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Anne Bisson, Céline Casenave, Simon Boudsocq, Tanguy Daufresne. Maximization of fertility transfers from rangeland to cropland: the contribution of control theory. *Journal of Theoretical Biology*, 2019, 469, pp.187-200. 10.1016/j.jtbi.2019.01.039 . hal-02066979

**HAL Id: hal-02066979**

**<https://hal.science/hal-02066979>**

Submitted on 13 Mar 2019

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## Accepted Manuscript

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PII: S0022-5193(19)30032-3  
DOI: <https://doi.org/10.1016/j.jtbi.2019.01.039>  
Reference: YJTBI 9830



To appear in: *Journal of Theoretical Biology*

Received date: 17 July 2018  
Revised date: 16 November 2018  
Accepted date: 21 January 2019

Please cite this article as: Anne Bisson, Céline Casenave, Simon Boudsocq, Tanguy Daufresne, Maximization of fertility transfers from rangeland to cropland: the contribution of control theory., *Journal of Theoretical Biology* (2019), doi: <https://doi.org/10.1016/j.jtbi.2019.01.039>

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**Highlights**

- A simple meta-ecosystem model is proposed to represent mixed farming system
- Optimization of livestock-induced nutrient transfer from rangeland to cropland improves crop production
- Control theory provides tools that take into account the variability over time of agricultural practices
- Time-varying livestock management can lead to higher crop production

# Maximization of fertility transfers from rangeland to cropland: the contribution of control theory.

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## Abstract

In traditional mixed farming systems, soil fertility in cropland relies on the transfer of fertility from rangeland through the transfer of manure produced by livestock that grazes in rangeland. In this work, we introduce a simple meta-ecosystem model in which the mixed farming system is represented by a cropland sub-system connected to a rangeland sub-system by nutrient fluxes. The livestock plays the role of nutrient-pump from the rangeland sub-system to the cropland sub-system. We use this model to study how spatial organization and practices of livestock management such as the control of grazing pressure and night corralling can help optimize both nutrient transfers and crop production.

We argue that addressing the optimization of crop production requires different methods, depending on whether the agricultural practice in focus is constant or variable over time. We first used classical optimization methods at equilibrium to address optimization when the grazing pressure was as-

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sumed to be constant over time. Second, we address optimization for a more realistic configuration of our model, where grazing pressure was assumed to vary over the course of a year. In this case, we used methods developed in the field of the control theory. Classical methods showed the existence of an optimal level of constant grazing pressure that maximizes the transfers from rangeland to cropland, leading to the maximization of crop production. Control methods showed that by varying the grazing pressure adequately an additional gain of production is possible, with higher crop production and lower nutrient transfer from rangeland to cropland. This additional gain arises from the fact that the requirement of nutrient by crops is variable along the year. Consequently, a constant adjustment of the grazing pressure allows a better match between nutrient transfer and nutrient requirement over time, leading to a substantial gain of crop biomass. Our results provide new insights for a “smarter” management of fertility transfers leading to higher crop production with less rangeland surface.

*Keywords:* meta-ecosystem, fertility transfer, crop production, control theory

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## 1. Introduction

In the context of demographic growth and economic changes, modern agriculture is facing an increasing demand in crop production. The crop production strongly relies on soil fertility which is usually maintained by addition of nutrient from external sources. In intensive high-yield agriculture, soil fertility relies on addition of synthetic fertilizers (Tilman et al., 2002). In traditional mixed farming systems, exploited by most smallholders in developing

8 countries (Food and Agriculture Organization of the United Nations., 2011),  
 9 the addition of fertilizers from external sources is usually too costly (Dugué  
 10 et al., 2011) and soil fertility relies on the addition of organic matter from  
 11 animal manure (Powell et al., 1996, Manlay et al., 2004). Therefore, increas-  
 12 ing crop production relies mostly on the optimization of these agricultural  
 13 practices, which are the driving forces of the agro-ecosystem functioning.

14 Indeed, the current practice is to use livestock as a “nutrient pump”,  
 15 displacing nutrient from rangeland to cropland. The terminology “biological  
 16 pump” comes from Longhurst and Glen Harrison (1989) which defines the  
 17 transfer of carbon and nitrogen from the euphotic zone through the downward  
 18 flow of various organic matters (see also the terminology “whale pump” of  
 19 Roman and McCarthy (2010) and “nutrient pump” of Bisson et al., in review).  
 20 Livestock is herded in rangeland during the day where it feeds and ingests  
 21 nutrient that it partly transfers to cropland by excreting manure during night  
 22 corraling. Many agricultural practices affect the transfer of nutrient. We  
 23 focused on three key practices, namely, the rangeland to cropland surface  
 24 ratio, the grazing pressure, and the fraction of nutrient consumed by livestock  
 25 in rangeland that is effectively transferred to cropland. The first one may  
 26 be referred to as an “organizational” practice since it transforms the spatial  
 27 organization of the landscape. It has strong implications for the balance  
 28 of nutrient at landscapes scale. The two later practices may be referred to  
 29 as “interconnection” practices because they directly determine the fluxes of  
 30 nutrient between rangeland and cropland.

31 Here we propose to study these key practices through the use of a math-  
 32 ematical model, a classical approach in theoretical ecology (DeAngelis, 1992,

33 Loreau, 2010). Several dynamical systems have already been proposed to  
 34 help optimize the grazing in agro-ecosystems (e.g. Noy-Meir (1975), Wood-  
 35 ward et al. (1993). In the vein of these works, some recent studies suggest  
 36 that theoretical ecology provides some valuable theoretical tools to address  
 37 agronomic questions (Diegues Cameroni and Fort, 2017, Fort et al., 2017).

38 To take into account the patchiness of the landscape with regard to nu-  
 39 trient, and more specifically the source-sink dynamics between rangeland  
 40 and cropland, we based our approach on the meta-ecosystem theory (Loreau  
 41 et al., 2003). We define sources and sinks according to Loreau et al. (2013)  
 42 where a source (sink) sub-system is an exporter (importer) of nutrient within  
 43 the whole agro-ecosystem. By construction, our model shows sources-sink  
 44 properties. Sources-sink dynamics simply result from the nutrient transfer  
 45 by livestock from rangeland to cropland. This theory provides a convenient  
 46 framework to address source-sink dynamics and its consequences for primary  
 47 production in natural landscapes (Loreau et al., 2013). It has been proved  
 48 recently to be applicable to agro-ecosystems (Bisson et al., in review). We  
 49 built a simple meta-ecosystem model with two sub-systems (rangeland and  
 50 cropland) to assess the optimization of crop production through the three  
 51 agricultural practices introduced earlier.

52 We looked for configurations and management strategies that maximized  
 53 crop production. Depending on whether the practices are of the “organiza-  
 54 tional” or “interconnection” types, the optimization of crop production with  
 55 respect to these practices require different methods. Therefore, for “organi-  
 56 zational” practices, that do not change over time (at least over one year), the  
 57 optimization can be addressed through the use of “classical” methods, such

as sensitivity analysis, numerical exploration or minimization of an objective function. However, such methods are not well-adapted to study practices of livestock management that may change over a year. For “interconnection” practices, we advocate for the use of methods of control theory. Control theory is a mathematical framework dedicated to the control of systems (Astrom and Murray, 2008, Levine, 2010, Corriou, 2004). It has already been used for agronomic and environmental issues such as the reduction of the soil sodicity (Mau and Porporato, 2016), the optimization of cutting frequency of grassland (Chen, 1986, Chen and Wang, 1988), or robust management of natural resource (Anderies et al., 2007). Its objective is to propose some mathematical expressions for the inputs of a system that makes the controlled system behave as desired. The input values adapt over time according to the system state and to the constraints on the parameters values, creating a feedback loop. In this paper, we first address the optimization of crop production by using a classical method of optimization at equilibrium. Second, we no longer study the values at equilibrium but take advantage of the transient dynamics to lead the system at a given state: we address optimization of crop production when grazing pressure can vary over the course of the year with the use of methods developed in the control theory.

## 2. Ecological model of the agro-ecosystem

We model the agro-ecosystem as a simple meta-ecosystem composed of a cropland sub-system of surface  $s_c$  and a rangeland sub-system of surface  $s_r$  (figure C.1). Each sub-system ( $z$ ), with  $z = c$  for cropland and  $z = r$  for rangeland is composed of two compartments: plants ( $P_z$ ) and inorganic



82 nutrient ( $N_z$ ). Both  $P_z$  and  $N_z$  compartment represent stocks of nutrient. For  
 83 the sake of simplicity and genericity, only one limiting nutrient (for instance  
 84 nitrogen, phosphorus or potassium) of primary production is represented in  
 85 this model. We are aware that considering multiple nutrients instead of  
 86 a single, and considering specificities of nutrient biogeochemical cycles (for  
 87 instance, the symbiotic fixation for nitrogen) would carry out more outcomes.  
 88 However, it would also bring complexity that would prevent the detection of  
 89 generic mechanisms.

90 [Figure 1 about here.]

91 We used a classic formalism in ecology (DeAngelis, 1992, Loreau, 2010).  
 92 The equations of the meta-ecosystem model are:

$$\begin{cases} \dot{P}_r &= u_r P_r N_r (1 - \frac{P_r}{K_r}) - m_r P_r - d_r P_r \\ \dot{N}_r &= -u_r P_r N_r (1 - \frac{P_r}{K_r}) - e_r N_r + i_r + m_r P_r + \alpha d_r P_r \\ \dot{P}_c &= u_c P_c N_c (1 - \frac{P_c}{K_c}) - m_c P_c \\ \dot{N}_c &= -u_c P_c N_c (1 - \frac{P_c}{K_c}) - e_c N_c + i_c + m_c P_c + \underbrace{(1 - \alpha) d_r P_r}_{T} \frac{s_r}{s_c} \end{cases} \quad (1)$$

93 The units and definition of the model parameters are summarized in ta-  
 94 ble C.1. In each sub-system ( $z$ ), plants take-up nutrient from the inorganic  
 95 nutrient compartment  $N_z$  following a modified logistic growth with a max-  
 96 imal growth rate  $u_z$  and a carrying capacity  $K_z$ . Nutrient is recycled back  
 97 to the  $N_z$  compartment at a constant rate  $m_z$  through plant mortality and  
 98 mineralization. Nutrient input in each sub-system is a constant flux of inor-  
 99 ganic nutrient  $i_z$ . Losses of inorganic nutrient  $N_z$  are mainly due to erosion,  
 100 leaching and are modelled by a linear term with a constant rate  $e_z$ .

101 In this model, we only consider livestock as a vector of nutrient from the  
 102 rangeland sub-system to the cropland sub-system, therefore livestock is not  
 103 explicitly represented. The rate of grazing pressure  $d_r$  (in  $\text{d}^{-1}$ ) is proportional  
 104 to the amount of plants  $P_r$ . We can consider the grazing pressure  $d_r$  as an  
 105 aggregated parameter of livestock biomass (in  $\text{kgN ha}^{-1}$ ) multiplied by the  
 106 consumption coefficient of grass biomass by livestock (in  $\text{ha kgN}^{-1} \text{d}^{-1}$ ).  
 107 The flux of nutrient from compartment  $P_r$  carried by livestock from the  
 108 rangeland sub-system is split in two parts: a fraction  $\alpha$  is recycled directly  
 109 in the inorganic compartment of the rangeland sub-system ( $N_r$ ), the other  
 110 part  $(1 - \alpha)$  is transferred to the inorganic compartment of the cropland  
 111 sub-system ( $N_c$ ). The term  $T = (1 - \alpha)d_r P_r$  is the one which connects both  
 112 sub-systems.

113 In the cropland sub-system, harvest is not modelled as a continuous pro-  
 114 cess along the year but as a punctual event; it consists in the removal of a  
 115 percentage  $h$  of the plant compartment in the cropland sub-system every 365  
 116 days.

117 [Table 1 about here.]

118 The meta-ecosystem model has a particular structure hereafter named  
 119 “cascade structure” (Seibert and Suarez, 1990). Indeed, the rangeland sub-  
 120 system ( $r$ ), composed of the equations of  $P_r$  and  $N_r$ , does not depend on the  
 121 dynamics of the cropland sub-system ( $c$ ), that is composed of the equations  
 122 of  $P_c$  and  $N_c$ . This cascade structure enables to study independently the two  
 123 sub-systems. Moreover, the two sub-systems have the same organization and

both can be written under the following form:

$$\begin{cases} \dot{P} = uPN(1 - P/K) - mP - dP \\ \dot{N} = -uPN(1 - P/K) - eN + i + mP + \beta dP \end{cases} \quad (2)$$

- with  $P = P_r$  and  $N = N_r$  for the state variables and  $u = u_r$ ,  $K = K_r$ ,  $m = m_r$ ,  $e = e_r$ ,  $i = i_r$ ,  $\beta = \alpha$  and  $d = d_r$  for the parameters of the rangeland sub-system,
- with  $P = P_c$  and  $N = N_c$  for the state variables and  $u = u_c$ ,  $K = K_c$ ,  $m = m_c$ ,  $e = e_c$ ,  $i = i_c + T \frac{s_r}{s_c}$ ,  $\beta = 0$  and  $d = 0$  for the parameters of the cropland sub-system.

### 2.1. Equilibrium points of the generic sub-system model and their stability

Let us study the generic sub-system (2). For biological relevance, both plant biomass and inorganic nutrient stocks have to be positive ( $P, N \geq 0$ ). Furthermore the model is built such that plant biomass cannot exceed the carrying capacity  $K$ . We therefore only look at the equilibrium points that are inside the set  $[0, K] \times \mathbb{R}^+$ : it represents the set of  $(P, N)$  values of the dynamical system (2) that are biologically acceptable.

We show that the system (2) always admits one and only one stable equilibrium point whose value depends on the parameters. By definition, the equilibrium points  $(P^*, N^*)$  of the system (2) are the solutions of the following equations:

$$\begin{cases} uPN(1 - P/K) - mP - dP = 0 \\ -uPN(1 - P/K) - eN + i + mP + \beta dP = 0 \end{cases} \quad (3)$$

The trivial equilibrium point  $E_0$  corresponds to the case where there is no plant in the sub-system.  $E_0$  exists whatever the parameter values and is

144 given by:

$$E_0 := \left( P^* = 0, N^* = \frac{i}{e} \right) \quad (4)$$

145 The equilibrium point  $E_0$  is locally stable if and only if  $\frac{i}{e} < \frac{m+d}{u}$  (see Ap-  
146 pendix A.1) that is if the inputs of nutrient and the growth rate are not high  
147 enough to balance inorganic losses and plant mortality.

148 Let us now consider the case where  $P \neq 0$ . In this case, if  $(\beta - 1)d < 0$ ,  
149 then the system (3) can be written:

$$\begin{cases} P = K \frac{uN - c - d}{uN} =: F_1(N) \\ P = \frac{eN - i}{(\beta - 1)d} =: F_2(N) \end{cases} \quad (5)$$

150 Finding the solution of (5) amounts to find the intersection points of the  
151 curves  $F_1$  and  $F_2$ , which is done graphically in the sequel. The two functions  
152  $F_1$  and  $F_2$  are differentiable on  $\mathbb{R}^+ \setminus \{0\}$  and we have,  $\forall N \in \mathbb{R}^+ \setminus \{0\}$ :

$$F_1'(N) = \frac{K(m+d)}{uN^2} > 0 \quad \text{and} \quad F_2'(N) = \frac{e}{(\beta - 1)d} < 0 \quad (6)$$

153 We deduce from (6) that  $F_1$  is strictly increasing whereas  $F_2$  is decreasing on  
154  $\mathbb{R}^+ \setminus \{0\}$ . Moreover we have:  $\lim_{N \rightarrow 0^+} F_1(N) = -\infty$ ,  $\lim_{N \rightarrow +\infty} F_1(N) = K$ ,  
155  $F_2(0) = -\frac{i}{(\beta - 1)d} > 0$  and  $\lim_{N \rightarrow 0^+} F_2(N) = -\infty$ .

156  $F_1$  and  $F_2$  have therefore one and only one intersection point on  $\mathbb{R}^+ \setminus \{0\}$ .  
157 Let us now find the conditions for which this intersection point is biologically  
158 acceptable. We have:  $F_1(N) = 0 \Leftrightarrow N = \frac{m+d}{u}$  and  $F_2(N) = 0 \Leftrightarrow N = \frac{i}{e}$ .

159 The relative position of  $\frac{m+d}{u}$  and  $\frac{i}{e}$  of the N-axis determines if the in-  
160 tersection point is above or below the N-axis. Indeed, as we can see on  
161 the figure C.2, if  $\frac{i}{e} > \frac{m+d}{u}$  then the intersection point of  $F_1$  and  $F_2$  is  
162 above the N-axis: it is therefore a biologically acceptable equilibrium point

163  $E_1 := (P_1^*, N_1^*) \in \mathbb{R}^+ \times [0, K]$ . On the contrary, if  $\frac{i}{e} < \frac{m+d}{u}$  then the inter-  
 164 section of  $F_1$  and  $F_2$  is below the N-axis and it does not exist any positive  
 165 equilibrium point on  $[0, K] \times \mathbb{R}^+$ . Note that in this case the trivial equilibrium  
 166 point  $E_0$  is stable.

167 [Figure 2 about here.]

168 The analytic expression of the positive equilibrium point  $E_1$  can be ob-  
 169 tained by solving the equation  $F_1(N) = F_2(N)$  which leads to a second  
 170 degree polynomial equation (see Appendix A.1). The positive equilibrium  
 171 point  $E_1 := (P_1^*, N_1^*)$  is finally given by:

$$E_1 := \left( P^* = \frac{1}{2}(K - a\frac{i}{e} + a\frac{\sqrt{\Delta}}{u}), N^* = \frac{1}{2}\left(\frac{i}{e} + \frac{K}{a} + \frac{\sqrt{\Delta}}{u}\right) \right) \quad (7)$$

172 for  $(\beta - 1)d \neq 0$  with  $a = \frac{e}{(\beta-1)d}$  and  $\Delta = u^2(\frac{i}{e} + \frac{K}{a})^2 - 4u\frac{K}{a}(m+d)$  and only  
 173 exists if  $\frac{i}{e} > \frac{m+d}{u}$ . The analysis of its stability shows that when it exists,  $E_1$   
 174 is always locally stable (see Appendix A.1).

175 In the particular case where  $(\beta - 1)d = 0$ ,  $F_2(N)$  is a vertical line with a  
 176  $x$  coordinate  $N = \frac{i}{e}$ . The equilibrium point  $E_1$  becomes:

$$E_1 := \left( P^* = K \left( 1 - \frac{e(m+d)}{ui} \right), N^* = \frac{i}{e} \right) \quad (8)$$

177 Finally, there are two cases: the case where  $\frac{i}{e} > \frac{m+d}{u}$ , for which only  $E_1$   
 178 is stable and for which living conditions are met for plants to settle and the  
 179 case where  $\frac{i}{e} < \frac{m+d}{u}$ , for which only  $E_0$  is stable and for which plants cannot  
 180 survive.

### 181 3. Methods: optimization of the system at equilibrium and contri- 182 bution of control theory

183 In this section, we detail the optimization methods used to maximize the  
184 flux of crop production. This flux is represented by the term:

$$C = s_c P_{c,t_h}, \quad (9)$$

185 where  $s_c$  is the surface of the cropland sub-system and  $P_{c,t_h}$  the crop biomass  
186 in the cropland sub-system at the end of the year, at harvest time  $t_h$ .

187 In this paper, we will first apply a classical approach used in ecology,  
188 which consists in finding the constant values of parameters that optimize  
189 one or several quantities at equilibrium. We hereafter refer to the classical  
190 method of optimization at equilibrium with constant parameters simply as  
191 the optimization at equilibrium. The values of the optimal parameters will  
192 be obtained analytically if possible or numerically in other cases. To use  
193 this approach we have to assume that the biomass of plant has reached an  
194 equilibrium when the harvest occurs.

195 The optimization at equilibrium is well adapted to “organizational” prac-  
196 tices which are constant over time or at least fixed for a whole year, as it  
197 is the case for the rangeland to cropland surface ratio. These practices can  
198 be mathematically represented by setting constant values to parameters. On  
199 the other hand, this optimization at equilibrium is not adapted to study “in-  
200 terconnection” practices that may change over a year as it is the case for  
201 herbivory pressure. It is the very purpose of the control theory to take into  
202 account the fact that some input variables can vary over time. Secondly,  
203 we will thus study how performances of the agro-ecosystem may change if

204 grazing pressure change over the year by applying some methods developed  
205 in control theory.

### 206 3.1. Optimization at equilibrium method (constant parameters)

207 The cropland sub-system dynamics directly (and so the value of  $P_c$  and  $N_c$   
208 at equilibrium) depends on the rangeland sub-system management through  
209 the inputs term  $i = i_c + T \frac{s_r}{s_c}$  where  $T = (1 - \alpha)d_r P_r$ . Regarding the “in-  
210 terconnection” practices, we will focus on livestock management, which is a  
211 driver of the amount of nutrient per unit of surface that is transferred each  
212 day by livestock from the rangeland to the cropland sub-system:

$$T = (1 - \alpha)d_r P_r \quad (10)$$

213 Two parameters of the rangeland sub-system, the grazing pressure  $d_r$  and the  
214 recycled fraction  $\alpha$  are under farmer influence and may be used as driving  
215 forces to optimize the transfer  $T$  (and maximize the crop production). Both  
216  $d_r$  and  $\alpha$  influence crop production  $C$  through the term  $T$ . Thanks to the  
217 cascade structure of the meta-ecosystem model, we can study this indirect  
218 impact in three steps. First we will study the variation of crop production  
219  $C$  with  $T$ . Second, we will focus on the variation of  $T$  with  $\alpha$  and  $d_r$ .  
220 Finally we will also study the variation of crop production  $C$  with respect to  
221 the “organizational” practice in focus: the ratio between the surfaces of the  
222 cropland sub-system and the rangeland sub-system  $s_r$  and  $s_c$ . This ratio has  
223 an impact on both the crop biomass in cropland sub-system (in  $\text{kgN ha}^{-1}$ )  
224 and the crop production at the scale of the agro-ecosystem (in  $\text{kgN}$ ).

225 *3.2. Control method (varying herbivory pressure)*

226 The aim is to propose a mathematical expression of the grazing pressure  
 227  $d_r$  that makes the controlled system behave as desired. We want the crop  
 228 production  $C$  to reach a given value  $\tilde{C}$ , which corresponds to a value  $\tilde{P}_c = \frac{\tilde{C}}{s_c}$   
 229 of the crop biomass at harvest time. The mathematical expression of  $d_r$   
 230 will depend on the current state of the system and will take into account  
 231 the constraints on the parameters values: in our case,  $d_r$  is positive and  
 232  $\alpha \in [0, 1]$ . Contrary to the previous optimization at equilibrium (section  
 233 3.1), we do not assume that crop biomass has reached equilibrium when  
 234 the harvest occurs. Following the cascade structure of the system, we use a  
 235 control strategy composed of two feedback control loops (see figure C.3): a  
 236 first loop controlling  $P_c$  with  $T$ , the amount of nutrient transferred from the  
 237 rangeland to the cropland sub-system, and a second one controlling  $T$  with  
 238 the grazing pressure  $d_r$ . We use feedback linearizing control laws (Isidori,  
 239 1995) for the two loops and we choose some control parameters values that  
 240 fulfill the constraints on the inputs.

241 [Figure 3 about here.]

242 *3.2.1. First loop: controlling  $P_c$  with  $T$*

243 Let first design the feedback control law of the first loop. Remind that  
 244  $\tilde{P}_c$  is the value of  $P_c$  that we expect  $P_c$  to reach at the end of the year: it  
 245 is the set-point. Assume that  $T$  can be modified by the farmer all over the  
 246 year:  $T$  is the control input.



247 Consider the functions  $G_1$  and  $G_2$  defined by:

$$G_1(P_c, N_c) = u_c P_c N_c (1 - P_c/K_c) - m_c P_c, \quad (11)$$

$$G_2(P_c, N_c) = -u_c P_c N_c (1 - P_c/K_c) - e_c N_c + i_c + m_c P_c, \quad (12)$$

248 that are such that system (23) can be written:

$$\begin{cases} \dot{P}_c = G_1(P_c, N_c) \\ \dot{N}_c = G_2(P_c, N_c) + T \frac{s_r}{s_c} \end{cases} \quad (13)$$

249 The equation of  $P_c$  has a relative degree equal to 2 with respect to the  
250 control input  $T$ : it means that we need to derivate two times  $P_c$  to make  
251 appear explicitly the control input  $T$  in the equation of  $P_c$ . We thus get:

$$\ddot{P}_c = G_1 \partial_1 G_1 + \left( G_2 + T \frac{s_r}{s_c} \right) \partial_2 G_1. \quad (14)$$

252 To obtain a second order dynamics for  $P_c$ , that is, an equation of the form  
253  $\ddot{P}_c + 2\xi\omega\dot{P}_c + \omega^2 P_c = \omega^2 \tilde{P}_c$  where  $\omega$  is the cutoff frequency and  $\xi$  the damping  
254 factor, we have to consider the following expression of  $T$ :

$$T = \frac{u - G_1 \delta_1 G_1 - G_2 \delta_2 F}{\frac{s_r}{s_c} \delta_2 G_1} \quad (15)$$

255 with  $u = -2\xi\omega\dot{P}_c + \omega^2(\tilde{P}_c - P_c)$ . The expression (15) of  $T$  is called a “lin-  
256 earizing control law” as it enables to get a linear dynamics of the closed loop  
257 system. To be biologically acceptable, the value of  $T$  has to be positive; we  
258 so define the quantity  $\tilde{T}$  by:

$$\tilde{T} = \max \left( 0, \frac{-2\xi\omega G_1 + \omega^2(\tilde{P}_c - P_c) - G_1 \delta_1 G_1 - G_2 \delta_2 F}{\frac{s_r}{s_c} \delta_2 G_1} \right) \quad (16)$$

### 259 3.2.2. Second loop: controlling $T$ with the grazing pressure $d_r$

260 Contrary to what was assumed in the previous paragraph,  $T$  can not be  
 261 directly modified by the farmer.  $\tilde{T}$  has to be viewed as a set-point for  $T$ ,  
 262 that is a value that we expect  $T$  to be close of at any time. Note that, unlike  
 263 the first loop, the value of the set-point  $\tilde{T}$  varies over the time.  
 264 There are 2 driving forces in the agro-ecosystem which may control the value  
 265 of  $T$ :  $d_r$  and  $\alpha$ . In the sequel, we assume that  $\alpha$  is fixed and we consider  
 266  $d_r$  as the control input. The objective of the second control loop is therefore  
 267 to control  $T$  with  $d_r$ . The quantity  $T$  is given by  $T = (1 - \alpha)d_r P_r^*$ . After  
 268 derivation, we get:

$$\dot{T} = (1 - \alpha)\dot{d}_r P_r + (1 - \alpha)d_r \dot{P}_r \quad (17)$$

269 The equation of  $T$  is therefore of relative degree equal to 1 with respect to  
 270 the control input  $d_r$ . To obtain a first order dynamics for  $T$  in closed loop,  
 271 that is, an equation of the form  $\dot{T} = k(\tilde{T} - T) = k(\tilde{T} - (1 - \alpha)d_r P_r)$  with  $k$   
 272 a control parameter, we have to use the following control law for  $d_r$ :

$$\dot{d}_r = \frac{k(\tilde{T} - (1 - \alpha)d_r P_r) - (1 - \alpha)d_r \dot{P}_r}{(1 - \alpha)P_r}. \quad (18)$$

273 In practice, we add a saturation to this control law in order to ensure  
 274 that the applied value of  $d_r$  is realistic, that is non negative. The applied  
 275 control law is so:

$$d_r = \max(0, d) \text{ with } \dot{d} = \frac{k(\tilde{T} - (1 - \alpha)d P_r) - (1 - \alpha)d \dot{P}_r}{(1 - \alpha)P_r}. \quad (19)$$

### 276 3.2.3. Choice of the control parameters values

277 The values of the control parameters ( $\omega$ ,  $\xi$  and  $k$ ) are chosen to make the  
 278 system stabilize after about 365 days. To ensure that, we consider the tow  
 279 loops independently.

280 The first loop is the slowest one: it will impose its stabilization time to  
 281 the whole system. The closed loop equation of  $P_c$  is a second order one,  
 282 with parameters  $\xi$  and  $\omega$ . For such systems, an approximation of the 1%  
 283 settling time, that is the time necessary for the response to reach and remain  
 284 in the interval  $[0.99\tilde{P}_c, 1.01\tilde{P}_c]$  (with  $\tilde{P}_c$  the set-point), is given by Goodwin  
 285 and Sin (1984)  $\frac{4.6}{\xi\omega}$ . The value of  $\xi$  will also determine whether the output  
 286 will oscillate before stabilizing or not. For values of  $\xi$  greater than 0.7, the  
 287 response is very damped, there is almost no oscillations. For this reason, we  
 288 chose to take a value of  $\xi$  equal to 0.9. To get a 1% settling time smaller  
 289 than 365 days, we have to consider a value of  $\omega$  that is such that:

$$\frac{4.6}{\xi\omega} < 365 \Leftrightarrow \omega > \frac{4.6}{365\xi} \simeq 0.014 \text{ (with } \xi = 0.9) \quad (20)$$

290 We chose to take a value of  $\omega$  equal to 0.015 that is a little bit greater  
 291 than 0.014: it ensures that the system will have reached the set-point at the  
 292 harvest time. For  $\omega = 0.015$  and for  $\xi = 0.9$ , we indeed have  $\frac{4.6}{\xi\omega} \simeq 341$  days.

293 The second loop, which is the internal loop, has to be faster than the first  
 294 one in order to ensure the stability of the first loop. In general, we try to  
 295 have a settling time at least 1000 times faster in the internal loop, which in  
 296 our case corresponds to a maximum value of 0.34 days. The equation of  $T$  is  
 297 a first order system with time constant  $\frac{1}{k}$ . For such systems, the 1% settling  
 298 time is about  $\frac{5}{k}$  (Corriou, 2004). We therefore have:

$$\frac{5}{k} < 0.34 \Leftrightarrow k > \frac{5}{0.34} \simeq 14.7 \quad (21)$$

299 We chose to take a value of  $k$  equal to 20 that is a little bit greater than 14.7.

300 So finally, the values of the control parameters used for the control strat-

egy are the following ones:

$$\omega = 0.015, \xi = 0.9 \text{ and } k = 20. \quad (22)$$

## 4. Results

*4.1. Cropland sub-system as a sink: the greater the transfers, the greater the crop production*

We first focus on the sub-system (c) that corresponds to cropland and look at the variation of the crop production with respect to the nutrient transfer  $T$ .

Remind that the equations of the cropland sub-system are of the form (2) with  $P = P_c$  and  $N = N_c$  for the state variables and  $u = u_c$ ,  $K = K_c$ ,  $m = m_c$ ,  $e = e_c$ ,  $i = i_c + T \frac{s_r}{s_c}$ ,  $\beta = 0$  and  $d = 0$  for the parameters (with  $T = (1 - \alpha)d_r P_r$ ):

$$\begin{cases} \dot{P}_c &= u_c P_c N_c (1 - P_c / K_c) - m_c P_c \\ \dot{N}_c &= -u_c P_c N_c (1 - P_c / K_c) - e_c N_c + i_c + m_c P_c + T \frac{s_r}{s_c} \end{cases} \quad (23)$$

From section 2.1, the cropland sub-system always admits one stable equilibrium point which depends on the parameter values and on the value  $P_r^*$  reached by  $\bar{P}_r$  at equilibrium. If  $\frac{i_c + T^* \frac{s_r}{s_c}}{e_c} \leq \frac{m_c}{u_c}$ , this equilibrium point is given by:

$$E_{0,c} := \left( P_c^* = 0, N_c^* = \frac{i_c + T^* \frac{s_r}{s_c}}{e} \right) \quad (24)$$

and if  $\frac{i_c + T^* \frac{s_r}{s_c}}{e_c} > \frac{m_c}{u_c}$ , it is given by:

$$E_{1,c} := \left( P_c^* = K_c \left( 1 - \frac{m_c}{u_c} \frac{e_c}{i_c + (1 - \alpha)d_r P_r^* \frac{s_r}{s_c}} \right), N_c^* = \frac{i_c + T^* \frac{s_r}{s_c}}{e_c} \right) \quad (25)$$

As explained in section 3, the parameters  $d_r$  and  $\alpha$  impact on the crop production  $C$  through the quantity  $T^* = (1 - \alpha)d_r P_r^*$ . We therefore study the variation of  $C$  with respect to  $T^*$  in order to determine the value of  $T^*$  that maximizes  $C$ .

[Figure 4 about here.]

From section 2.1, we know that, when  $\frac{i_c + T^* \frac{s_r}{s_c}}{e_c} > \frac{m_c}{u_c}$ ,  $P_c^*$  is given by the intersection point between the curve of  $F_1 : N_c \mapsto K_c \frac{u_c N_c - m_c}{u_c N_c}$  and the vertical line of equation  $N_c = \frac{i_c + T^* \frac{s_r}{s_c}}{e_c}$ . When  $T^*$  increases, the curve of  $F_1$  stays at the same place whereas the vertical line is shifted to the right (figure C.4). Consequently the equilibrium point moves to the right on the curve of  $F_1$ . As  $F_1$  increases,  $P_c^*$  increases.

This can also be shown analytically because we have:

$$\frac{dP_{1,c}^*}{dT^*} = \frac{d}{dT^*} \left( K_c \left( 1 - \frac{m_c}{u_c} \frac{e_c}{i_c + T^* \frac{s_r}{s_c}} \right) \right) = \frac{K_c m_c s_r}{u_c s_c} \frac{e_c}{(i_c + T^* \frac{s_r}{s_c})^2} > 0. \quad (26)$$

Finally, as  $\frac{dC}{dT^*} = s_c \frac{dP_{1,c}^*}{dT^*}$ , we show that the crop production  $C$  is increasing with  $T^*$  and therefore reaches its maximal value for the maximal value  $T_{max}^*$  that  $T^*$  can take in  $\left[ \max(0, \frac{s_r}{s_c} (\frac{e_c u_c}{u_c} - i_c)), +\infty \right)$ . However, it is important to notice that the relationship between  $C := s_c P_{1,c}^*$  and  $T^*$  is nonlinear. The closer  $P_{1,c}^*$  is to  $K_c$ , the less profitable is the increase of  $T^*$ . For example, as we can see on the figure C.4, for a same increase of  $T^*$  between  $T_2^*$  and  $T_3^*$  and between  $T_1^*$  and  $T_2^*$  (i.e  $T_3^* - T_2^* = T_2^* - T_1^*$ ), the increase of crop biomass  $P_c^{*,3} - P_c^{*,2}$  is inferior to  $P_c^{*,2} - P_c^{*,1}$ . This non-linearity in the efficiency of the inputs impacts the optimization of the cropland sub-system to rangeland sub-system surfaces ratio (see 4.3).

339 4.2. Rangeland sub-system as a source: maximizing the transfers without  
340 jeopardizing plant biomass

341 After studying the variation of crop production with  $T^*$ , we now look at  
342 the variation of  $T^*$  with the two driving forces  $\alpha$  and  $d_r$ ; we search for the  
343 values of  $\alpha$  and  $d_r$  that maximize the amount  $T^* = (1 - \alpha)d_r P_r^*$  of transferred  
344 nutrient.

345 For that, we now focus on the rangeland sub-system ( $r$ ) only. Remind  
346 that the equations of this sub-system are written under the form (2) with  
347  $P = P_r$  and  $N = N_r$  for the state variables and  $u = u_r$ ,  $K = K_r$ ,  $m =$   
348  $m_r$ ,  $e = e_r$ ,  $i = i_r$ ,  $\beta = \alpha$  and  $d = d_r$  for the parameters:

$$\begin{cases} \dot{P}_r &= u_r P_r N_r (1 - P_r / K_r) - m_r P_r - d_r P_r \\ \dot{N}_r &= -u_r P_r N_r (1 - P_r / K_r) - e_r N_r + i_r + m_r P_r + \alpha d_r P_r \end{cases} \quad (27)$$

349 From the section 2.1 and similarly to the cropland sub-system, the range-  
350 land sub-system always admits one stable equilibrium, which depends on the  
351 parameter values. If  $\frac{i_r}{e_r} \leq \frac{m_r + d_r}{u_r}$ , this equilibrium point without plant is given  
352 by

$$E_{0,r} = \left( P_r^* = 0, N_r^* = \frac{i_r}{e_r} \right) \quad (28)$$

353 and if  $\frac{i_r}{e_r} > \frac{m_r + d_r}{u_r}$ , it is given by:

$$E_{1,r} = \left\{ \left( P_r^* = \frac{1}{2} \left( K_r - \frac{i_r}{e_r} a + \frac{\sqrt{\Delta}}{u_r} a \right), N_r^* = \frac{1}{2} \left( \frac{i_r}{e_r} + \frac{K_r}{a} + \frac{\sqrt{\Delta}}{u_r} \right) \right) \right\} \quad (29)$$

354 with  $\Delta = u_r^2 \left( \frac{i_r}{e_r} + \frac{K_r}{a} \right)^2 - 4u_r \frac{K_r}{a} (m_r + d_r)$  and  $a = \frac{e_r}{(\alpha - 1)d_r} < 0$  if  $(\alpha - 1)d_r \neq 0$ .

355 The case  $(\alpha - 1)d_r = 0$  corresponds to an absence of grazing pressure or a  
356 case where all the uptake by livestock is recycled in the rangeland sub-system.

In this case, the equilibrium point  $E_{1,r}$  becomes:

$$E_{1,r} = \left\{ \left( P_r^* = K_r \left( 1 - \frac{e_r(m_r+d_r)}{u_r i_r} \right), N_r^* = \frac{i_r}{e_r} \right) \right\} \quad (30)$$

Let's now look at how the nutrient transfer  $T^* = (1 - \alpha)d_r P_r^*$  varies with  $\alpha$  and  $d_r$ .

#### 4.2.1. Minimizing the recycled fraction ( $\alpha$ ) to increase the transferred amount ( $T^*$ )

The first driving force that we consider is the parameter  $\alpha$ , which represents the percentage of the grazed part of  $P_r$  which, after transformation by livestock, is recycled to the inorganic nutrient compartment  $N_r$ . The other fraction  $(1 - \alpha)$  is exported to the cropland sub-system.  $\alpha$  can vary from 0 (all nutrient is transferred to the cropland sub-system) to 1 (all nutrient is recycled within the rangeland sub-system). We are looking for the value of  $\alpha$  that maximizes the amount  $T^*$  of nutrient transferred.

When  $\frac{i_r}{e_r} \leq \frac{m_r+d_r}{u_r}$ , there is no positive equilibrium point whatever the value of  $\alpha$ :  $P_r^* = 0$  for all  $\alpha \in [0, 1]$ . If we consider the case where  $\frac{i_r}{e_r} > \frac{m_r+d_r}{u_r}$ , as depicted in figure C.5,  $P_{1,r}^*$  and  $N_{1,r}^*$  increase with  $\alpha$ . Indeed when  $\alpha$  increases from 0 to 1, the curve of  $F_1$  does not move but the slope of  $F_2$  goes from  $-\frac{e_r}{d_r}$  to  $-\infty$  by rotating around the point  $(\frac{i_r}{e_r}, 0)$  clockwise (see figure C.5a). The intersection point between  $F_1$  and  $F_2$  moves therefore to the right on the curve of  $F_1$ : thus  $P_{1,r}^*$  and  $N_{1,r}^*$  increase with  $\alpha$ . The mathematical proof is given in Appendix B.1

[Figure 5 about here.]

Let us consider the function  $\alpha \in [0, 1] \mapsto T^* = (1 - \alpha)d_r P_r^*$  and let us denote  $T_{max}^*$  the maximal value taken by  $T^*$  on  $[0, 1]$ .

380 If  $\frac{i_r}{e_r} \leq \frac{m_r+d_r}{u_r}$ , then  $P_r^* = 0$  for all  $\alpha \in [0, 1]$ . The function  $\alpha \in [0, 1] \mapsto$   
 381  $T^* = (1 - \alpha)d_r P_r^*$  is therefore null on  $[0, 1]$  and  $T_{max}^* = 0$ .

382 Let us now consider the case where  $\frac{i_r}{e_r} > \frac{m_r+d_r}{u_r}$ . We showed earlier  
 383 that the quantity of biomass produced in the rangeland sub-system ( $P_{1,r}^*$ )  
 384 increases with  $\alpha$ . The less we transfer, the higher this biomass is. However,  
 385 we can wonder how the quantity  $T^* = (1 - \alpha)d_r P_{1,r}^*$  varies with  $\alpha$ . Indeed,  
 386 whereas  $P_{1,r}^*$  increases with  $\alpha$ , the quantity  $1 - \alpha$  decreases. This could lead  
 387 to a trade-off between the biomass produced  $P_{1,r}^*$  and the fraction  $(1 - \alpha)$  of  
 388 nutrient transferred to the cropland sub-system.

389 From (27), we can show that the amount of transferred nutrient  $T$  is  
 390 written:

$$T^* = (1 - \alpha)d_r P_{1,r}^* = i_r - e_r N_{1,r}^* \quad (31)$$

391 From this equation (31), we deduce that  $T^*$  is maximal for the minimal  
 392 value of  $N_{1,r}^*$ . Yet, we saw that both  $N_{1,r}^*$  and  $P_{1,r}^*$  increase with  $\alpha$ . As a  
 393 consequence, the quantity  $T^*$  is maximal for  $\alpha = 0$ : it corresponds to the  
 394 case where all nutrient is transferred to the cropland sub-system. So there is  
 395 no trade-off and we have

$$T_{max}^* = T(\alpha = 0) = d_r P_{1,r}^* \quad (32)$$

396 Finally, we showed that it is necessary to minimize the recycled fraction  $\alpha$   
 397 of livestock uptake into rangeland sub-system to maximize the total amount  
 398 of nutrient transferred to the cropland sub-system.



399 *4.2.2. Maximizing the transferred amount  $T^*$  with an intermediate grazing*  
 400 *pressure  $d_r$*

401 The second driving force that we can consider in this system is the graz-  
 402 ing pressure  $d_r$ , a parameter which takes into account the size of the herd  
 403 multiplied by the time spent by the herd on the rangeland sub-system. The-  
 404 oretically  $d_r$  can vary from 0 (no livestock) to  $\infty$ . In practice, there exists a  
 405 bound for  $d_r$  which corresponds to the maximal herd size that can graze on  
 406 the rangeland sub-system multiplied by the maximum of time that the herd  
 407 can spend in the rangeland sub-system. The grazing pressure is also bounded  
 408 by the capacity of rangeland to produce enough biomass to feed the livestock,  
 409 and by the resistance of rangeland to overgrazing. In the model, the con-  
 410 ditions of existence of the positive equilibrium  $E_{1,r}$  that lead to a non-zero  
 411 value of the transfer  $T^*$  require that  $d_r$  must be smaller than  $\frac{u_r i_r}{e_r} - m_r$ . If  
 412  $d_r \geq \frac{u_r i_r}{e_r} - m_r$ , there is not enough nutrient left in the rangeland sub-system  
 413 to maintain the plant biomass: in that case of overgrazing, the rangeland  
 414 sub-system goes to the equilibrium point  $E_{0,r}$ .

415 [Figure 6 about here.]

416 We are looking for the value of  $d_r$  in  $(0, \frac{u_r i_r}{e_r} - m_r)$  that maximizes the  
 417 transferred amount  $T^*$ . We set  $\alpha$  to 0, since it is the value of  $\alpha$  for which  
 418  $T^*$  is maximal whatever the value of  $d_r$ . However, we obtain similar results  
 419 for a non-zero value of  $\alpha$ . As in section 4.2.1, we deduce from the expression  
 420 (31) of  $T$  that  $T^*$  is maximal for the minimal value of  $N_r^*$ . Moreover, we  
 421 know that  $T^* = 0$  for  $d_r = 0$  and for  $d_r = \frac{u_r i_r}{e_r} - m_r$  (case of overgrazing  
 422 that implies  $P_r^* = 0$ ). As  $T^*$  is a continuous and positive function of  $d_r$  on

423  $(0, \frac{u_r i_r}{e_r} - m_r)$ , we deduce that the amount of nutrient transferred depends  
 424 on a trade-off between the grazing pressure and the plant biomass in the  
 425 rangeland sub-system .

426 Therefore, there exists a value  $d_r^{opt} \in (0, \frac{u_r i_r}{e_r} - m_r)$  of  $d_r$  for which  $T^*$  is  
 427 maximal on  $(0, \frac{u_r i_r}{e_r} - m_r)$ . It also means that  $T^*$  and  $N_r^*$  are not strictly  
 428 monotonic on  $(0, \frac{u_r i_r}{e_r} - m_r)$ . Figure C.6 shows the variation of  $P_r^*$ ,  $N_r^*$  and  $T^*$   
 429 with  $d_r \in (0, \frac{u_r i_r}{e_r} - m_r)$  for  $\alpha = 0$ . We see that on the interval  $(0, \frac{u_r i_r}{e_r} - m_r)$ ,  $P_r^*$   
 430 decreases with  $d_r$ . Indeed, when  $d_r$  increases from 0 to  $\frac{u_r i_r}{e_r} - m_r$ , the curve of  
 431  $F_1$  moves to the right and the slope of  $F_2$  goes from  $-\infty$  to  $\frac{e_r^2}{(\alpha-1)(u_r i_r - m_r e_r)} < 0$   
 432 by rotating anticlockwise around the point  $(\frac{i_r}{e_r}, 0)$  (see figure C.5b). The  
 433 intersection point between  $F_1$  and  $F_2$  moves therefore to the left on the curve  
 434 of  $F_1$ : so  $P_{1,r}^*$  decreases with  $d_r$ . However, as the curve of  $F_1$  is shifted to the  
 435 right when  $d_r$  increases, the variation of  $N_{1,r}^*$  is not easily noticeable. We see  
 436 on Figure C.6 and C.5 that  $N_r^*$  first decreases and then increases with  $d_r$ : it  
 437 therefore admits a global minimum on  $d_r^{opt} \in (0, \frac{u_r i_r}{e_r} - m_r)$ .

438 As we deduced earlier from equation (31), we also observe that the value  
 439  $d_r^{opt} \in (0, \frac{u_r i_r}{e_r} - m_r)$  of  $d_r$  which minimizes  $N_r^*$  is the same as the one which  
 440 maximizes the transfer  $T^* = (1 - \alpha)d_r P_r^*$  from the rangeland sub-system to  
 441 the cropland sub-system.

442 Among the set of grazing pressure values that do not compromise the  
 443 plant production in the rangeland sub-system, there is a value that optimizes  
 444 the amount of nutrient transferred to the cropland sub-system. Moreover,  
 445 we observe that this value  $d_r^{opt}$  is a low value among the set of possible values  
 446 of grazing pressure  $(0, d_r^{max})$ .

447 4.3. *Surface ratio of cropland:rangeland sub-systems affects the trade-off be-*  
 448 *tween plan biomass in cropland sub-system and production at the scale*  
 449 *of the agro-ecosystem*

450 We now look at the variation of the crop production with respect to the  
 451 respective surfaces of cropland and rangeland sub-systems ( $s_c$  for cropland  
 452 sub-system and  $s_r$  for rangeland subsystem). At the agro-ecosystem scale,  
 453 the maximization of crop production  $C$  does not depend on the absolute value  
 454 of the surface of the subsystems but depends on the relative surface area of  
 455 each subsystem in relation to the total surface area of the agro-ecosystem.  
 456 When the surface of cropland sub-system  $s_c$  increases (at the expense of  
 457 the rangeland sub-system), a positive effect comes from the increase of the  
 458 surface devoted to crop production (equation 9) whereas a negative effect  
 459 comes from the dilution effect on the amount of transferred nutrient ( $T^* \frac{s_r}{s_c}$ ).

460 Let  $\gamma = \frac{s_c}{s_r + s_c}$  be the fraction of the agro-ecosystem surface  $S := s_r + s_c$   
 461 occupied by the cropland sub-system and  $1 - \gamma = \frac{s_r}{s_r + s_c}$  the fraction occupied  
 462 by the rangeland sub-system. The fraction occupied by cropland/rangeland  
 463 impacts the crop production directly as  $C := s_c P_c^* = \gamma S P_c^*$  and indirectly  
 464 through the quantity  $T^* \frac{s_r}{s_c} = T^* \frac{1-\gamma}{\gamma}$  on which depends the value  $P_c^*$ .

465 We are looking for the value of  $\gamma$  in  $(0, 1]$  that maximizes the crop pro-  
 466 duction  $C$ . We can show (see Appendix B.2 for mathematical proofs) that  
 467 there exist two different situations depending on if the inputs  $i_c$  are greater  
 468 or smaller than a threshold value  $i^+$  defined as:

$$i^+ = \frac{e_c m_c + \sqrt{e_c m_c (e_c m_c + 4u_c T)}}{2u_c} > 0. \quad (33)$$

469 If  $i_c > i^+$ , then the inputs  $i_c$  are high enough to ensure a good production  
 470 in the cropland sub-system without any transfer. In that case, the transfer

is no more interesting for crop production, and it becomes more profitable to cultivate the maximum available surface ( $\gamma = 1$ ). The maximal value  $C_{max}$  of  $C$  is then given by:

$$C_{max} = SP_{1,c}^*(\gamma = 1) = SK_c \left( 1 - \frac{m_c e_c}{u_c i_c} \right). \quad (34)$$

If  $i_c < i^+$ , then the inputs  $i_c$  are not high enough and there exists an optimal value  $\gamma^{opt}$  of  $\gamma$  for which the crop production  $C$  is maximal. In that case, the optimal value  $\gamma^{opt}$  depends on the trade-off between the crop biomass and the surface of the cropland sub-system (see figure C.7).

[Figure 7 about here.]

$\gamma^{opt}$  is given by:

$$\gamma^{opt} = \frac{T}{P^{opt} + e_c N^{opt} + T - i_c} \text{ with } N^{opt} = \frac{K_c m_c}{u_c (K_c - P^{opt})} \quad (35)$$

where  $P^{opt}$ , the value of  $P_c^*$  for which the crop production  $C$  is maximal, is expressed:

$$P^{opt} = K_c \left[ 1 - \frac{-e_c m_c + \sqrt{e_c m_c [e_c m_c + u_c (T - i_c)]}}{u_c (T - i_c)} \right] \in [0, K_c]. \quad (36)$$

The maximal value  $C_{max}$  of  $C$  is then given by:

$$C_{max} = \gamma^{opt} SP^{opt}. \quad (37)$$

When atmospheric depositions and other inputs independent of livestock are not sufficiently high in the cropland sub-system compared to the capacity of transfer of the rangeland sub-system, it is adequate to keep both rangeland and cropland surfaces in the agro-ecosystem to optimize crop production.

4.4. *Changing functioning of the agro-ecosystem by changing grazing pressure over the year*

In this section, we give the results obtained with the control strategy described in section 3.2, that corresponds to the case where the grazing pressure can vary over the year.

4.4.1. *A same production with less transfer, a higher production with as much?*

[Figure 8 about here.]

We first look at the maximal crop production value that the system can reach with the control strategy. Figures C.8a, C.8b and C.8c represent the average stock of plant in rangeland over a year ( $\frac{1}{t_h} \int_0^{t_h} P_r(t)dt$ ), the stock of plant in cropland at harvest time ( $P_{c,t_h}$ ) and the total amount of nutrient transferred over a year ( $\int_0^{t_h} T(t)dt$ ) with respect of the value of the set-point  $\tilde{P}_c$ . The values obtained with the optimization at equilibrium (that is with a constant value  $d_r = d_r^{opt}$ ) are represented by the horizontal straight lines. The black and gray vertical straight lines mark the values of  $P_c^{opt}$  and of the maximal crop production value reached with the control strategy respectively. The set-point  $\tilde{P}_c$  is reached in all the cases until  $\tilde{P}_c = 1.085 \times P^{opt}$ . Above this point the feedback control fails to make  $P_c$  reach the set-point: the cropland sub-system “collapses” and we get  $P_c = 0$ . As the value of the set-point  $\tilde{P}_c$  increases from 0 to  $1.085 \times P^{opt}$ , the average stock of plant in rangeland ( $\frac{1}{t_h} \int_0^{t_h} P_r(t)dt$ ) decreases and the total amount of nutrient transferred over a year ( $\int_0^{t_h} T(t)dt$ ) increases. By simulations, we observe that the total amount of nutrient transferred over a year obtained for the highest crop production

511 value ( $\tilde{P}_c = 1.085 \times P^{opt}$ ) with the control strategy is equal to the total  
 512 amount of nutrient transferred over a year ( $t_h \times T_{max}^*$ ) obtained with the  
 513 optimization at equilibrium (that is for a constant value of  $d_r = d_r^{opt}$ ).

#### 514 4.4.2. Comparison of scenarios

515 Let us now compare the different optimization strategies, through the  
 516 simulated trajectories of 3 different scenarios.

517 The first scenario (1) consists in the application of the optimal values  
 518 of  $d_r$ ,  $\alpha$  and  $\gamma$  that have been calculated with the classical approach of  
 519 optimization at equilibrium as constant values. We set  $\alpha = 0$  (see section  
 520 4.2.1) and get the optimal value of  $d_r^{opt}$  by simulation (see section 4.2.2). From  
 521 the so-obtained value of  $T^*$ , we compute the optimal value  $\gamma^{opt}$  (see equation  
 522 35) of  $\gamma$  that determines the surfaces  $s_r$  and  $s_c$  for which  $C$  is maximal (see  
 523 equation 34). This scenario is our “control sample” to be compared with  
 524 two other scenarios (2) and (3) obtained with the control theory for different  
 525 values of the set point  $\tilde{P}_c$ . For scenario (2),  $\tilde{P}_c$  is defined as the optimal value  
 526 of crop biomass obtained with the optimization at equilibrium ( $\tilde{P}_c = P^{opt}$ )  
 527 and for scenario (3),  $\tilde{P}_c$  is defined as the maximal value reached numerically  
 528 with the feedback control (see C.8).  $\tilde{P}_c$  is equal to 1.085 times the optimal  
 529 value obtained with the optimization at equilibrium ( $\tilde{P}_c = 1.085 P^{opt}$ ). There  
 530 is no difference in the parametrization of the scenarios (1), (2) and (3) except  
 531 for the expression of the grazing pressure  $d_r$ .

532 Figures C.9a, C.9b, C.9c and C.9d represent the dynamics of plant and  
 533 inorganic nutrient in the cropland sub-system and the rangeland sub-system  
 534 over five years for the 3 scenarios. Scenarios (1), (2) and (3) are plotted in  
 535 black lines, dotted black lines and grey lines respectively.

In the scenario (1), only the stocks of the cropland sub-system change over the year and the equilibrium point is reached each year. These changes are due to the harvest event that occurs at the end of each year. In this scenario, both scenarios 2 and 3 reach their set-point  $\tilde{P}_c$ . We observe that the growth of plants is slower with feedback than without feedback (figure C.9b). Note that in scenario (2) and (3), we are no longer looking for the solution at equilibrium but instead we used the transient dynamics to reach the set-point at the time of the harvest.

With the feedback control, the dynamics of nutrient in the rangeland sub-system becomes dependent of the state of the cropland sub-system and varies over time following the dynamics of  $d_r$ . In both scenarios, levels of plant and inorganic nutrient in the rangeland sub-system are always higher than for scenario 1. Considering that the end of the year matches with the harvest event, we observe in scenarios (2) and (3) that the value of  $d_r$  is minimal in the middle of the year (figure C.9e). The low value of  $d_r$  promotes the increase of plant biomass in the rangeland sub-system  $P_r$  and as a consequence, the increase of the inorganic nutrient level  $N_r$  (figure C.9b,d). Note that in the scenario (3), the value of  $d_r$  is a bit higher than  $d_r^{opt}$  but much lower than the value above which there is overgrazing ( $\frac{u_r i_r}{e_r} - m_r$ ).

These higher values of  $P_r$  and  $N_r$  allow a higher transfer of nutrient for the growth of cropland plants during the second part of the year. Indeed, as depicted in figure C.9f the amount of nutrient transferred over time decreases slightly at the beginning of the year and then increases to be maximal at the end of the year. The calculation of the total amount of nutrient transferred during a year gives 237.3 kgN y<sup>-1</sup> for scenario (1), and respectively 216.1 and

234.7 kgN y<sup>-1</sup> for scenarios (2) and (3).

In scenario (2) and (3), losses are lower in the rangeland sub-system and higher in the cropland sub-system compared to scenario (1). At the scale of the agro-ecosystem the lowest losses are obtained for scenario (2) (249.1 kgN y<sup>-1</sup>). Losses are a bit higher in scenario (3) (254.1 kgN y<sup>-1</sup>) than in scenario (1) (253.7 kgN y<sup>-1</sup>). In scenario (3), higher losses associated with a larger biomass at the end of the year are due to the fact that loss rates are much lower in the rangeland sub-system than in the cropland sub-system, and that the average biomass (over a year) in the cropland sub-system is lower in scenario (3) than in scenario (1).

[Figure 9 about here.]

Hence, by constantly adjusting the grazing pressure over a year, one can promote a higher crop production, thanks to the transient dynamics. Indeed, as we showed, bringing nutrient to crops at the right time when they require it for growth maximizes the overall crop production over a year.

## 5. Discussion

We addressed the optimization of crop production in a mixed farming system, using a simple meta-ecosystem model. Our purpose was to find the optimal values of three quantities of interest to maximize crop production, namely, the fraction of the agro-ecosystem occupied by the cropland sub-system  $\gamma$ , the grazing pressure in rangeland  $d_r$  and the fraction of nutrient transferred to cropland by livestock  $(1 - \alpha)$ . We first used the classical method of optimization at equilibrium to maximize the production when



these quantities are constant over time. Second, we used methods from the control theory to optimize crop production when the grazing pressure  $d_r$  can be constantly adjusted,  $\alpha$  and  $\gamma$  being kept constant.

With the optimization at equilibrium, our results suggest that maximizing crop production is obtained by maximizing the flux of nutrient transferred by livestock from rangeland to cropland. To maximize this flux, the following conditions must be met:

1. the fraction  $\alpha$  of nutrient ingested by livestock and recycled within the rangeland must be minimal, the best strategy being to transfer all the nutrient ingested by livestock to cropland.
2. the grazing pressure  $d_r$  cannot be higher than a threshold value that leads to the overgrazing of the rangeland, with the extirpation of plants at steady-state. Hence, there is a trade-off between exporting as much nutrient as possible from cropland and avoiding overgrazing.
3. Once the driving forces  $d_r$  and  $\alpha$  related to livestock are optimized, it is possible to determine the optimal value of the rangeland to cropland surface ratio  $\gamma/(1-\gamma)$ . This ratio strongly depends on the quantity of inputs independent from livestock in the cropland sub-system  $i_c$  compared to the capacity of transfer of the rangeland sub-system  $T$ .

The control theory allows a further gain of optimization, based on a better fit of the driving forces to the transient dynamics of the system. By constantly adjusting the grazing pressure over time, we showed that it is possible to obtain the same yearly crop production with a lower yearly amount of nutrient transferred by livestock. We even show that it is possible to reach higher values of crop biomass.

Our model allows a better understanding of the mechanisms that lead to optimization. First, we note that the maximal flux of nutrient transferred corresponds to the case where the stock of inorganic nutrient in the rangeland is minimal, a result similar to what has been shown in previous models (Boudsocq et al., 2009, Loreau, 1998). Indeed, these studies showed that primary productivity was maximized when the inorganic resource was minimized. Since  $T^*$  is proportional to the primary productivity obtained in rangeland, our result is consistent with these other nutrient cycling models. This arises from the fact that the inputs of nutrient in the rangeland sub-system are independent on plant biomass. Second, note that there are two output fluxes of nutrient from the rangeland sub-system: one in organic form, through the grazing of plant biomass followed by an exportation to cropland, and one in inorganic form through erosion and leaching of the inorganic stock. Therefore, maximizing the flux of nutrient transferred by livestock consists in minimizing the inorganic losses in favors of the organic output.

With our model, when the grazing pressure is adjusted over time, we observe that to maximize the crop biomass at the moment of harvest, one must boost the growth of crops during the second part of the year. This unexpected result comes from the fact that both the plant growth function and the mortality rate stay constant all along the year. In reality, the deposition of manure on the one hand, and the growth of plants, on the other hand, are decoupled over the year. For instance, in West Africa, manure deposition occurs during the dry season, when the livestock is corralled at night in the cropland, whereas growth occurs during the wet season. During the dry

season, the livestock is kept in rangeland to protect the crops (Manlay et al., 2004, Guerin and Roose, 2015).

With the control method, we obtain a grazing pressure curve (figure C.9e) and a plant biomass curve in the rangeland sub-system (figure C.9b) whose shapes are similar to the ones obtained by Chen and Wang (1988). In their work, these authors used optimal control theory to determine the maximal potential productivity of grassland under grazing over a year. In our study, we showed that maximizing the integral of the transfers over a year does not guarantee to get the maximal crop production at harvest time. Indeed, for the same yearly average, a time-varying transfer can lead to greater crop production than a constant one. Note that using time-varying inputs to improve the performance of a system is a well-known practice (see for example (Ruan and Chen, 1996) in the case of a fermentation process).

Thanks to the control method which allows to implement a time-varying grazing pressure and with the set of parameters used in our simulation, we succeeded to reach a crop production (set-point  $\tilde{P}_c$ ) 1.085 times higher (8.5% increase) than the optimal crop production obtained with optimization at equilibrium. Above this value, the grazing pressure calculated by the feedback linearizing control law (equation 19) does not allow plants to subsist in the rangeland sub-system (overgrazing) and the dynamical system does not reach the desired crop production. In both the “classical” scenario (1) and the scenario where time-varying grazing pressure is applied and the optimal crop production value is reached (3), the total annual amount of transferred nutrient was almost the same. It suggests that the control on  $d_r$  has almost no effect on the transfer capacity of the rangeland sub-system.

659 In this study, we only applied the control theory to the grazing pressure  $d_r$ .  
 660 Yet, we could have applied it as well to the fraction of nutrient recycled in the  
 661 rangeland sub-system over a year  $\alpha$ . In this case, as suggested by simulations  
 662 (not shown), we would observe similar dynamics but with a lower maximal  
 663 value of crop production. This result is partly due to the constraints on the  
 664 value of the fraction of recycled nutrient that is bounded between 0 and 1,  
 665 but it also arises from the fact that to maximize nutrient transfer, it is more  
 666 profitable to minimize  $\alpha$  than to maximize  $d_r$ .

667 We could as well apply the control theory on both  $d_r$  and  $\alpha$ . In this  
 668 case, possibilities of control are numerous, and some of them might allow a  
 669 higher crop production  $\tilde{P}_c$  than the one obtained with only one parameter  
 670 as control input. However, in the simulations we performed, the recycled  
 671 fraction always went to 0 after a few years (simulations not shown), which  
 672 finally amounts to control the system with the grazing pressure as the only  
 673 control input.

674 In the paper, we assume that farmers have the full control on the three  
 675 driving forces in focus. We know however that depending on the context,  
 676 some constraints may restrict this control. Initially, our model was built to  
 677 represent traditional mixed farming systems. In these systems,  $\alpha$  is not really  
 678 a driving-force: it depends on the time passed (day/night) in the rangeland  
 679 by livestock and on its metabolism. However the model is generic enough to  
 680 be used for other systems or at different scales. For instance in areas where  
 681 the crop production and the livestock production are decoupled at the scale  
 682 of the farms (cattle fattening), our model can be used to couple them at  
 683 the scale of the landscape. In that case, the rangeland sub-system is in fact

a field in which fodder is cultivated and used to feed the livestock:  $\alpha$  can therefore be considered as a full leverage.

The model presented in this paper is rather simple, which serves our capability to extract the basic mechanisms underlying the optimization process, but which bears some limitations. For instance, we assumed that the input of nutrient to the rangeland is independent on plant biomass. This may be relevant for phosphorus but not for nitrogen, which is often subject to symbiotic fixation by legumes, plants that are present in most rangeland. The quantity of nitrogen fixed depends on the biomass of legumes, which relieves the assumption of independence between nitrogen inputs and plant biomass. Without this assumption, some of the results presented earlier may not hold.

Overall, we are fully aware of the theoretical nature of our results. The results obtained by the study of our model are qualitative more than quantitative. These results are not meant to be straightforward recommendations to farmers but rather, to allow a better understanding of the different possibilities that exist to optimize crop production. These results also show the relevance of both the meta-ecosystem and the control theory to address optimization in agricultural systems. Indeed, agricultural systems are by essence highly dynamics, and are usually very patchy in terms of biogeochemistry, with source-sink dynamics. Our results suggest that both the patchiness and the dynamics provide a set of unexplored ways to optimize the production of crops, meat, or other goods.

## 706 Acknowledgments

707 This research was supported by the LABEX AGRO – Agropolis Fondation  
708 (Project 1605-039 ECOW) and the French National Research Agency (ANR)  
709 through the CERAO project (ANR 13AGRO0002)

## 710 Appendix A. Model

### 711 Appendix A.1. Equilibrium points of generic model (2) and their stability

#### 712 • Equilibrium points

713 Consider the case where  $(\beta - 1)d \neq 0$  and let us solve the equation  
714  $F_1(N) = F_2(N)$  with  $F_1(N) = K \frac{uN-m-d}{uN}$ ,  $F_2(N) = a(N - \frac{i}{e})$  and  $a = \frac{e}{(\beta-1)d}$ .  
715 We have, for all  $N \in \mathbb{R}^+ \setminus \{0\}$ :

$$F_1(N) = F_2(N) \Leftrightarrow \mathcal{P}(N) = 0 \quad (\text{A.1})$$

716 with  $\mathcal{P}(N) = uN^2 - uN(\frac{i}{e} + \frac{K}{a}) + \frac{K}{a}(m+d)$ . The discriminant of  $\mathcal{P}$  is written:

$$\Delta = u^2 \left( \frac{i}{e} + \frac{K}{a} \right)^2 - 4u \frac{K}{a}(m+d); \quad (\text{A.2})$$

717 in the case where  $\frac{i}{e} > \frac{m+d}{u}$  and as  $a < 0$  we have  $\Delta > 0$ . Therefore, there  
718 exists two roots that are given by:

$$N^* = \frac{1}{2} \left( \frac{i}{e} + \frac{K}{a} \right) \pm \frac{\sqrt{\Delta}}{2u} \quad (\text{A.3})$$

719 the corresponding value of  $P^* = s(N^* - \frac{i}{e})$  being:

$$P^* = \frac{K}{2} - \frac{ia}{2e} \pm a \frac{\sqrt{\Delta}}{2u}. \quad (\text{A.4})$$

Among these two points, only one is positive in the sense  $(P^*, N^*) \in (\mathbb{R}^+)^2$ ;  
it is given by:

$$E_1 = \left( \frac{1}{2} \left( K - \frac{ia}{e} + a \frac{\sqrt{\Delta}}{u} \right), \frac{1}{2} \left( \frac{i}{e} + \frac{K}{a} + \frac{\sqrt{\Delta}}{u} \right) \right) \quad (\text{A.5})$$

In the case where  $(\beta - 1)d = 0$ , system (3) leads to  $N = \frac{i}{e}$  and  $uP \frac{i}{e} (1 - \frac{P}{K}) -$   
 $mP - dP = 0 \Leftrightarrow P = 0$  or  $P = K \left( 1 - \frac{e(m+d)}{ui} \right)$ .

• Stability of the equilibrium points:

The Jacobian matrix of the system (2) at the point  $E_0$  is given by:

$$J(E_0) = J \left( 0, \frac{i}{e} \right) = \begin{pmatrix} u \frac{i}{e} - m - d & 0 \\ -u \frac{i}{e} + m + \beta d & -e \end{pmatrix} \quad (\text{A.6})$$

This matrix has two eigenvalues that are  $u \frac{i}{e} - m - d$  and  $-e < 0$ . As a  
consequence,  $E_0$  is stable if and only if  $u \frac{i}{e} - m - d < 0 \Leftrightarrow \frac{m+d}{u} > \frac{i}{e}$ .

The equilibrium point  $E_1$  exists when  $\frac{i}{e} > \frac{m+d}{u}$ ; it is such that:

$$uN_1^* \left( 1 - \frac{P_1^*}{K} \right) = m + d \text{ and } uP_1^* N_1^* \left( 1 - \frac{P_1^*}{K} \right) = i + mP_1^* + \beta dP_1^* - eN_1^*. \quad (\text{A.7})$$

By using these relationships, we get the following expression of the Jacobian  
matrix of system (2) at the point  $E_1$ :

$$J(E_1) = \begin{pmatrix} -\frac{uP_1^* N_1^*}{K} & uP_1^* \left( 1 - \frac{P_1^*}{K} \right) \\ \frac{uP_1^* N_1^*}{K} + (\beta - 1)d & -uP_1^* \left( 1 - \frac{P_1^*}{K} \right) - e \end{pmatrix} \quad (\text{A.8})$$

The real part of the eigenvalues of a  $2 \times 2$  matrix are strictly negative if and  
only if the trace of the matrix is strictly negative and the determinant of the  
matrix is strictly positive. Here we have:

$$\text{Tr}(J(E_1)) = -\frac{uP_1^* N_1^*}{K} - uP_1^* \left( 1 - \frac{P_1^*}{K} \right) - e \quad (\text{A.9})$$

734 which is always strictly negative, because  $N_1^* > 0$  and  $0 < P_1^* < K$ . More-  
 735 over, after simple calculations, we have:

$$\det(J(E_1)) = \frac{uP_1^*}{K} \left( i + \frac{e}{a}(2P_1^* - K) \right) \quad (\text{A.10})$$

736 The equilibrium point  $E_1$  is therefore stable if and only if:

$$\det(J(E_1)) > 0 \Leftrightarrow i + \frac{e}{a}(2P_1^* - K) > 0 \Leftrightarrow P_1^* < \frac{1}{2} \left( K - i\frac{a}{e} \right) \quad (\text{as } a < 0), \quad (\text{A.11})$$

737 which is always true as  $P_1^* := \frac{1}{2}(K - \frac{ia}{e} + a\frac{\sqrt{\Delta}}{u})$  and  $a < 0$ . When it exists,  
 738 the equilibrium point  $E_1$  is therefore always stable.

## 739 Appendix B. Results

### 740 Appendix B.1. Impact of $\alpha$ on $P_{1,r}^*$ and $N_{1,r}^*$

741  $N_{1,r}^*$  is solution of the equation  $F_1(N) = F_2(N, \alpha)$  where  $F_1 : N \mapsto$   
 742  $K_r \frac{u_r N - m_r - d_r}{u_r N}$  is a strictly increasing concave function defined on  $\mathbb{R}^+ \setminus \{0\}$ ,  
 743 and  $F_2(N, \alpha) = a(\alpha)(N - \frac{i_r}{e_r})$  with  $a(\alpha) = \frac{e_r}{(\alpha-1)d_r} < 0$ . It only exists if  
 744  $\frac{i_r}{e_r} > \frac{m_r + d_r}{u_r}$ , so we only consider this case. We then have:

$$\frac{d}{d\alpha} (F_1(N_{1,r}^*)) = \frac{d}{d\alpha} (F_2(N_{1,r}^*, \alpha)) \Leftrightarrow \frac{dN_{1,r}^*}{d\alpha} F_1'(N_{1,r}^*) = \frac{dN_{1,r}^*}{d\alpha} \frac{\partial F_2}{\partial N}(N_{1,r}^*, \alpha) + \frac{\partial F_2}{\partial \alpha}(N_{1,r}^*, \alpha). \quad (\text{B.1})$$

745 As  $\frac{\partial F_2}{\partial N} = a(\alpha)$  and  $\frac{\partial F_2}{\partial \alpha} = a'(\alpha)(N - \frac{i_r}{e_r})$  with  $a'(\alpha) = \frac{-e_r}{(\alpha-1)^2 d_r} < 0$ , we get:

$$\frac{dN_{1,r}^*}{d\alpha} = \frac{a'(\alpha)(N_{1,r}^* - \frac{i_r}{e_r})}{F_1'(N_{1,r}^*) - a(\alpha)} \quad (\text{B.2})$$

746 As  $a(\alpha) < 0$  and  $a'(\alpha) < 0$ , we know that  $N_{1,r}^* < \frac{i_r}{e_r}$ , so  $a'(\alpha)(N_{1,r}^* - \frac{i_r}{e_r}) > 0$ .

747 Moreover,  $F_1'(N_{1,r}^*) = \frac{K_r(m_r + d_r)}{u_r(N_{1,r}^*)^2} > 0$  and  $a(\alpha) < 0$ , so  $F_1'(N_{1,r}^*) - a(\alpha) > 0$ .

748 We so have  $\frac{dN_{1,r}^*}{d\alpha} > 0$ .



749 The derivative of  $P_{1,r}^*$  with respect to  $\alpha$  can then be deduced; we indeed  
 750 have  $P_{1,r}^* = F_1(N_{1,r}^*)$  which leads to:

$$\frac{dP_{1,r}^*}{d\alpha} = \frac{dN_{1,r}^*}{d\alpha} \underbrace{F_1'(N_{1,r}^*)}_{>0} > 0. \quad (\text{B.3})$$

## 751 Appendix B.2. Impact of $\gamma$ on $C$

### 752 Impact of $\gamma$ on $P_{1,c}^*$

753 The equilibrium point  $E_{1,c} = (P_{1,c}^*, N_{1,c}^*)$  exists if and only if  $\frac{i_c + T \frac{1-\gamma}{\gamma}}{e_c} > \frac{m_c}{u_c}$   
 754 and for  $\gamma \in (0, 1]$ . We have:

$$\frac{i_c + T \frac{1-\gamma}{\gamma}}{e_c} > \frac{m_c}{u_c} \Leftrightarrow \begin{cases} \gamma < \frac{T}{\frac{m_c e_c}{u_c} - i_c + T} & \text{if } \frac{m_c e_c}{u_c} - i_c + T > 0 \\ \gamma > \frac{T}{\frac{m_c e_c}{u_c} - i_c + T} & \text{if } \frac{m_c e_c}{u_c} - i_c + T < 0 \end{cases} \quad (\text{B.4})$$

755 If  $\frac{m_c e_c}{u_c} - i_c + T > 0$ , then  $\frac{T}{\frac{m_c e_c}{u_c} - i_c + T} > 0$  and  $\frac{T}{\frac{m_c e_c}{u_c} - i_c + T} \leq 1 \Leftrightarrow \frac{m_c e_c}{u_c} - i_c \geq 0$ .

756 If  $\frac{m_c e_c}{u_c} - i_c + T < 0 \Rightarrow \frac{m_c e_c}{u_c} - i_c < -T < 0$ , then  $\frac{T}{\frac{m_c e_c}{u_c} - i_c + T} < 0$ .

757 So  $P_{1,c}^*$  only exists for values of  $\gamma$  included in a domain  $\Omega_\gamma$  given by:

- 758 •  $\Omega_\gamma = \left(0, \frac{T}{\frac{e_c}{u_c} m_c - i_c + T}\right] \subset [0, 1]$  if  $\frac{i_c}{e_c} \leq \frac{m_c}{u_c}$  ;
- 759 •  $\Omega_\gamma = (0, 1]$  if  $\frac{i_c}{e_c} > \frac{m_c}{u_c}$ .

760 It is given by  $P_{1,c}^* = K_c \left(1 - \frac{m_c}{u_c} \frac{e_c}{i_c + T \frac{1-\gamma}{\gamma}}\right)$ . We so have, for all  $\gamma \in \Omega_\gamma$ :

$$\frac{dP_{1,c}^*}{d\gamma} = -K_c \frac{m_c e_c T}{u_c \gamma^2} \frac{1}{(i_c + T \frac{1-\gamma}{\gamma})^2} < 0. \quad (\text{B.5})$$

761 The function  $\gamma \mapsto P_{1,c}^*$  is therefore strictly decreasing on  $\Omega_\gamma$ .

### 762 Impact of $\gamma$ on $C$

763 The crop production defined by  $C = \gamma S P_c^*$  is positive only when  $P_c^* = P_{1,c}^*$ ,

that is when  $P_{1,c}^*$  exists, i.e.  $\forall \gamma \in \Omega_\gamma$ . After simple computations, we can show from (3) that  $C$  is given on  $\Omega_\gamma$  by:

$$C := \gamma S P_{1,c}^* = \frac{T}{e_c N_{1,c}^* + T - i_c} S P_{1,c}^* \quad (\text{B.6})$$

with

$$N_{1,c}^* = \frac{m_c}{u_c} \frac{K_c}{K_c - P_{1,c}^*}. \quad (\text{B.7})$$

We then get:

$$\forall \gamma \in \Omega_\gamma, \frac{dC}{d\gamma} = S \frac{dP_{1,c}^*}{d\gamma} \frac{T}{(e_c N_{1,c}^* + T - i_c)^2} \frac{\rho(K_c - P_{1,c}^*)}{u_c (K_c - P_{1,c}^*)^2} \quad (\text{B.8})$$

where  $\rho(x) = a_2 x^2 + a_1 x + a_0$  with  $a_2 = u_c(T - i_c)$ ,  $a_1 = 2e_c K_c m_c > 0$  and  $a_0 = -e_c K_c^2 m_c < 0$ . The discriminant of polynomial  $\rho$  is given by  $a_1^2 - 4a_2 a_0 = 4e_c K_c^2 m_c [e_c m_c + u_c(T - i_c)]$  which leads to the following cases:

- if  $\frac{e_c m_c}{u_c} + T < i_c$ , then  $\rho(x) < 0, \forall x \in \mathbb{R}$  and  $\frac{dP_{1,c}^*}{d\gamma} < 0$  for  $\gamma \in [0, 1]$  so  $\frac{dC}{d\gamma} > 0$  for all  $\gamma \in [0, 1]$  and  $C$  is therefore maximal for  $\gamma = 1$ .
- if  $\frac{e_c m_c}{u_c} + T > i_c$ , then the polynomial  $\rho$  has two roots. After simple computations (not detailed here for simplicity), we can show that, whatever the value of  $a_2$  is, among these two roots only the root given by  $x^+ = \frac{-a_1 + \sqrt{a_1^2 - 4a_2 a_0}}{2a_2}$  corresponds to a value  $P^{opt} = K_c - x^+$  of  $P_{1,c}^*$  that belongs to  $[0, K_c]$ . We also easily show that for all  $P_{1,c}^*(\gamma) \in (0, P^{opt})$ ,  $\frac{dC}{d\gamma} < 0$  and for all  $P_{1,c}^*(\gamma) \in (P^{opt}, K_c)$ ,  $\frac{dC}{d\gamma} > 0$ . By denoting  $\gamma_0 = \frac{T}{\frac{e_c m_c}{u_c} - i_c + T}$  the value of  $\gamma$  such that  $P_{1,c}(\gamma_0) = 0$ , and because  $\frac{dP_{1,c}^*}{d\gamma} < 0$  for  $\gamma \in \left[0, \min\left(\frac{T}{\frac{e_c m_c}{u_c} - i_c + T}, 1\right)\right]$ , we can finally deduce the variations table of the function  $\gamma \in (0, \gamma_0] \mapsto C$  that is given in figure C.10.

[Figure 10 about here.]

We then have to consider two cases. If  $\gamma_0 > 1$  and  $P_{1,c}^*(\gamma = 1) > P^{opt}$ , then the maximal value of  $C$  on  $(0, 1]$  is reached for  $\gamma = 1$ . If  $\gamma_0 < 1$  or if  $\gamma_0 > 1$  and  $P_{1,c}^*(\gamma = 1) < P^{opt}$ , then the maximal value of  $C$  on  $(0, 1]$  is reached for the value  $\gamma^{opt}$  of  $\gamma$  such that  $P_{1,c}^*(\gamma = \gamma^{opt}) = P^{opt}$ . From (B.6) and (B.7),  $\gamma^{opt}$  is given by:

$$\gamma^{opt} = \frac{T}{e_c \frac{K_c m_c}{u_c(K_c - P^{opt})} + T - i_c} \quad (\text{B.9})$$

with:

$$P^{opt} = K_c - \frac{-a_1 + \sqrt{a_1^2 - 4a_2a_0}}{2a_2} = K_c \left[ 1 - \frac{e_c m_c}{e_c m_c + \sqrt{e_c m_c [e_c m_c + u_c(T - i_c)]}} \right] \quad (\text{B.10})$$

We finally get two cases:  $C$  is maximal either for  $\gamma = 1$ , or for  $\gamma = \gamma^{opt}$ .

Let us now express the different conditions that lead to these cases in terms of values of  $i_c$ .

Case 1:  $C$  is maximal for  $\gamma = 1$  if  $\frac{e_c m_c}{u_c} + T < i_c$  OR if  $\frac{e_c m_c}{u_c} + T > i_c$ ,  $\gamma_0 > 1$  and  $P_{1,c}^*(\gamma = 1) > P^{opt}$ . We first have  $\gamma_0 > 1 \Leftrightarrow \frac{T}{\frac{e_c m_c}{u_c} - i_c + T} > 1 \Leftrightarrow T > \frac{e_c m_c}{u_c} - i_c + T \Leftrightarrow \frac{e_c m_c}{u_c} < i_c$ . For the condition  $P_{1,c}^*(\gamma = 1) > P^{opt}$ , we can show that:

$$P_{1,c}^*(\gamma = 1) > P^{opt} \Leftrightarrow 0 < \underbrace{u_c i_c^2 - e_c m_c i_c - e_c m_c T}_{\mathcal{P}(i_c)}. \quad (\text{B.11})$$

The polynomial  $\mathcal{P}$  has two roots, one negative and one positive:

$$i^- = \frac{e_c m_c - \sqrt{e_c m_c (e_c m_c + 4u_c T)}}{2u_c} < 0 \text{ and } i^+ = \frac{e_c m_c + \sqrt{e_c m_c (e_c m_c + 4u_c T)}}{2u_c} > 0, \quad (\text{B.12})$$

799 which leads to  $\mathcal{P}(i_c) > 0 \Leftrightarrow i_c > i^+$  or  $i_c < i^-$ . As we only consider positive  
 800 values of  $i_c$ , the condition  $P_{1,c}^*(\gamma = 1) > P^{opt}$  is simply rewritten  $i_c > i^+$ .  
 801 After computations, we can show that:

$$\frac{e_c m_c}{u_c} < i^+ < \frac{e_c m_c}{u_c} + T. \quad (\text{B.13})$$

802 The conditions for which  $C$  is maximal at  $\gamma = 1$  can be therefore reduced to  
 803  $i_c > i^+$ .

804 Case 2:  $C$  is maximal for  $\gamma = \gamma^{opt}$  if  $e_c m_c + u_c(T - i_c) > 0$  and  $\gamma_0 < 1$  OR  
 805 if  $e_c m_c + u_c(T - i_c) > 0$ ,  $\gamma_0 > 1$  and  $P_{1,c}^*(\gamma = 1) < P^{opt}$ . In the same way as  
 806 for case 1, we can show that these conditions reduced to  $i_c < i^+$ .

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913 **Appendix C. Figures**

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962		$0.08$ , $m_c = 0.05$ , $h = 90\%$ . We moreover define $\alpha = 0$ , $\gamma^{opt} =$	
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967 **Figures**

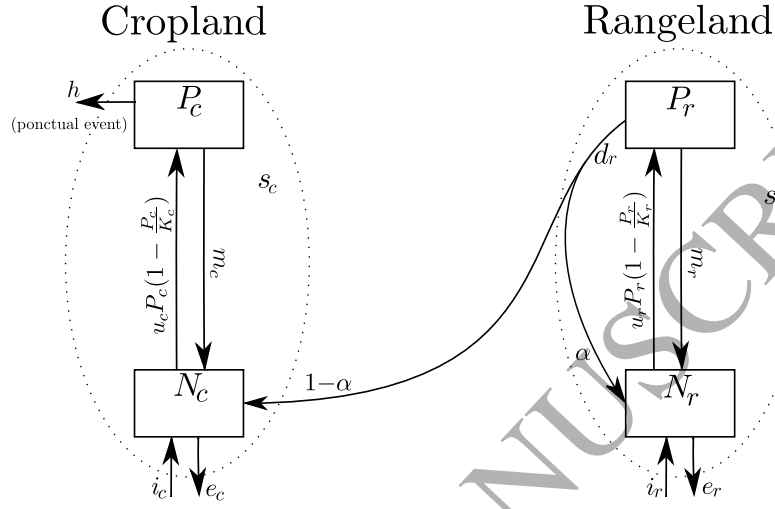


Figure C.1: Model of the main limiting nutrient cycle in a simplify agro-ecosystem. Representation of all stocks and fluxes presents in the model. See the table C.1 for the parameters description and units.

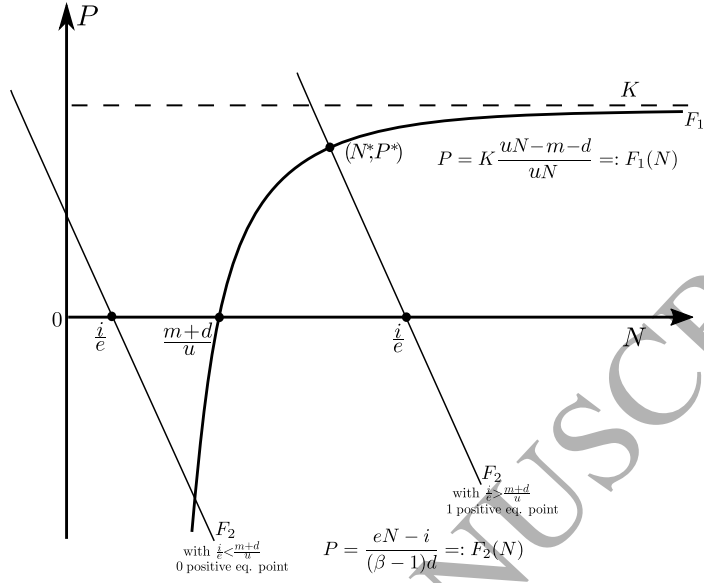


Figure C.2: Graphical determination of the equilibrium points for the generic sub-system model.

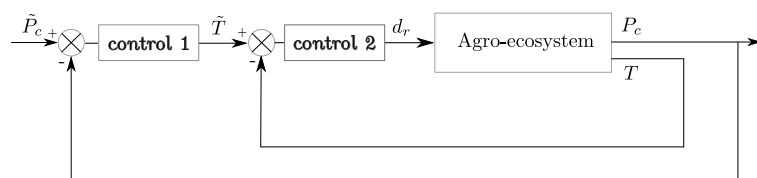


Figure C.3: Scheme of the two loops control strategy.

53

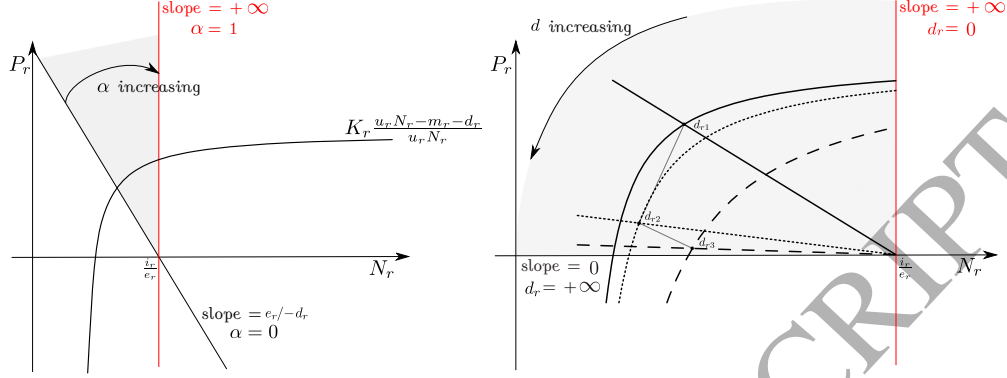


Figure C.5: (left) Evolution of  $F_2(N) = \frac{eN-i}{(\alpha-1)d_r}$  when  $\alpha$  goes from 0 to 1. (right) Evolution of  $F_2(N) = \frac{eN-i}{(\alpha-1)d_r}$  when  $d_r$  goes from 0 to  $+\infty$ . Graphical method of determination of equilibrium points for 3 values of  $d_r$  with  $d_r^1 \leq d_r^2 \leq d_r^3$ .  $P_{1,r}^*$  decreases with  $d_r$  but the variation of  $N_{1,r}^*$  is not monotonous.



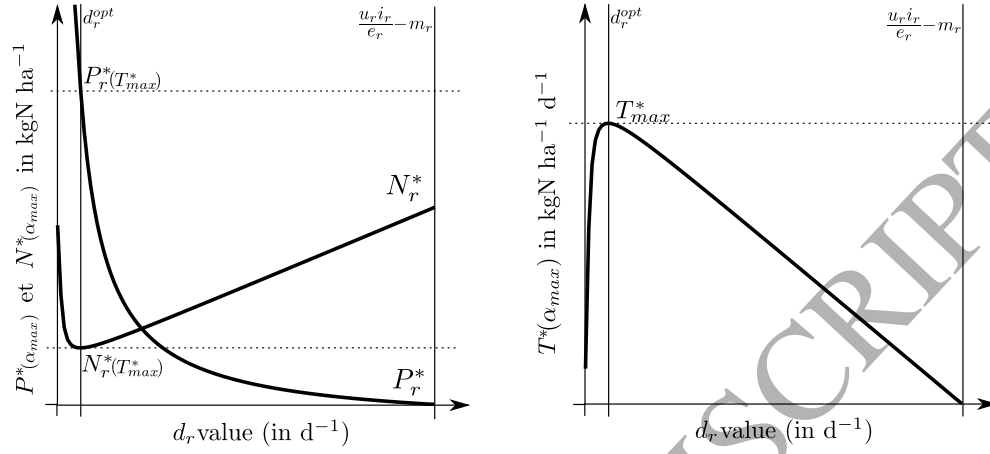


Figure C.6: Variation of  $P^*$  and  $N^*$  (left) and  $T$  (right) with  $d_r$  (and for  $\alpha_{max} = 0$ )

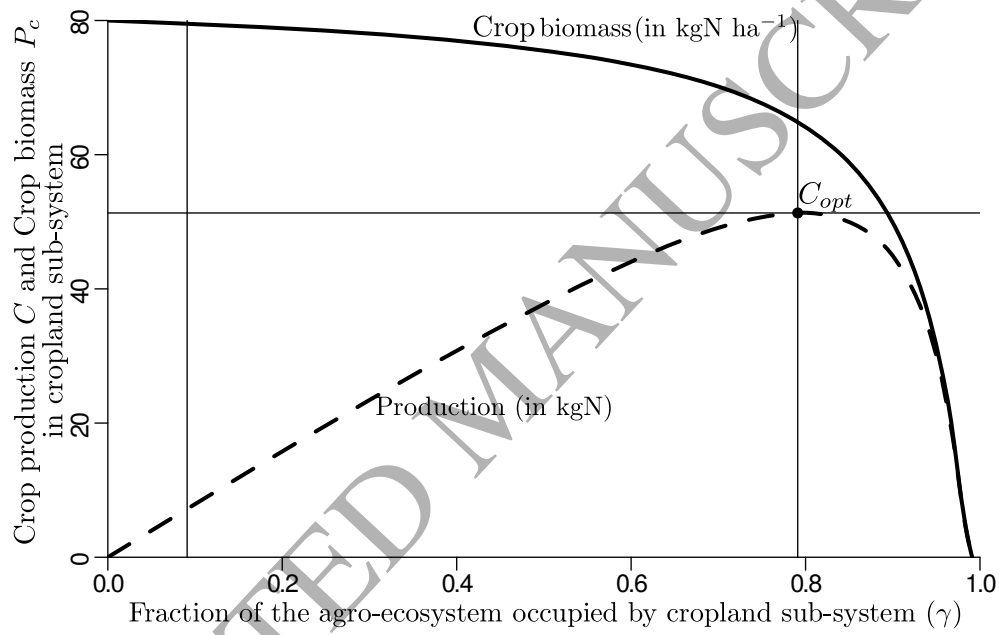


Figure C.7: Variation of crop biomass  $P_c^*$  and the production  $C^*$  of the cropland sub-system with the fraction of the agro-ecosystem surface occupied by the cropland sub-system ( $\gamma$ )

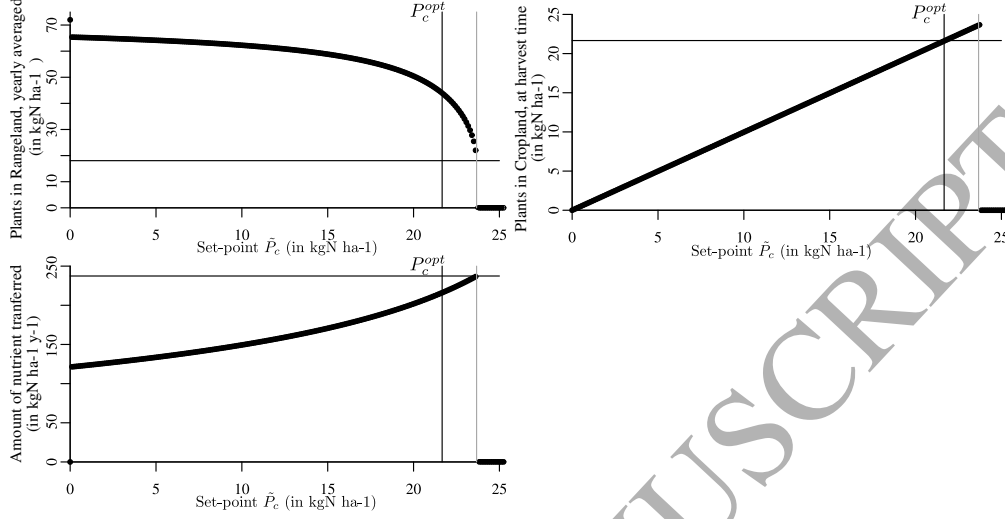


Figure C.8: Yearly averaged stock of plants in rangeland (in kgN h<sup>-1</sup>), stock of plants in cropland at harvest time (value of  $P_{c,t_h}$ , in kgN ha<sup>-1</sup>) and total amount of nutrient transferred from rangeland to cropland over a year ( $\int_0^{t_h} T(t)dt$ ) with respect of the value of the set-point  $\tilde{P}_c$ . The set of parameters used is  $u_r = 0.08$ ,  $u_c = 0.01$ ,  $K_r = 80$ ,  $K_c = 40$ ,  $e_r = 0.08$ ,  $e_c = 0.4$ ,  $i_r = 0.8$ ,  $i_c = 0.05$ ,  $m_r = 0.08$ ,  $m_c = 0.05$ ,  $h = 90\%$ . We moreover define  $\alpha = 0$  and  $\gamma = \gamma^{opt} = 0.131$  from the optimization at equilibrium with  $d_r^{opt} = 0.036$ . The black horizontal straight lines represent the values obtained with the optimization at equilibrium such as  $P_c^{opt} = 21.7$  (black vertical straight line). The gray vertical straight line represents the maximal value of  $\tilde{P}_c$  reached with the control strategies.

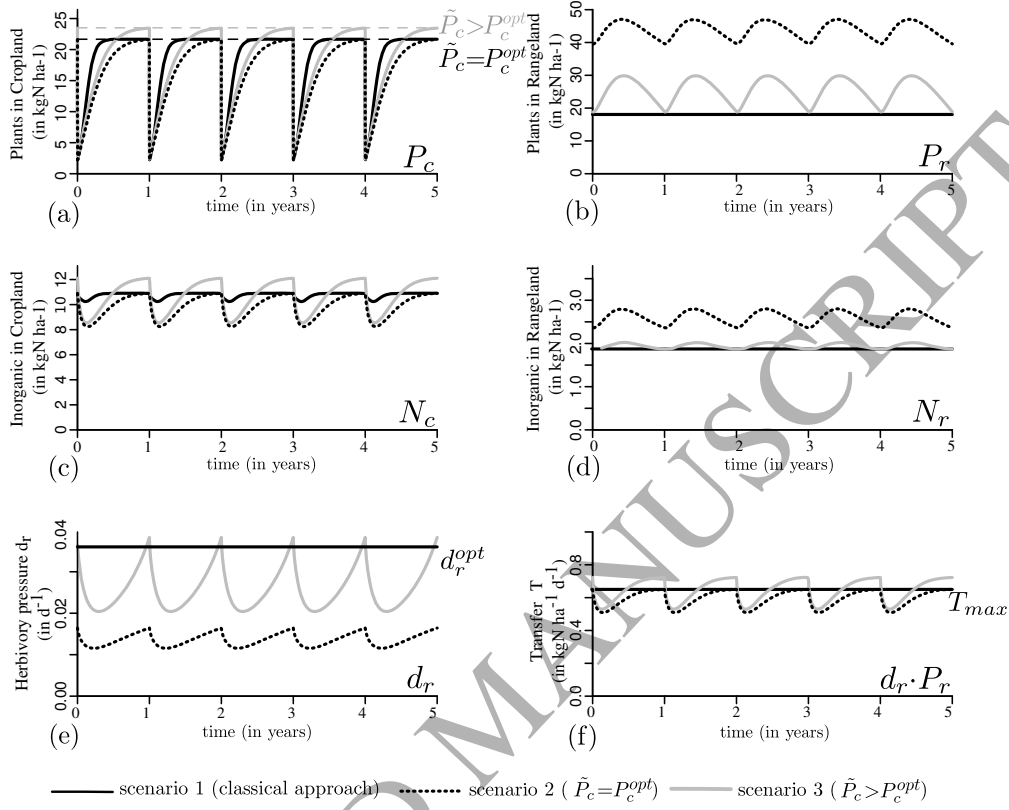


Figure C.9: Dynamic over time of stocks of plants (a) and (b) and inorganic nutrient (c) and (d) in the cropland sub-system and the rangeland sub-system respectively. Dynamic of grazing pressure  $d_r$  (e) and transfer of inorganic nutrient  $T$  (f). The optimal scenario obtained with the optimization at equilibrium (scenarios 1), and the scenario obtained with the control theory (scenario 2),  $\tilde{P}_c = P_c^{opt}$  and scenario 3,  $\tilde{P}_c > P_c^{opt}$  are plotted in black lines, dotted black lines and grey lines respectively. In the subfigure (a), black and gray dashed lines respectively represent the value of the set point  $\tilde{P}_c = P_c^{opt}$  and  $\tilde{P}_c > P_c^{opt}$ . The set of parameters used is  $u_r = 0.08$ ,  $u_c = 0.01$ ,  $K_r = 80$ ,  $K_c = 40$ ,  $e_r = 0.08$ ,  $e_c = 0.4$ ,  $i_r = 0.8$ ,  $i_c = 0.05$ ,  $m_r = 0.08$ ,  $m_c = 0.05$ ,  $h = 90\%$ . We moreover define  $\alpha = 0$ ,  $\gamma^{opt} = 0.131$ ,  $d_r^{opt} = 0.036$  (value of  $d_r$  for scenario (1) only) and get  $d_r^{max} = 0.72$ ,  $T_{max} = 0.65$ .

$\gamma$	0	$\gamma^+$	$\gamma_0$
$dP/d\gamma$		—	—
$P$	$K$	$\rightarrow p^+$	$\rightarrow 0$
$dC/d\gamma$		+	—
$C$	0	$\rightarrow C^+$	$\rightarrow 0$

Figure C.10: Variations table of the function  $\gamma \mapsto C$  in the case when  $\frac{e_c m_a}{u_c} + T - i_c > 0$ .

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972 **Tables**

variable	dimensions	description
$P_z$	$\text{kgN ha}^{-1}$	Stock of nutrient in the plant compartment
$N_z$	$\text{kgN ha}^{-1}$	Stock of nutrient in the inorganic nutrient compartment
parameter	dimensions	description
$u_z$	$\text{ha kgN}^{-1} \text{ d}^{-1}$	nutrient uptake rate of plants
$K_z$	$\text{kgN ha}^{-1}$	carrying capacity of plant
$m_z$	$\text{d}^{-1}$	mortality rate of plants
$e_z$	$\text{d}^{-1}$	losses (ex: leaching) of inorganic nutrient
$i_z$	$\text{kgN ha}^{-1} \text{ d}^{-1}$	inputs of inorganic nutrient (ex: atmospheric deposition)
$s_z$	ha	surface of the sub-system
$\gamma$		fraction of agro-ecosystem occupied by cropland
$d_r$	$\text{d}^{-1}$	grazing pressure in rangeland
$\alpha$		fraction of uptake by livestock recycled into rangeland
$1 - \alpha$		fraction of uptake by livestock transferred to cropland
$h$	%	harvest in cropland in the end of each year
$T$	$\text{kgN ha}^{-1} \text{ d}^{-1}$	nutrient transferred from rangeland to cropland $T = (1 - \alpha)d_r P_r$ , (see equation 10)

Table C.1: Nomenclature of model parameters. The subscript  $z$  stands for either  $r$  or  $c$  depending on the sub-system considered (rangeland or cropland).