

Multifractal analysis of different hydrological products of X-band radar in Greater Paris

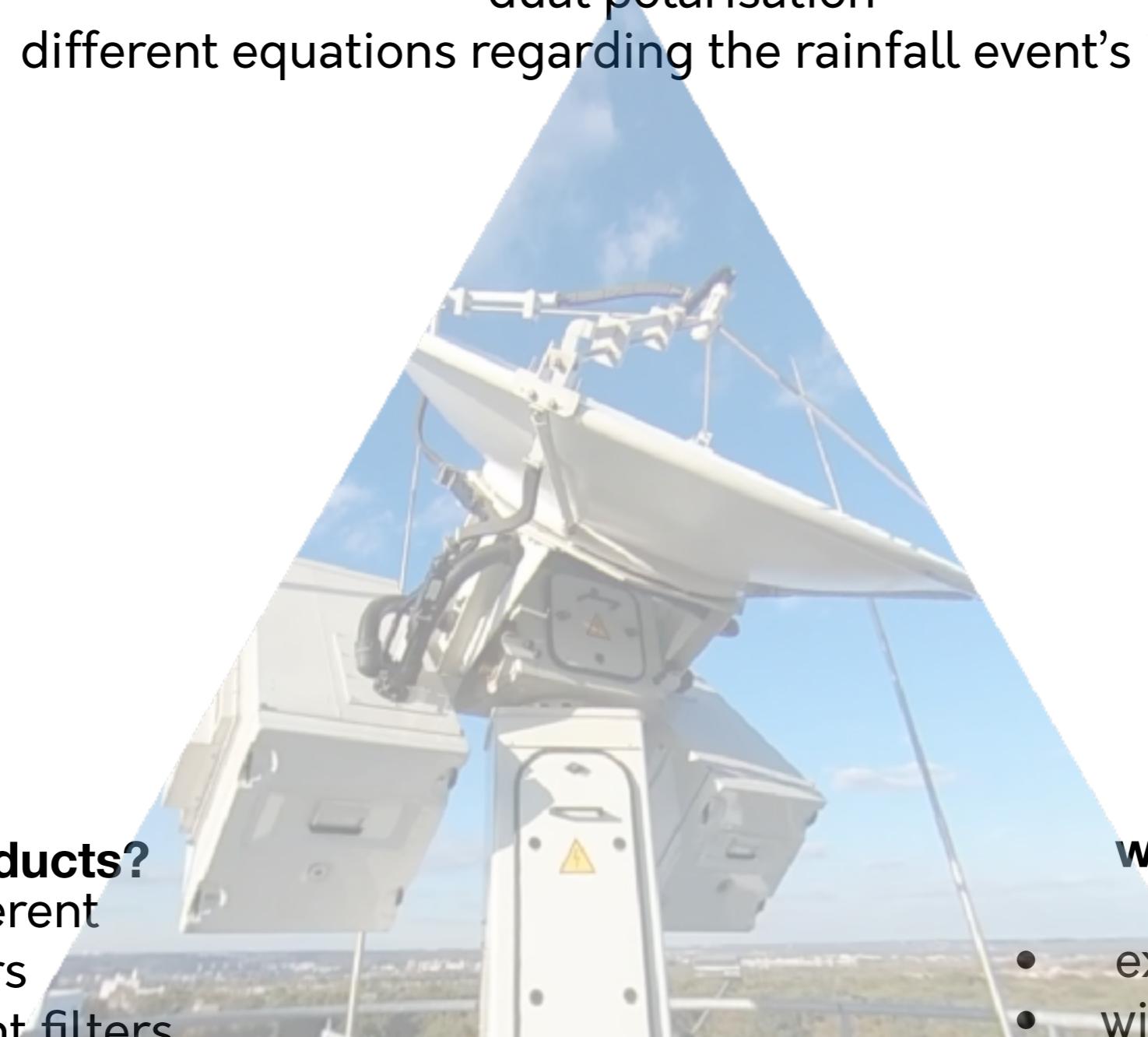
I. Skouri-Plakali (1), I. Paz(1,2), A. Ichiba (1), A. Gires (1), I. Tchiguirinskaia (1), D. Schertzer (1)

(1) HM&Co, Ecole des Ponts ParisTech, UPE, Champs-sur-Marne, France (ilektra.skouri@enpc.fr, +33 (0)164153607)
(2) Instituto Militar de Engenharia, Rio de Janeiro, Brazil

**HS7.7/NH1.18 Hydrometeorologic variability: spatio-temporal scales and probability of extremes
EGU2018-12758**

why this kind of radar?

- more precise
- dual polarisation
- different equations regarding the rainfall event's intensity

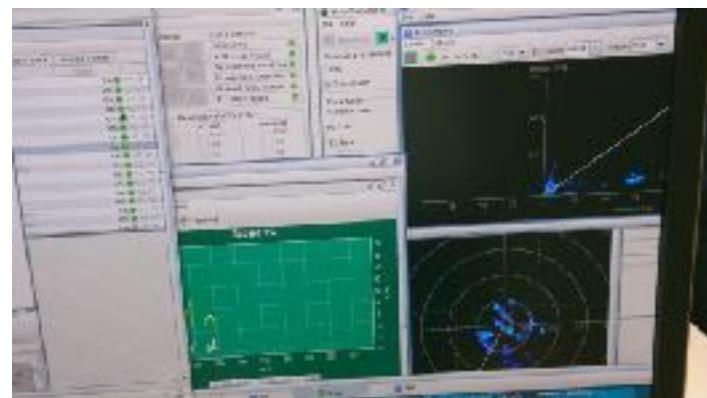
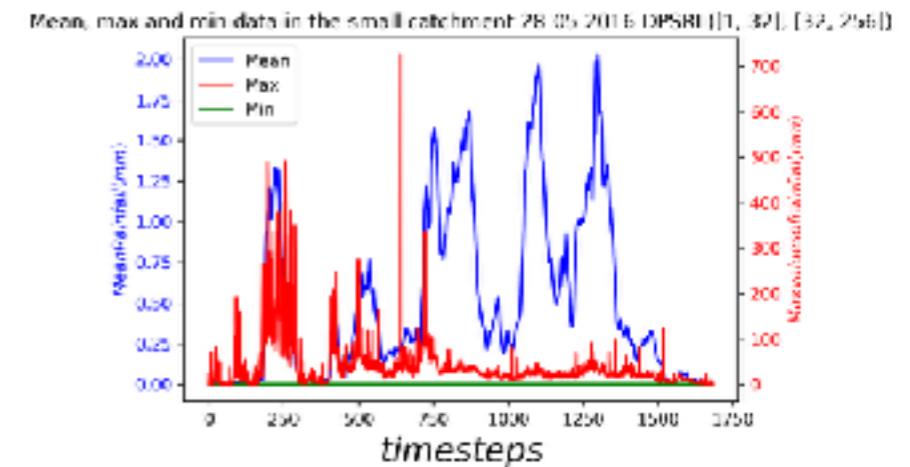
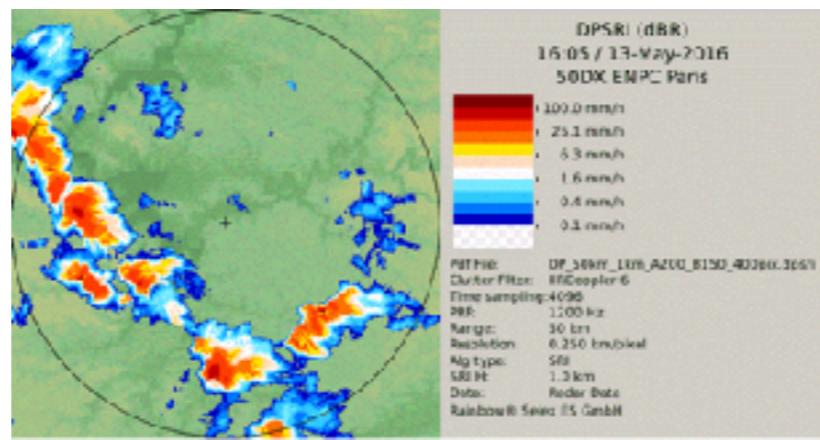
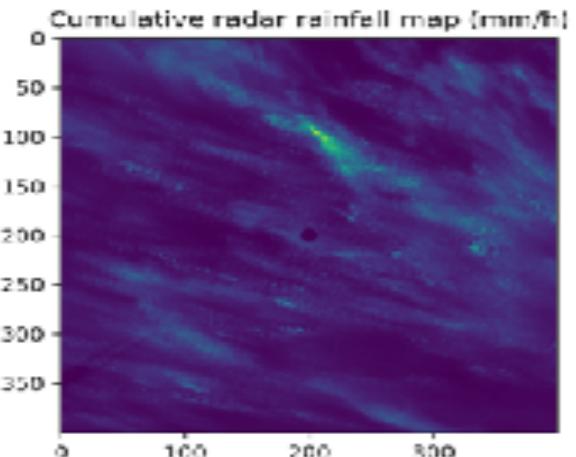


why different products?

- by using different parameters
- by using different filters
- to find out which one approaches more the reality

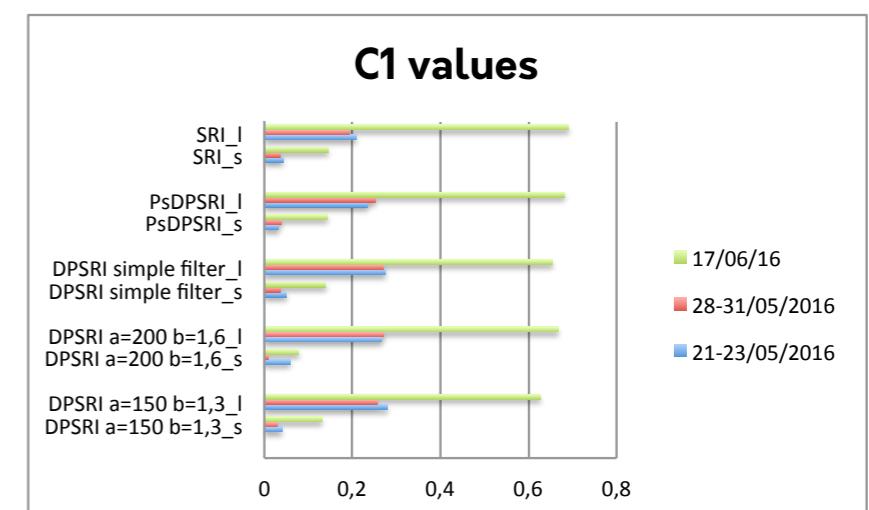
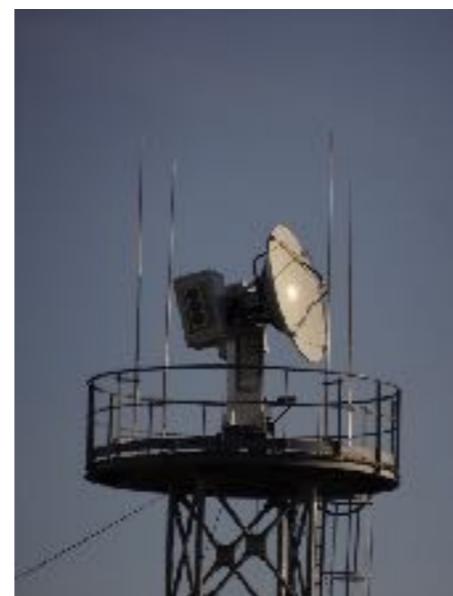
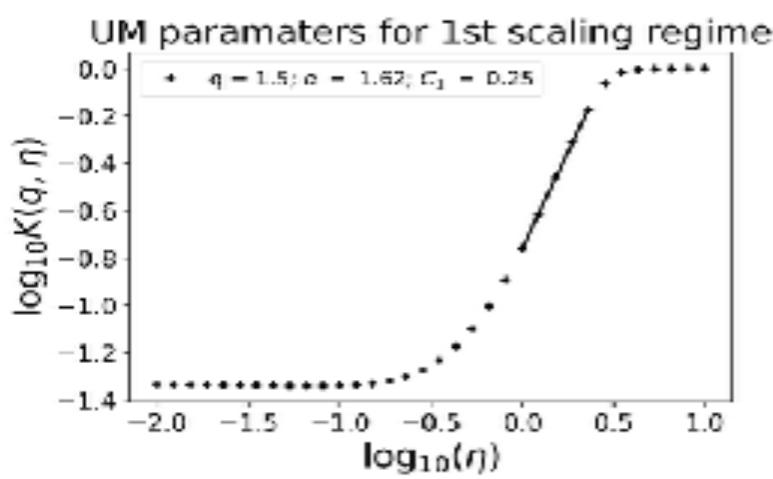
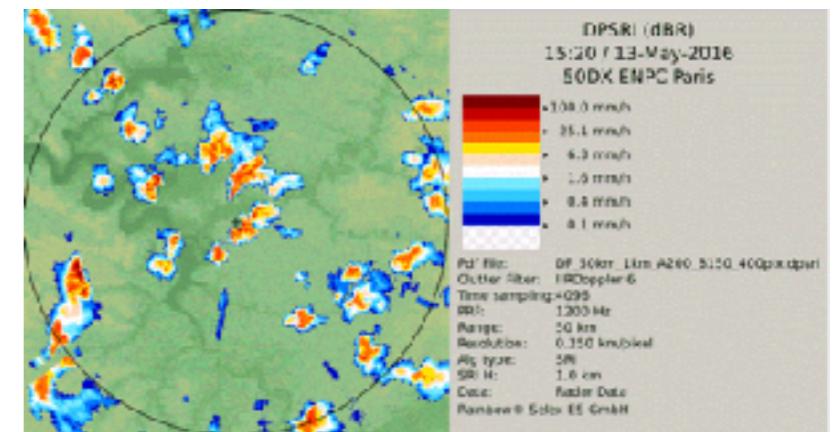
why multifractal analysis?

- extreme variability
- wide range of scales
- UM: only 3 parameters to describe the phenomenon



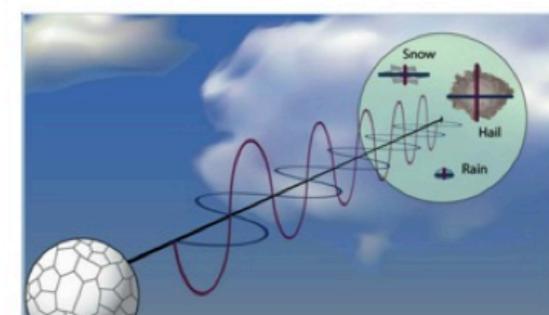
Thank you for your attention !

PICO spot 5b.10



ENPC dual polarimetric X-band radar

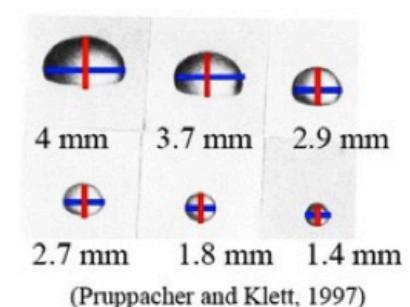
- Oblate drops->differential phase shift (ΦDP) between the horizontal and vertical axis. The specific differential phase shift (=grad ΦDP) is used for rainfall rate computation.
- Rainfall estimations
 - Strong KDP ($> 1^\circ/\text{km}$) -> directly $R = c \left(\frac{KDP}{f} \right)^d$ f:frequency of the radar signal
 - Low KDP -> Marshall-Palmer relation $Z=aR^b$
- ΦDP signal noisy -> smoothed (FIR and median filter) before calculating KDP so almost independent from attenuation and partial beam blockage
- no absolute calibration



Horizontally (blue) and vertically (red) polarized pulse, emitted by a dual-polarization radar (lower left)

(Copyright © 2013 NOAA, NSSL, Norman, Oklahoma, USA)

Figure 1: Schematic illustration of the dual polarization technique given that large drops are oblate (courtesy Selex)



Spectral analysis

Spectral analysis: the first indication of the scaling behaviour of the field

Spectral slope β calculated
 $E(k) \approx k^{-\beta}$, where k : wave number

Double Trace Method

- generalization of TM technique introduced in order to estimate UM parameters (α, C_1)
- 1st step: arise the conservative field to the power η at the highest resolution, then the obtained field is normalized. $\langle (R_{\lambda}^{(n)})^q \rangle = Z^{K(q,\eta)}$
- the statistical moment for the different resolution λ , for different η and different moments q $K(q,\eta) = \eta^{\alpha} K(q)$
- 2nd step: the estimation of the UM parameters, represent the $K(q,\eta)$ as a function of λ in log-log plot for different η and for fixed q , this allows to estimate the value of α [1]

UM theory

Universal Multifractals parameters fully define statistics across scales^[2,3,4]

- $\alpha \in [0,2]$ (multifractality index): the variability of intermittency with respect to intensity level
- C_1 (mean intermittency): mean inhomogeneity of the field at large and small scales ($C_1=0$ for homogeneous fields)
- H : the degree of non-conservation, the scale dependency of the mean field ($H=0$ considered in our case)

C_1 and α estimated with the **Double Trace Moment** (DTM) technique [1]

K(q): statistical moment scaling function

$$K(q) - qH = \begin{cases} \frac{C_1}{\alpha - 1}(q^\alpha - q) & \alpha \neq 1 \\ C_1 q \log(q) & \alpha = 1 \end{cases}$$

Choice of the events

The project aimed to compare the data between X-band and C-band radar to study the impact of small-scale rainfall variability over the Bièvre catchment (in SW of Paris) which is 40km away from the radar. The purpose of this work is to analyse and inter-compare 5 (where was available) different products of X-band radar with pulsed emissions for simple and dual polarization.

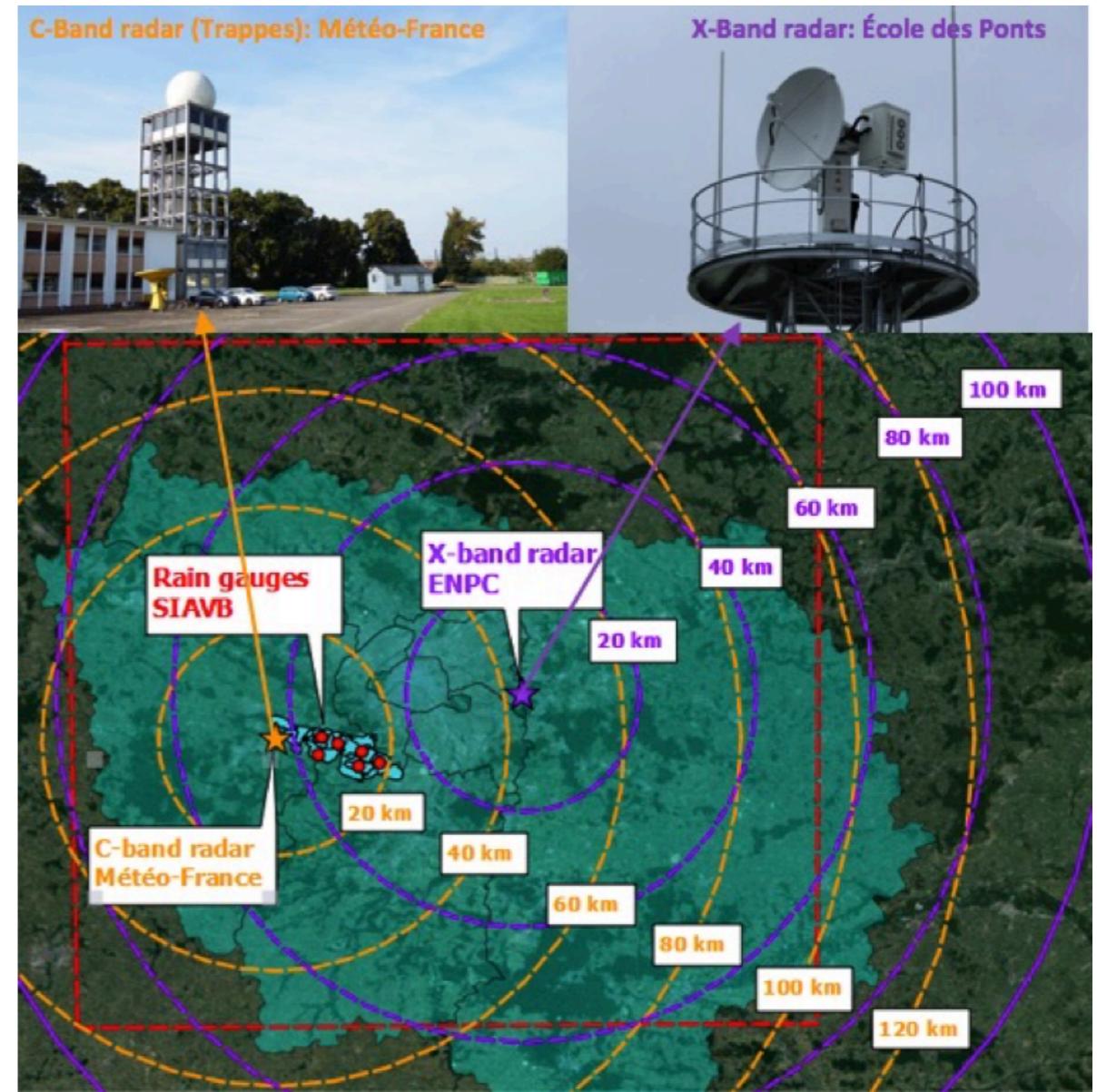


Figure 2: The location of X-band radar of ENPC, C-band of Météo France and of the catchment

DATA

The signal of KDP is noisy so the use of a smoothed signal is used to compute KDP. Two different filters are used: the Finite Impulse Response (FIR) and the median one. The data were processed with standard Rainbow software.

Table 1: Different radar products (-SP:Simple Filter, -PS:Pseudo)

| Product | Dual Polarization | Data used for products | Required parameters | Equation used | Filter |
|----------|-------------------|------------------------|---------------------|-------------------|--------|
| DPSRI | ✓ | dBZ, ZDR, KDP | a=150 b=1.3 | Z=aR ^b | FIR |
| DPSRI | ✓ | dBZ, ZDR, KDP | a=200 b=1.6 | Z=aR ^b | FIR |
| DPSRI-SP | ✓ | dBZ, ZDR, KDP | a=150 b=1.3 | Z=aR ^b | Median |
| DPSRI-PS | ✓ | dBZ, ZDR, KDP | a=150 b=1.3 | Z=aR ^b | FIR |
| SRI | ✗ | dBZ | a=200 b=1.6 | Z=aR ^b | FIR |

DATA

Table 2: Rainfall events and their characteristics

| Event | Date | Start time (UTC) | End time (UTC) | Duration (in h) |
|--------------|---------------|-------------------------|-----------------------|------------------------|
| E1 | 12-13/09/2015 | 06:05:00 | 18:55:00 | 36.83 |
| E2 | 12/09/2015 | 06:05:00 | 01:15:00 | 20.25 |
| E3 | 13/09/2015 | 05:45:00 | 18:55:00 | 13.17 |
| E4 | 16/09/2015 | 00:05:00 | 16:50:00 | 16.75 |
| E5 | 05-06/10/2015 | 09:10:00 | 16:05:00 | 30.92 |
| E6 | 21-23/05/2016 | 12:25:00 | 23:50:00 | 59.42 |
| E7 | 28-31/05/2016 | 08:25:00 | 00:00:00 | 87.58 |
| E8 | 17/06/2016 | 00:00:00 | 21:15:00 | 21.25 |

Multifractal Phase Transition

theoretical $K(q)$ and $c(\gamma)$ are convex but the empirical can be linear beyond a specific threshold-> multifractal phase transition (extreme values)

2 kinds of multifractal phase transition caused by 2 different phenomena : divergence of the statistical moment and the size of samples.

Divergence of statistical moment

The strong variability of the process at small scale compared to the one at large scale, induce to the divergence of order q larger than the value q_D (q_D : divergence moment order): $E(\varepsilon_\lambda^q) \rightarrow \infty$

So, for $q > q_D$ the statistical moment function becomes: $K(q) = \infty$

Following the Legendre transform we can obtain: $K(q) = \gamma_{max}(q - q_D) + K(q_D) \quad \text{for } q > q_D$

$$c(\gamma) = q_D(\gamma - \gamma_D) + c(\gamma_D) \quad \text{for } \gamma > \gamma_D$$

When the upper limit of the UM model is the divergence of the statistical moment, the divergence order is the solution of : $D = \frac{C_1}{\alpha - 1} \frac{q_D^\alpha - q_D}{q_D - 1}$

can not be detected if the sample size of the studied field is not enough large

Sample size

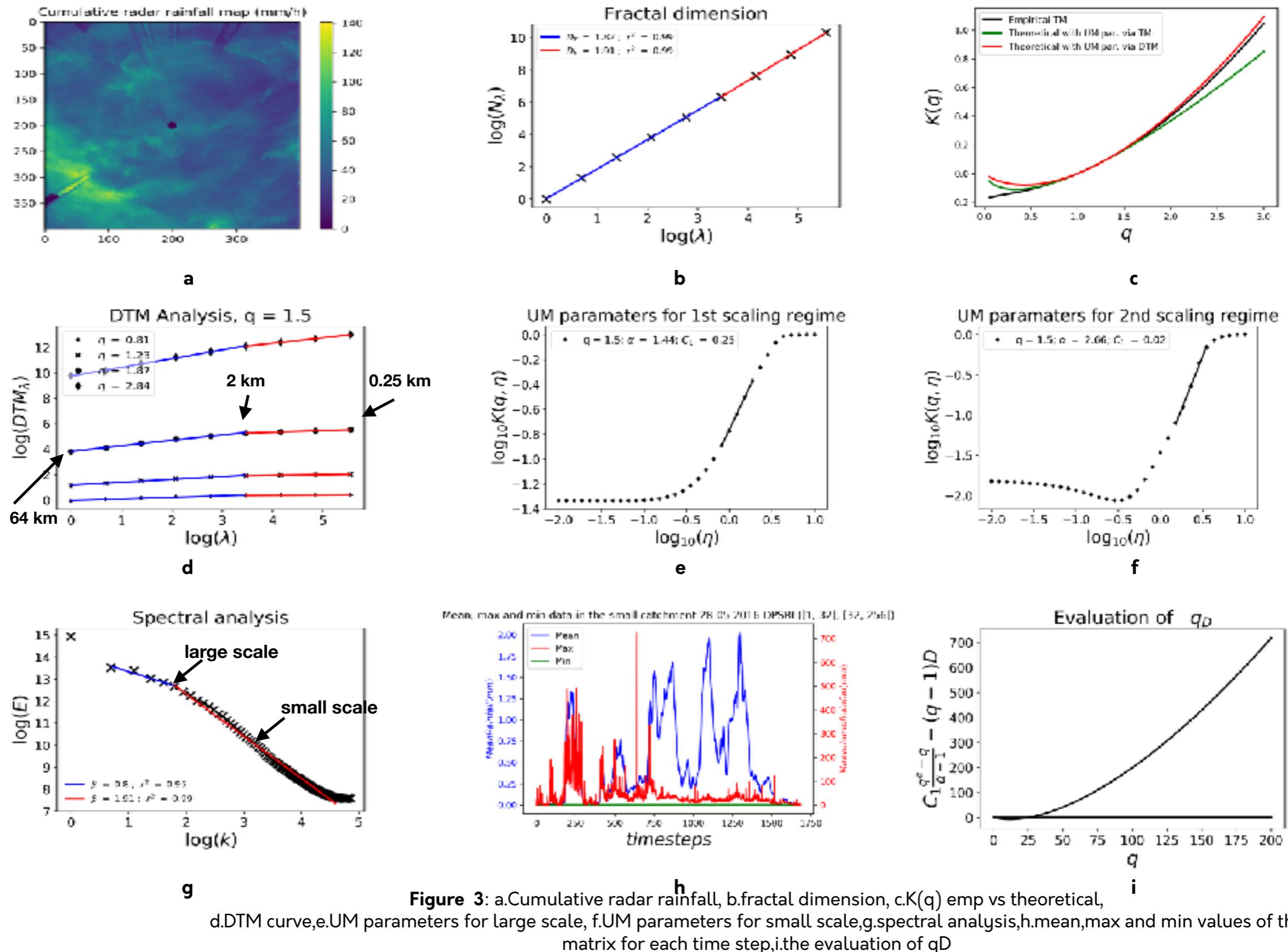
maximal theoretical observable singularity γ_s , $c(\gamma)$ bounded so $K(q)$ linear (Legendre transformation)

$$K(q) = \gamma_s(q - q_s) + K(q_s) \quad \text{for } q > q_s$$

$$q_s = \left(\frac{D + D_s}{C_1} \right)^{1/\alpha}$$

Ds:dimension of sampling
Ns :samples
D: dimension

$$D_s = \frac{\log N_s}{\log \lambda}$$



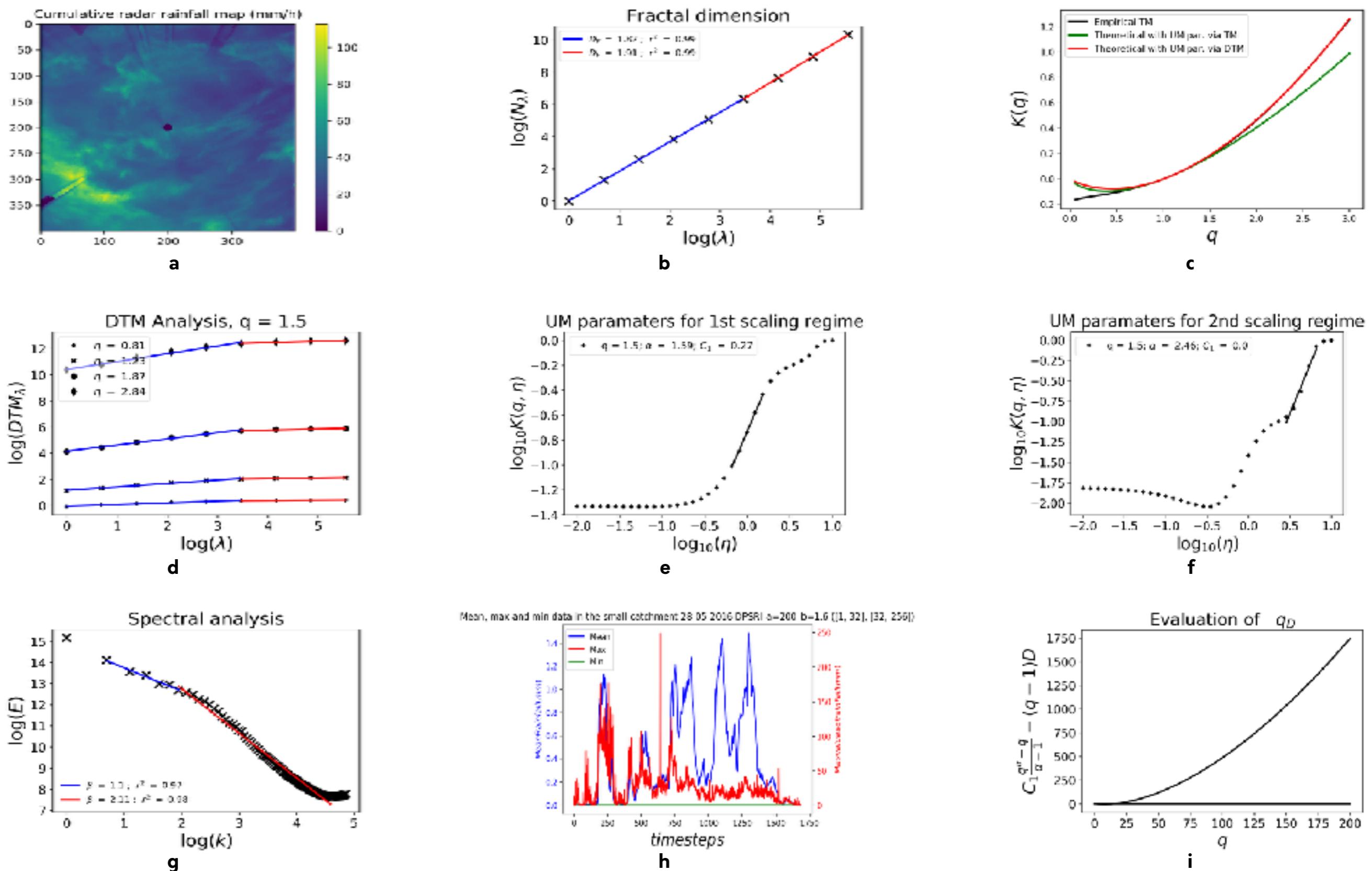


Figure 4: a.Cumulative radar rainfall, b.fractal dimension, c. $K(q)$ emp vs theoretical, d.DTM curve,e.UM paramters for large scale, f.UM paramters for small scale,g.spectral analysis,h.mean,max and min values of the matrix for each time step,i.the evaluation of q_D

28-31/05/2016 DPSRI simple filter a=150 b=1.3

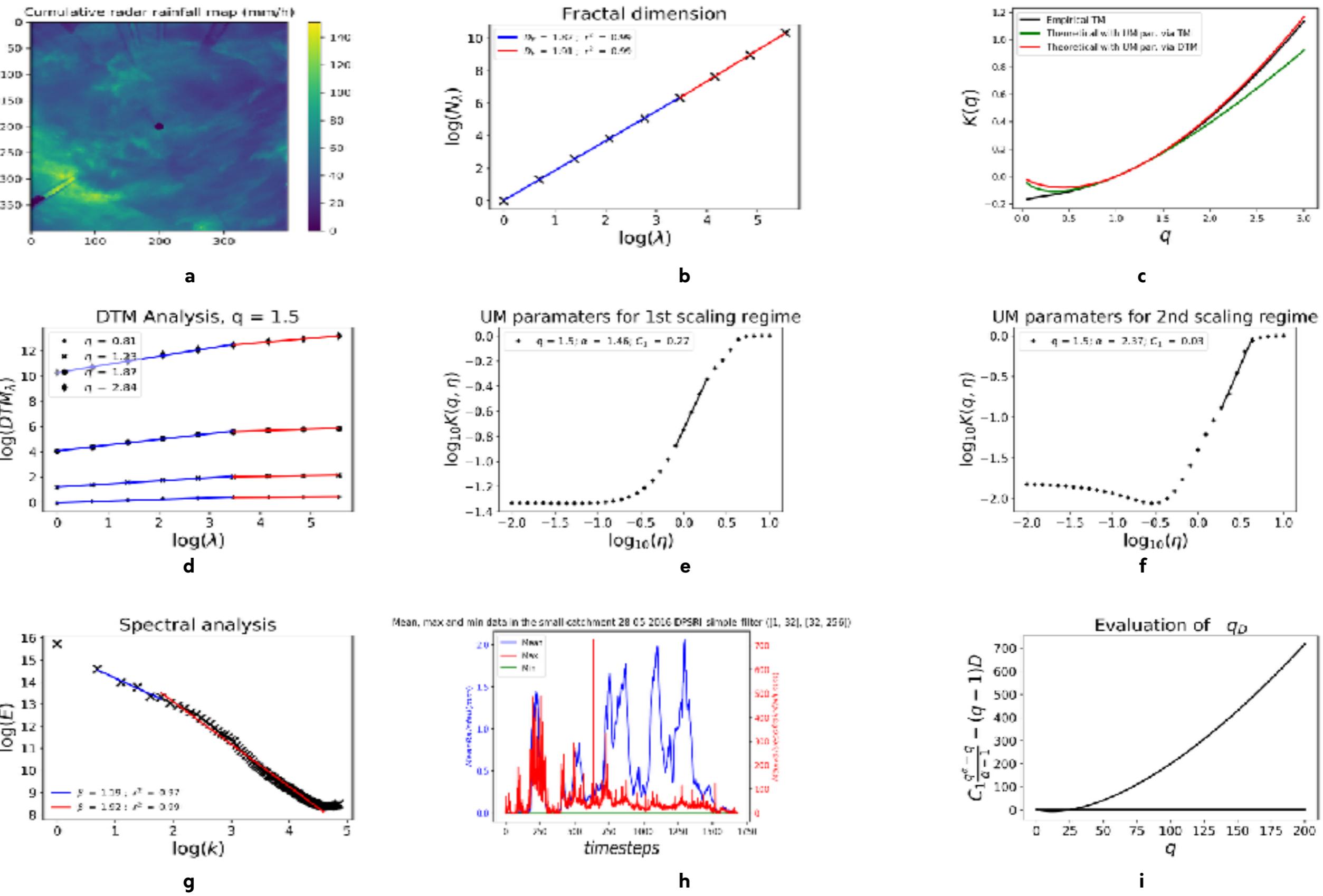


Figure 5: a.Cumulative radar rainfall, b.fractal dimension, c. $K(q)$ emp vs theoretical, d.DTM curve,e.UM parameters for large scale, f.UM parameters for small scale,g.spectral analysis,h.mean,max and min values of the matrix for each time step,i.the evaluation of q_D

28-31/05/2016 Pseudo DPSRI a=150 b=1.3

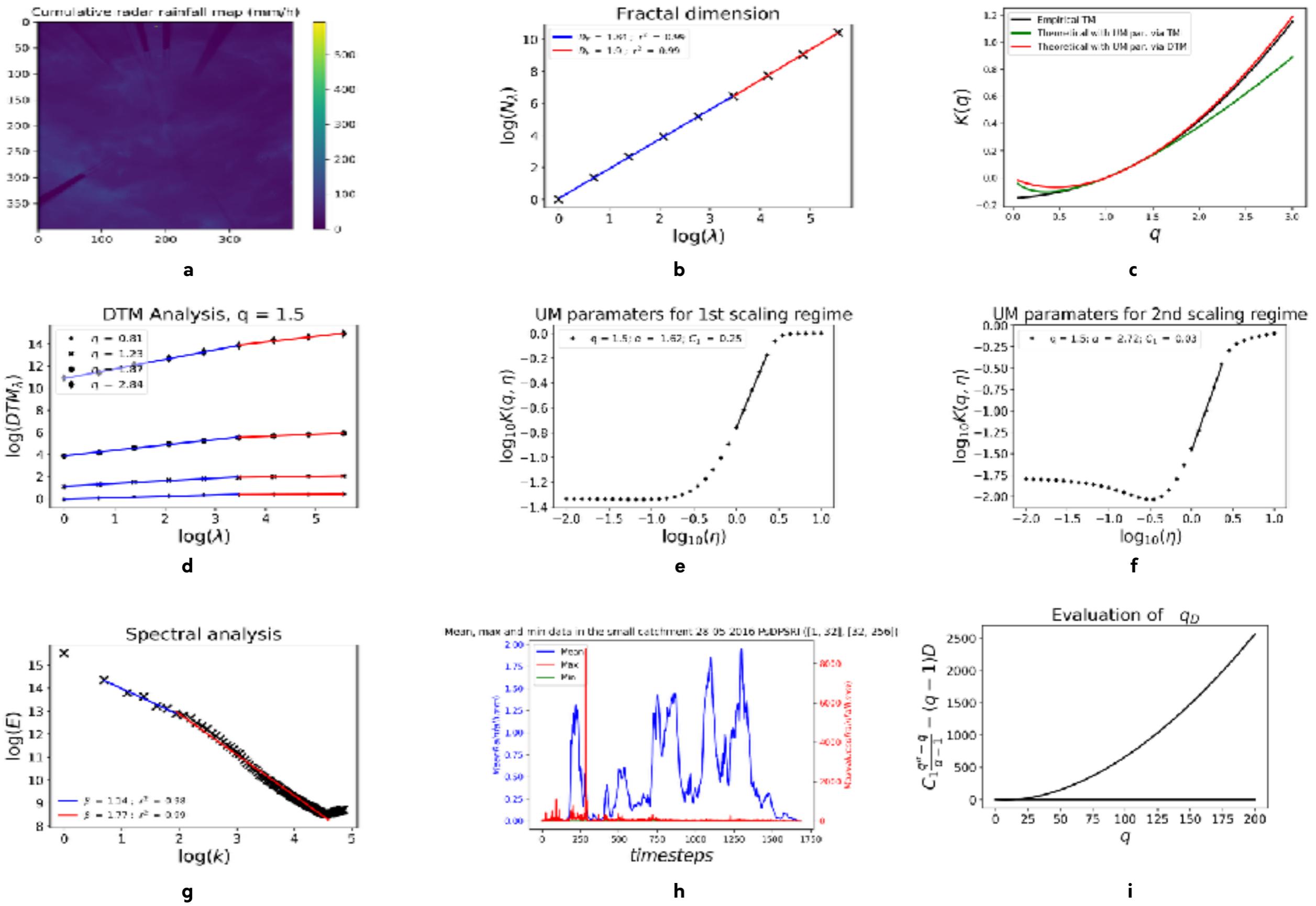


Figure 6: a.Cumulative radar rainfall, b.fractal dimension, c. $K(q)$ emp vs theoretical, d.DTM curve,e.UM parameters for large scale, f.UM parameters for small scale,g.spectral analysis,h.mean,max and min values of the matrix for each time step,i.the evaluation of q_D

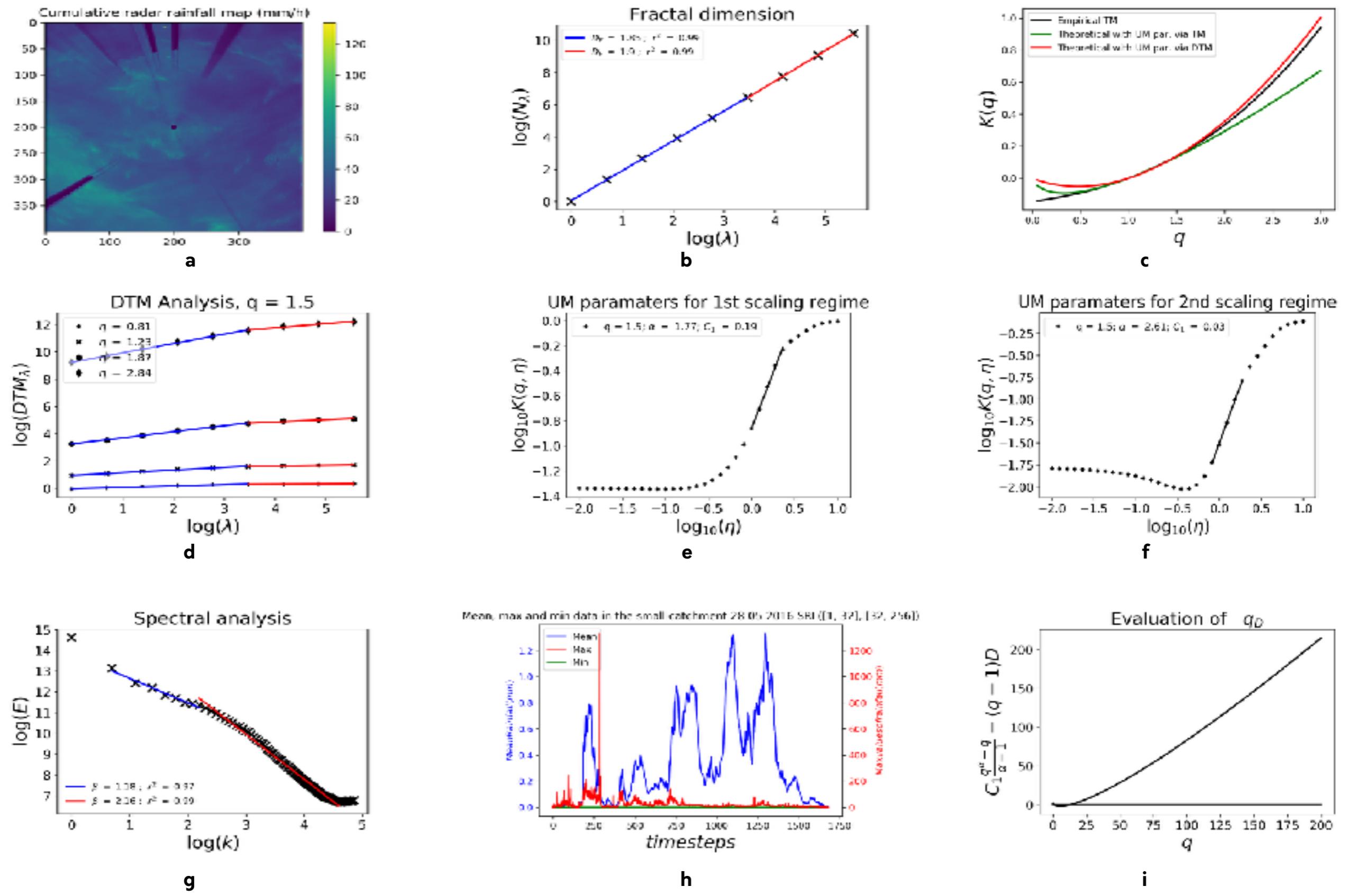


Figure 7: a.Cumulative radar rainfall, b.fractal dimension, c. $K(q)$ emp vs theoretical, d.DTM curve,e.UM parameters for large scale, f.UM parameters for small scale,g.spectral analysis,h.mean,max and min values of the matrix for each time step,i.the evaluation of q_D

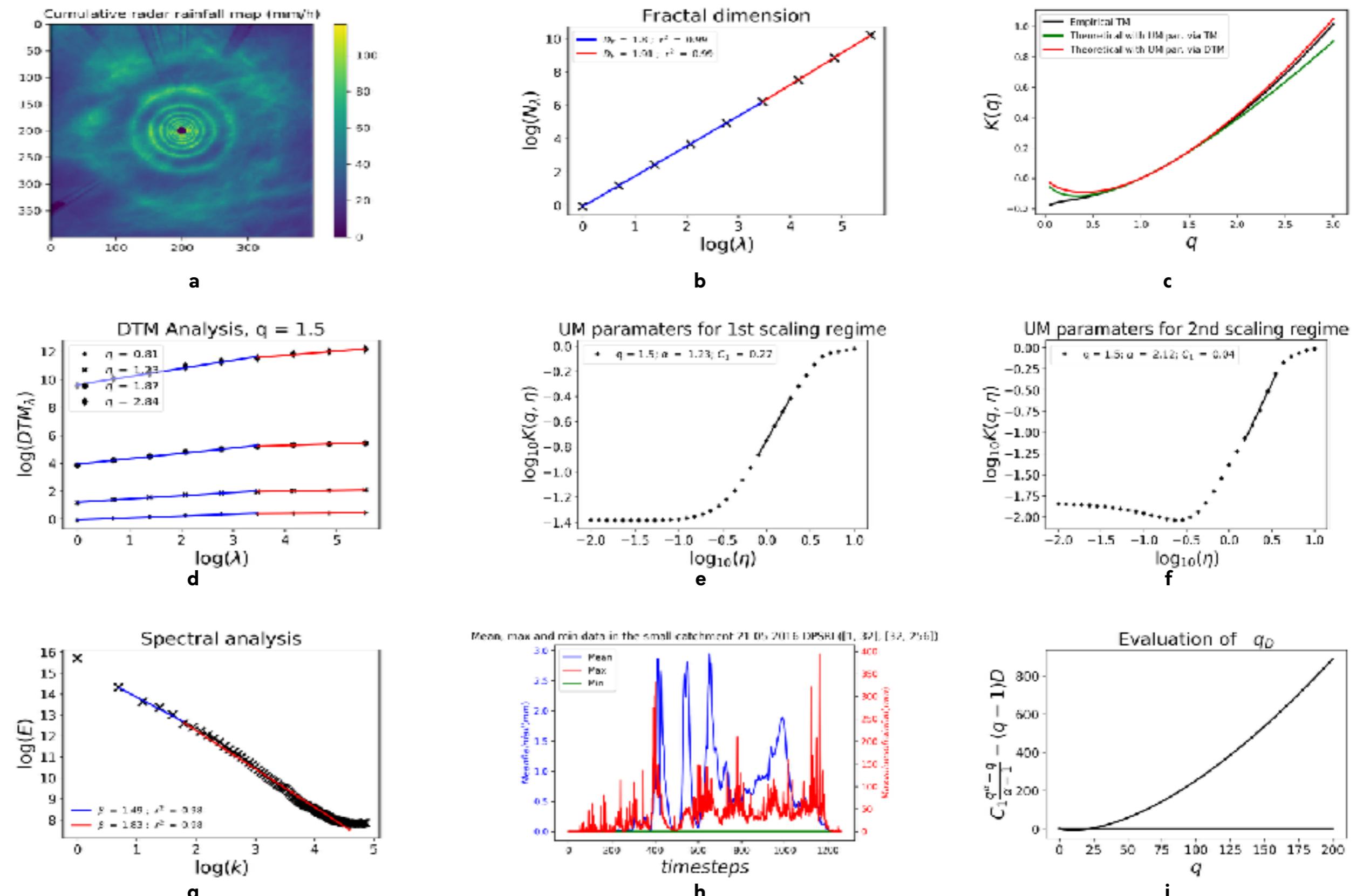


Figure 8: a.Cumulative radar rainfall, b.fractal dimension, c. $K(q)$ emp vs theoretical, d.DTM curve,e.UM parameters for large scale, f.UM parameters for small scale,g.spectral analysis,h.mean,max and min values of the matrix for each time step,i.the evaluation of q_D

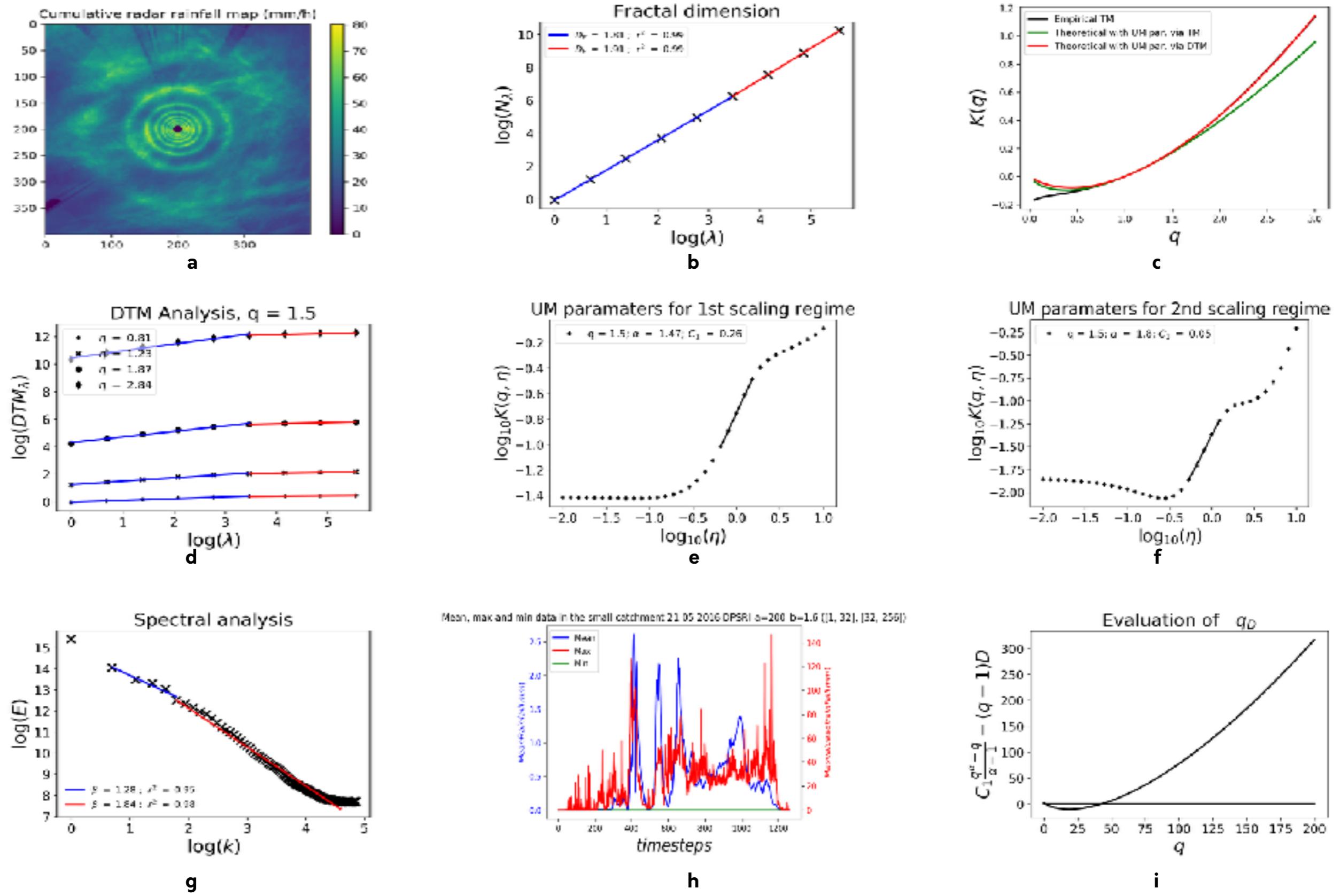


Figure 9: a.Cumulative radar rainfall, b.fractal dimension, c. $K(q)$ emp vs theoretical, d.DTM curve,e.UM parameters for large scale, f.UM parameters for small scale,g.spectral analysis,h.mean,max and min values of the matrix for each time step,i.the evaluation of q_D

21-23/05/2016 DPSRI simple filter a=150 b=1.3

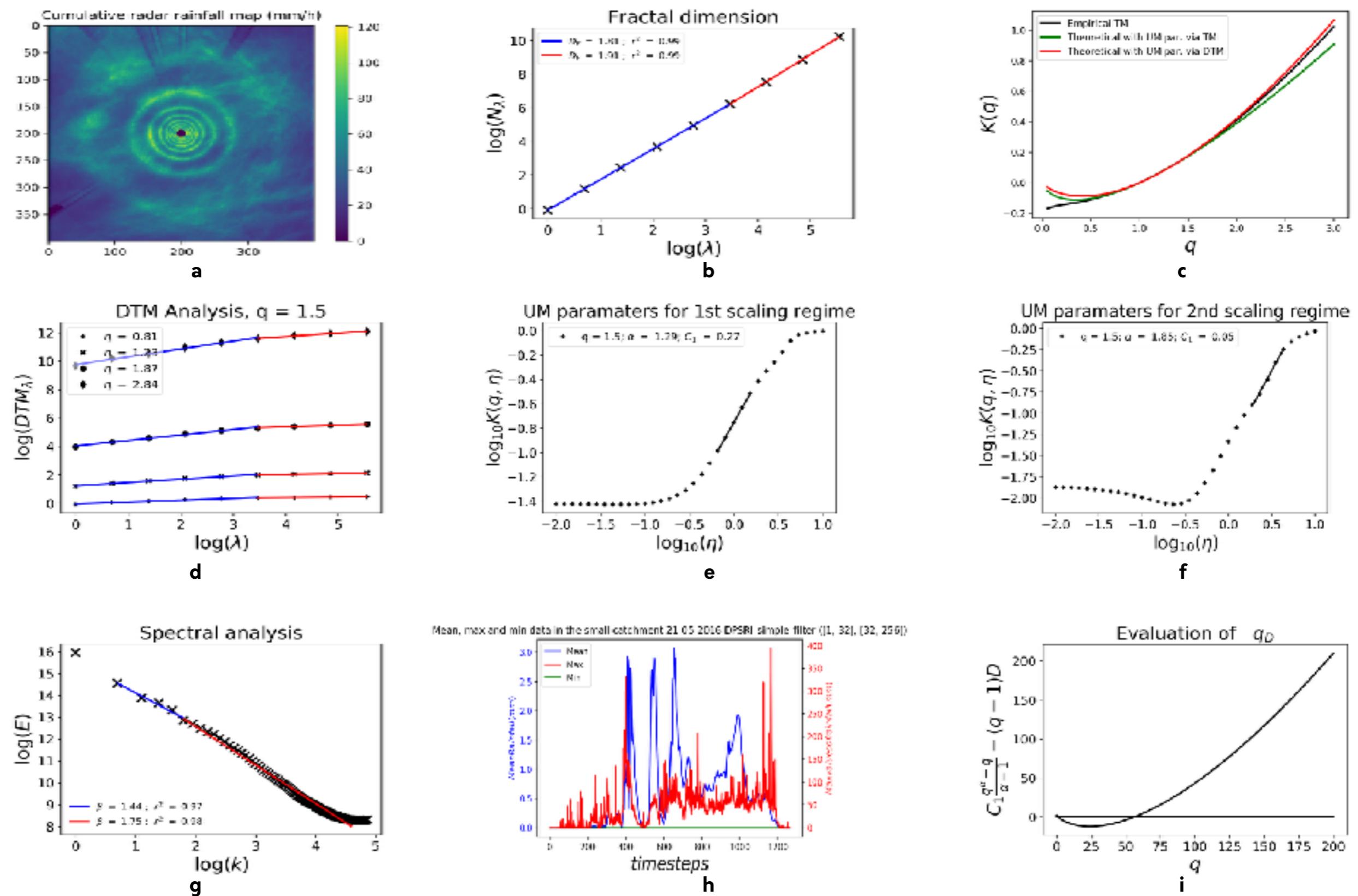


Figure 10: a.Cumulative radar rainfall, b.fractal dimension, c. $K(q)$ emp vs theoretical, d.DTM curve,e.UM parameters for large scale, f.UM parameters for small scale,g.spectral analysis,h.mean,max and min values of the matrix for each time step,i.the evaluation of q_D

21-23/05/2016 Pseudo DPSRI a=150 b=1.3

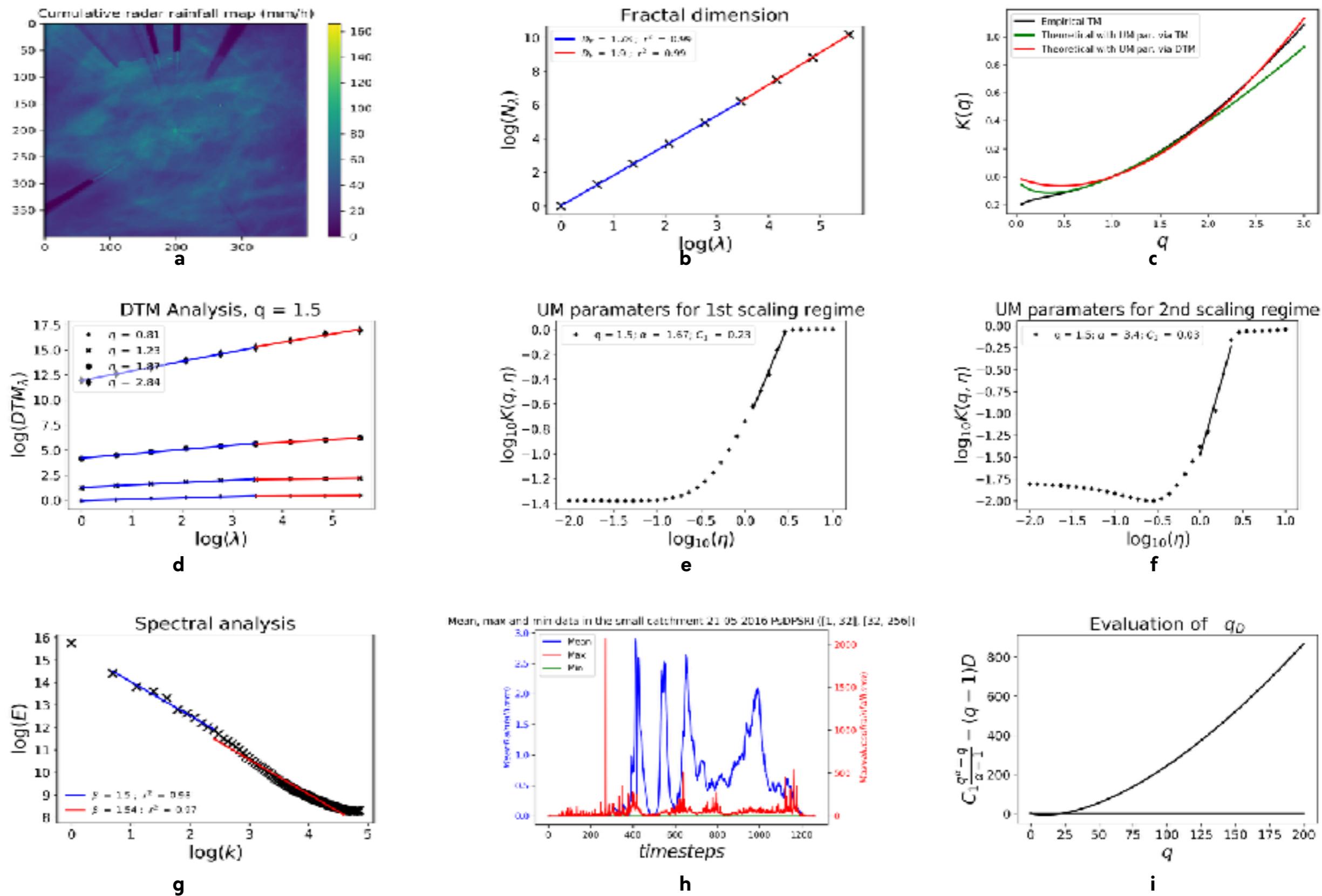


Figure 11: a.Cumulative radar rainfall, b.fractal dimension, c. $K(q)$ emp vs theoretical, d.DTM curve,e.UM parameters for large scale, f.UM parameters for small scale,g.spectral analysis,h.mean,max and min values of the matrix for each time step,i.the evaluation of q_D

21-23/05/2016 SRI a=200 b=1.6

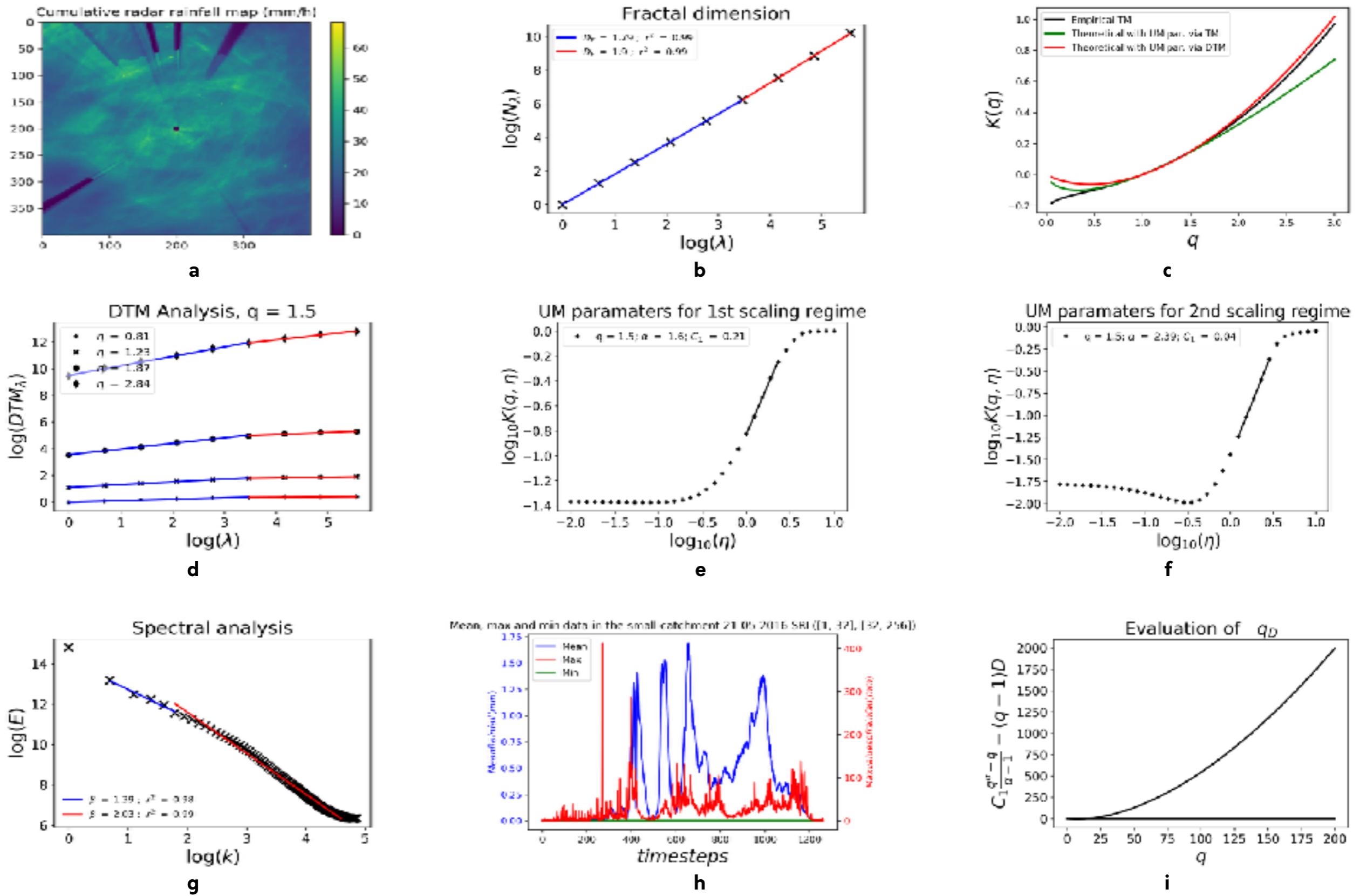


Figure 12: a.Cumulative radar rainfall, b.fractal dimension, c. $K(q)$ emp vs theoretical, d.DTM curve,e.UM parameters for large scale, f.UM parameters for small scale,g.spectral analysis,h.mean,max and min values of the matrix for each time step,i.the evaluation of q_D

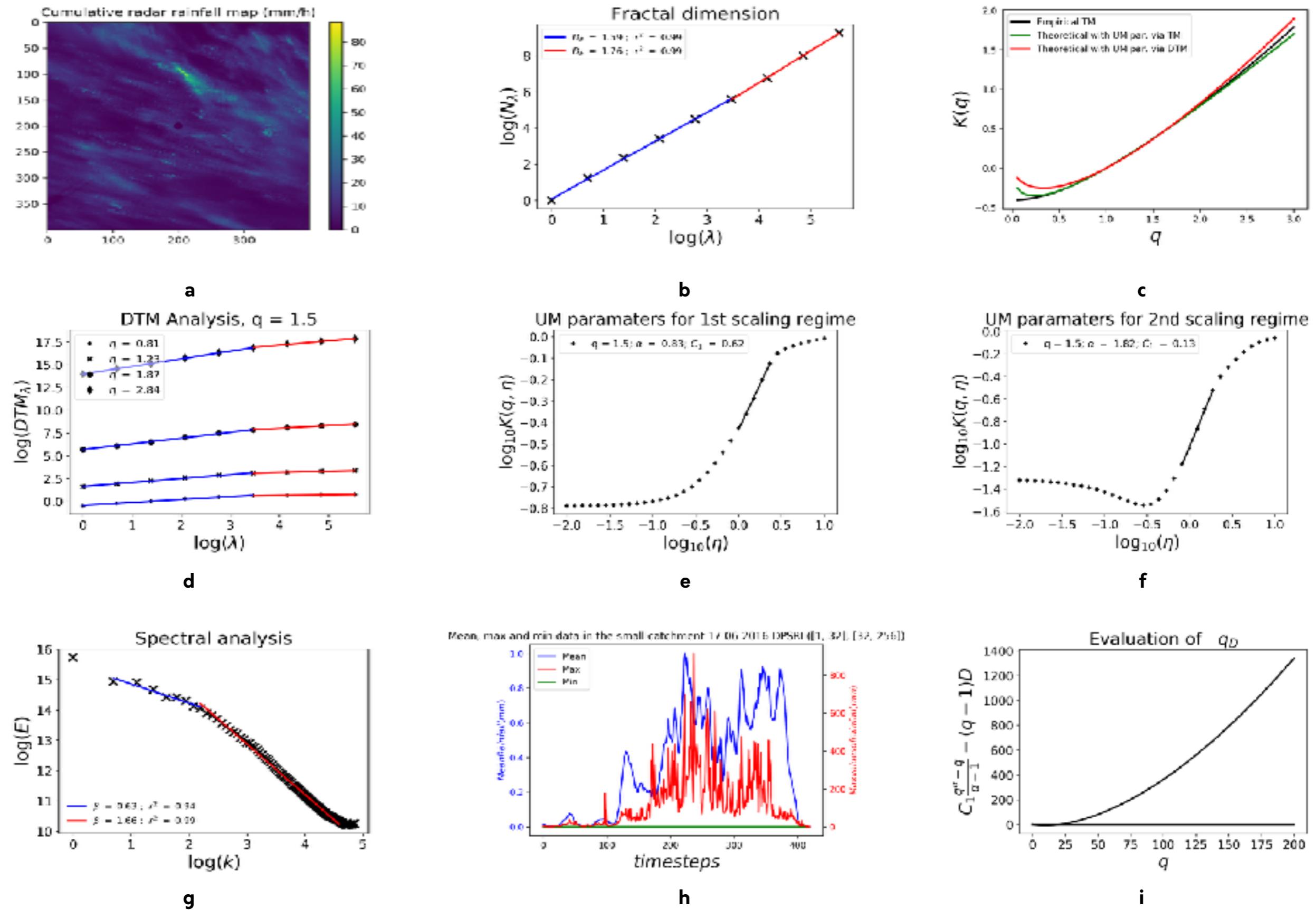


Figure 13: a.Cumulative radar rainfall, b.fractal dimension, c. $K(q)$ emp vs theoretical, d.DTM curve,e.UM parameters for large scale, f.UM parameters for small scale,g.spectral analysis,h.mean,max and min values of the matrix for each time step,i.the evaluation of q_D

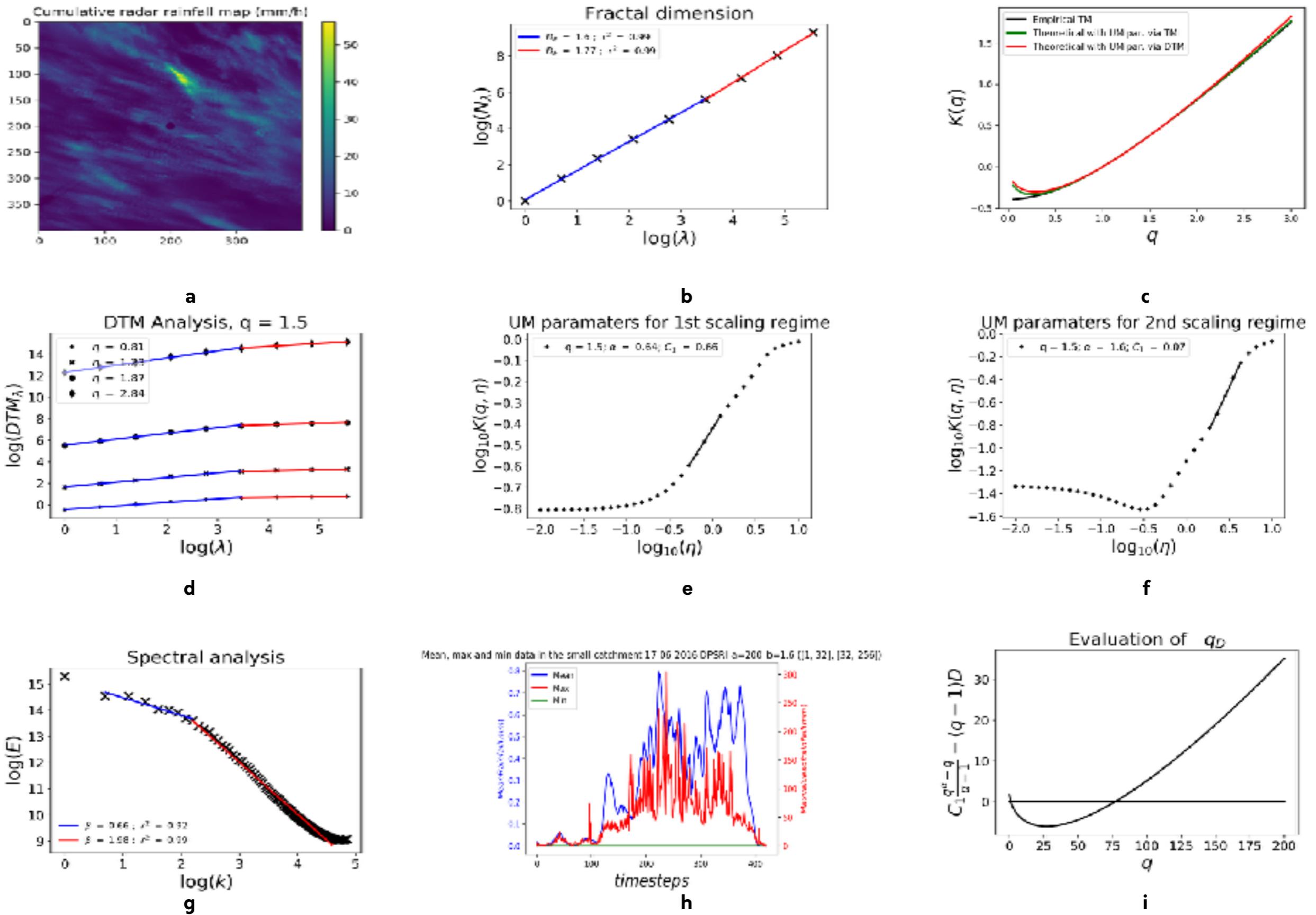


Figure 14 : a.Cumulative radar rainfall, b.fractal dimension, c. $K(q)$ emp vs theoretical, d.DTM curve,e.UM parameters for large scale, f.UM parameters for small scale,g.spectral analysis,h.mean,max and min values of the matrix for each time step,i.the evaluation of q_D

17/06/2016 DPSRI simple filter a=150 b=1.3

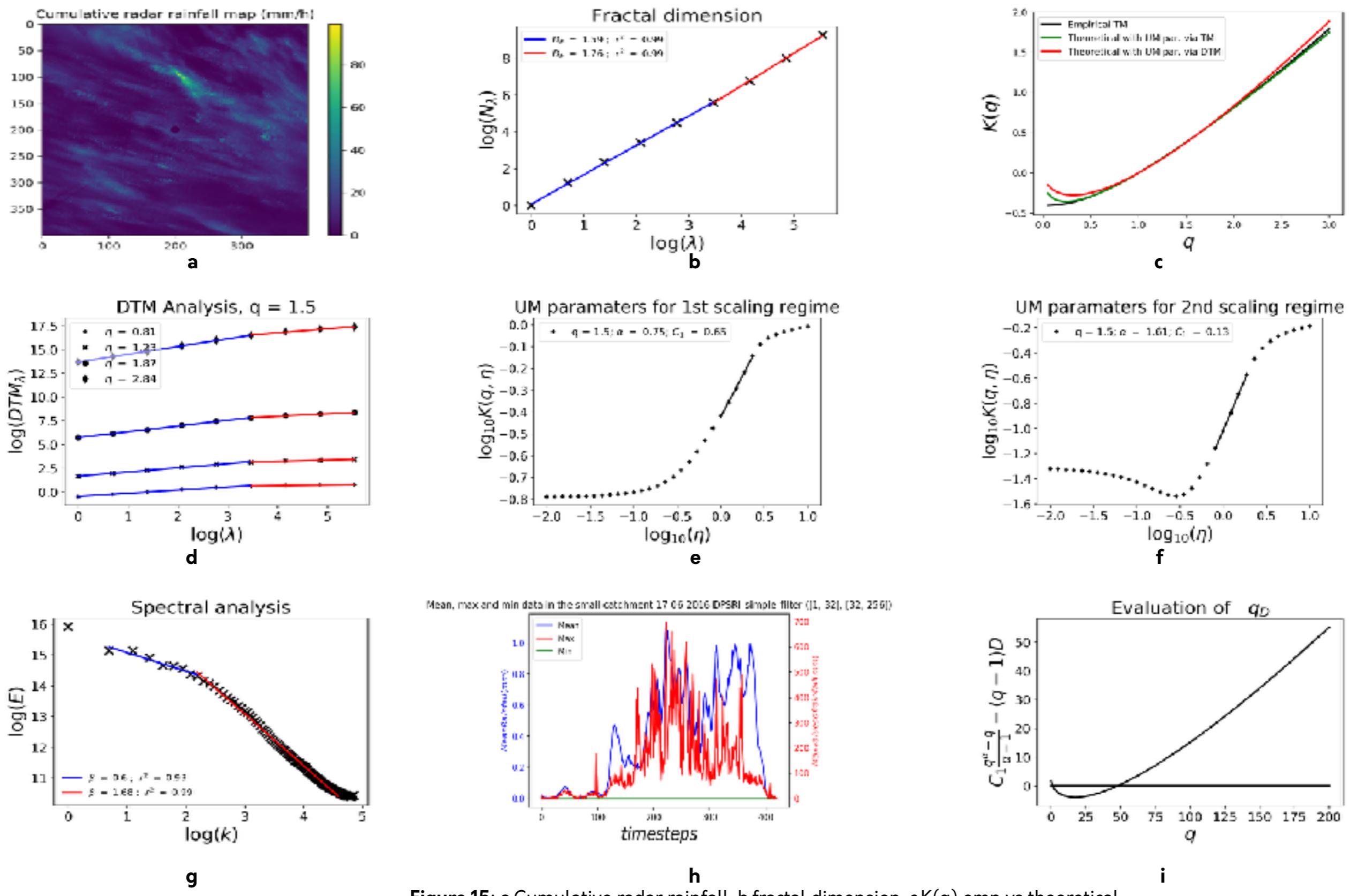


Figure 15: a.Cumulative radar rainfall, b.fractal dimension, c. $K(q)$ emp vs theoretical, d.DTM curve,e.UM parameters for large scale, f.UM parameters for small scale,g.spectral analysis,h.mean,max and min values of the matrix for each time step,i.the evaluation of q_D

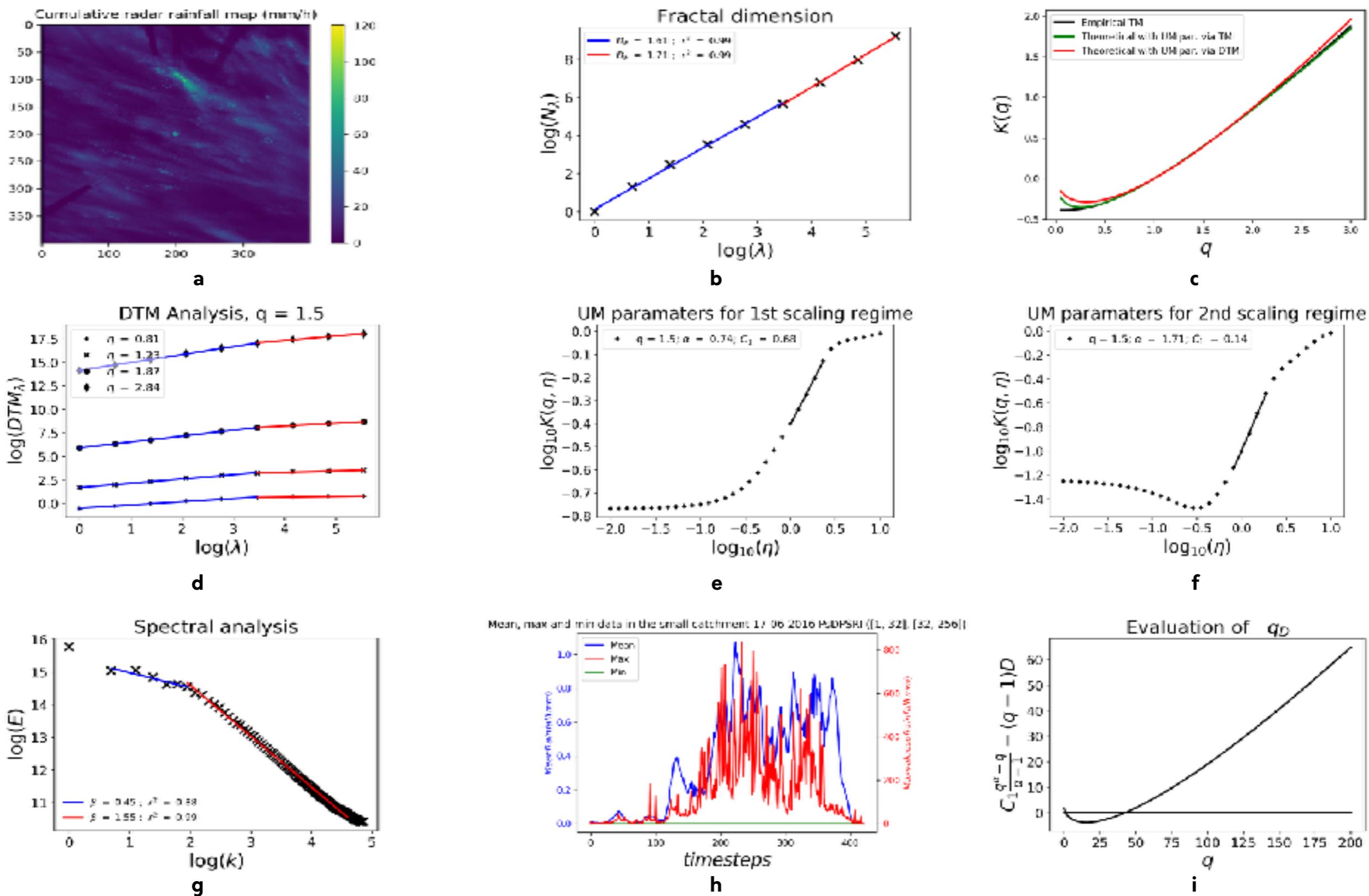


Figure 16: a.Cumulative radar rainfall, b.fractal dimension, c. $K(q)$ emp vs theoretical, d.DTM curve,e.UM parameters for large scale,f.UM parameters for small scale,g.spectral analysis,h.mean,max and min values of the matrix for each time step,i.the evaluation of q_D

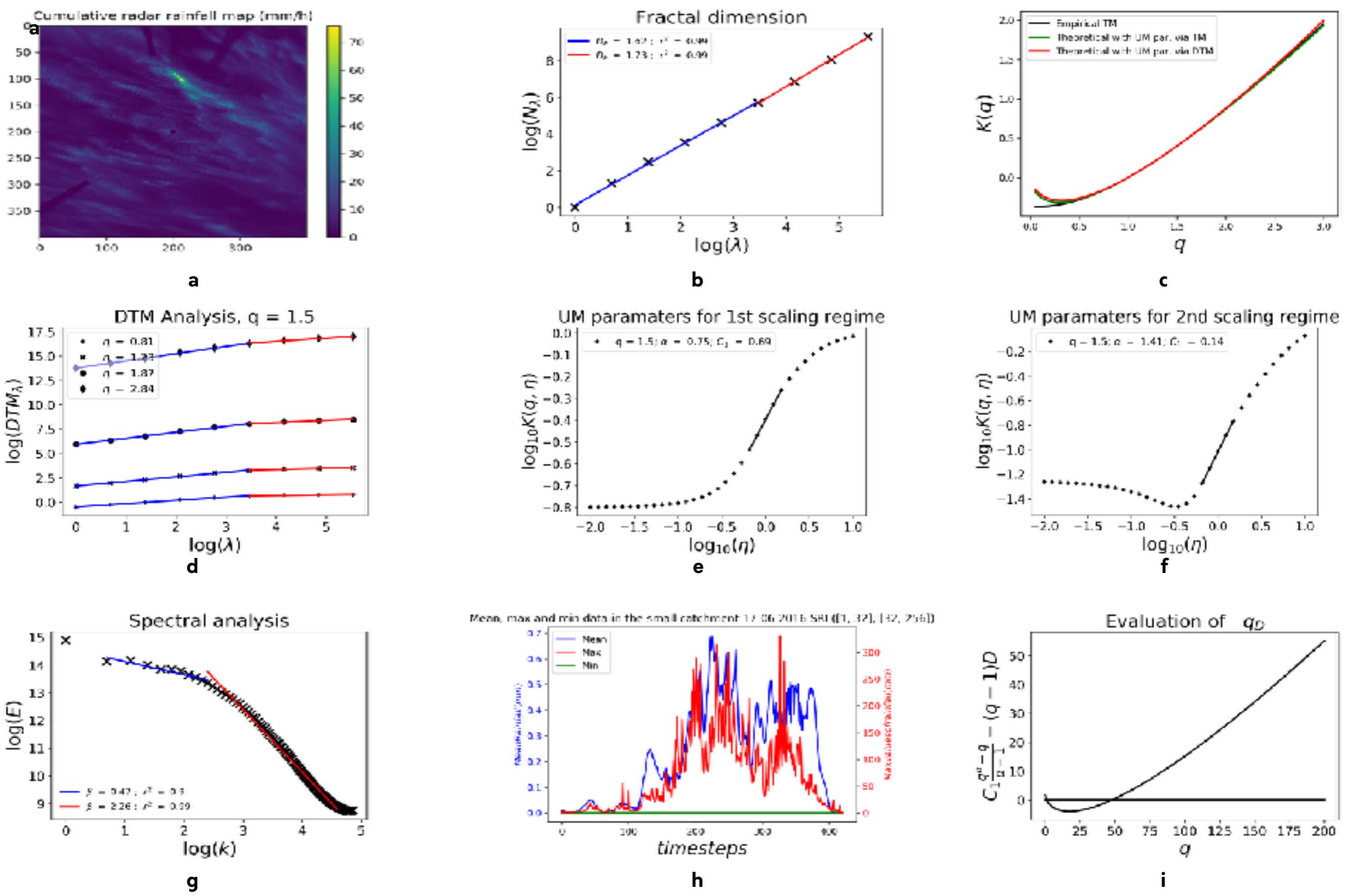


Figure 17: a.Cumulative radar rainfall, b.fractal dimension, c. $K(q)$ emp vs theoretical, d.DTM curve,e.UM parameters for large scale, f.UM parameters for small scale,g.spectral analysis,h.mean,max and min values of the matrix for each time step,i.the evaluation of q_D

Results

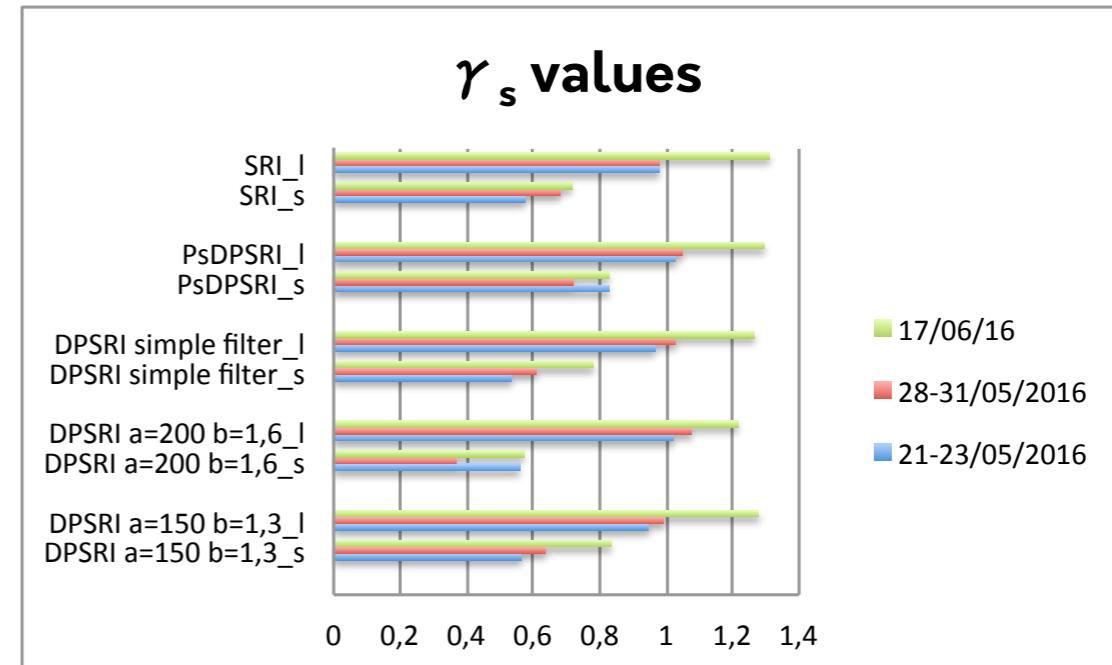
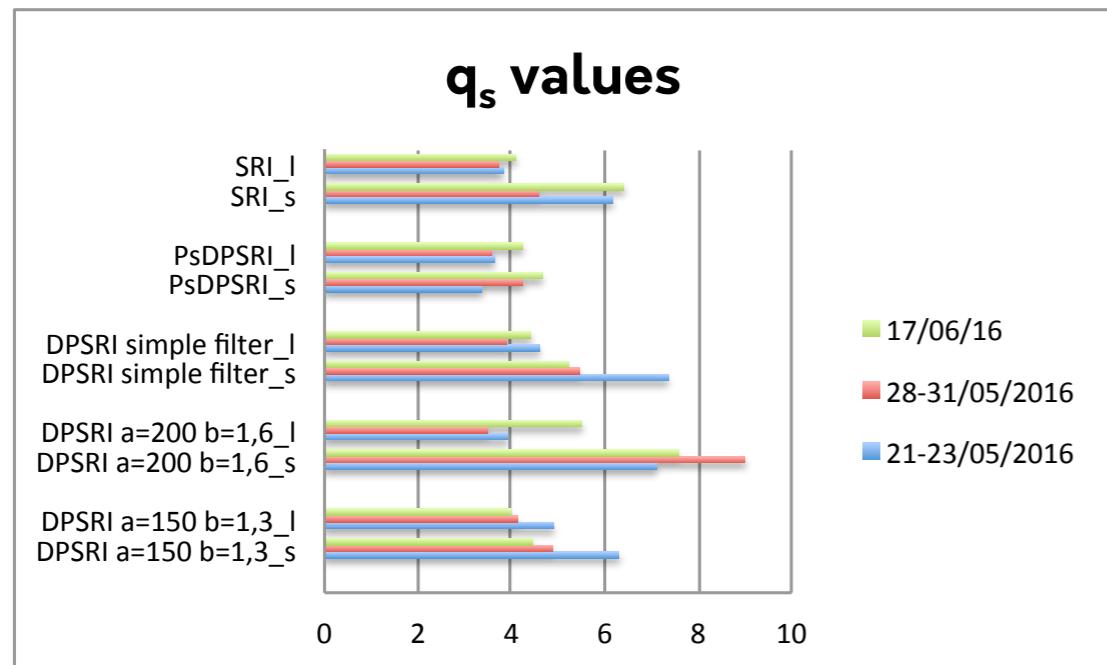
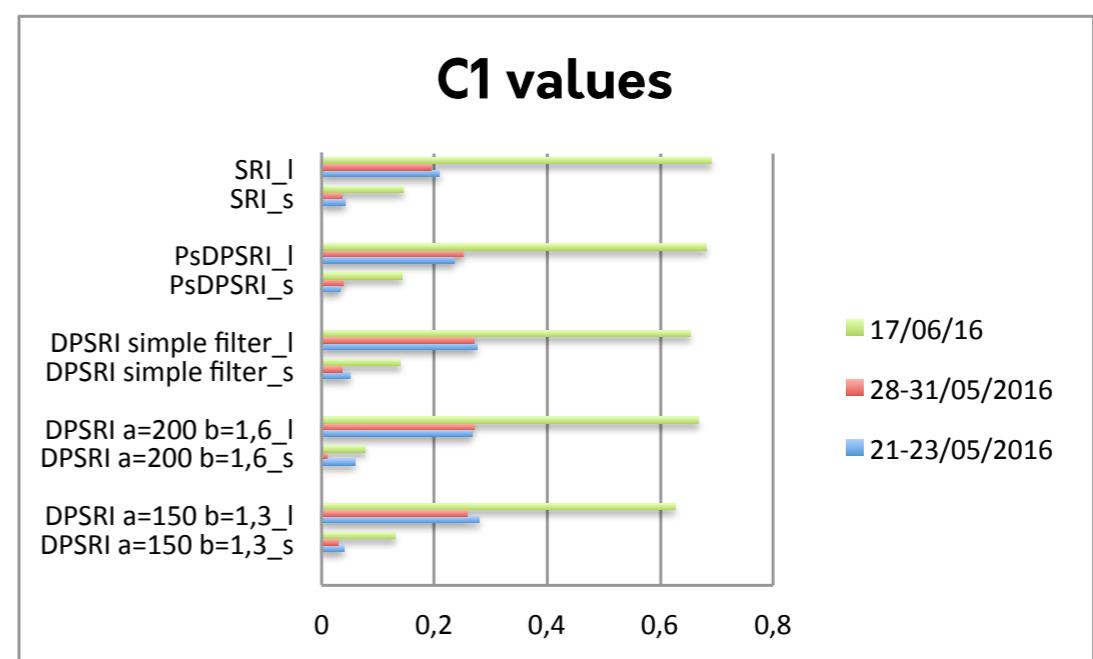
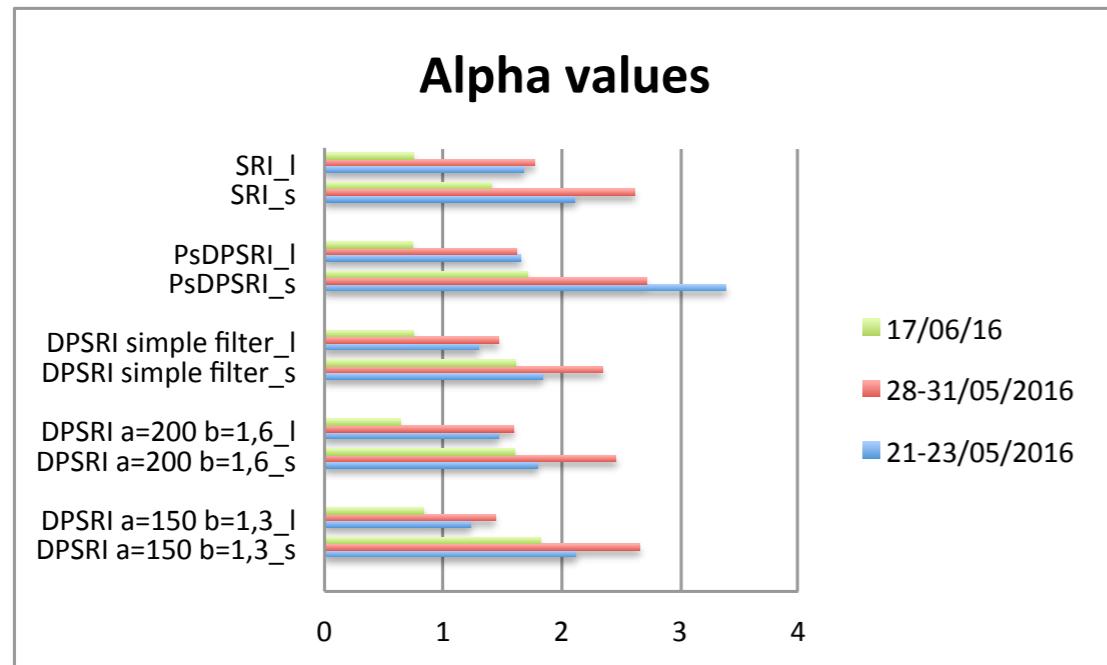


Figure 18: a.The alpha values of all the events for each product,b.the C1 values of all the events for each product,c.the q_s values of all the events for each product,d.the γ_s values of all the events for each product

Results

- With the X-band radar, a scale break can be seen in 2km. It means that small scale behaviour is not accessible with C-band radars
- The scale break is less pronounced for some events (i.e. 28-31/05/2016) and for some products (Pseudo DPSRI)
- SRI seems to produce data with higher homogeneity (especially in large scales)
- α stronger for SRI , DPSRI $a=200$ $b=1.6$ and PseudoDPSRI for large scales while for small is more complicated because of the results
- Event 17/06/2016 very strong multifractality (>1.6)

Future work

- Analysing more events of lower and higher intensity
- Studying the Multifractal Phase Transition and combining the results we have so far
- Comparing with other types of radar data and with rain gauge measurements
- Use them in urban hydrological models (Multi Hydro) to find out which filter is better to use

Acknowledgements

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References

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