

Local Minkowski measures for geometric characterization of stochastic spatial structures

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Many physical properties (eg. porosity, tortuosity, flow properties) of a granular media rely on the size and shape of the particles. In order to quantify them the Minkowski functionals are widely used. They play an important role in geometric characterization (in \mathbb{R}^2 and \mathbb{R}^3 Minkowski functionals coincide up to normalisation with standard geometric parameters). Nevertheless, facing the complex spatial structure, the global functionals appear to be not discriminating enough. For example, images in fig.1 of borosilicate glass display different structures possessing the same global area, perimeter and Euler-Poincaré characteristic. One of the possible generalisation of the Minkowski functionals allowing the finer geometrical characterisation and description of spatial heterogeneity of structures is *local Minkowski measures*. For a particular distribution of the random set B , the local Minkowski measures could be understood as the restrictions to B of area, perimeter, Euler-Poincaré measure, see figure 2. The study of the mean Minkowski measures could be found at [Schneider, 1993]. The asymptotic behaviour in Boolean model were described by [Molchanov, 1997]. The numerical approximation of local Minkowski measures were suggested in [Legland et al., 2007]. But the analysis of the distribution of these (random) local measures as a whole seems to be missing in the literature.

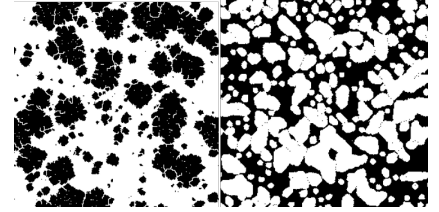


Figure 1: Borosilicate glass images with the same global Minkowski functionals. [Schulz, 2003]

At this stage, we study the random Minkowski measures of typical stochastic geometrical models. An application of the local measures to the characterisation of the elongation of a typical grain in the Boolean model is presented. Our objective is to provide a fuller description of the mathematical properties of local Minkowski measures, find an analytical expression in hypothesis of typical stochastic models, suggest the unbiased estimators for the characterisation of real spatial structures.

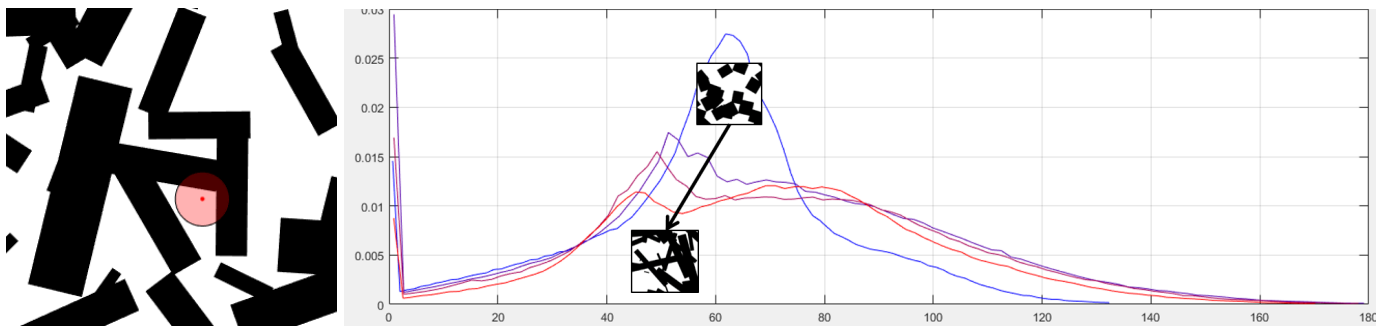


Figure 2: (a) A realisation of Boolean model with rectangular grains analysed locally in the neighbourhood of a uniform random point (in red). (b) Probability density functions for the local perimeter measure of four Boolean models with rectangular grains of growing sizes ratio.

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