

Supplemental Materials: Multi-stable free states of an active particle from a coherent memory dynamics

V. Bacot^{1,2,3}, S. Perrard⁴, M. Labousse^{5,*}, Y. Couder¹ and E. Fort²

¹Matière et Systèmes Complexes, CNRS UMR 7057, Université Paris Diderot, Sorbonne Paris Cité, 75013 Paris, France.

²Institut Langevin, CNRS UMR 7587, ESPCI Paris and PSL University, 75005 Paris, France.

³LadHyX, CNRS UMR 7646, École Polytechnique, 91128 Palaiseau, France.

⁴Laboratoire de Physique Statistique, CNRS UMR 8550 ENS and PSL University, 75005 Paris, France.

⁵Gulliver, CNRS UMR 7083, ESPCI Paris and PSL University, 75005 Paris, France.

*email: matthieu.labousse@espci.psl.eu

Supplemental Figure 1 Influence of the numerical time step Δt at high memory. Numerical simulations of the free walk at high memory ($C = 1.1 \text{ m/s}^3$, $\eta = 5.5 \text{ s}^{-1}$ and $\text{Me} = 80$). (a) $\Delta t = T_{\text{Far}}$ (experimental time step corresponding to the bouncing period). (b) $\Delta t = T_{\text{Far}}/2$. (c) $\Delta t = T_{\text{Far}}/3$.

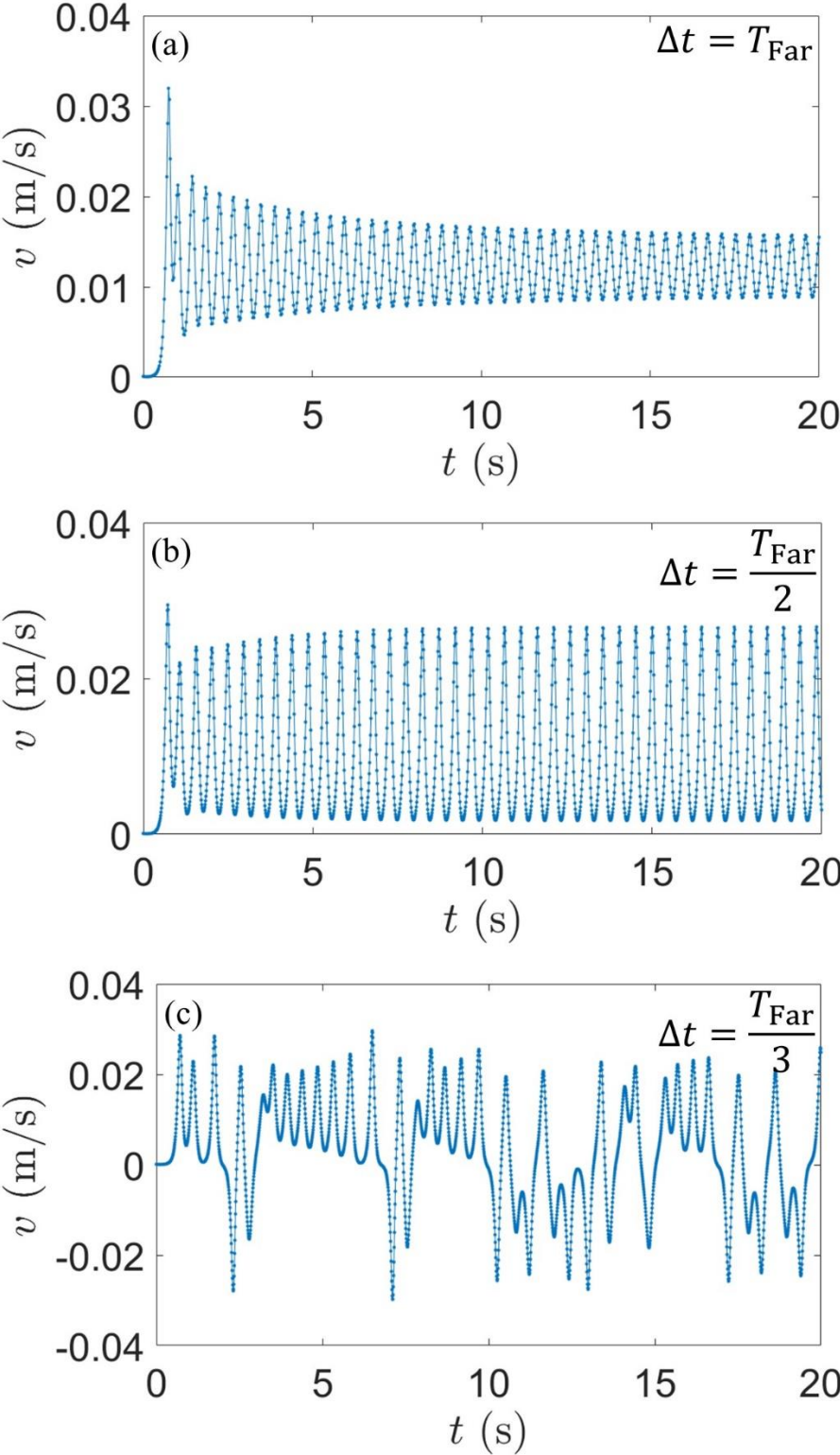
Supplemental Figure 2 Numerical time evolution of the drop speed in the three regimes; (a) stable $\text{Me} = 10$, (b) limit cycle $\text{Me} = 80$, (c) chaos $\text{Me} = 400$ corresponding to the circles in Fig. 3(a) for $C = 1.1 \text{ m/s}^3$ et $\eta = 6.2 \text{ s}^{-1}$.

Supplemental Figure 3 Numerical simulations of the permanent speed oscillation regime. Color map of order parameter $\chi = \min\{\Delta v/v_f, 1\}$ for $C = 3.8 \text{ m/s}^3$ as a function of the memory parameter Me and of the effective friction coefficient η . Three distinct regimes are observed: constant speed ($\chi = 0$), stable oscillations ($0 < \chi < 1$) and chaotic regime ($\chi = 1$). This phase diagram is not qualitatively different from Fig. 3a.

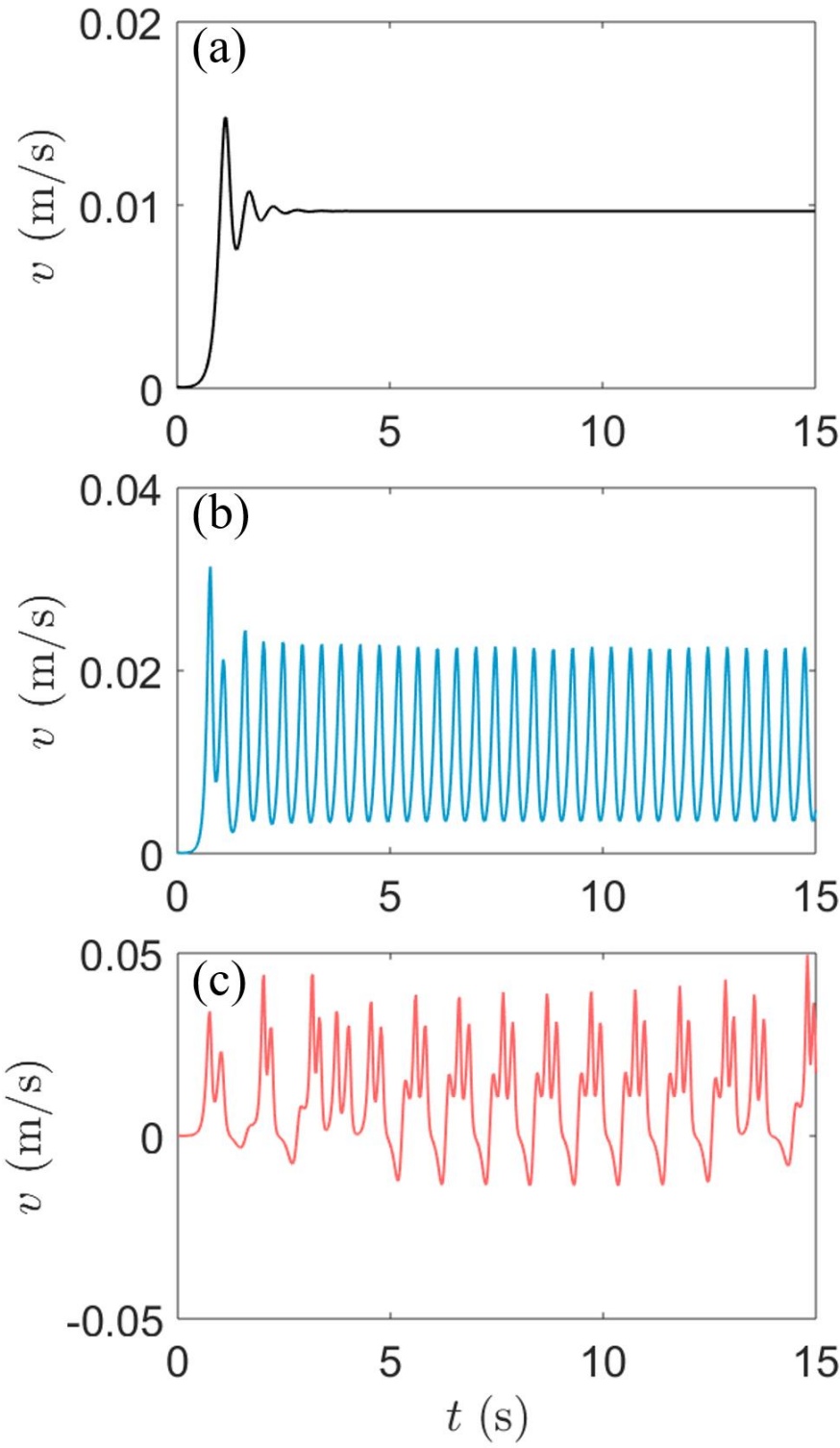
Supplemental Figure 4 Coexistence of several attractors in numerical simulation. Time evolution of the drop speed at $\text{Me} = 70$, $C = 1.1 \text{ m/s}^3$ et $\eta = 6.2 \text{ s}^{-1}$. (a) The memory parameter is set abruptly to $\text{Me} = 70$ à $t = 0$. The speed converges to a limit cycle. (b) The memory parameter is increased progressively to $\text{Me} = 70$: $0 \text{ s} < t < 75 \text{ s}$, $\text{Me} = 40$ then $75 \text{ s} < t < 112.5 \text{ s}$, $\text{Me} = 60$ and $112.5 \text{ s} < t < 150 \text{ s}$, $\text{Me} = 70$. The speed converges to a stable solution.

Supplemental Movie 1 Reconstructed wavefield h/h_0 of the experimental two speed regimes at $\text{Me} = 45$. The movie corresponds to the time evolution of the paths in Fig. 3c (Left: constant speed, right: oscillating speed). The rate is 5 frames per second, and the duration between two successive frames correspond to the bouncing period. Colorbar range is: left $[-9.2 \text{ } 12.18]$, right $[-8.5; 11.2]$. The black circle corresponds to the instantaneous drop position. The absolute directions are ill-defined and we align the two paths to emphasize the qualitative difference of the wavefield dynamics between the two situations.

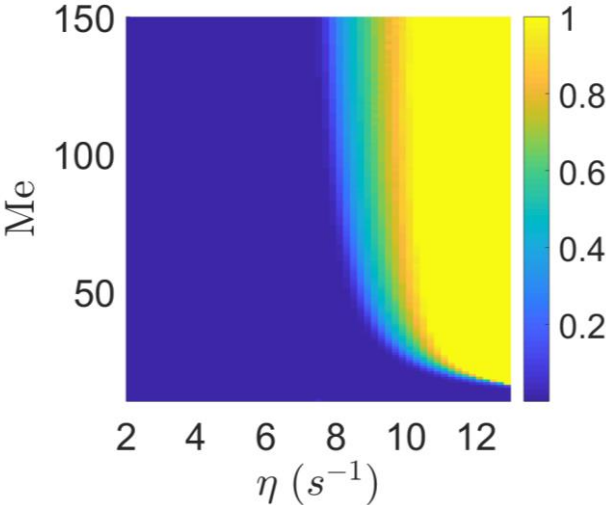
Supplemental Figure 1



Supplemental Figure 2



Supplemental Figure 3



Supplemental Figure 4

