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Approximation SEM-DG pour les problèmes d'ondes elasto-acoustiques

Hélène Barucq¹, Henri Calandra², Aurélien Citrain^{3,1}, Julien Diaz¹ and Christian Gout³

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² TOTAL SA, CSTJF, Pau, France.

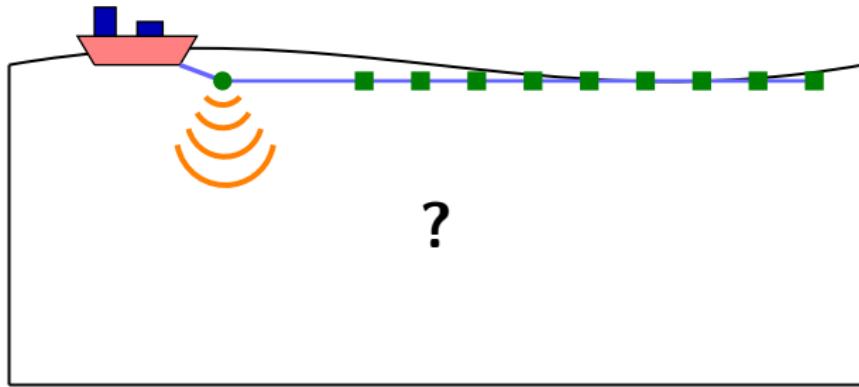
³ INSA Rouen-Normandie Université, LMI EA 3226, 76000, Rouen.

JOSO 2019

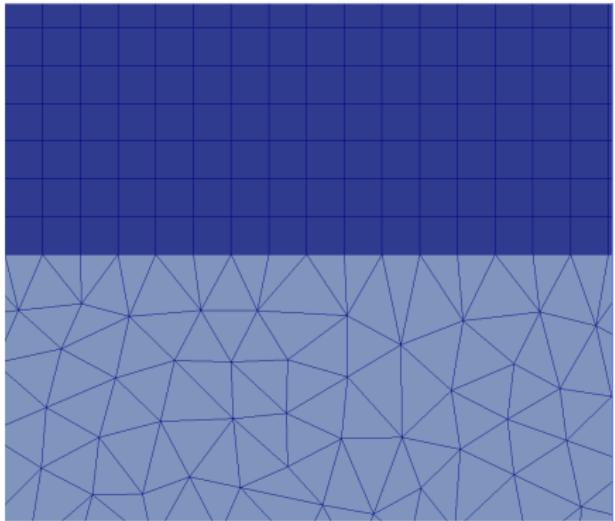
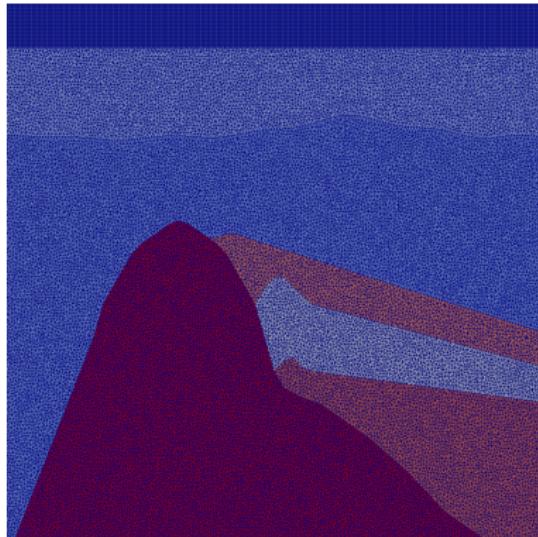


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Seismic imaging

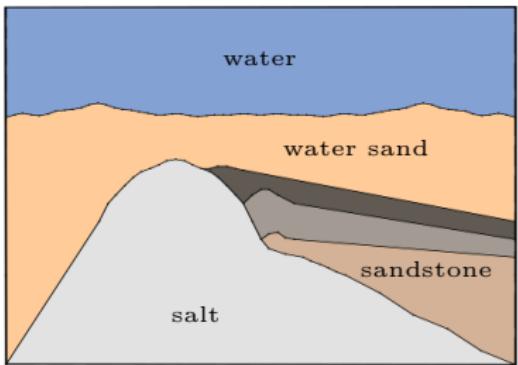
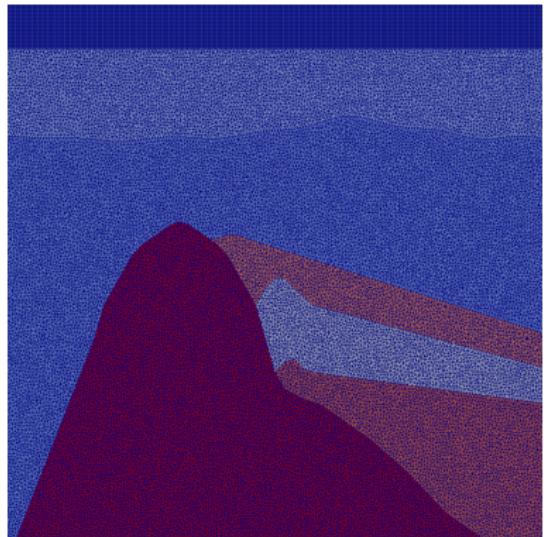


Why using hybrid meshes?



- Useful when the use of unstructured grid is non-sense (e.g. medium with a layer of water).
- Well suited for the coupling of numerical methods in order to reduce the computational cost and improve the accuracy.

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Elastodynamic system

$x \in \Omega \subset \mathbb{R}^d$, $t \in [0, T]$, $T > 0$:

$$\begin{cases} \rho(x) \frac{\partial \nu}{\partial t}(x, t) = \nabla \cdot \underline{\sigma}(x, t), \\ \frac{\partial \underline{\sigma}}{\partial t}(x, t) = \underline{\underline{C}}(x) \underline{\underline{\epsilon}}(\nu(x, t)). \end{cases}$$

With:

- $\rho(x)$ the density,
- $\underline{\underline{C}}(x)$ the elasticity tensor,
- $\underline{\underline{\epsilon}}(x, t)$ the deformation tensor,
- $\nu(x, t)$, the wavespeed,
- $\underline{\sigma}(x, t)$ the strain tensor.

Software written in **Fortran** for wave propagation simulation in the **time domain**

Features

Simulation:

- on various types of meshes (**unstructured triangles and tetrahedra**),
 - on **heterogeneous media** (**acoustic, elastic and elasto-acoustic**).
-
- **Discontinuous Galerkin** (DG) based on **unstructured triangles and unstructured tetrahedra**,
 - with various time-schemes : **Runge-Kutta (2 or 4), Leap-Frog,**
 - with **multi-order computation(p-adaptivity)**...

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- 3 DG/SEM coupling
- 4 DGSEM vs DG
- 5 3D extension

1 Numerical Methods

- Discontinuous Galerkin Method (DG)
- Spectral Element Method (SEM)
- Advantages of each method

Discontinuous Galerkin Method

Use discontinuous functions :



mesh



continuous

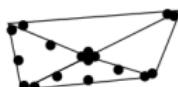


discontinuous

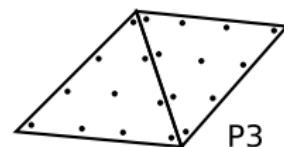
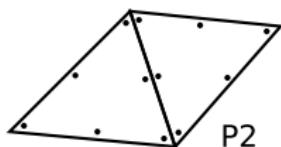
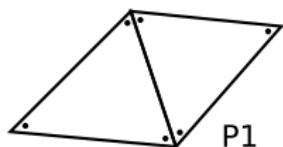
h adaptivity :



p adaptivity :

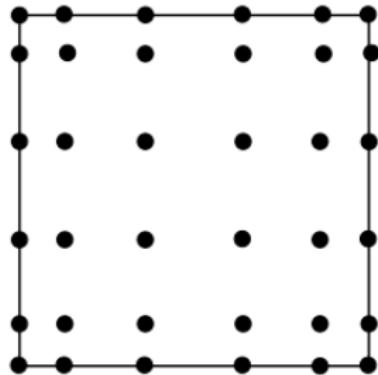


Degrees of freedom necessary on each cell :



General principle

- Finite Element Method (FEM) discretization + Gauss-Lobatto quadrature,
- Gauss-Lobatto points as degrees of freedom (gives us exponential convergence on L^2 -norm).



- $\int f(x)dx \approx \sum_{j=1}^{N+1} \omega_j f(\xi_j),$
- $\varphi_i(\xi_j) = \delta_{ij}.$

DG

- Element per element computation (*hp*-adaptivity).
- Time discretization quasi explicit (block diagonal mass matrix).
- Simple to parallelize.
- Robust to brutal changes of physics and geometry

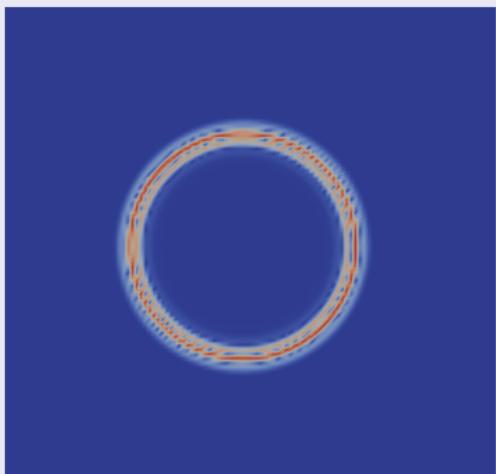
SEM

- Couples the flexibility of FEM with the accuracy of the pseudo-spectral method.
- Simplifies the mass and stiffness matrices (mass matrix diagonal).

- 2 Comparison DG/SEM on structured quadrangle mesh
- Description of the test cases
 - Comparative tables

Description of the test cases

Physical parameters



P wavespeed	1000 m.s^{-1}
Density	1 kg.m^{-3}

Second order Ricker Source in P wave
($f_{\text{peak}} = 10\text{Hz}$)

General context

- Acoustic homogeneous medium.
- Four different meshes : **10000 cells, 22500 cells, 90000 cells, 250000 cells.**
- CFL computed using **power iteration** method.
- Leap-Frog time scheme.
- **Eight threads parallel execution with OpenMP.**

Comparative tables

- Error computed as the difference between an analytical and a numerical solution for each method.
- Three cases considered : DG without penalization terms, DG with penalization terms and SEM.

	CFL	L2-error	CPU-time	Nb of time steps
DG($\alpha = 0$)	3.18e-3	2e-1	5.13	629
SEM	4.9e-3	5e-2	0.80	409

Figure: DG not penalized and SEM comparison on the 10000 cells case

	CFL	L2-error	CPU-time(s)	Nb of time steps
DG($\alpha = 0$)	2.12e-3	7e-1	18.11	943
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Figure: DG not penalized and SEM comparison on the 20000 cells case

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DG

- Element per element computation (*hp*-adaptivity).
- Time discretization quasi explicit (block diagonal mass matrix).
- Simple to parallelize.
- Robust to brutal changes of physics and geometry

SEM

- Couples the flexibility of FEM with the accuracy of the pseudo-spectral method.
- Simplifies the mass and stiffness matrices (mass matrix diagonal).
- **Reduces the computational costs on structured quadrangle cells in comparison with DG**

3

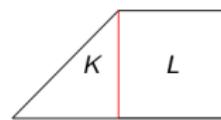
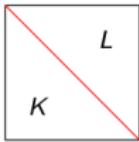
DG/SEM coupling

- Hybrid meshes structures
- Variational formulation

Hybrid meshes structures

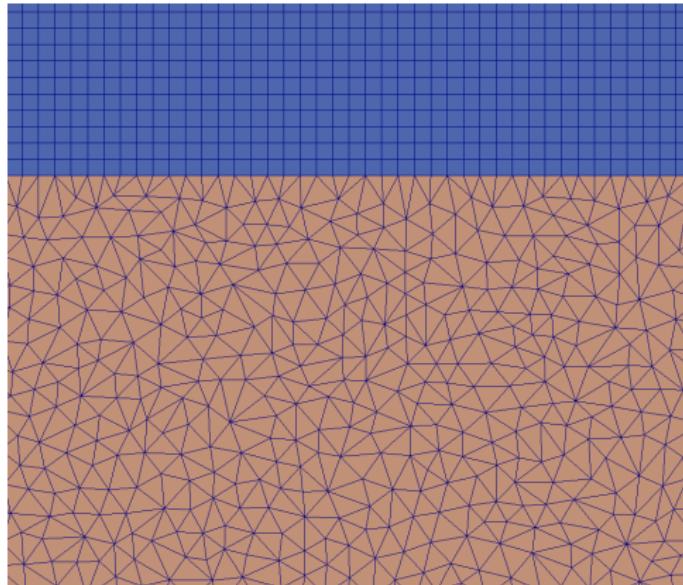
- Aim at coupling P_k and Q_k structures.
- Need to extend or split some structures (e.g. neighbour indexes).
- Define new face matrices:

$$M_{ij}^{K,L} = \int_{K \cap L} \phi_i^K \phi_j^L, \quad M_{ij}^{K,L} = \int_{K \cap L} \psi_i^K \psi_j^L, \quad M_{ij}^{K,L} = \int_{K \cap L} \phi_i^K \psi_j^L.$$



Global context

- Domain in two parts : $\Omega_{h,1}$ (**structured quadrangles + SEM**), $\Omega_{h,2}$ (**unstructured triangles + DG**).



Definitions

- Jump and average.

$$[[u]] \cdot \mathbf{n} = (u_{K_2} - u_{K_1}) \cdot \mathbf{n}$$

$$\{\{u\}\} = \frac{1}{2}(u_{K_2} + u_{K_1})$$

Variational formulation

SEM variational formulation :

$$\begin{cases} \int_{\Omega_{h,1}} \rho \partial_t v_1 \cdot w_1 = - \int_{\Omega_{h,1}} \sigma_1 \cdot \nabla w_1 + \int_{\Gamma_{out,1}} (\sigma_1 \mathbf{n}_1) \cdot w_1, \\ \int_{\Omega_{h,1}} \partial_t \sigma_1 : \xi_1 = - \int_{\Omega_{h,1}} (\nabla(C\xi_1)) \cdot v_1 + \int_{\Gamma_{out,1}} (C\xi_1 \mathbf{n}_1) \cdot v_1. \end{cases}$$

DG variational formulation :

$$\begin{cases} \int_{\Omega_{h,2}} \rho \partial_t v_2 \cdot w_2 = - \int_{\Omega_{h,2}} \sigma_2 \cdot \nabla w_2 + \int_{\Gamma_{out,2}} (\sigma_2 \mathbf{n}_2) \cdot w_2 + \int_{\Gamma_{int}} \{\{\sigma_2\}\} [[w_2]] \cdot \mathbf{n}_2, \\ \int_{\Omega_{h,2}} \partial_t \sigma_2 : \xi_2 = - \int_{\Omega_{h,2}} (\nabla(C\xi_2)) \cdot v_2 + \int_{\Gamma_{out,2}} (C\xi_2 \mathbf{n}_2) \cdot v_2 + \int_{\Gamma_{int}} \{\{v_2\}\} [[C\xi_2]] \cdot \mathbf{n}_2. \end{cases}$$

Variational formulation

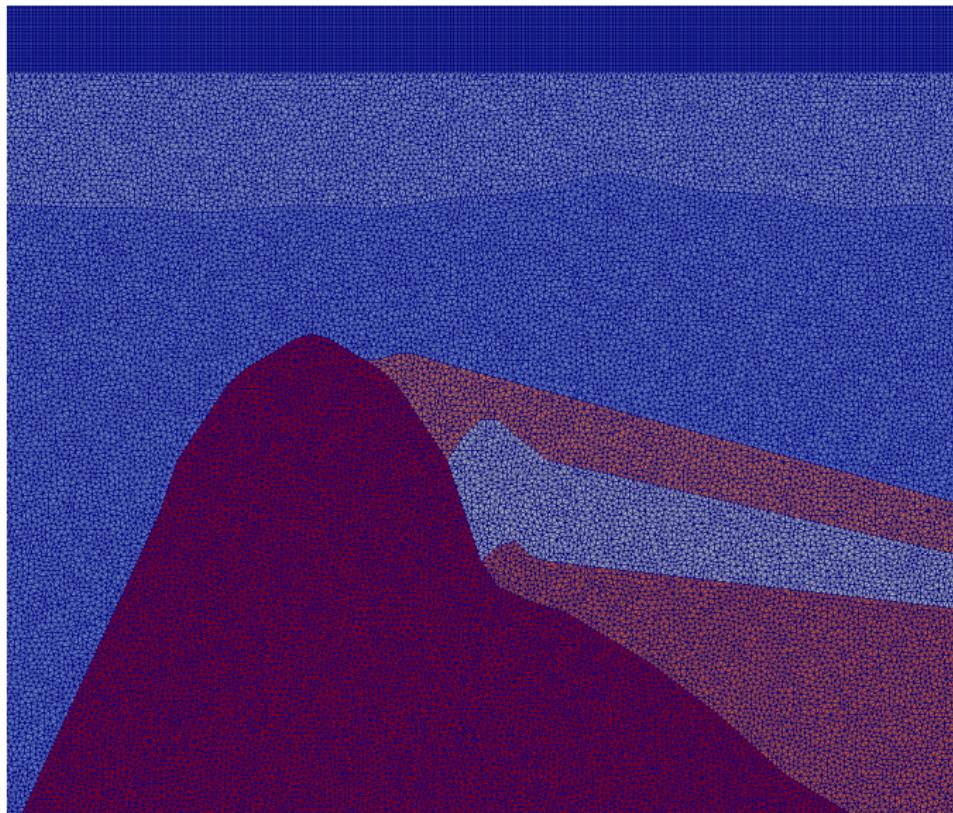
$$\left\{ \begin{array}{l} \int_{\Omega_{h,1}} \rho \partial_t v_1 \cdot w_1 + \int_{\Omega_{h,2}} \rho \partial_t v_2 \cdot w_2 = - \int_{\Omega_{h,1}} \sigma_1 \cdot \nabla w_1 - \int_{\Omega_{h,2}} \sigma_2 \cdot \nabla w_2 \\ \\ + \int_{\Gamma_{out,1}} (\sigma_1 \mathbf{n}_1) \cdot w_1 + \int_{\Gamma_{out,2}} (\sigma_2 \mathbf{n}_2) \cdot w_2 + \int_{\Gamma_{int}} \{\{\sigma_2\}\} [[w_2]] \cdot \mathbf{n}_2 \\ \\ + \int_{\Gamma_{1/2}} [[\sigma w]] \cdot \mathbf{n}, \\ \\ \int_{\Omega_{h,1}} \partial_t \sigma_1 : \xi_1 + \int_{\Omega_{h,2}} \partial_t \sigma_2 : \xi_2 = - \int_{\Omega_{h,1}} (\nabla(C\xi_1)) \cdot v_1 - \int_{\Omega_{h,2}} (\nabla(C\xi_2)) \cdot v_2 \\ \\ + \int_{\Gamma_{out,1}} (C\xi_1 \mathbf{n}_1) \cdot v_1 + \int_{\Gamma_{out,2}} (C\xi_2 \mathbf{n}_2) \cdot v_2 + \int_{\Gamma_{int}} \{\{\sigma_2\}\} [[w_2]] \cdot \mathbf{n}_2 \\ \\ + \int_{\Gamma_{1/2}} [[(C\xi)v]] \cdot \mathbf{n}. \end{array} \right.$$

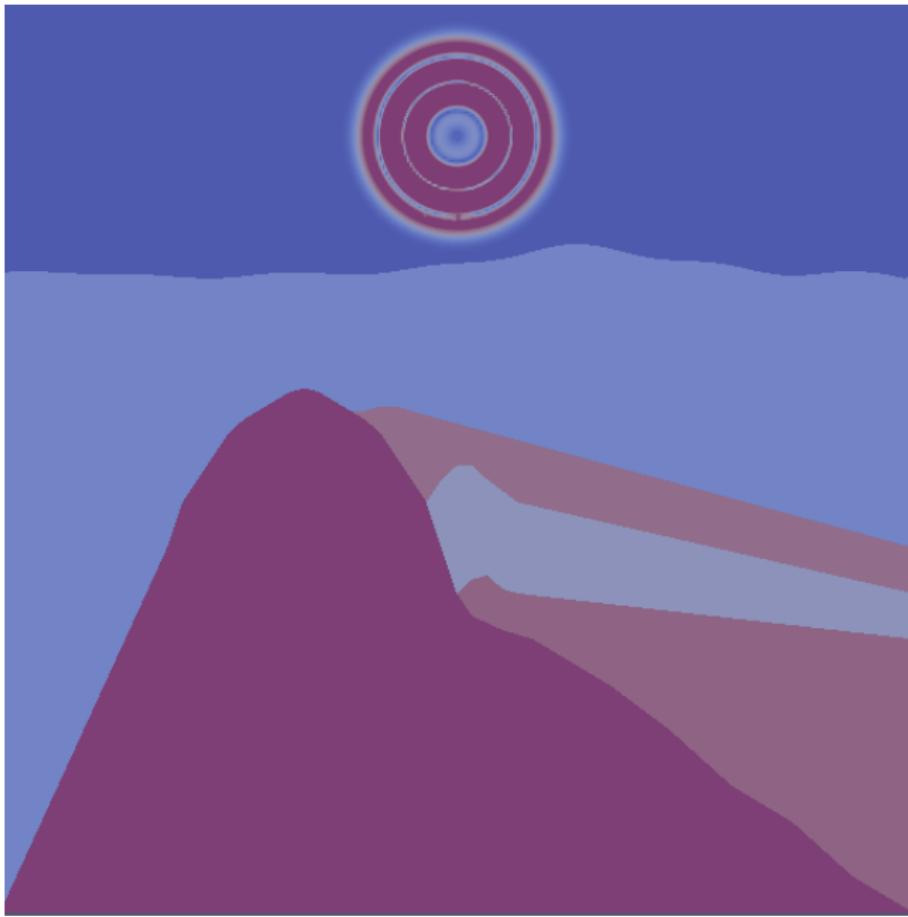
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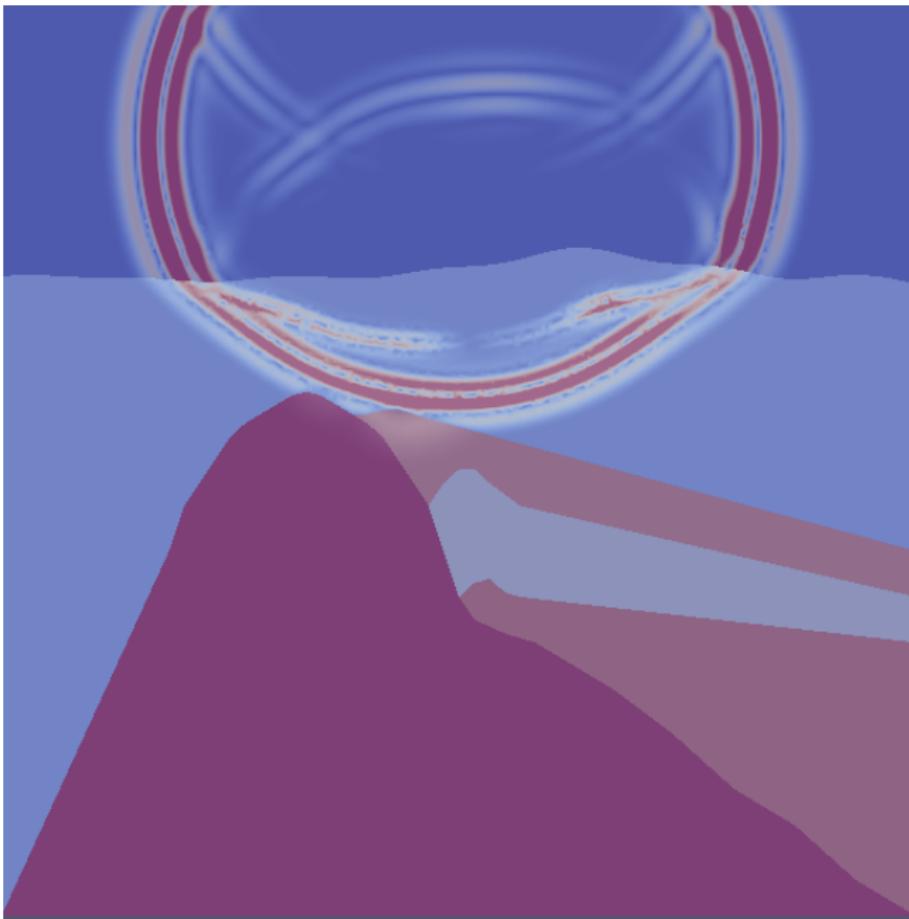
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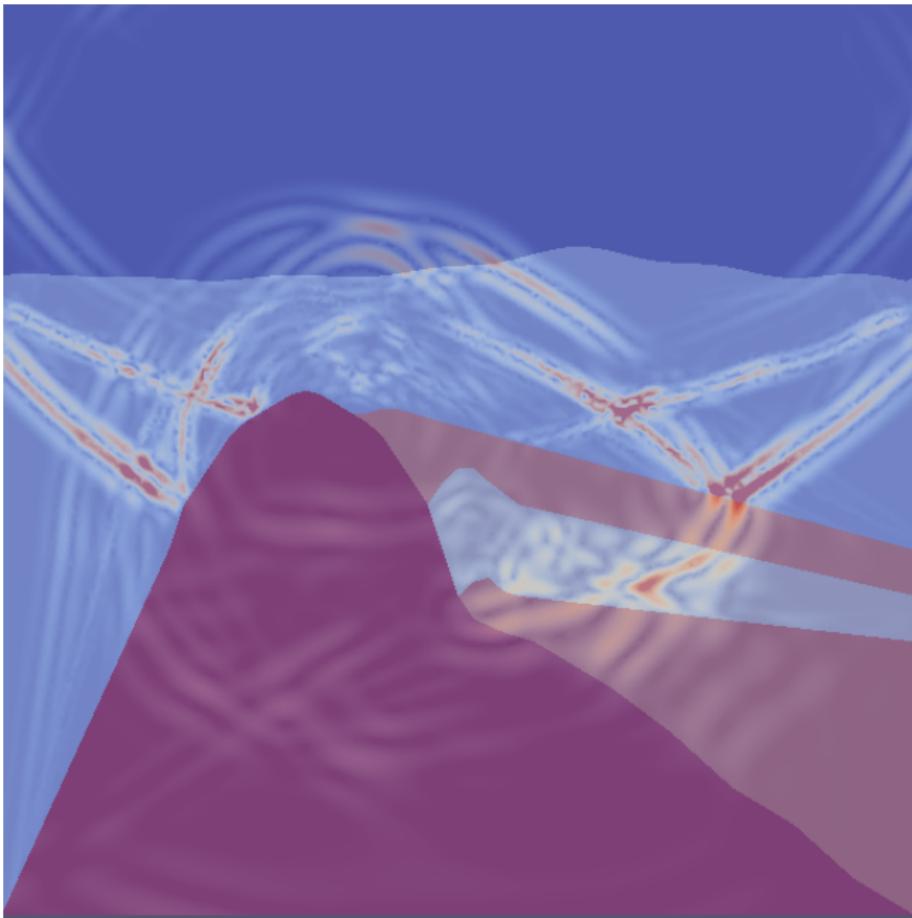
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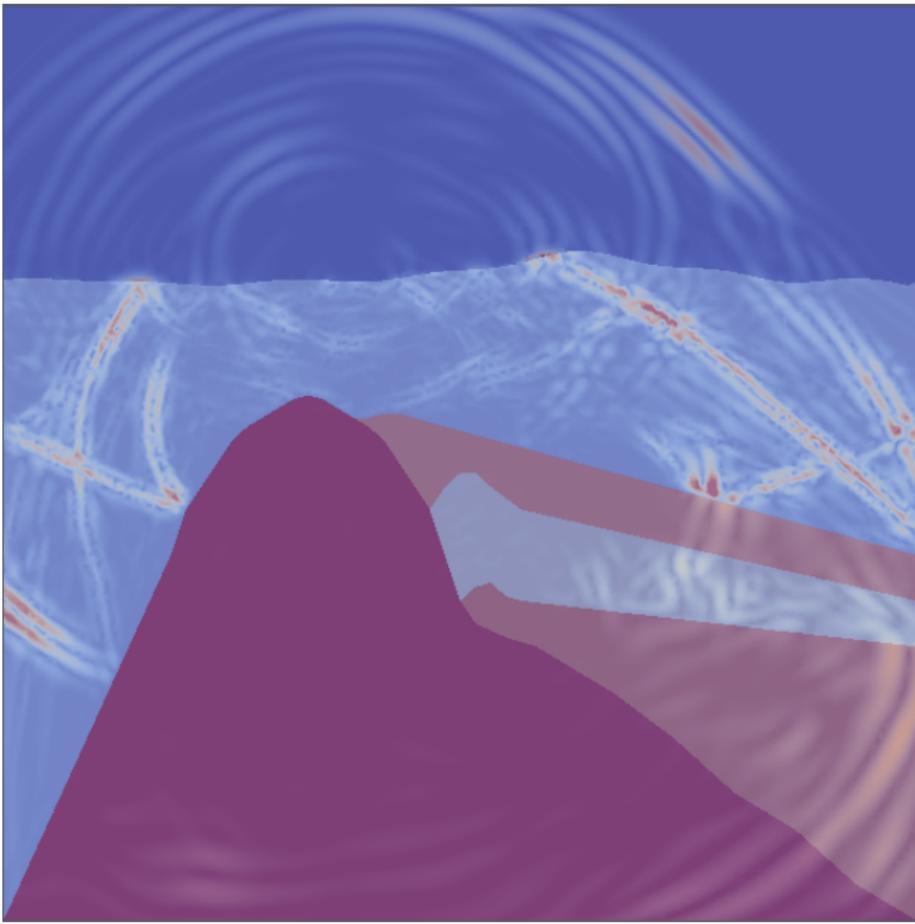
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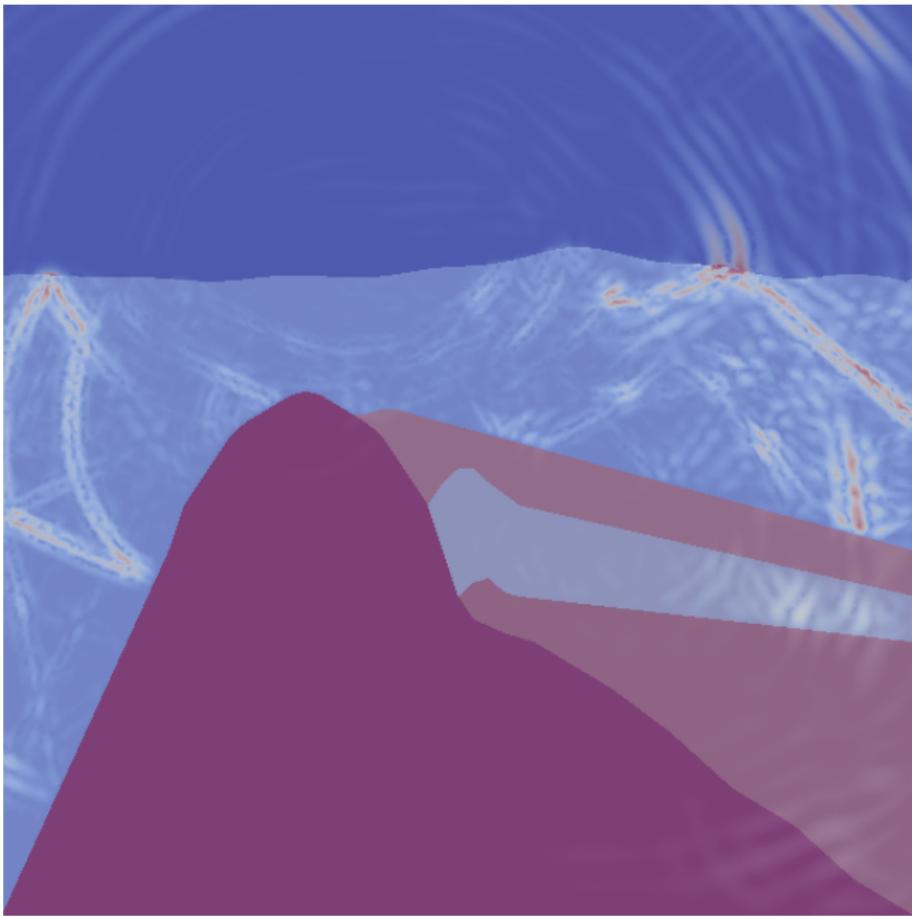












- Two dimensional Mesh with 74969 cells : 53969 triangles and 21000 quadrangles.

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- Elastic isotropic case with order 2 on triangles (DG) and order 8 on the quadrangles (DG or SEM), point source at the center of the 3000x3000 domain.

	CFL	CPU-time	Nb of time steps
DG($\alpha = 0.5$)	1e-4	4148	10000
DG/SEM	1e-4	2645	10000

Figure: DG and DG/SEM comparison on an elastic isotropic case

- Two dimensional Mesh with 74969 cells : 53969 triangles and 21000 quadrangles.
- Elastic isotropic case with order 2 on triangles (DG) and order 8 on the quadrangles (DG or SEM), point source at the center of the 3000x3000 domain.
- "Salt dome" case with order 2 on the triangles (DG) and order 8 on the quadrangles (DG or SEM), point source on the top layer of water.

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DG/SEM	3e-5	22260	33333

Figure: DG and DG/SEM comparison on a "salt dome" case

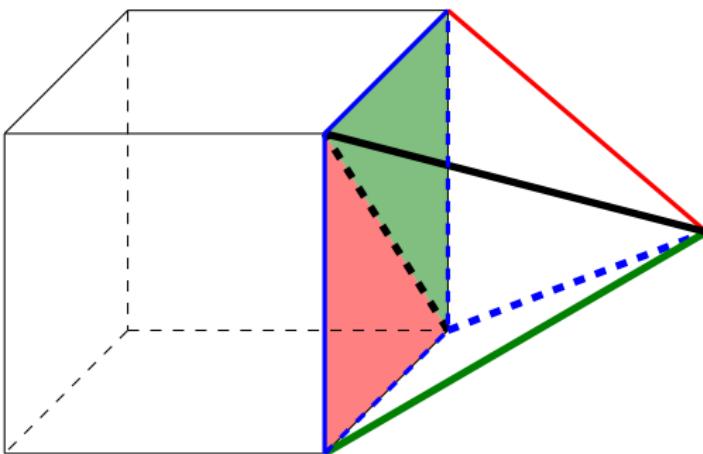


Figure: Hexa/Tet boundary configuration

- Only deal with a simple case of 3D hybrid meshes : one hexahedron has only two tetrahedra as neighbour.
- Extend SEM in 3D (basis functions...).
- Require introducing a new matrix which handles the rotation cases between two elements.

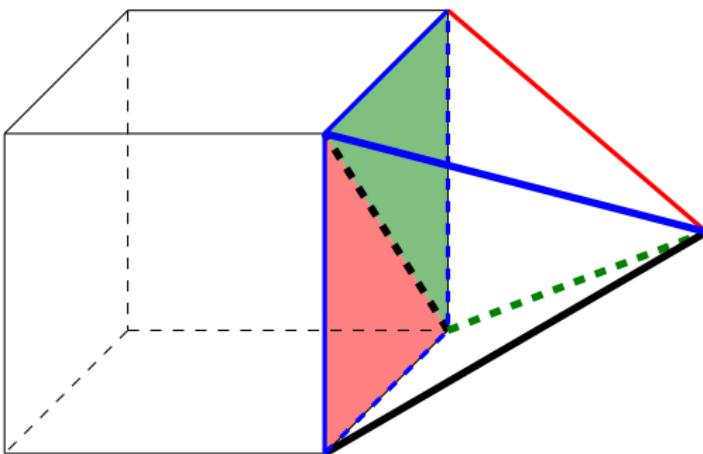


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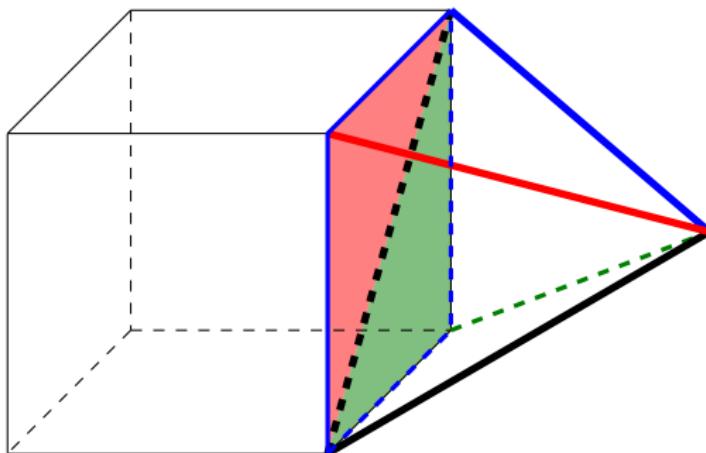


Figure: Hexa/Tet boundary configuration

- Only deal with a simple case of 3D hybrid meshes : one hexahedron has only two tetrahedra as neighbour.
- Extend SEM in 3D (basis functions...).
- Require introducing a new matrix which handles the rotation cases between two elements.

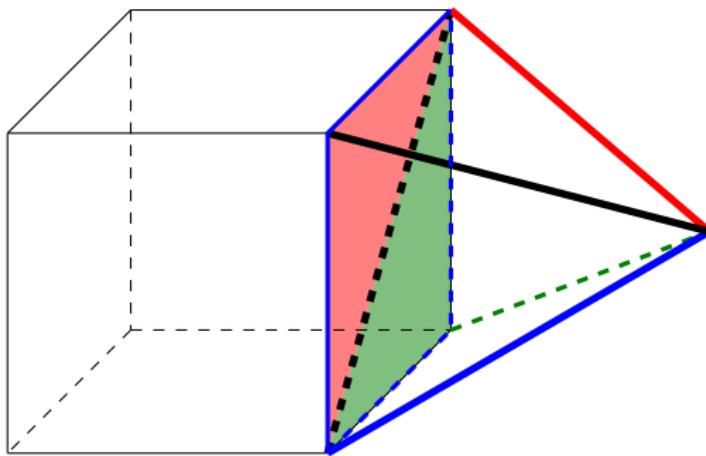
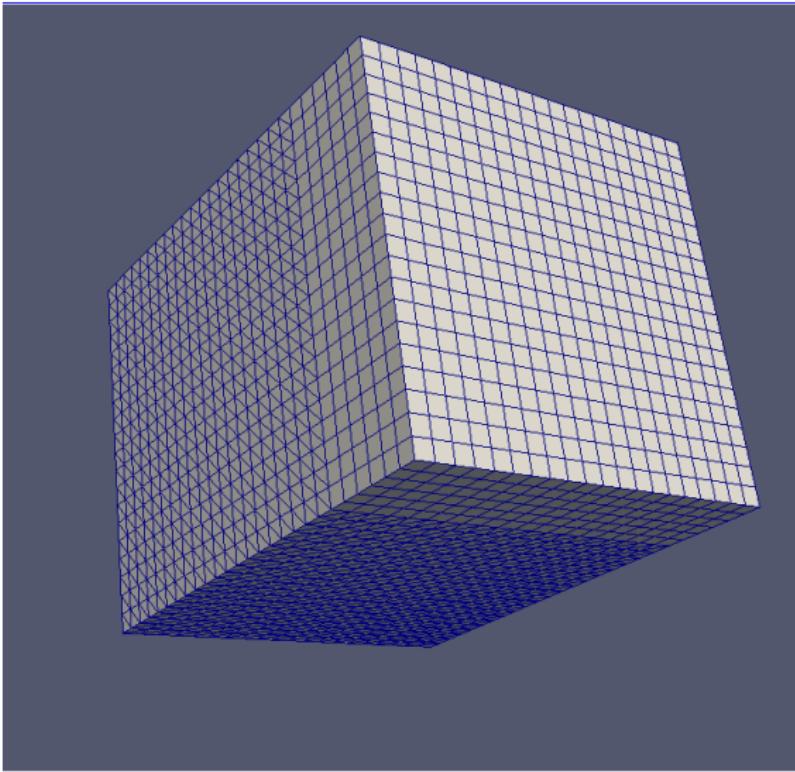
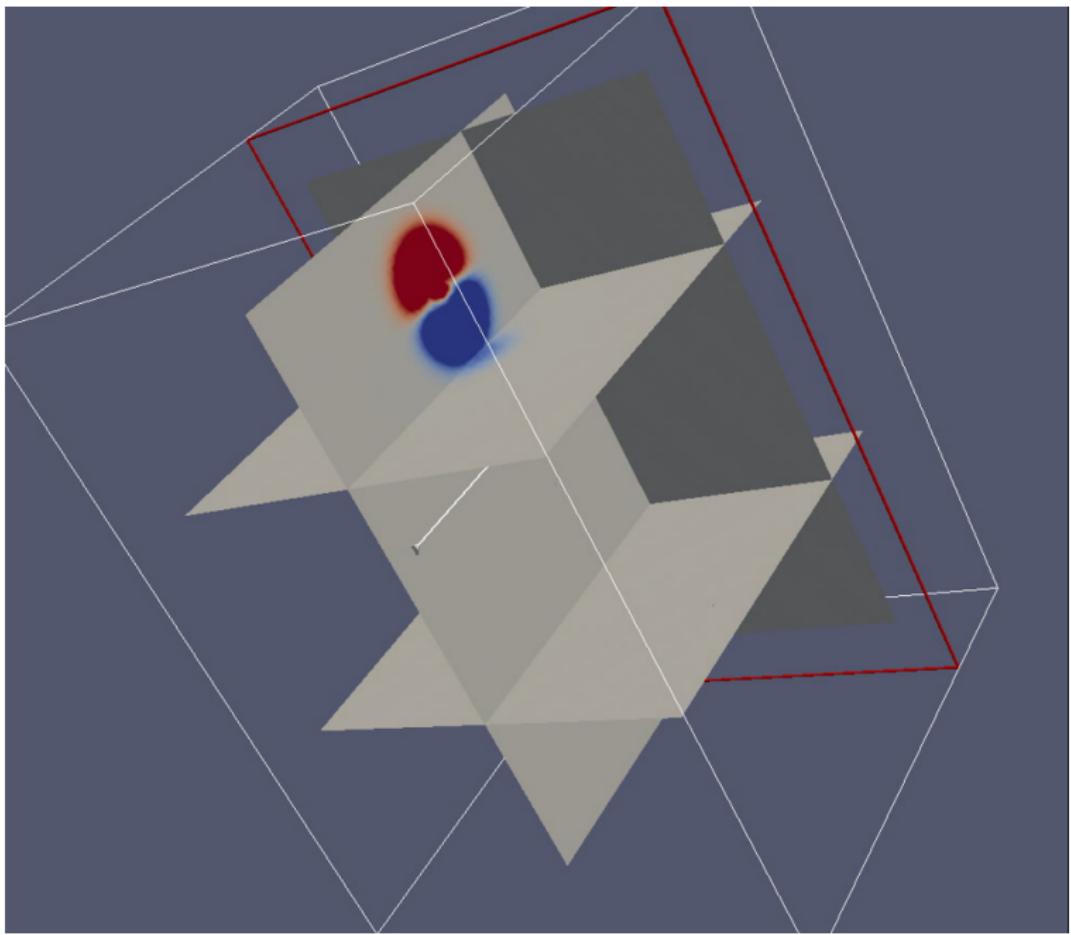
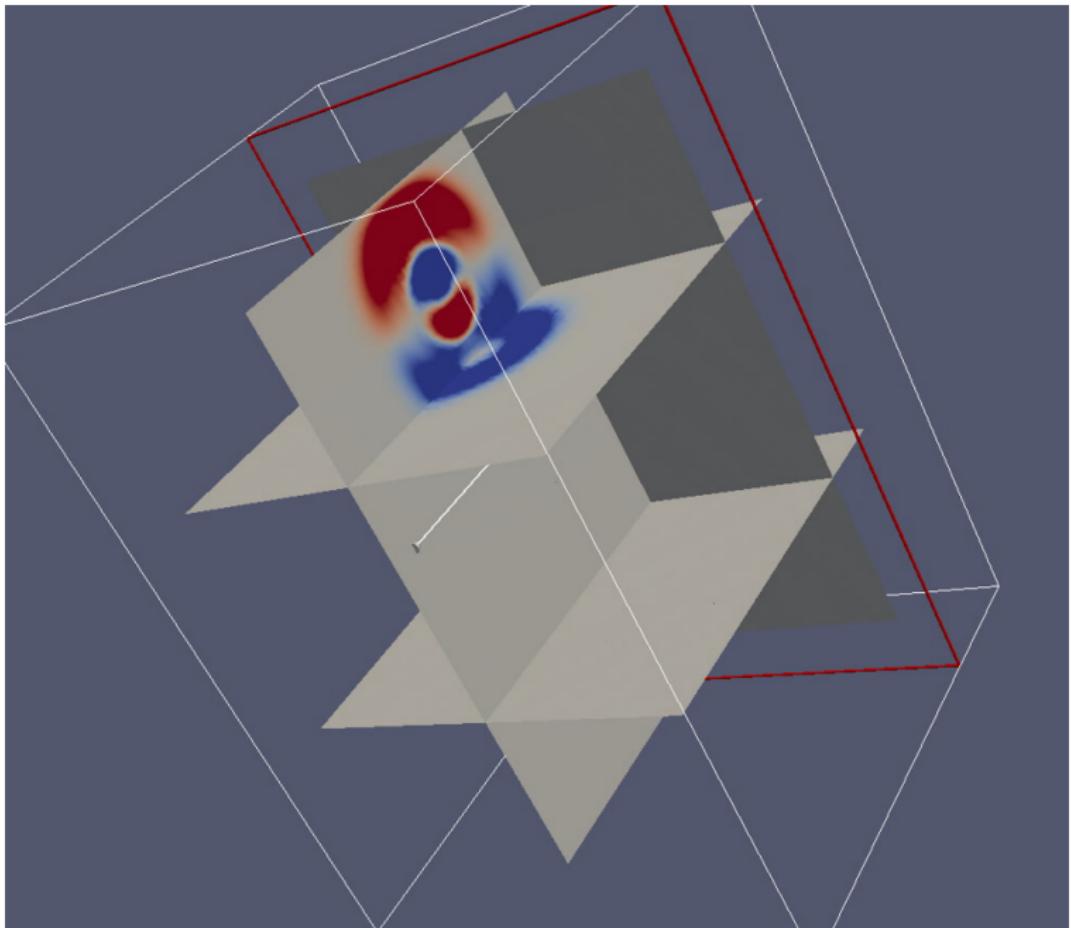


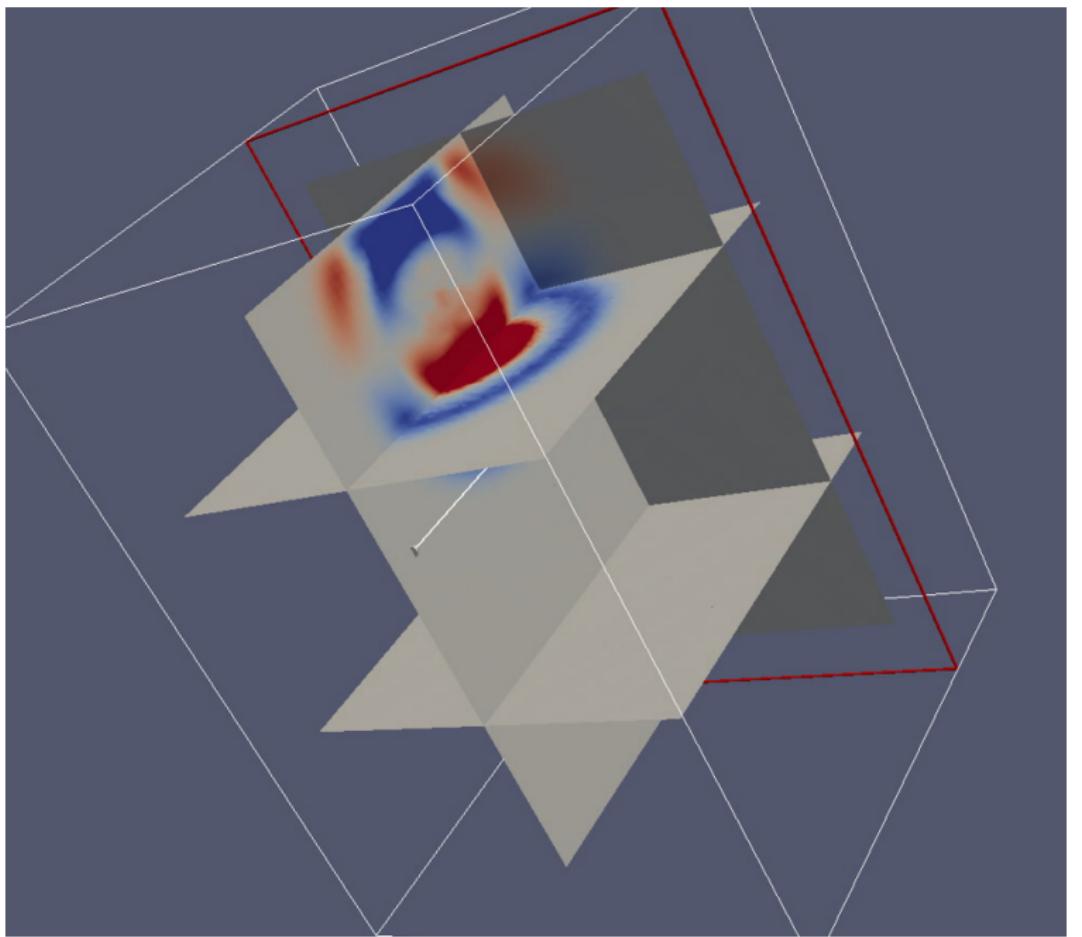
Figure: Hexa/Tet boundary configuration

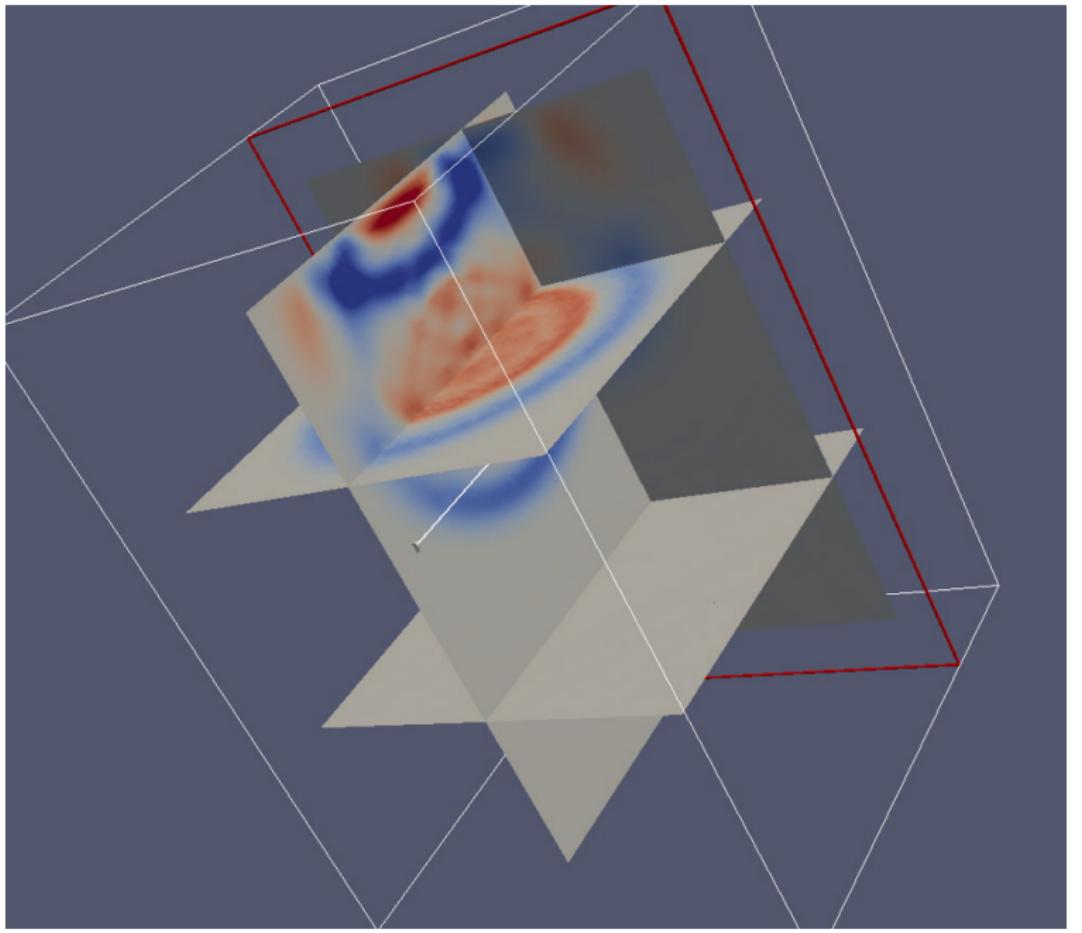
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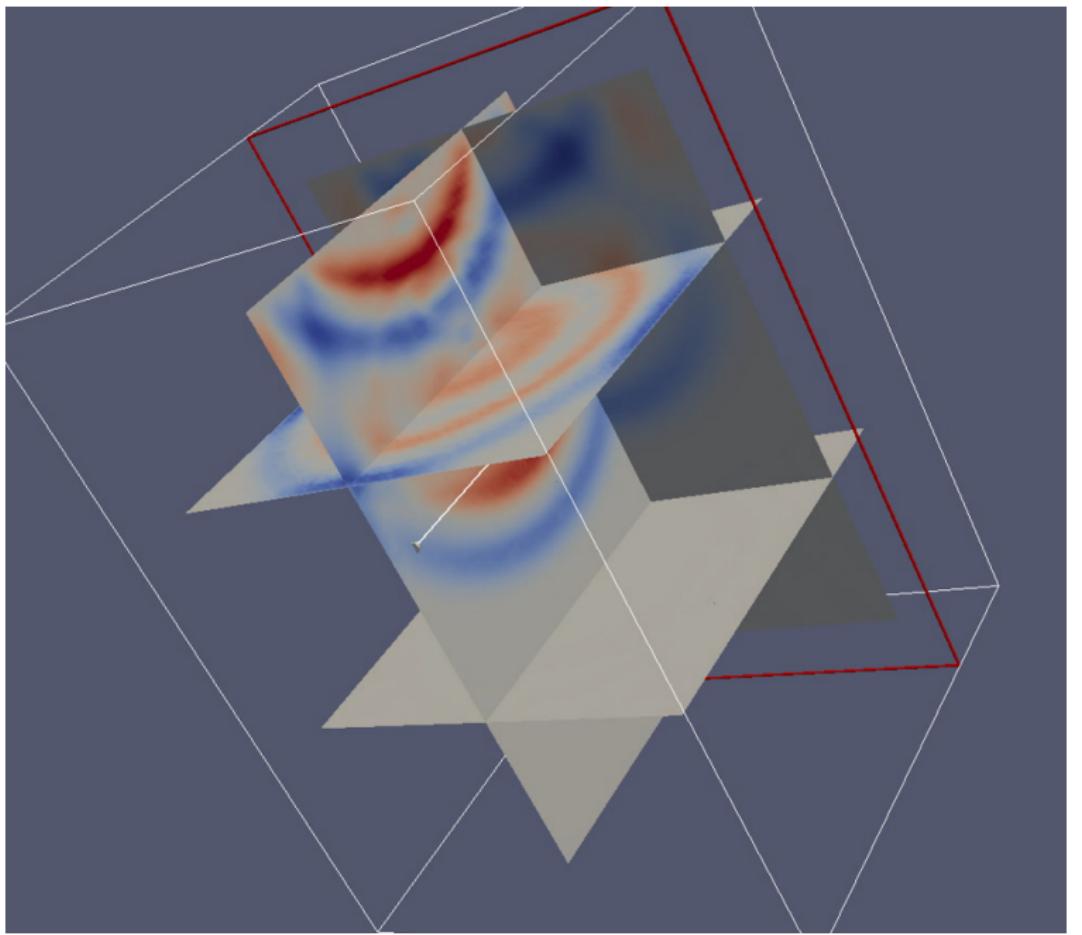


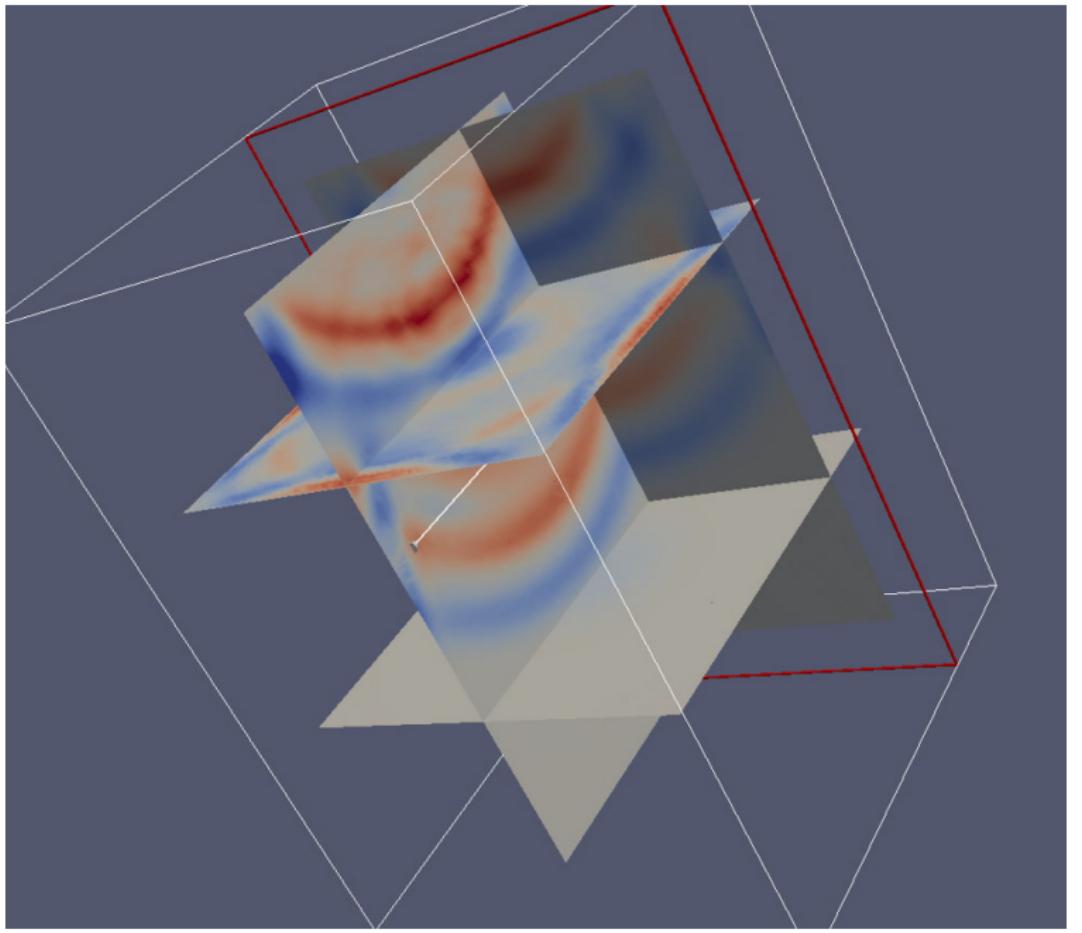


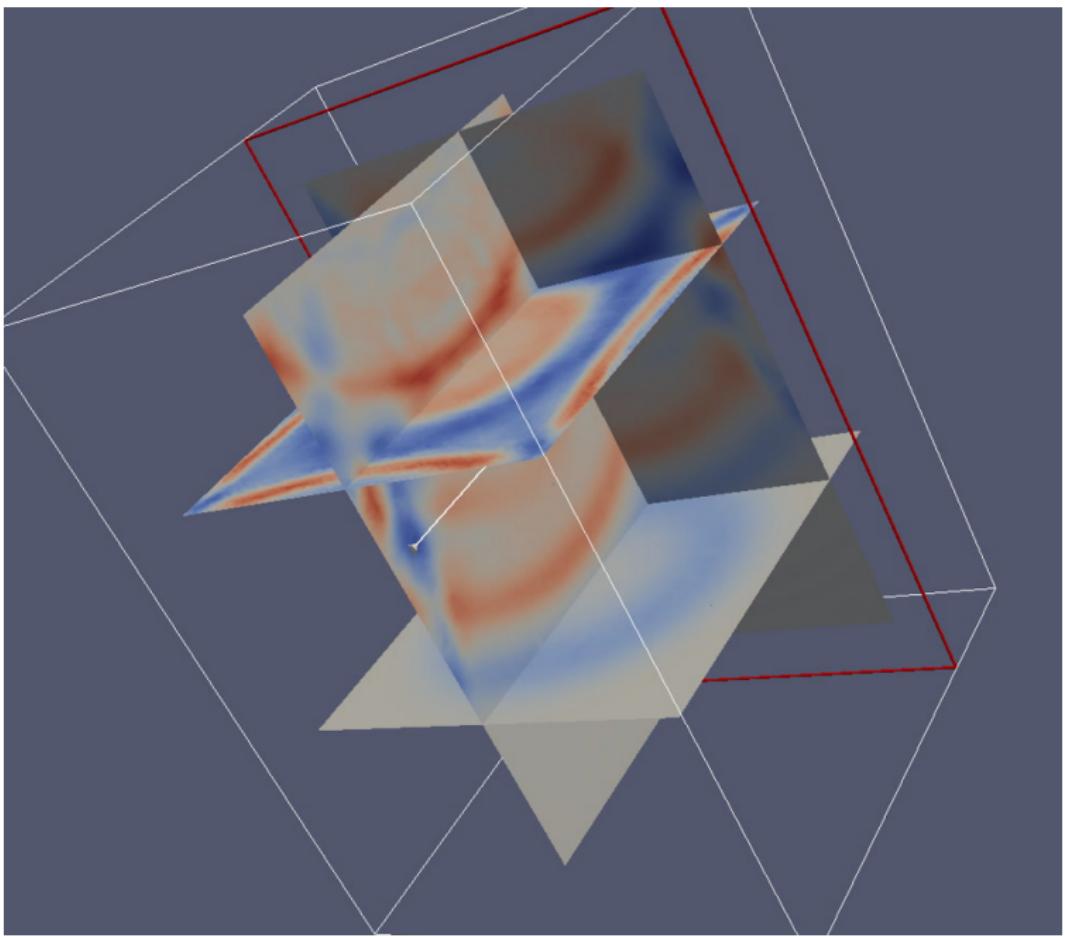












Conclusion

- ① SEM is more efficient on structured quadrangle mesh than DG
- ② Build a variational formulation for DG/SEM coupling and find a CFL condition that ensures stability
- ③ Show the utility of using hybrid meshes and method coupling (reduce computational cost,...)

Perspectives

- Implement DG/SEM coupling on the code (2D) ✓
- Develop DG/SEM coupling in 3D ✓
- Develop PML in the hexahedral part ✓
- Add a local time-stepping scheme

Thank you for your attention !

Questions?

Error-order graphic

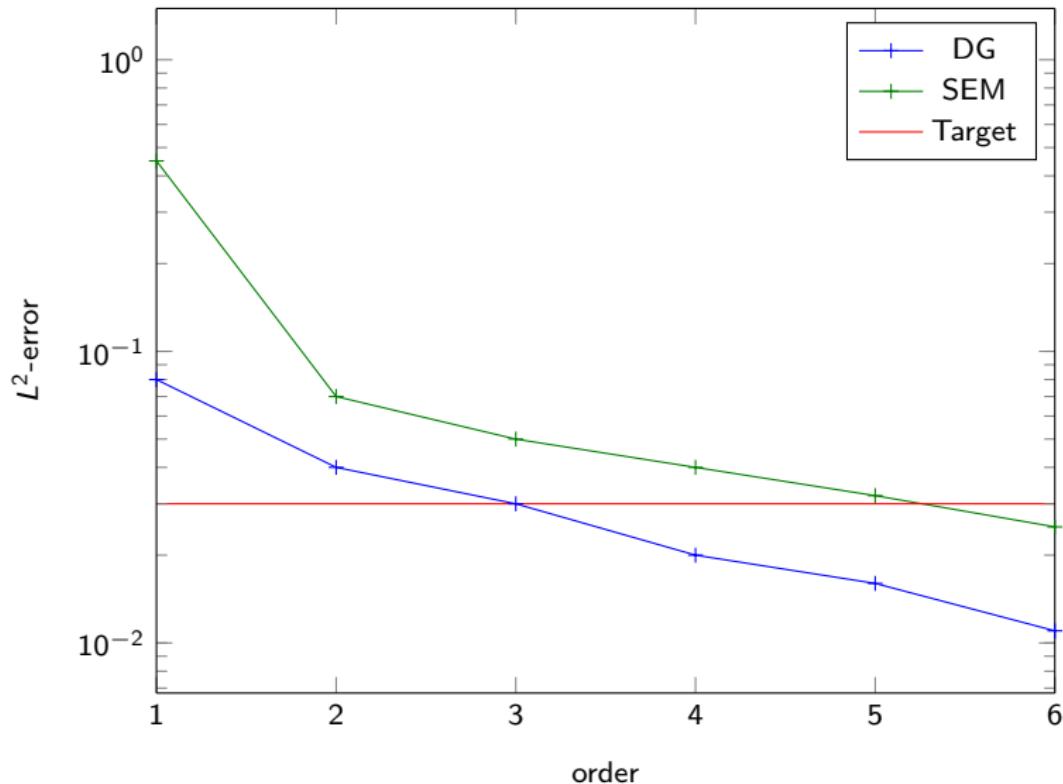


Figure: L^2 -error comparison on a 10000 cells mesh

Error-order graphic

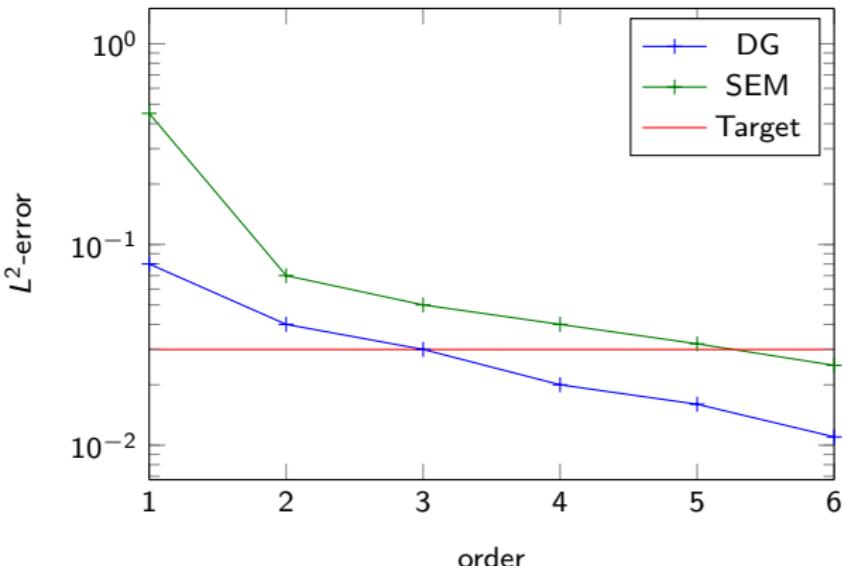


Figure: L^2 -error comparison on a 10000 cells mesh

	CFL	$L^2\text{-error}$	CPU-time	Nb of time steps
DG	2e-3	3e-2	7.93	1000
SEM(order five)	2.13e-3	3e-2	9.06	943

Figure: SEM and DG comparison with fixed error