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### ► **To cite this version:**

Mohamed Othman Latrous, Amélie Ponchet-Durupt, Nassim Boudaoud, Magali Bosch, Zohra Cherfi-Boulanger. Integration model for condition based maintenance and a multivariate control chart : A comparative study univariate vs multivariate. Congrès Lambda Mu 21 “ Maîtrise des risques et transformation numérique : opportunités et menaces ”, Oct 2018, Reims, France. hal-02074945

**HAL Id: hal-02074945**

**<https://hal.science/hal-02074945>**

Submitted on 21 Mar 2019

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## Modèle d'intégration de la maintenance conditionnelle et carte de contrôle multivariée : Une analyse comparative univarié vs multivarié

### Integration model for condition based maintenance and a multivariate control chart: A comparative study univariate vs multivariate

#### Exemple :

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#### Résumé

Afin d'atteindre les performances attendues d'un système industriel, des solutions sont souvent implémentées pour remédier aux problèmes qui peuvent nuire au bon fonctionnement du processus de production. Ceci étant, Il est fréquent que les différents services (qualité, maintenance...) agissent indépendamment les uns des autres. Pourtant, avec la digitalisation, la quantité d'information récoltée au niveau du système industriel est devenue de plus en plus importante ce qui encourage son exploitation en adoptant une approche intégrée. L'objet de cet article est de proposer une réponse à cette problématique en proposant un modèle qui s'appuie sur l'utilisation des cartes de contrôle multivariées au lieu des cartes univariées. Le gain par unité de temps sera considéré comme un indicateur de performance.

#### Summary

In order to reach the expected manufacturing process performance, common solutions are often implemented to solve problems that may affect the flow of production. It is noticed that the different services (quality, maintenance) act in an independent way to come up with disjoint solutions. With the digitalization of the industrial domain, information and data issued from the manufacturing process witnessed a growth in size and became more available, which encourages its exploitation under an integrated framework. The purpose of this article is to suggest an answer to this problem with a model based on multivariate control charts and not on the commonly used univariate control charts, interlinked with condition based maintenance. The profit per unit of time is considered an indicator of performance.

#### Introduction

The industrial domain is well known for its constant battles over market shares, and its tendency to increase benefits acquiring a dominant position as the best provider of a service or product. To stay ahead of the competition, industrial structures tend to get the focus on good image and customer satisfaction. Common solutions are often implemented in shop floors like quality improvement programs, production optimization planning or maintenance optimal scheduling in order to get closer to the expected performance (Ben Daya and Duffuaa 1995). Maintenance, Production and Quality of the industrial process are often managed in order to meet customer's expectations by treating each component as independent. However, just a focused interest on each of the different elements of an industrial system tend to cover the real problem; that the whole process work in an integrated way based on a constant exchange of information (Hadidi et al., 2012)(Bassem Bouslah 2015). Each team tries to solve their problems disregarding their mutual interest which is optimizing overall system performance. Researchers took keen interest in the concept of integration since it helps improving the performance of the manufacturing system. As maintenance is considered as a "necessary evil" to industrial specialists, flaws detection and failure prevention represents a constant challenge to ensure overall equipment efficiency (Kisi et al. 2015). An important ascertainment, is the distinction between two different approaches to consider maintenance policies. Either traditional maintenance actions are carried out with a predetermined schedule based on how much time the machine have been operational, we talk about systematic Time base Maintenance (TBM) (Kim et al. 2016). Or, another way to proceed with maintenance actions is to monitor the system and use the collected information reflecting the state of machines, which represents condition based maintenance (CBM) policies (Alaswad and Xiang

2017). In recent work, interest in condition based maintenance policies has grown and studies took interest in the concept using machine information to model its degradation level.

Literature abounds with integration models suggesting improvement to classical disjoint optimization models. One area of improvement is integrating Statistical process control with maintenance planning (Hadidi et al., 2012). The link between maintenance and quality control is easily perceptible, as faulty equipment is ought to produce items with non-satisfactory quality requirements (Mtiba 2017). So, in order to get a closer image of the actual state of the process, researchers suggested the use of control charts to monitor and keep track of the eventual process shifts that can lead to non-quality items (Ben Daya and Rahim 1999; Cassady et al. 2000; Liu et al. 2013). And, yet again different approaches are perceived in the literature as presented integration solutions available vary between using univariate control charts and multivariate control charts (Montgomery 2009). Most of the proposed integration solutions rely on univariate control charts by monitoring one quality characteristic as an indicator of the shifts that can occur on the manufacturing process (Ben-Daya 1999; Panagiotidou and Tagaras 2010; Makis 2008). But, the performance of such control charts (X-bar which is usually used) have been proved quite limited especially if multiple correlated quality characteristics are considered (Lowry And Montgomery 1995; Bersimis 2007). So, using just X-bar control chart for a single variable doesn't represent an accurate process monitoring solution in a multivariate framework (Ardakan et al. 2016). Multivariate statistical process control was introduced on 1947 to provide a better detection in case of monitoring multiple correlated variables and is getting increased attention by research specialist in industrial engineering (Montgomery 2009). The research work proposed in this paper is encouraged by the emergence of the condition based maintenance policies and multivariate control charts for

process for quality control which represent a research area to investigate furthermore. The proposed solution is based on the work of (Panagiotidou and Tagaras 2010) which is considered as a starting point. This study allows us to emphasize the importance of multivariate SPC over traditional univariate SPC which is widely used in the literature of integrated solutions.

This work differs from the existing solutions of integration of statistical process control and CBM as it focuses on investigating the impact of using multivariate process control compared to the use of univariate process control. Authors in (Kisi et al. 2015) introduced a hidden Markov model for condition based maintenance and multivariate control charts where they used  $T^2$  control chart for raw data analysis for a specific grinding system. Authors in (Ardakan et al. 2016) presented a model joining MEWMA control chart and maintenance planning. Their maintenance policy is a mix of reactive maintenance and systematic maintenance where the reactive maintenance actions are triggered automatically by the control chart alarm. The proposed approach relies on delivering a comparative study between the use of univariate control charts and univariate control charts under an integrated framework CBM – SPC.

The next section gives insight about the integrated elements in the proposed model, SPC and CBM and the link between them. Section 3 explains the model definition and section 4 regroups the obtained results. Section 5 concludes this study by summarizing the studied problem and results followed by the perspectives of this research work.

## Overview of SPC and CBM

### 1. Process monitoring through control charts

Satisfying customer's expectations is a constant challenge as industrial specialists try to manage both quantity but more importantly the quality of the product. Statistical process control is of the popular tools to track and detect deviations in the process performance in term of quality monitoring. Univariate control chart provide a control and monitoring solution, but it remains limited as it only tracks one single quality characteristic (Harris et al. 2016). Due to the complexity of manufacturing processes, using such simple solutions remains limited and no longer suitable. If we need to monitor multiple variables, using individual control chart for each variable becomes counterproductive when there is a correlation between the monitored variables. Multivariate control chart have been developed in order to control variability of correlated multiple process characteristics. Hotelling was the first to deliver a first glimpse on multivariate quality control and such concept have been constantly improved since then (Montgomery 2009). One of the common multivariate control chart is

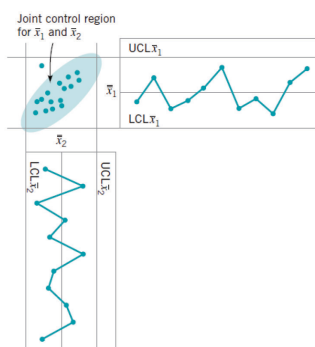


Figure 1. Univariate vs Multivariate detection for correlated variables (Montgomery 2009)

Hotelling  $T^2$ . Since it is designed for multivariate statistical process control, it provides a greater sensitivity to out of

control points in a multivariate framework compared to using multiple univariate  $\bar{X}$ -bar charts (refer to Figure 1).

### 2. Condition Based Maintenance vs Systematic maintenance

Time based maintenance (TBM) or systematic maintenance schedules are typically determined based on a probabilistic model of system failure and based on historical failure data.

On the other hand, condition-based maintenance (CBM) revolves around planning maintenance actions according to the state of the system through condition monitoring. Condition based maintenance concept gained popularity because of its reliance on stochastic deterioration models which describes at best the behavior of manufacturing systems. In a recent work, authors (Kim et al., 2016) led a head to head comparative study between TBM and CBM. They reached the conclusion that beside the high requirements to set up condition based maintenance in an industrial environment, such policy grants a steady condition level of the machines with increased reliability and reduced number of failures and critical errors. Nevertheless, TBM knows a widespread usage across the literature of integration modeling because of its simplicity of implementation and the growing complexity of the developed integrated models (Cassady et al. 2000; Bouslah et al., 2015). We highlight different kinds of CBM methods and one of the most popular is data driven methods which rely on collected data from the manufacturing process (Kisi et al. 2015). Other alternatives approaches are introduced for condition monitoring like the rate of defective produced items (Rivera-Gomez et al., 2013; Kouki et al. 2014).

### 3. Integration of Multivariate SPC with CBM

Manufacturing processes are witnessing a growing digitalization, thus data is becoming more available. It is important to exploit overflow of information, on one hand, with the use of multivariate methods, on the other hand, with integrating the use of statistical process control along with condition based maintenance. CBM is based on collecting various information about the status of manufacturing process through condition monitoring, followed by a decision making process based on an estimation of the deterioration level or failure rate of the system (Rivera-Gomez et al., 2013; Deloux 2008). One of the widely used condition monitoring approaches is data driven methods. And often, such method is quite demanding in term of infrastructure (sensors, data cleaning, etc.). Despite the fact that such methods have the ability to deliver an accurate interpretation of the system, it still lack reliability as it relies on the quality of the collected data and delivers no tangible physical understanding. Among other data driven methods, SPC have been considered for condition monitoring. Since the concept revolves around graphic process monitoring, it delivers a tangible image to the state of the process and witnessed a growing interest among the research work on condition monitoring and integration of SPC to maintenance planning (Ardakan et al. 2016).

The proposed approach revolves around the use of the  $T^2$  multivariate control charts to assess the system condition and deliver early detection of the shifts that the process may withstand.

## Model definition

### 1. Problem description

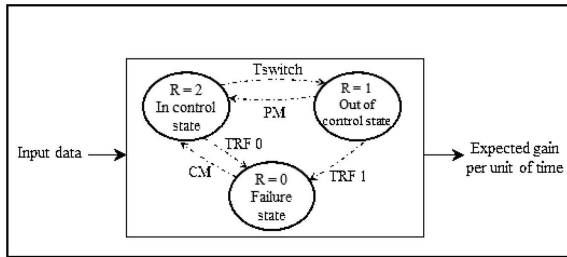


Figure 2. SADT representation of the renewal process under study

The system under study is a single imperfect machine subjects to random failures. An imperfect process is a process who can shift spontaneously from the in control state to the out of control state if not maintained. The in control state being the stage of production where the system performs according expectations and production items with good quality and the out of control state grants a higher probability of producing non-conform items.

The process has 3 states (refer to Figure 2)

- State 2 : In control operational state
- State 1 : Out of control operational state
- State 0 : Failure state

The system is initially considered as good as new producing quality conform items. The process may shift to the out of control state. Disregarding its operating state ( $R=1, 2$ ), and like any manufacturing process, the system is subject to random breakdowns which induce heavy losses. Corrective maintenance (CM) actions are performed in case of process failure ( $R=0$ ). As for the eventual shifts that may occur on the process, preventive maintenance (PM) actions are carried out to recover the system to the in control state. The model under consideration is a renewal process which restarts upon the execution of maintenance action which is supposed perfect in this study. Perfect maintenance action recover the process to the as good as new state ( $R=2$ ). The times of shift between each state ( $TRF_0, TRF_1, T_{switch}$ ) are random variables generated following a known probability density functions  $\varphi_0(t), \varphi_1(t)$  and  $\varphi(t)$  and their respective cumulative distribution functions  $\Phi_0(t), \Phi_1(t)$  and  $\Phi(t)$ . To identify process quality shift, non-periodic inspections are carried out to monitor the state of the system.

The inspection times are predetermined following the constant integrated hazard rate inspection policy (refer to eq. (1)) introduced by Munford (1981) and its usage is popular in the literature (Rivera-Gomez et al., 2013; Ben Daya 1999). The hazard rate of the time to quality shift is

$$h(t) = \varphi(t) (1 - \Phi(t)) \quad \{1\}$$

with  $\varphi(t)$  being the probability density functions of the process quality shift and  $\Phi(t)$  its respective cumulative distribution functions. The aim of the method is to obtain non periodic inspection times where the system probability of failure is the same between all inspection dates. The constant hazard rate policy is used to generate the quality sampling times given Eq (2) below:

$$\int_{t_{i-1}}^{t_i} h(t) dt = \int_{t_i}^{t_{i+1}} h(t) dt \quad \forall i = 2, \dots, s - 1 \quad \{2\}$$

In order to use this policy, the first interval of inspection is needed ( $t_1, t_2$ ) as well as the number of inspections  $s$ .

### 2. Integration strategy SPC and CBM

What follows regroupes the notations and the model assumptions as well as an overview of the system functioning.

#### 2.1. Assumptions:

- The times to failure ( $TRF_0, TRF_1$ ) and shifts ( $T_{switch}$ ) from state in control to out of control duration probability density are known;
- Each inspection of the system is assumed to be perfect; it delivers perfect information of the manufacturing process;
- Each system inspection and quality control sampling require stopping the system;
- The durations of maintenance actions are known and constant;
- The related costs to maintenance actions, quality control and system inspection are known and constant;
- Maintenance actions, sampling and inspections do not require setup time, they are executed right away;
- Maintenance actions are considered perfect and they recover the system to an as good as new state (AGAN) ( $R=2$ );
- The expected gains under the state out of control are significantly lower than the in control state gains as we suppose that the lower quality items are still sold at a reduced price.

#### 2.2. Model description:

The system initially in control  $R=2$ , may encounter one of the 3 events:

- i. Reaching inspection time at  $T_{inspec}$

A quality sampling (using control chart) takes place with duration  $D_{qc}$ . If an out of control signal is detected by the control chart, the system is declared statically out of control.

A perfect investigation of the system for a duration  $D_{inv}$  takes place to check if it is a false alarm (Error I) or not

- Case 1: Affirmative shift to the out of control state and then system undergoes a perfect preventive maintenance action, the system is As Good As New (AGAN) and the cycle ends.
- Case 2: It is a false alarm, the system is declared in an in control state ( $R=2$ ) and no maintenance actions are performed.

- ii. Natural shift from in control state ( $R=2$ ) to out of control state ( $R=1$ ) at  $T_{switch}$

The events that may happen then are:

- Failure ( $R=0$ ) at  $TRF_1$ : the process undergoes a perfect corrective maintenance action, the system is then AGAN and the production cycle ends.
- Inspection time at  $T_{inspec}$  a quality inspection takes place:
  - Case 1: A shift to the out of control state is detected and the system is declared statistically out of control with an alarm (Alarm=1) and a perfect investigation of the system takes place. The system undergoes preventive maintenance actions and the production cycle ends.
  - Case 2: If the control chart doesn't detect the actual state of the process (Error II – non detection) and no action performed, the system remains functioning until the next event occurs (failure or another inspection)

- iii. Reaching time of systematic preventive maintenance action ( $T_{m0}$ )

The system undergoes a preventive maintenance action and the production cycle ends. As explained previously, the purpose of this model is to emphasize the performance of using multivariate control chart in comparison to multiple univariate control charts.

### 2.2.1. Case univariate

The univariate case is summarized using a traditional X-bar control chart for each of the monitored variables. Each quality characteristic  $x_{ij}$  follows a normal distribution of mean  $\mu_{ij}$  (with  $i = 1, 2$  and  $j = 1, 2, \dots, p$ ) and a standard deviation  $\sigma_j$ . The mean depends on the state in which the system is in. Out of control mean  $\mu_{1j}$  is a deviation from the in control mean  $\mu_{2j}$  by  $\delta\sigma_j$  (Eq. 3), with  $\delta$  being the shift magnitude.

$$\mu_{1j} = \mu_{2j} + \delta\sigma_j \quad \{3\}$$

Sampling is proceeded according to the predetermined inspection intervals and the size of the sample is  $n$ .

The control limits of each control chart are calculated as follows:

$$UCL_j = \mu_{2j} + \frac{k\sigma_j}{\sqrt{n}} \quad \{4\}$$

$$LCL_j = \mu_{2j} - \frac{k\sigma_j}{\sqrt{n}} \quad \{5\}$$

With  $k$  represents the control limit parameter  $k$  and  $UCL_j$  and  $LCL_j$  are respectively the upper and lower control limits of the characteristic  $x_{ij}$ . Next step is plotting the sample mean on the control chart and if the point exceeds in any of the  $p$  control charts, an alarm is triggered.

The probability of false alarms (Error I) (refer to Eq. 5) and non-detection (Error II) (refer to Eq. 6) are as follow:

$$\alpha = 2 F(-k) \quad \{6\}$$

$$\beta = 2 F(k - \delta\sqrt{n}) - F(-(k + \delta\sqrt{n})) \quad \{7\}$$

With  $F$  represent the cumulative density function of the normal distribution.

Note that a shift of the process doesn't forcibly induce a shift of all the  $p$  monitored characteristic. Thus, each characteristic  $x_{ij}$  have a  $1/p$  probability to shift its mean from  $\mu_{2j}$  to  $\mu_{1j}$ .

### 2.2.2. Case multivariate

To build a Hotelling's  $T^2$  control chart, it is assumed that historical samples data are available. It is considered that  $p$  processes quality characteristics are monitored and represented by a  $p$ -dimensional vector  $X \sim N(\mu, \Sigma)$  (with  $i = 1, 2$ ) a statistical multivariate normal distribution with  $\mu_i$  representing the mean vector and  $\Sigma$  the covariance matrix (Montgomery 2009).

The available data is a set of  $m$  subgroup samples with  $n$  observations. Firstly, an estimation of the samples means  $\bar{x}_j$  of each variable observations and the samples variances and covariances matrix  $S$  is conducted.

$$X = \begin{bmatrix} \bar{x}_1 \\ \vdots \\ \bar{x}_j \\ \vdots \\ \bar{x}_p \end{bmatrix} \quad \bar{x}_j = \frac{\sum_{i=1}^n x_{ij}}{n} \quad j = 1, 2, \dots, p \quad \{8\}$$

$$S = \begin{bmatrix} \bar{s}_1^2 & \bar{s}_{12} & \dots & \bar{s}_{1p} \\ & \bar{s}_2^2 & \dots & \bar{s}_{2p} \\ & & \ddots & \\ & & & \bar{s}_p^2 \end{bmatrix} \quad \bar{s}_{jk} = \frac{\sum_{i=1}^n (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)}{n-1} \quad j = 1, 2, \dots, p \quad \{9\}$$

Then calculate the statistic  $T^2$

$$T^2 = n(X - \bar{X})' S^{-1} (X - \bar{X}) \quad \{10\}$$

with the upper control limits UCL given by Eq. (11) and the lower control limits LCL being zero.

$$UCL = \frac{p(m+1)(n-1)}{mn-m-p+1} F_{\alpha, p, mn-m-p+1} \quad \{11\}$$

Where  $\bar{X}$  represents the mean vector of the monitored characteristics and  $F_{\alpha, p, mn-m-p+1}$  is the upper 100  $\alpha$  percentile of the F distribution with  $(p, mn - m - p + 1)$  degrees of freedom.

### 2.2.3. Performance analysis criteria

The main output of this model is the expected profit per unit of time (EPT). It represents the difference between the profits of selling good and bad quality items and the cost issued from the manufacturing process, thus, the aim is increase the profit per unit of time.

EPT is issued following the given formula:

$$EPT = \frac{E(P)}{E(T)} \quad \{12\}$$

Where  $E(P)$  is the expected profit of a cycle and the  $E(T)$  is the expected cycle time.

A reminder is that a cycle of the considered process starts with a manufacturing process as good as new and following the scenarios presented in §III.2 will restarts always after maintenance actions either corrective or preventives.

The system operates under states in control and out of control with respective expected durations  $E(Ic)$  and  $E(Oc)$ . Quality control sampling actions require a duration  $D_{qc}$  and checking the system condition requires a duration  $D_{inv}$ . Maintenance actions durations are not negligible and they require a  $D_{pm}$  time units per preventive maintenance actions and  $D_{cm}$  time units for corrective maintenance actions. All the quality control and system investigation as well as maintenance actions generate costs that are regrouped on what follows

- $C_{cm}$  : cost per corrective maintenance action
- $C_{pm}$  : cost per preventive maintenance action
- $C_{inv}$  : cost per system investigation
- $C_{insp}$  : Cost per quality sampling setup
- $C_{sample}$  : unit cost of sample
- $C_{qc}$  : cost per quality control actions

Note that cost of sampling depends on the size of the sample  $n$  and is computed as follows

$$C_{qc} = C_{insp} + n C_{sample} \quad \{13\}$$

The expected cycle time and expected profit are calculated as follows:

$$E(T) = E(Ic) + E(Oc) + E(Qc)D_{qc} + (D_{inv} + D_{pm})p_{pm} + D_{inv}E(\alpha) + D_{cm}p_{cm} + D_{pm}p_{pm}(Tm0) \quad \{14\}$$

With  $p_{pm}$  and  $p_{cm}$  being respectively the probabilities of performing PM and CM on a cycle. And  $p_{pm}(Tm0)$  is the probability to reach the critical age  $Tm0$  in a cycle.

$$E(P) = G_{in}E(Ic) + G_{out}E(Oc) - C_{qc}E(Qc) - (C_{inv} + C_{pm})p_{pm} - C_{cm}p_{cm} - C_{inv}E(\alpha) - C_{pm}p_{pm}(Tm0) \quad \{15\}$$

With  $E(Qc)$  representing the expected number of quality control and  $E(\alpha)$  the expected number of false alarms per cycle.

$G_{in}$  and  $G_{out}$  represent the respective gains the system provide under states in control and out of control.

For further understanding of the mathematical model used in this work, we refer reader to the work of (Panagiotidou and Tagaras 2010) which represents the ground work of the proposed model.

## Results analysis

### 1. Simulation approach

Simulation is used to reproduce the behavior of the studied system and obtain the results of this model.

Without loss of generality, the simulation model will be used to calculate, for a specific optimal configuration, the expected cost EPT. The aim of the simulation is not to optimize the expected cost but to reflect the behavior of the model and emphasize the differences between the



cases of using univariate control charts and multivariate T<sup>2</sup> control chart. Thus, the obtained results are issued from 10000 simulation runs.

The times to failure (TRF<sub>0</sub>, TRF<sub>1</sub>) and shifts (T<sub>switch</sub>) from state in control to out of control are generated following Weibull distribution of known shape (β<sub>0</sub>, β<sub>1</sub>, β<sub>sw</sub>) and scale parameters (η<sub>0</sub>, η<sub>1</sub>, η<sub>sw</sub>).

- TRF<sub>0</sub> ~ Weibull (β<sub>0</sub>, η<sub>0</sub>)
- TRF<sub>1</sub> ~ Weibull (β<sub>1</sub>, η<sub>1</sub>)
- T<sub>switch</sub> ~ Weibull (β<sub>sw</sub>, η<sub>sw</sub>)

As the machine is highly likely to deteriorate faster under the out control state, the random variables above are generated for each simulation run granting a time to failure under the state out of control equal or lower than under the in control time. In order to ensure that, the random variables are generated under the conditions β<sub>0</sub> = β<sub>1</sub> and η<sub>0</sub> ≥ η<sub>1</sub> (Panagiotidou and Tagaras 2010).

## 2. Effect of magnitude of shift

Monitoring quality characteristics shifts can be easily detected through univariate control charts. But, the monitoring is affected by the shifts magnitude.

In (Panagiotidou and Tagaras 2010), authors used a factorial design of experiment to variate the input data of their model. They also proceeded by SPC and maintenance parameters optimization.

In this work, the input data of the design of experiment is used to highlight the effect of using multivariate control charts vs univariate control charts. The data set used is issued from the design of experiment and allows us to variate the input data of our model; time to failure and shifts distribution parameters, durations and costs of the manufacturing process as well as the profit under both states in and out of control.

For simulation, two quality characteristics are considered for monitoring. Thus, a set of data is generated following a multinomial distribution of mean vector μ<sub>2</sub> =  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$  for the in control state and μ<sub>1</sub> for the out of control state calculated with the Eq. (3) for each monitored characteristic with a shift magnitude δ = 0.5.

The covariances matrix used for the further calculus is

$$\Sigma = \begin{bmatrix} 1 & 0,9 \\ 0,9 & 1 \end{bmatrix}$$

Figure 3 highlights the impact of varying the magnitude of shift on the expected profit. The data used to obtain the results in Figure 3 are issued from line 29 of the DoE in (Panagiotidou and Tagaras 2010).

With higher shift magnitude, the performance of a Multivariate T<sup>2</sup> control chart is quite similar to multiple univariate X bar control charts.

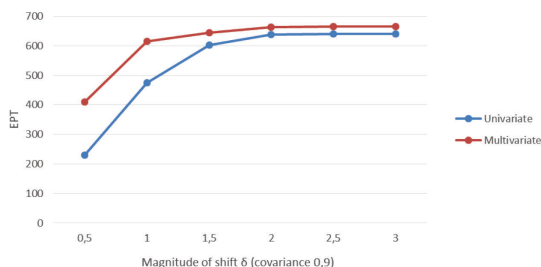


Figure 3. Magnitude shift effect on EPT

But the difference can be noticed with a magnitude of shift between 0.5 and 1.5. As individual monitoring of quality characteristics no longer reflects the actual state of the process which triggers false alarms and unnecessary system inspections which impact the overall system profit.

## 3. Expected profit

As presented previously in Eq. (14), the expected profit of the process relies on high performance with less false alarms and process inspections. Thus, process shift detection has an important impact on the process profit. But detections must be accurate in order to reduce the false alarms.

DoE input data from (Panagiotidou and Tagaras 2010) is used to highlight the effect of using multivariate control charts over multiple univariate control charts for expected profit improvement.

Data set	C <sub>inv</sub>	C <sub>qc</sub>	C <sub>pm</sub>	C <sub>cm</sub>	D <sub>inv</sub>	D <sub>qc</sub>	D <sub>pm</sub>	D <sub>cm</sub>
7	100	11	400	1500	0,5	0,1	1	2
18	100	11	400	1500	0,5	0,1	1	2
29	100	11	400	1000	0,5	0	1	1
32	100	3	800	1500	0,5	0,1	2	2

Table 1 Input variables for the displayed results

Overall EPT obtained values showed that multivariate control charts outperform always the use of multiple individual univariate control charts for each monitored variables.

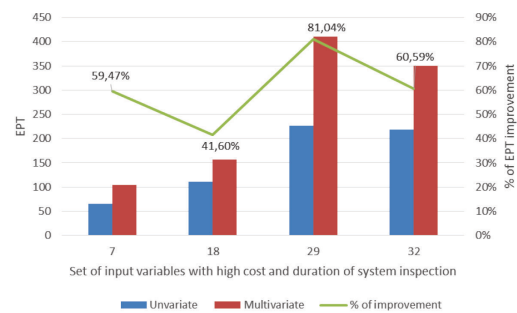


Figure 4. EPT obtained for sets of input values with high cost and duration of inspections

Figure 4 reflects the improvement in the expect profit with a multivariate quality control with high correlation between the variables 0.9. Table 1 represents the input variables considered for the displayed results which are characterized by a high cost and duration of system inspection. The obtained gain is justified by a lower rate of false alarm which results is lesser process inspection and as a result, a cost reduction of the overall manufacturing process. Also a difference in the time where the system is out of control before the detection of shifts is noticed as the multivariate T<sup>2</sup> allows a faster detection of the shifts that may occur on the process.

Thus, Multivariate control charts give a better assessment of the equipment condition and their usage to monitor the condition of the process surpasses in case of correlation between the variables. We remind the reader that, the considered shifts that the system undergoes are supposed of a low magnitude (δ = 0.5).

## Conclusion

Condition based maintenance is taking a leap in industrial engineering as it is witnessing an increasing interest in the concept. Condition monitoring represents a keystone to reflect with accuracy the state of the system. The aim of this work is to highlight the aspect of using control charts for condition monitoring and more specifically multivariate control charts. The model developed is quite simple as it relies on a 3 state model with transition probabilities following a known distribution but it allows us to reflect the importance that multivariate control charts have to deal with the monitoring of multiple correlated variables. Simulation is used to reflect the result and effect on the expected cost of the manufacturing system. This comparative study helped us assess the importance of exploring integrating multivariate control charts with a noticeable effect compared to univariate control charts which are quite popular in the literature.

In this study, a  $T^2$  control chart and multiple Xbar control charts are used and this work doesn't cover optimization as the input data is issued from the literature. Those results provide a window to the area of integrated SPC-CBM improvement. Other assumptions are quite restrictive as the nature of the renewal process restarts upon maintenance action.

It would be interesting to push the use of multivariate control charts to diversify maintenance actions. For example, studying the effect of varying costs, durations on different types of maintenance actions depending on the monitored characteristics as well as delivering confidence intervals of our expected profit which would allow us to refine our solution.

As far as this model is concerned, it covers only detection of a shift to the out of control state, thus, identification of the cause of the shift may provide additional information to be used for the maintenance policy and reduce inspection duration and costs.

One important point also is the consideration of incipient shifts, as such concept is not treated in the literature under an integrated framework. Thus, its consideration can be of a great insight to our research as the main goal is to reflect the system state and prevent degradation in order to optimize the performance and reduce the overall cost of the manufacturing process.

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