d'Artois - Ecole des Mines de Douai.
- [8] Sadok, A., R. Zentar, and N. Abriak. 2016. "Genetic Programming for Granular Compactness Modelling". *European Journal of Environmental and Civil Engineering*: 1–13.